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LIA Tuition Centre, Meppur.

Class: 12th (Matriculation) **Mathematics - 2022-23** **Date:17.11.22**

Max. Marks: 50 **Chapter: 7, 8 and 9** **Time: 90mins.**

Second Mid Term - 2022

I. Choose the best answer **10 x 1 =10**

1. The abscissa of the point on the curve f(x) = $\sqrt{8-2x}$ at which the slope of the tangent is -0.25?
   (a) -8 (b) -4 (c) -2 (d) 0
2. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
   (a) $y = 0$ (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \frac{1}{\sqrt{3}}$
3. What is the value of limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$?
   (a) 0 (b) 1 (c) 2 (d) ≤
4. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x} = ?$
   (a) $e^{x^2+y^2}$ (b) 2xu (c) x^2u (d) y^2u
5. If $f(x,y) = e^{xy}, then \frac{\partial^2 f}{\partial x \partial y} =$
   (a) xye^{xy} (b) $(1 + xy)e^{xy}$ (c) $(1 + y)e^{xy}$ (d) $(1 + x)e^{xy}$
6. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is …….
   (a) $12x_0 + dx$ (b) $12x_0dx$ (c) $6x_0dx$ (d) $6x_0 + dx$
7. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$
   (a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$ (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$
8. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$, then n is.....
   (a) 10 (b) 5 (c) 8 (d) 9
9. If $\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{8}$, then a is.....
   (a) 4 (b) 1 (c) 3 (d) 2
10. The volume, when the curve y = $\sqrt{3+x^2}$ from x = 0 to x = 4 is rotated about x – axis is ……
    (a) $100\pi 4$ (b) $\frac{100}{9}\pi$ (c) $\frac{100}{3}\pi$ (d) $\frac{100}{\pi}$

II. Answer any five questions only (Qn.no.17 Compulsory) **5 x 2 =10**

11. Compute the value of "c" satisfied by the Rolle's theorem for the function $f(x) = \sqrt{x} - \frac{x}{3}, x \in [0,9]$.
12. Write the Maclaurin Series expansion of the functions $f(x) = \sin x$
13. Find the local extremum of the function $f(x) = x^4 + 32x$
14. If $w(x,y) = x^3 - 3xy + 2y^2, x, y \in R,$ find the linear approximation for w at $(1,-1)$.

15. Show that $F(x,y)=\dfrac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

16. Evaluate $\int_0^{\frac{\pi}{2}}\left(\sin^2 x+\cos^4 x\right)dx$

17. Evaluate $\int_0^1 x^3(1-x)^4\,dx$

III. Answer any 5 questions only (Qn.no.24 is compulsory) **5 x 3 = 15**

18. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

19. Prove, using mean value theorem, that $|\sin\alpha-\sin\beta|\le|\alpha-\beta|, \alpha,\beta\in R$.

20. Let $w(x,y)=xy+\dfrac{e^y}{y^2+1}$ for all $(x,y)\in R^2$. Calculate $\dfrac{\partial^2 w}{\partial y\partial x}$ and $\dfrac{\partial^2 w}{\partial x\partial y}$.

21. If $v(x,y)=\dfrac{x^2+y^2}{x+y}$, Prove that $x\dfrac{\partial v}{\partial x}+y\dfrac{\partial v}{\partial y}=1$.

22. Evaluate $\int_{-\log 2}^{\log 2} e^{-|x|}dx$.

23. Evaluate $\int_0^{\frac{\pi}{2}}\begin{vmatrix}\cos^4 x & 7\\ \sin^5 x & 3\end{vmatrix}dx$.

24. Find the area of the region bounded between the parabola $y^2=4ax$.

IV. Answer all the questions **5 x 3 =15**

25. For the function $f(x)=4x^3+3x^2-6x+1$find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. **(OR)**
A hollow cone with a base radius of a cm and a height of b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times the volume of cone.

26. Sketch the curve $y=f(x)=x^3-6x-9$ **(OR)**
Prove that $g(x,y)=x\log\left(\dfrac{y}{x}\right)$is homogeneous. What is the degree? Verify Euler's theorem for g.

27. Evaluate $\int_0^{\pi}\dfrac{x}{1+\sin x}dx$ **(OR)**
Find the area of region bounded by $y=\cos x, y=\sin x,$ the lines $x=\dfrac{\pi}{4}$ and $x=\dfrac{5\pi}{4}$.