

SECOND MIDTERM EXAM - 2022

CLASS : 12

TIME : 1.30 HRS

MATHEMATICS

MARKS : 50

I. Answer all the Questions.

10 x 1 = 10

1. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is
- a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$
2. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is
- a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$
3. If we measure the side of a cube to be 4cm with an error of 0.1 cm, then the error in our calculation of the volume is
- a) 0.4 cu.cm b) 0.45 cu.cm c) 2 cu.cm d) 4.8 cu.cm
4. If $f(x) = \frac{x}{x+1}$ then its differential is given by
- a) $\frac{-1}{(x+1)^2} dx$ b) $\frac{1}{(x+1)^2} dx$ c) $\frac{1}{x+1} dx$ d) $\frac{-1}{x+1} dx$
5. The value of $\lim_{x \rightarrow \infty} \frac{e^x}{x^m}$, $M \in N$ is
- a) 0 b) ∞ c) 1 d) -1
6. The value of $\int_{-1}^2 |x| dx$ is
- a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$
7. The area between $y^2 = 4x$ and its latus rectum is
- a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$
8. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
- a) $\tan^{-1} \frac{3}{4}$ b) $\tan^{-1} \frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$
9. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
- a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31
10. Let $F(x, y) = x^3 + y^2 x + 7$ for all $(x, y) \in R^2$. Then the value of $\frac{\partial F}{\partial x}(-1, 3)$ is
- a) 18 b) -12 c) 7 d) 12

II. Answer any 5 questions. (Q.No.17 is compulsory)

5 x 2 = 10

11. If the volume of a cube of side length x is $V = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.

12. Evaluate $\lim_{x \rightarrow 0^+} x \log x$
13. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x=2, dx=0.1$
14. Let $g(x, y) = 2y + x^2, x = 2r - S, y = r^2 + 2S, r, S \in R$ find $\frac{\partial g}{\partial r}$
15. Evaluate $\int_0^3 (3x^2 - 4x + 5) dx$
16. Evaluate $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x) dx$
17. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$
- III. Answer any five questions. (Q.No.24 is compulsory) 5 x 3 = 15**
18. Write the Maclaurin series expansion of the function $f(x) = \sin x$
19. Find the local extremum of the function $f(x) = x^4 + 32x$
20. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using calculator.
21. If $u(x, y) = x^2y + 3xy^4, x = e^t$ and $y = \sin t$ find $\frac{du}{dt}$ and evaluate it at $t=0$
22. Evaluate $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1 + \sec^2 x} dx$
23. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$
24. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.
- IV. Answer all the questions 3 x 5 = 15**
25. a) Find the angle between $y = x^2$ and $y = (x-3)^2$ (OR)
b) Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.
26. a) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ (OR)
b) Evaluate $\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx$
27. a) A sphere is made of ice having radius 10cm. Its radius decreases from 10cm to 9.8cm. Find approximations for the following.
i) change in the volume ii) change in the surface area (OR)
b) Prove that among all the rectangles of the given area square has the least perimeter.