Tsi12M

Tenkasi District Common Examinations
Common Second Mid Term Test - November 2022

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Standard 12
MATHEMATICS

Time: 1.30 Hrs.

Marks: 45

Part - A

Answer the all questions: 10×1=10

1) The volume of a sphere is increasing in volume at the rate of 3π cm³/sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm.

   1) 3 cm/s   2) 2 cm/s   3) 1 cm/s   4) $\frac{1}{2}$ cm/s

2) The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is

   1) 1   2) $\sqrt{2}$ 3) $\frac{3}{2}$ 4) 2

3) The point of inflection of the curve $y = (x-1)^3$ is
   1) (0, 0)   2) (0, 1)   3) (1, 0)   4) (1, 1)

4) If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

   1) $e^{x^2+y^2}$ 2) $2xu$ 3) x^2u 4) y^2u

5) If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to
   1) $z-x$ 2) $y-z$ 3) $x-z$ 4) $y-x$

6) Find the value of dy: $y = \frac{(1-2x)^3}{3-4x}$

   1) $\frac{2(1-2x)^2(8x-7)}{(3-4x)^2}dx$ 2) $\frac{2(1-x)(8-x)}{3x-4}$

   3) $\frac{2(1-x^2)(8-x)}{3x-4}$ 4) $\frac{2(1-x^2)(8x-7)}{(3x-4)^2}$

7) The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

   1) $\frac{\pi a^3}{16}$ 2) $\frac{3\pi a^4}{16}$ 3) $\frac{3\pi a^2}{8}$ 4) $\frac{3\pi a^4}{8}$

8) The area between $y^2 = 4x$ and its latus rectum is

   1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $\frac{8}{3}$ 4) $\frac{5}{3}$

9) The value of $\int_{-1}^{2} |x| \, dx$ is

   1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) $\frac{5}{2}$ 4) $\frac{7}{2}$

10) Find the value of $\int_0^{\infty} x^5 e^{-3x} \, dx$

   1) $\frac{5}{3}$ 2) $\frac{5}{3^6}$ 3) $\frac{5!}{3^6}$ 4) 0

Part - B

Answer the all questions. Question number 16 compulsory: **4×2=8**

11) If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

12) Evaluate: $\lim\limits_{x \to 0^+} x \log x$

13) If $u(x, y) = \dfrac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{3}{2}u$.

14) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\dfrac{\partial U}{\partial x} + \dfrac{\partial U}{\partial y} + \dfrac{\partial U}{\partial z}$.

15) The time T, taken for a complete oscillation of a single pendulum with length ℓ, is given by the equation $T = 2\pi\sqrt{\dfrac{\ell}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of ℓ.

16) Evaluate: $\int_0^1 x^5(1 - x^2)^5 \, dx$

Part - C

Answer the all questions. Question number 22 compulsory: **4×3=12**

17) Find the local extremum of the function $f(x) = x^4 + 32x$.

18) Write down the Taylor series expansion, of the function log x about x = 1 upto three non-zero terms for x>0.

19) Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

20) Evaluate: $\int_2^3 \dfrac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}} \, dx$

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21) If $\int_0^\infty e^{-\alpha x^2} x^3 \, dx = 32$, $\alpha>0$, find α.

22) If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\dfrac{dg}{dt}$ is equal to

Part - D   *Tenkasi Dist.*

Answer the all questions: **3×5=15**

23) a) If we blow air into a baloon of spherical shape at a rate of 1000 cm³ per second, at what rate the radius of the baloon changes when the radius is 7 cm? Also compute the rate at which the surface area changes.

     (OR)

   b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

24) a) Sketch the curve $y = f(x) = x^2 - x - 6$.

     (OR)

   b) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

25) a) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major - axis 20 cm and minor - axis 10 cm about its major - axis. Find its volume using integration.

     (OR)

   b) $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.