CHAPTER 1

Std - 12

APPLICATION OF MATRICES AND DETERMINANTS

formula

THEOREM 1.1 For every square matrix A of order n, A(adj A) = (adj A)A = |A| I_n

THEOREM 1.2 If a square matrix has an inverse, then it is unique.

THEOREM 1.3 Let A be square matrix of order n. Then A^{-1} exists if and only if A is non – singular.

THEOREM 1.4 If A is non - singular, then

(i) $|A^{-1}| = \frac{1}{|A|}$ (ii) $(A^T)^{-1} = (A^{-1})^T$ (iii) $(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}$, where λ is a non zero scalar.

THEOREM 1.5 (Left cancellation law)

Let A, B and C be square matrices of order n. If A is a non - singular and AB = AC, then B = C.

THEOREM 1.6 (Right cancellation law)

Let A, B and C be square matrices of order n. If A is non – singular and BA = CA, then B = C.

THEOREM 1.7 (Reversal law for inverse)

If A and B are non – singular matrices of the same order, then the product AB is also non- singular and $(AB)^{-1} = B^{-1}A^{-1}$

THEOREM 1.8 (Law of double inverse)

If A is non – singular, then A^{-1} is also non – singular and $(A^{-1})^{-1} = A$

THEOREM 1.9 If A is a non - singular square matrix of order n , then

(i) $(\text{adj } A)^{-1} = \text{adj}(A^{-1}) = \frac{1}{|A|} A$
(ii) $|\text{adj } A| = |A|^{n-1}$
(iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$
(iv) $(\text{adj}\lambda A) = \lambda^{n-1}\,\text{adj}(A)$ where λ is a non - zero scalar
(v) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$
(vi) $(\text{adj } A)^T = \text{adj}(A^T)$

THEOREM 1.10 If A and B are any two non – singular square matrices of order n , then

adj(AB) = adj(B) adj (A)

Adjoint $\text{adj}A = [A_{ij}]^T$

Inverse $A^{-1} = \frac{1}{|A|}\, adj\ A$; where $|A| \neq 0$

(i) $A^{-1} = \pm \frac{1}{\sqrt{adj A}}$ (adj A) (ii) $A^{-1} = \pm \frac{1}{\sqrt{adj A}}$ adj (adj A)

A matrix A is orthogonal $AA^T = A^T A = I$

A matrix A is a orthogonal if and only if A is a non – singular and $A^{-1} = A^T$

Methods to solve the system of linear equations AX = B

(i) Matrix inversion method : $X = A^{-1} B$, $|A| = 0$

(ii) Cramer's rule: $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}, \Delta \neq 0$

(iii) Gaussian elimination method

(iv) Rank method

$\rho(A) = \rho(A, B) = 3$	Consistent	One solution
$\rho(A) = \rho(A, B) < 3$	Consistent	Many solution
$\rho(A) \neq \rho(A, B)$	Inconsistent	No solution

The homogenous system of linear equation A X = 0

(i) Has a trivial solution $|A| \neq 0$

(ii) Has a non - trivial solution , $|A| = 0$

CHAPTER 2

COMPLEX NUMBERS

PROPERTY 1 (The commutative property under addition)

For all complex numbers z_1 and z_2, prove that $z_1 + z_2 = z_2 + z_1$

PROPERTY 2 (Inverse property under multiplication)

The multiplicative inverse of a nonzero complex number z = x + iy, is

$\frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

PROPERTY 3 For all two complex numbers z_1 and z_2, prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

PROPERTY 4 $\overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$ where x_1, x_2, y_1 and $y_2 \in \mathbb{R}$

PROPERTY 5 Z is purely imaginary if and only if $z = -\overline{z}$

PROPERTY 6 (Triangle inequality)

For any two complex number z_1 and z_2, prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

PROPERTY 7

For any complex number z_1 and z_2, prove that $|z_1 z_2| = |z_1||z_2|$

PROPERTY 8

If $z = r(\cos\theta + i\sin\theta)$, then $z^{-1} = \frac{1}{r}(\cos\theta - i\sin\theta)$

PROPERTY 9

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ then

$z_1z_2 = r_1r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

PROPERTY 10

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ then

$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$

PROPERTIES OF COMPLEX CONJUCATES

If z = x + iy then $\overline{z} = x - iy$

(1) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(2) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

(3) $\overline{z_1 z_2} = \overline{z_1} . \overline{z_2}$

(4) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$; $\overline{z_2} \neq 0$

(5) $\text{Re}(z) = \frac{z+\overline{z}}{2}$

(6) Im (z) = $\frac{z-\bar{z}}{2i}$

(7) $\overline{(z^n)}$ = $(\bar{z})^n$

(8) If z is real then z = $\bar{z}$

(9) If z is purely imaginary if z =- $\bar{z}$

(10) $\bar{\bar{z}}$ = z

PROPERTIES OF MODULUS OF A COMPLEX NUMBER

(1) $|z| = |\bar{z}|$

(2) $|z_1 + z_2| \leq |z_1| + |z_2|$

(3) $|z_1 z_2| = |z_1||z_2|$

(4) $|z_1 - z_2| \geq ||z_1| - |z_2||$

(5) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(6) $|z^n| = |z|^n$

(7) Re(z) ≤ | z|

(8) Im(z) ≤ |z|

SQUARE ROOT : $\sqrt{a+ib} = \pm\left(\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$

POLAR FORM : z = r (cos θ + i sin θ)

GENERAL RULE FOR DETERMINING ARGUMENT:

SECOND QUADRANT		FIRST QUADRANT	
sin θ + ive	$\theta = \pi - \alpha$	$\theta = \alpha$	Sin θ +ive
Cos θ –ive			Cos θ + ive
Sin θ - ive	$\theta = -\pi + \alpha$	$\theta = -\alpha$	Sin θ - ive
Cos θ – ive			Cos θ +ive
THIRD QUADRANT		FOURTH QUADRANT	

n[th] roots of complex numbers:

$$z^{1/n} = r^{1/n}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right) \quad k = 0, 1, 2, 3, \ldots$$

CHAPTER 3

THEORY OF EQUATIONS

Vieta's formula for polynomial equations of degree 2 $\Longrightarrow$ $x^2 + (\alpha + \beta)\,x + \alpha\beta = 0$.

Vieta's formula for polynomial equations of degree 3

$$\Longrightarrow x^3 - (\alpha+\beta+\gamma)\,x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)\,x - \alpha\beta\gamma = 0.$$

Vieta's formula for polynomial equations of degree n > 3

$$\begin{aligned}
\sum \alpha &= \alpha+\beta+\gamma+\delta\\
\sum \alpha\beta &= \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta\\
\sum \alpha\beta\gamma &= \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta\\
\sum \alpha\beta\gamma\delta &= \alpha\beta\gamma\delta
\end{aligned}$$

THE FUNDAMENTAL THEOREM OF ALGEBRA:

A polynomial of degree $n \geq 1$ has at least one root in C.

COMPLEX CONJUCATE ROOT THEOREM:

Imaginary (non - real complex) roots occur as conjugate pairs, if the coefficients of the polynomial are real

RATIONAL ROOT THEOREM:

- Let $a_n x^n + \ldots + a_1 x + a_0 = 0$ with $a_n \neq 0$ and $a_0 \neq 0$ be a polynomial with integer coefficients.
- If p/q, with (p, q) = 1, is a root of the polynomial, then p is a factor of a_0 and q is a factor of a_n
- Methods to solve some special types of polynomial equations like polynomials having only even powers , partly factored polynomials , polynomials with sum of the coefficients is zero, reciprocal equations.

DESCARTES RULE:

If p is the number of positive roots of a polynomial P(x) and s is the number of sign changes in coefficients of P(x), then s p – is a nonnegative even integer.

CHAPTER 4

INVERSE TRIGONOMETRIC FUNCTIONS

PROPERTY 1

(i) $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta$, if $\theta \in [-0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta$, if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \backslash \{0\}$

(v) $\sec^{-1}(\sec\theta) = \theta$, if $\theta \in [-0, \pi] \backslash \left\{\frac{\pi}{2}\right\}$

(vi) $\cot^{-1}(\cot\theta) = \theta$, if $\theta \in (-0, \pi)$

PROPERTY 2

(i) $\sin(\sin^{-1} x) = x$, if $x \in [-1,1]$

(ii) $\cos(\cos^{-1} x) = x$, if $x \in [-1,1]$

(iii) $\tan(\tan^{-1} x) = x$, if $x \in \mathbb{R}$

(iv) $\text{cosec}(\text{cosec}^{-1} x) = x$, if $x \in \mathbb{R} \backslash (-1, 1)$

(v) $\sec(\sec^{-1} x) = x$, if $x \in \mathbb{R} \backslash (-1, 1)$

(vi) $\cot(\cot^{-1} x) = x$, if $x \in \mathbb{R}$

PROPERTY 3 (RECIPROCAL INVERSE IDENTITIES)

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}\, x$, if $x \in \mathbb{R} \backslash (-1, 1)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec x$, if $x \in \mathbb{R} \backslash (-1, 1)$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} cot^{-1}x & ,if\ x > 0 \\ -\pi + cot^{-1}x & ,if\ x < 0 \end{cases}$

PROPERTY 4 (REFLECTION IDENTITIES)

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, if $x \in [-1,1]$

(ii) $\tan^{-1}(-x) = -\tan^{-1} x$, if $x \in \mathbb{R}$

(iii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \backslash (-1, 1)$

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, if $x \in [-1, 1]$

(v) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, if $x \in \mathbb{R}$

(vi) $\sec^{-1}(-x) = \pi - \sec^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \backslash (-1, 1)$

PROPERTY 5 (CO FUNCTION INVERSE IDENTITIES)

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, if $x \in \mathbb{R}$

(iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$

PROPERTY 6

(i) $sin^{-1}x + sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$, where either $x^2 + y^2 \leq 1$ or $xy < 0$

(ii) $sin^{-1}x - sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$, where either $x^2 + y^2 \leq 1$ or $xy > 0$

(iii) $cos^{-1}x + cos^{-1}y = cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$, if $x + y \geq 0$

(iv) $cos^{-1}x - cos^{-1}y = cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$, if $x \leq y$

(v) $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

(vi) $tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$

PROPERTY 7

(i) $2\,tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $|x| < 1$

(ii) $2\,tan^{-1}x = cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $x \geq 0$

(iii) $2\,tan^{-1}x = sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x| \leq 1$

PROPERTY 8

(i) $sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2sin^{-1}x$ $, if\ |x| \leq \frac{1}{\sqrt{2}}\ or -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

(ii) $sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2cos^{-1}x$ $, if\ \frac{1}{\sqrt{2}} \leq x \leq 1$

PROPERTY 9

(i) $sin^{-1}x = cos^{-1}\sqrt{1-x^2}$, $if\ 0 \leq x \leq 1$

(ii) $sin^{-1}x = -cos^{-1}\sqrt{1-x^2}$, $if -1 \leq x < 0$

(iii) $sin^{-1}x = tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, $if -1 < x < 1$

(iv) $cos^{-1}x = sin^{-1}\sqrt{1-x^2}$, $if\ 0 \leq x \leq 1$

(v) $cos^{-1}x = \pi - sin^{-1}\sqrt{1-x^2}$, $if -1 \leq x < 0$

(vi) $tan^{-1}x = sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$, if $x > 0$

PROPERTY 10

(i) $3\,sin^{-1}x = sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

(ii) $3cos^{-1}x = cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$

CHAPTER 5

TWO DIMENSIONAL ANALYTICAL GEOMETRY

THEOREM 1

The circle passing through the points of intersection of the line lx + my + n = 0 and the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ the circle of the form $x^2 + y^2 + 2gx + 2fy + c + \lambda(lx + my + n) = 0, \lambda \in \mathbb{R}^1$

THEOREM 2

The equation of a circle with (x_1, y_1) and (x_2, y_2) as extremities of one of the diameters of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

THEOREM 3

The position of a point $P(x_1, y_1)$ with respect to a given circle $x^2 + y^2 + 2gx + 2fy + c = 0$ in the plane containing the circle is outside or on or inside the circle according as

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is $\begin{cases} > 0, & or \\ = 0, & or \\ < 0, & \end{cases}$

THEOREM 4 From any point outside the circle $x^2 + y^2 = a^2$ two tangent can be drawn.

THEOREM 5 The sum of the focal distances of any points on the ellipse is equal to length of the major axis.

THEOREM 6 Three normal can be drawn to a parabola $y^2 = 4ax$ from a given point, one of which is always real.

TANGENT AND NORMAL

CURVE	EQUATION	EQUATION OF TANGENT	EQUATION OF NORMAL
CIRCLE	$x^2 + y^2 = a^2$	(i) Cartesian form $xx_1 + yy_1 = a^2$ (ii) parametric form $x\cos\theta + y\sin\theta = a$	(i) Cartesian form $xy_1 - yx_1 = 0$ (ii) parametric from $x\sin\theta - y\cos\theta = 0$
PARABOLA	$y^2 = 4ax$	(i) $yy_1 = 2a(x + x_1)$ (ii) $yt = x + at^2$	(i) $xy_1 + 2y = 2ay_1 + x_1y_1$ (ii) $y + xt = at^3 + 2at$
ELLIPSE	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	(i) $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ (ii) $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$	(i) $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 - b^2$ (ii) $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$
HYPERBOLA	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(i) $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ (ii) $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$	(i) $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$ (ii) $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

CONDITION FOR THE SINE y = mx + c TO BE A TANGENT TO THE CONICS

CONIC	EQUATION	CONDITION TO BE TANGENT	POINT OF CONTACT	EQUATION OF TANGENT
CIRCLE	$X^2 + y^2 = a^2$	$c^2 = a^2(1 + m^2)$	$\left(\frac{\mp am}{\sqrt{1+m^2}}, \frac{\pm a}{\sqrt{1+m^2}}\right)$	$y = mx \pm \sqrt{1 + m^2}$
PARABOLA	$Y^2 = 4ax$	$c = \frac{a}{m}$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$
ELLIPSE	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c^2 = a^2m^2 + b^2$	$\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$	$y = mx \pm \sqrt{a^2m^2 + b^2}$
HYPERBOLA	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c^2 = a^2m^2 - b^2$	$\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$	$y = mx \pm \sqrt{a^2m^2 - b^2}$

PARAMETRIC FORMS

CONIC	PARAMETRIC EQUATIONS	PARAMETER	RANGE OF PARAMETER	ANY POINT ON THE CONIC
CIRCLE	$x = a\cos\theta$ $y = a\sin\theta$	θ	$0 \le \theta \le 2\pi$	' θ ' or $(a\cos\theta, b\sin\theta)$
PARABOLA	$x = at^2$ $y = 2at$	t	$-\infty < t < \infty$	' t ' or $(at^2, 2at)$
ELLIPSE	$x = a\cos\theta$ $y = b\sin\theta$	θ	$0 \le \theta \le 2\pi$	' θ ' or $(a\cos\theta, b\sin\theta)$
HYPERBOLA	$x = a\sec\theta$ $y = b\tan\theta$	θ	$-\pi \le 0 \le \pi$ Except $\theta = \pm\frac{\pi}{2}$	' θ ' or $(a\sec\theta, b\tan\theta)$

PARABOLA

EQUATION	VERTICES	FOCUS	AXIS OF SYMMETRY	EQUATION OF DIRECTRIX	LENGTH OF LATUS RECTUM
$(y - k)^2 = 4a(x - h)$	(h, k)	(h + a, 0 + k)	y = k	x = h - a	4a
$(y - k)^2 = -4a(x - h)$	(h, k)	(h –a, 0 + k)	y = k	x = h + a	4a
$(x - h)^2 = 4a(y - k)$	(h, k)	(0 + h, a + k)	x = h	y = k -a	4a
$(x - h)^2 = -4a(y - k)$	(h, k)	(0 +h , -a +k)	x = h	y = k + a	4a

PARAMETRIC FORMS

Identifying the conic from the general equation of conic

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

The graph of the second degree equation is one of a circle, parabola, an ellipse, a hyperbola, a point, an empty set, a single line or a pair of lines. When,

1) $A = C = 1$, $B = 0$, $D = -2h$, $E = -2k$, $F = h^2 + k^2 - r^2$ the general equation reduces to $(x - h)^2 + (y - h)^2 = r^2$, which is a circle .

2) $B = 0$ and either A or $C = 0$, the general equation yields a parabola under study, at this level

3) $A \neq C$ and A and C are of the same sign the general equation yields an ellipse.

4) $A \neq C$ and A and C are of the opposite signs the general equation yields a hyperbola

ELLIPSE

EQUATION	CENTRE	MAJOR AXIS	VERTICES	FOCI
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $a^2 > b^2$ a) Major axis parallel to the x – axis foci are c units right and c units left of centre , where $c^2 = a^2 - b^2$	(h , k)	Parallel to the x - axis	(h – a , k) (h + a , k)	(h – c , k) (h + c , k)
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ $a^2 > b^2$ a) Major axis parallel to the y – axis foci are c units right and c units left of centre , where $c^2 = a^2 - b^2$	(h , k)	Parallel to the y - axis	(h, k – a) (h, k + a)	(h , k – c) (h , k + c)

HYPERBOLA

a) Transverse axis parallel to the x – axis	a) Transverse axis parallel to the x- axis The equation of a hyperbola with centre C (h, k) and transverse axis parallel to the x- axis is given by $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$ The coordinates of the vertices are A(h+a, k) and A'(h – a , k) . the coordinates of the foci are S(h + c , k) and S'(h – c , k) where $c^2 = a^2 + b^2$ The equations of directrices are x = $\pm\frac{a}{e}$
b) Transverse axis parallel to the y – axis	b) Transverse axis parallel to the y- axis The equation of a hyperbola with centre C (h , k) and transverse axis parallel to the y- axis is given by $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1.$ The coordinates of the vertices are A(h, k+a) and A'(h , k - a) . the coordinates of the foci are S(h, k+c) and S'(h, k-c) where $c^2 = a^2 + b^2$ The equations of directrices are y = $\pm\frac{a}{e}$

CHAPTER 6

VECTOR ALGEBRA

THEOREM 1

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, *then*

$$(\vec{a} \times \vec{b}) . \vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

THEOREM 2

For any three vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, $(\vec{a} \times \vec{b}) . \vec{c} = \vec{a} . (\vec{b} \times \vec{c})$ the scalar triple product of three non – zero vectors is zero if and only if the three vectors are coplanar.

THEOREM 3

The position of a point P (x_1 , y_1) with respect to a given circles $x^2 + y^2 + 2gx + 2fy + c = 0$ in the plane containing the circle is outside or on or inside the circle according as

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is $\begin{cases} > 0, & or \\ = 0, & or \\ < 0 \end{cases}$

THEOREM 4

The scalar triple product of three non – zero vectors is zero, if and only if the three vector are coplanar

THEOREM 5

Three vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if, there exist scalars r, s, t ∈ ℝ such that at least one of them is non – zero and $r\vec{a} + s\vec{b} + t\vec{c} = 0$

THEOREM 6

If $\vec{a}, \vec{b}, \vec{c}$ *and* $\vec{p}, \vec{q}, \vec{r}$ are two system of three vectors, and if $\vec{p} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$,

$\vec{q} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$ and $\vec{r} = x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$, then $[\vec{p}, \vec{q}, \vec{r}] = \begin{vmatrix} \vec{x_1} & \vec{y_1} & \vec{z_1} \\ \vec{x_2} & \vec{y_2} & \vec{z_2} \\ \vec{x_3} & \vec{y_3} & \vec{z_3} \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$

THEOREM 7

The vector triple product satisfies the following properties:

$(\vec{a_1} + \vec{a_2}) \times (\vec{b} \times \vec{c}) = \vec{a_1} \times (\vec{b} \times \vec{c}) + \vec{a_2} \times (\vec{b} \times \vec{c})$,

$(\lambda\vec{a}) \times (\vec{b} \times \vec{c}) = \lambda\left(\vec{a} \times (\vec{b} \times \vec{c})\right)$, $\lambda \in \mathbb{R}$

$\vec{a} \times \left((\vec{b_1} + \vec{b_2}) \times \vec{c}\right) = \vec{a} \times (\vec{b_1} \times \vec{c}) + \vec{a} \times (\vec{b_2} \times \vec{c})$,

$\vec{a} \times \left((\lambda\vec{b}) \times \vec{c}\right) = \lambda\left(\vec{a} \times (\vec{b} \times \vec{c})\right)$, $\lambda \in \mathbb{R}$

$\vec{a} \times \left(\vec{b} \times (\vec{c_1} + \vec{c_2})\right) = \vec{a} \times (\vec{b} \times \vec{c_1}) + \vec{a} \times (\vec{b} \times \vec{c_2})$,

$\vec{a} \times \left(\vec{b} \times (\lambda\vec{c})\right) = \lambda\left(\vec{a} \times (\vec{b} \times \vec{c})\right)$, $\lambda \in \mathbb{R}$

THEOREM 8

Three vectors $\vec{a}, \vec{b}, \vec{c}$ $we\ have$ $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\vec{c})\vec{b} - (\vec{a}\,.\vec{b})\vec{c}$

THEOREM 9 (JACOBI'S IDENTITY)

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ we have $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

THEOREM 10 (LAGRANGE'S IDENTITY)

For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ we have $(\vec{a} \times \vec{b})\,.\,(\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a}\,.\vec{c} & \vec{a}\,.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}\,.\vec{d} \end{vmatrix}$

THEOREM 11

The vector equation of a straight line passing through a fixed point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b},\ where\ t \in \mathbb{R}$

THEOREM 12

The parametric form of of vector equation of a line passing through two given points vector are $\vec{a}$ and $\vec{b}$ respectively is $\vec{r} = \vec{a} + t(\vec{b} - \vec{a}), where\ t \in \mathbb{R}$

Two lines are said to be **a coplanar** if their lie in the same plane.

Two lines in space are called **skew lines** if they are not parallel and do not intersect

THEOREM 13

The shortest distance between the two parallel lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{b}$ is given by

$\delta = \frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|}$ $where$ $|\vec{b}| \neq 0$

THEOREM 14:

The shortest distance between the two skew lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ is given by

$\delta = \frac{|(\vec{c}-\vec{a})\times(\vec{b}\times\vec{d})|}{|\vec{b}\times\vec{d}|}$ $where$ $|\vec{b} \times \vec{d}| \neq 0$

THEOREM 15

The equation of the plane at a distance p from the origin and perpendicular to the unit normal vector $\hat{d}$ is $\vec{r}\,.\,\hat{d} = p$

THEOREM 16

The general equation ax + by + cz + d = 0 of first degree in x , y , z represents a plane.

THEOREM 17

If three non – collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$ are given, then the vector equation of the plane passing through the given points in parametric form is $\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t(\vec{c} - \vec{a})$, where $\vec{b} \neq \vec{0}$, $\vec{c} \neq \vec{0}$ and s , t ∈ ℝ

THEOREM 18

The acute angle θ between the two plans $\vec{r}.\vec{n_1} = p_1$ *and* $\vec{r}.\vec{n_2} = p_2$ is given by $\theta = \cos^{-1}\left(\frac{|\vec{n_1}.\vec{n_2}|}{|n_1||n_2|}\right)$

THEOREM 19

The acute angle θ between the two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by $\theta = \cos^{-1}\left(\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$

THEOREM 20

The perpendicular distance from a point with position vector $\vec{u}$ to the plane $\vec{r}.\vec{n} = p$ is given by $\delta = \frac{|\vec{u}.\vec{n}-p|}{|\vec{n}|}$

THEOREM 21

The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

THEOREM 22

The vector equation of a plane which passes through the line of intersection of the planes $\vec{r}.\vec{n_1} = d_1$ *and* $\vec{r}.\vec{n_2} = d_2$ is given by $(\vec{r}.\vec{n_1} - d_1) + \lambda(\vec{r}.\vec{n_2} - d_2) = 0$ where λ ∈ ℝ

THEOREM 23

The position vector of the point of intersection of the straight line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r}.\vec{n} = p$ is $\vec{a} + \left(\frac{p-(\vec{a}.\vec{n})}{\vec{b}.\vec{n}}\right)$, provided $\vec{b}.\vec{n} \neq 0$

The shortest distance between the two skew lines is the length of the line segment perpendicular to both the skew lines.

A straight line which is perpendicular to a plane is called a normal to the plane.

PROPERTIES OF SCALAR AND VECTOR PRODUCTS

SCALAR PRODUCT	VECTOR PRODUCT
$\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert cos\ \theta$	$\vec{a}\ x\ \vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert sin\ \theta\ \hat{n}$
$\vec{a}.\vec{b} = 0$ i. $\vec{a}$ *is zero vector* $\vec{b}$ *any other vector* ii. $\vec{b}$ *is zero vector* $\vec{a}$ *any other vector* iii. $\vec{a}$ *and* $\vec{b}$ *are perpendicular*	$\vec{a} \times \vec{b} = 0$ i. $\vec{a}$ *is zero vector* $\vec{b}$ *any other vector* ii. $\vec{b}$ *is zero vector* $\vec{a}$ *any other vector* iii. $\vec{a}$ *and* $\vec{b}$ *are parallel*
$\vec{a}.\vec{a} = a^2$	$\vec{a} \times \vec{a} = \vec{0}$
$\vec{\imath}.\vec{\imath} = \vec{\jmath}.\vec{\jmath} = \vec{k}.\vec{k} = 1$ $\vec{\imath}.\vec{\jmath} = \vec{\jmath}.\vec{k} = \vec{k}.\vec{\imath} = 0$	$\vec{\imath} \times \vec{\imath} = \vec{\jmath} \times \vec{\jmath} = \vec{k} \times \vec{k} = \vec{0}$ $\vec{\imath} \times \vec{\jmath} = \vec{k}; \vec{\jmath} \times \vec{k} = \vec{\imath}; \vec{k} \times \vec{\imath} = \vec{\jmath}$
$\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$ $\vec{b} = b_1\vec{\imath} + b_2\vec{\jmath} + b_3\vec{k}$ $\vec{a}.\vec{b} = (a_1b_1 + a_2b_2 + a_3b_3)$	$\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$ $\vec{b} = b_1\vec{\imath} + b_2\vec{\jmath} + b_3\vec{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
$\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$	$\theta = \sin^{-1}\left(\frac{\lvert\vec{a} \times \vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$

A straight will lie on a plane of every point on the line , lie in the plane and the normal to the plane is perpendicular to the line .

CHAPTER 7

APPLICATION OF DIFFERENTIAL CALCULUS

DIFFERENTIAL CALCULUS BASIC FORMULAS

$\frac{d}{dx}[c] = 0$	$\frac{d}{dx}[\sec x] = sec\, x \tan x$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\frac{d}{dx}[\log x] = \frac{1}{x}$
$\frac{d}{dx}[uv] = uv' + vu'$	$\frac{d}{dx}[e^x] = e^x$
$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[\mathrm{cosec}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$
$\frac{d}{dx}[\tan x] = sec^2 x$	$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}[\cot x] = -cosec^2 x$	$\frac{d}{dx}[tan^{-1} x] = \frac{1}{1+x^2}$

VELOCITY & ACCELERATION

If distance x = f(t)

Velocity v = f'(t) or $\frac{dx}{dt}$, acceleration a = $\frac{dv}{dt}$ = f''(t) or $\frac{d^2x}{dt^2}$

(i) Initial velocity means velocity at t = 0
(ii) Initial acceleration means acceleration at t = 0
(iii) If the motion is upward, at the maximum height the velocity is zero.
(iv) If the motion is horizontal v = 0 when the particle cones to rest.

TANGENT $y - y_1 = m(x - x_1)$

NORMAL $y - y_1 = \frac{-1}{m}(x - x_1)$

If the two curves are parallel at (x_1, y_1) then $m_1 = m_2$

If the two curves are perpendicular at (x_1, y_1) , then $m_1m_2 = -1$

ANGLE BETWEEN TWO CURVES

$$\tan\varphi = \left|\frac{m_1 + m_2}{1 + m_1m_2}\right|$$

ROLLE'S THEOREM

Let f(x) be continuous on a closed interval [a, b] and differentiable on the open interval (a, b) let f(a) = f(b) , then there is at least one point c ∈ (a, b) where f'(c) = 0

LAGRANGE'S MEAN VALUE THEOREM

Let f(x) be continuous in a closed interval [a, b] and differentiable on the open interval (a , b) let f(a) = f(b) , then there exist at least one point c ∈ (a, b) where f'(c) = $\frac{f(b)-f(a)}{b-a}$

TAYLOR THEOREM

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \cdots + \frac{f''(a)}{n!}(x-a)^n + \cdots.$$

MACLAURIN'S THEOREM

$$f(x) = f(0) + \frac{f'(0)}{1!}(x) + \cdots + \frac{f''(0)}{n!}(x)^n + \cdots.$$

WORKING RULES

CRITICAL NUMBERS	STATIONARY POINT	INCREASING & DECREASING
If f(x) given (i) Find f'(x) (ii) Solve f'(x) = 0 and get critical number (CN) (If degree of f(x) is n . there are n – 1 critical numbers are possible)	If f(x) given (i) Find f'(x) (ii) Solve f'(x) = 0 and get C.N (iii) Put C.N in f(x) and get stationary points (S.P). (if degree of f(x) is n. there are n – 1 stationary points are possible)	If f(x) given (i) Find f'(x) (ii) Solve f'(x) = 0 and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of f'(x) in above limits If f'(x) > 0 increasing If f'(x) < 0 decreasing

WORKING RULES

MONOTONICITY	ABSOLUTE MAXIMUM/ MINIMUM IN [A , B]	CONCAVITY /CONVEXITY
If f(x) given (i) Find f'(x) (ii) Solve f'(x) = 0 and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of f'(x) in above limits If f'(x) > 0 Increasing If f'(x) < 0 Decreasing both sides of C.N If f'(x) has same sign then it is Monotonicity	If f(x) given (i) Find f'(x) (ii) Solve f'(x) = 0 and get C.N (iii) Put the values of end points [a, b] in f(x) i.e., find f(a) & f(b) and also find value of f(x) in C.N Compare all Maximum one is absolute maximum Minimum one is absolute minimum	If f(x) given (i) Find f'(x) (ii) Find f''(x) (iii) Solve f''(x) = 0 and get values of x. (iv) Fix the limits on both sides of x values (v) Check the sign of f''(x) in above limits If f''(x) > 0 concave upward If f''(x) < 0 concave downward

WORKING RULES

LOCAL MAXIMUM AND MINIMUM		POINT OF INFLECTION
FIRST DERIVATIVE	SECOND DERIVATIVE	
If f(x) given (i) Find f′(x) (ii) Solve f′(x) = 0 and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of f′(x) in above limits If f′(x) > 0 Increasing If f′(x) < 0 Decreasing If the results is : + ve to – ve to local maximum - ve to + ve to local minimum	If f(x) given (i) Find f′(x) (ii) Solve f′(x) = 0 and get C.N (iii) Find f′′(x) (iv) Check the sign of f′′(x) in critical number If the sign of If f′′(x) > 0 local minimum If f′′(x) < 0 local maximum $\int_0^{\infty} x^n e^{-ax} dx = \frac{\lfloor n}{a^{n+1}}$	If f(x) given (i) Find f′(x) (ii) Find f′′(x) (iii) Solve f′′(x) = 0 and get values of x. (iv) Fix the limits on both sides of x values (v) Check the sign of f′′(x) in above limits If f′′(x) > 0 concave upward If f′′(x) < 0 concave downward If a point has on both sides at the point the curve has a point of inflection

CHAPTER 8

DIFFERENTIAL & PARTIAL DERIVATIVES

PROPERTIES OF DIFFERENTIALS

Here we consider real – valued functions of real variable.

1) If f is a constant function , then df = 0
2) If f(x) = x identity function , then df = 1 dx
3) If f is differentiable and $c \in \mathbb{R}$, then d (cf) = c f'(x) dx
4) If f, g are differentiable , then d(f + g) = df + dg = f' (x) dx + g'(x) dx
5) If f, g are differentiable , then d(fg) = fdg + gdf = (f(x) g'(x) + f'(x)g(x)) dx
6) If f, g are differentiable , then d(f / g) = $\frac{g\,df - f\,dg}{g^2} = \frac{g(x)f'(x) - f(x)g\prime(x)}{g^2x}$ dx, where g(x) ≠ 0
7) If f, g are differentiable and $h = f \circ g$ is defined, then dh = f ' (g(x)) g'(x) dx .
8) If $h(x) = e^{f(x)}$, $then\ dh = e^{f(x)} f'(x)dx$
9) If f(x) > 0 for all x and g(x) = log (f(x)) , then $dg = \frac{f'(x)}{f(x)}dx$

Absolute error = actual value – approximate value

Relative error = $\frac{absolute\ error}{actual\ error}$

Percentage error = relative error × 100 (or) $\frac{absolute\ error}{actual\ error} \times 100$

Euler's Theorem $\implies$ $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = nu$

CHAPTER 9

APPLICATIONS OF INTEGERALS

BASIC INTEGRATION FORMULAS	
$\int \sin x\, dx = -\cos x + c$	$\int x^n\, dx = \frac{x^{n+1}}{n+1}\, dx$
$\int \cos x\, dx = \sin x + c$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\int \tan x\, dx = -\log(\cos x) + c$	$\int \frac{1}{x} dx = \log x + c$
$\int \cot x\, dx = \log(\sin x) + c$	$\int sec^2 x\, dx = tanx + c$
$\int \sec x \tan x\, dx = secx + c$	$\int e^x dx = e^x + c$
$\int cosec\, x \cot x\, dx = -cosec\, x + c$	
$\int cosec^2 x\, dx = -\cot x + c$	
$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + c$	
$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$	
$\int cosec\, x\, dx = -\log(cosec\, x + \cot x) + c$	

IMPORTANT PROPERTIES OF INTEGRAL

$\int_a^b f(x)dx = F(b) - F(a)$	$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
$\int_a^b f(x)dx = -\int_b^a f(x)dx$	$\int_0^a f(x)dx = \int_0^a f(a-x)dx$
$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx\ if\ (2a-x) = f(x)$	
$\int_0^{2a} f(x)dx = 0\ if\ (2a-x) = -f(x)$	
$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx\ if\ f\ is\ even$ $\int_{-a}^{a} f(x)dx = 0\ if\ f\ is\ odd$	

DEFINITE INTEGRAL AS THE LIMIT OF A SUM

$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{r=1}^{n} f\left(a + (b-a)\frac{r}{n}\right)$
$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{n} f\left(\frac{r}{n}\right) = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

BERNOULLI'S FORMULA

$$\int uvdx = uv_1 - u^2v_3 + u^3v_4 + \cdots.$$

REDUCTION FORMULAS

$$\int_0^{\frac{\pi}{2}} sin^n x\, dx = \int_0^{\frac{\pi}{2}} cos^n x\, dx = \begin{cases} \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \times \ldots \times \frac{1}{2} \times \frac{\pi}{2}\ , if\ n = 2, 4, 6 \\ \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \times \ldots \times \frac{2}{3}, if\ n = 3,5,7 \end{cases}$$

GAMMA FORMULAS

$$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx = (n-1)!$$

$$\int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$$

Volume of the solid of revolution

The volume of the solid of revolution about x – axis is V = $\pi \int_a^b y^2\, dx$.

The volume of the solid of revolution about y – axis is V = $\pi \int_a^b x^2\, dy$.

CHAPTER 10

DIFFERENTIAL EQUATION

1. A differential equation is any equation which contains at least one derivative of an unknown function , either ordinary derivative or partial derivative.
2. The order of a differential equation is the highest derivative present in the differential equation.
3. If a differential equation is expressible in a polynomial form , then the integral power of the highest order derivative appears is called the degree of the differential equation.
4. If a differential equation is not expressible to polynomial equation form having the highest order derivative as the leading term then that the degree of the differential equation is not defined.
5. If a differential equation contains only ordinary derivatives of one or more functions with respect to a single independent variable , it is said to be an ordinary differential equation (ODE).
6. An equation involving only partial derivatives of one or more function of two or more independent variables is called a partial differential equation (PDE).
7. The result of eliminating one arbitrary constant yields a first order differential equation and that of eliminating two arbitrary constants leads to a second order differential equation and so on.
8. A solution of a differential equation is an expression for the dependent variable in terms of the independent variable(s) which satisfies the differential equation.
9. The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution.
10. If we given particular values to the arbitrary constants in the general solution of differential equation , the resulting solution is called a Particular solution.

The order of a differential equation is the order of the highest order derivative occurring in it.

The degree of the differential equation is the degree of the highest order derivative which occur in it.

$e^{logA} = A$ $e^{mlogA} = A^m$ $e^{-mlog\,A} = \frac{1}{A^m}$

LINEAR DIFFERENTIAL EQUATION

If linear equation is in the form of $\frac{dy}{dx} + py = Q,$

then solution of $y\, e^{\int pdx} = \int Qe^{\int pdx}dx + c$

If linear equation is in the form of $\frac{dx}{dy} + px = Q,$

then solution of $x\, e^{\int pdy} = \int Qe^{\int pdy}dy + c$

CHAPTER 11

PROBABILITY

PROBABILITY MASS FUNCTION

The mathematical definition of discrete probability function p(x) is a function that satisfies the following properties

1. The probability that X can take a specific values x is p(x), i.e., P(X = x) = p(x) = p_x
2. p(x) is non – negative for all real x.
3. The sum of p(x) over all possible values of X is one. That is pi = 1 where j represents all possible values that X an have and pi is the probability at X = xi

If a_1, a_2 , ... , a_m , a , b_1, b_2 ,... b_n , b be the values of the discrete random variable X in ascending order then

(i) P(X ≥ a) = 1 – P (X < a)
(ii) P(X ≤ a) = 1 – P (X > a)
(iii) P (a ≤ X ≤ b) = P(X = a) + P (x = b1) + P (X = b2) + ... + P (X = bn) + P(X = b)

PROPERTIES OF CUMULATIVE DISTRIBUTION FUNCTION :

1. F(x) is a non - decreasing function of x.
2. 0 ≤ F(x) ≤ 1 , $-\infty < x < \infty$
3. $F(-\infty) = \lim_{x\to-\infty} F(x) = 0$
4. $F(\infty) = \lim_{x\to\infty} F(x) = 1$
5. F (X = x_n) = F(x_n) – F (x_{n-1})

PROPERTIES OF DISTRIBUTION FUNCTION:

1. F(x) is a non - decreasing function of x .
2. 0 ≤ F(x) ≤ 1 , $-\infty < x < \infty$
3. $F(-\infty) = \lim_{x\to-\infty} \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{\infty} f(x)dx = 0$
4. $F(\infty) = \lim_{x\to\infty} \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{\infty} f(x)dx = 0$
5. For all real constant a and b , a ≤ b , $P(a \le x \le b) = F(b) - F(a)$
6. $f(x) = \frac{d}{dx}F(x) = i.e., F'(x) = f(x)$

MATHEMATICAL EXPECTATION

$$E(X) = p_1x_1 + p_2x_2 + \cdots + p_nx_n = \sum_{i=1}^{n} p_ix_i = \sum_{i=1}^{n} p_i = 1$$

PROPERTIES:

1. E(c) = c where c is a constant
2. E(c X) = c E(x)
3. E(aX – b) = a E (x) + b
4. Var(X) = E(X^2) – [E (x)]^2
5. Var(X + c) = var(X)
6. Var(aX) = a^2Var(X)
7. Var(C) = 0

THEORETICAL DISTRIBUTION;

	FUNCTION	MEAN	VARIANCE	STANDARD DEVIATION
Binomial	P(X = x) = P(x) = $\begin{cases} nc_x\, , p^x q^{n-x}, x = 0,1,2,\dots,n \\ 0 \quad\quad elsewhere \end{cases}$	np	npq	$\sqrt{npq}$
Poisson	$P(X = x) = \frac{e^{-\lambda}\lambda^x}{\lfloor x}$, $x = 0, 1, 2, \dots for\ some \quad \lambda > 0$	λ	λ	$\sqrt{\lambda}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$; $-\infty < x < \infty$	μ	σ^2	σ

CHAPTER 12

DISCREATE MATHEMATICS

TRUTH TABLE

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

A non- empty set G, together with an operation * i.e., (G , *) is said to be a group if it satisfies the following axioms

1. **Closure axiom :** a , b ∈ G ⇒ a * b ∈ G
2. **Associative axiom :** $\forall\ a, b, c\ \in G$, (a* b) * c = a*(b* c)
3. **Identity axiom:** There exists an element $e\ \in G\ such\ that\ a * e = e * a = a\ ,\forall\ a\ \in G$
4. **Inverse axiom :** $\forall\ a\ \in G\ \ there\ exists\ an\ element\ a^{-1}\ \in G$

   $such\ that\ a^{-1} * a = a * a^{-1} = e$

e is called the identity element of G and a^{-1} is called the inverse of a in G

Idempotent laws $(i)\ p \vee p \equiv p$ $(ii)\ p \wedge p \equiv p$

Commutative laws $(i)\ p \vee q = q \vee p$ $(ii)\ p \wedge q = q \wedge p$

Associative laws $(i)\ p \vee (q \vee r) \equiv (p \vee q) \vee r$ $(ii)\ p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive laws $(i)\ p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$(ii) p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Identity laws $(i)\ p \vee T \equiv T\ \ and\ p \vee F \equiv p$

$(ii)\ p \wedge T \equiv P\ \ and\ p \wedge F \equiv F$

Complement laws $(i)\ p \vee \neg p \equiv T\ and\ p \wedge \neg p \equiv F$

$\neg T \equiv F$ and $\neg F \equiv T$

Involution law or double negation law $\neg(\neg p) = p$

De Morgan's law (i) $\neg(p \wedge q) \equiv \neg p\ \vee\ \neg q$ $(ii) \neg(p \vee q) \equiv \neg p \vee \neg q$

Absorption laws $(i) p \vee (p \wedge q) \equiv p$ $(ii)\ p \wedge (p \vee q) \equiv p$

IMPORTANT TRIGONOMETRY IDENTITIES

RECIPROCALS IDENTITIES

$cosec\ x = \frac{1}{\sin x}$

$sec\ x = \frac{1}{\cos x}$

$cotx = \frac{1}{\tan x}$

QUOTIENT ANGLES

$\tan x = \frac{\sin x}{\cos x}$

$\tan x = \frac{\cos x}{\sin x}$

PYTHAGOREAN IDENTITIES

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\mathrm{cosec}^2 x = 1 + \cot^2 x$

DOUBLE IDENTITIES

Sin2x = 2 sinxcosx

Cos2x = $\cos^2 x - \sin^2 x$

= $2\cos^2 x - 1$

= $1 - 2\sin^2 x$

tan2x = $\frac{2 \tan x}{1 - tan^2 x}$

ADDITION AND SUBTRACTION

Sin(x + y) = sinx cosy + cosx siny

Sin(x - y) = sinx cosy - cosx siny

cos(x + y) = cosx cosy - sinx siny

cos(x - y) = cosx cosy + sinx siny

1. Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact.

Solution:

$x^2+3y^2=12$	$x-y+4=0$
$\frac{x^2}{12}+\frac{y^2}{4}=1$	$y=x+4$
$a^2=12, b^2=4$	$m=1,\ c=4$

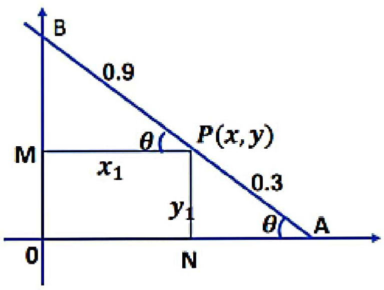
Condition: $a^2m^2+b^2=c^2=16$

Given line is a tangent to the ellipse.

Point of contact: $\left(-\frac{a^2m}{c},\frac{b^2}{c}\right)=(-3,1)$

2. A rod of length $1.2m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3m$ from the end in contact with x –axis is an ellipse. Find the eccentricity.

Solution:



$\cos\theta=\frac{x_1}{\frac{9}{10}}$ $\quad\sin\theta=\frac{y_1}{\frac{3}{10}}$

$\frac{x_1{}^2}{\frac{81}{100}}+\frac{y_1{}^2}{\frac{9}{100}}=1$

Ellipse : $\frac{x^2}{\frac{81}{100}}+\frac{y^2}{\frac{9}{100}}=1$

$a^2=\frac{81}{100}$, $b^2=\frac{9}{100}$

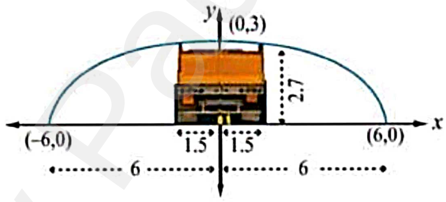
Eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}$

$e=\sqrt{1-\frac{\frac{9}{100}}{\frac{81}{100}}}=\sqrt{1-\frac{1}{9}}$

$e=\frac{2\sqrt{2}}{3}$

3. A semielliptical archway over a one-way road has a height of 3m and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

Solution:



Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$a=6, b=3\Rightarrow\frac{x^2}{36}+\frac{y^2}{9}=1$

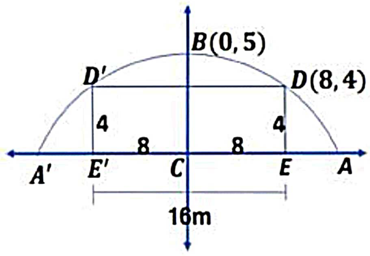
At $(1.5,\ y)\Rightarrow\frac{(1.5)^2}{36}+\frac{y^2}{9}=1$

$y=2.9\ m>2.7$ (*height of the truck*)

The truck will clear the archway

4. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately. How wide must the opening be?

Solution:



Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

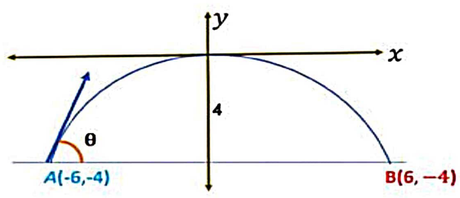
$b=5\Rightarrow\frac{x^2}{a^2}+\frac{y^2}{25}=1$

At $(8,4)\Rightarrow\frac{64}{a^2}+\frac{16}{25}=1\Rightarrow a=\frac{40}{3}$

Required width$=2a=\frac{80}{3}=26.66\ m$

5. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

Solution:



$Equation\ of\ parabola: x^2 = -4ay$

$At\ (-6,-4) \Rightarrow \quad (-6)^2 = -4a(-4)$

$4a = 9$

$Equation\ of\ parabola: x^2 = -9y$

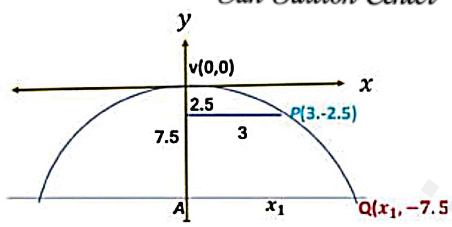
$D.w.r.to\ x,\ \frac{dy}{dx} = -\frac{2x}{9}$

$At\ (-6,-4) \Rightarrow \quad tan\theta = \frac{dy}{dx} = \frac{4}{3}$

$$\theta = tan^{-1}\left(\frac{4}{3}\right)$$

6. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

Solution:



$Equation\ of\ parabola: x^2 = -4ay$

$At\ (3,-2.5) \Rightarrow 9 = 10a$

$$a = \frac{9}{10}$$

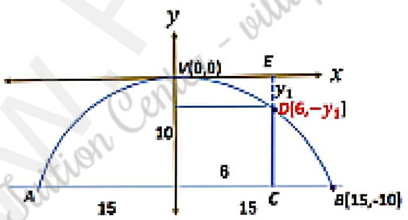
$Equation\ of\ parabola: x^2 = -4\left(\frac{9}{10}\right)y$

$At\ (x_1,-7.5) \Rightarrow {x_1}^2 = -4\left(\frac{9}{10}\right)(-7.5)$

${x_1}^2 = 27 \Rightarrow x_1 = 3\sqrt{3}\ m$

7. A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.

Solution:



$Equation\ of\ parabola: x^2 = -4ay$

$At\ (15,-10)\ \Rightarrow\ 225 = 40a$

$$a = \frac{225}{40}$$

$Equation\ of\ parabola: x^2 = -4\left(\frac{225}{40}\right)y$

$$x^2 = -\frac{225}{10}y$$

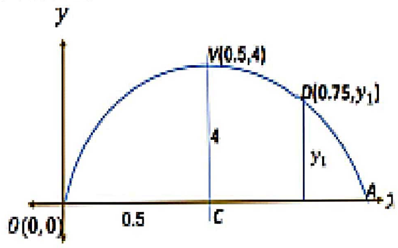
$At\ (6,\ -y_1) \Rightarrow 36 = -\frac{225}{10}(-y_1)$

$y_1 = 1.6$

$Required\ height = 10 - 1.6 = 8.4\ m$

8. At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

Solution:



$Vertex\ (h,k) = (0.5, 4)$

$Equation\ of\ parabola:$

$$(x - 0.5)^2 = -4a(y - 4)$$

$At\ (0,0) \Rightarrow\ 0.25 = 16a$

$$a = \frac{0.25}{16}$$

$Parabola: (x-0.5)^2 = -4 \times \frac{0.25}{16}(y-4)$

$At\ (0.75,\ y_1) \Rightarrow$

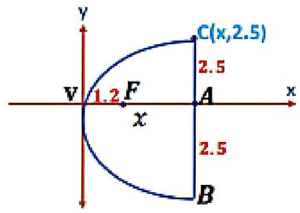
$$(0.75-0.5)^2 = -4 \times \frac{0.25}{16}(y_1 - 4)$$

$$y_1 = 3\,m$$

$Required\ height = \ 3\,m$

9. An engineer designs a satellite dish with a parabolic cross section. The dish is $5m$ wide at the opening, and the focus is placed $1.2m$ from the vertex (a) Position a coordinate system with the origin at the vertex and the x −axis on the parabola's axis of symmetry and find an equation of the parabola. (b) Find the depth of the satellite dish at the vertex.

Solution:



$Equation\ of\ parabola: y^2 = 4ax$

$(i)\ At\ a = 1.2 \Rightarrow y^2 = 4(1.2)x$

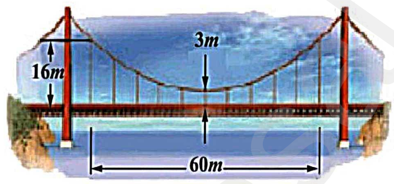
$$y^2 = 4.8x$$

$(ii)\ At\ (x, 2.5)\ \Rightarrow (2.5)^2 = 4.8x$

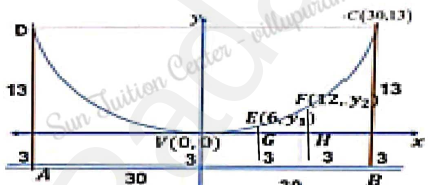
$$x = 1.3\,m$$

$Depth\ of\ the\ satellite\ =\ 1.3\,m$

10. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



Solution:



$Equation\ of\ parabola: x^2 = 4ay$

$At\ (30, 13) \Rightarrow (30)^2 = 4a(13)$

$$\frac{(30)^2}{13} = 4a$$

$Equation\ of\ parabola: x^2 = \frac{(30)^2}{13}y$

$At\ (6, y_1) \Rightarrow 6^2 = \frac{(30)^2}{13}y_1$

$$y_1 = 0.52\,m$$

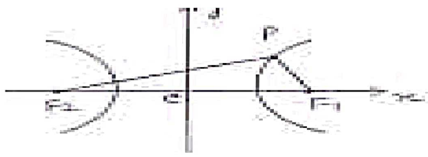
$Required\ height = 3 + 0.52 = 3.52\,m$

$At\ (12, y_2) \Rightarrow 12^2 = \frac{(30)^2}{13}y_2$

$$y_2 = 2.08\,m$$

$Required\ height = 3 + 2.08 = 5.08\,m$

11. Points A and B are $10km$ apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.



$2a = 6\ \Rightarrow a = 3\ \&\ 2c = 10 \Rightarrow c = 5$

$b^2 = c^2 - a^2 = 25 - 9 = 16$

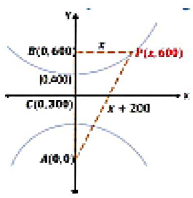
$Hyperbola: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

12. Two coast guard stations are located $600km$ apart at points $A(0,0)$ and $B(0, 600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is

200km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

Solution:



$2a = 200 \Rightarrow a = 100$

$2c = 600 \Rightarrow c = 300$

$b^2 = c^2 - a^2 = 300^2 - 100^2 = 80000$

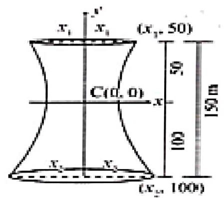
$Centre\ (h,k) = (0,300)$

$$Hyperbola: \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1$$

13. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150$m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

Solution:



$$Hyperbola: \frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$$

$$x = \frac{30}{44}\sqrt{44^2 + y^2}$$

$$At\ (x_1, 50) \Rightarrow x_1 = \frac{30}{44}\sqrt{44^2 + 50^2}$$

$$\Rightarrow x_1 = 45.41\ m$$

$The\ diameter\ of\ the\ top = 2x_1 = 90.82$

$$At\ (x_2, 100) \Rightarrow x_2 = \frac{30}{44}\sqrt{44^2 + 100^2}$$

$$\Rightarrow x_2 = 74.45\ m$$

$The\ diameter\ of\ the\ bottom\ 2x_2 = 148.9\ m$

14. Find the equation of the circle through the points $(1,0)$, $(-1,0)$ and $(0,1)$.

Solution:

$General\ equation\ of\ the\ circle$:

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(1,0) \Rightarrow 2g + c = -1$

$(-1,0) \Rightarrow -2g + c = -1$

$(0,1) \Rightarrow 2f + c = -1$

$Solving\ we\ get,\ c = -1,\ g = 0,\ f = 0$

$Equation\ of\ the\ circle: x^2 + y^2 = 1$

15. Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$ and $(3,2)$.

Solution:

$General\ equation\ of\ the\ circle$:

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(1,1) \Rightarrow 2g + 2f + c = -2$

$(2,-1) \Rightarrow 4g - 2f + c = -5$

$(3,2) \Rightarrow 6g + 2f + c = -13$

$Solving\ we\ get,\ c = 4,\ g = \frac{-5}{2},\ f = \frac{-1}{2}$

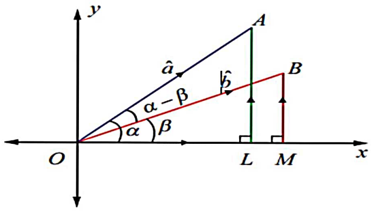
$Equation\ of\ the\ circle$:

$$x^2 + y^2 + 2(\frac{-5}{2})x + 2(\frac{-1}{2})y + c = 0$$

$$x^2 + y^2 - 5x - y + 4 = 0$$

16. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

Solution:



$\hat{a}=\cos\alpha\,\hat{\imath}+\sin\alpha\,\hat{\jmath}$

$\hat{b}=\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath}$

$\hat{a}\cdot\hat{b}=(\cos\alpha\,\hat{\imath}+\sin\alpha\,\hat{\jmath})\cdot(\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath})$

$\hat{a}\cdot\hat{b}=\cos\alpha\cos\beta+\sin\alpha\sin\beta$............①

By definition,

$\hat{a}\cdot\hat{b}=|\vec{a}||\vec{b}|\cos(\alpha-\beta)=\cos(\alpha-\beta)$...... ②

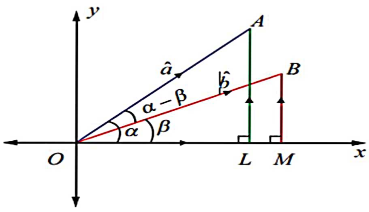
From ① & ② *we get,*

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

17. Prove by vector method that

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

Solution:



$\hat{a}=\cos\alpha\,\hat{\imath}+\sin\alpha\,\hat{\jmath}$

$\hat{b}=\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath}$

$\hat{b}\times\hat{a}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \cos\beta & \sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}$

$\hat{b}\times\hat{a}=(\sin\alpha\cos\beta-\cos\alpha\sin\beta)\hat{k}$.......①

By definition,

$\hat{b}\times\hat{a}=|\vec{b}||\vec{a}|\sin(\alpha-\beta)\hat{k}$

$\hat{b}\times\hat{a}=\sin(\alpha-\beta)\hat{k}$........②

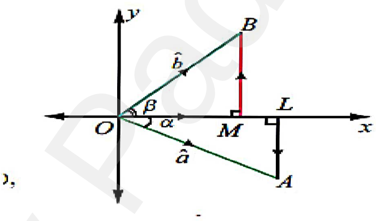
From ① & ② *we get*

$\mathrm{Sin}(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

18. By vector method, prove that

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

Solution:



$\hat{a}=\cos\alpha\,\hat{\imath}-\sin\alpha\,\hat{\jmath}$

$\hat{b}=\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath}$

$\hat{a}\cdot\hat{b}=(\cos\alpha\,\hat{\imath}-\sin\alpha\,\hat{\jmath})\cdot(\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath})$

$\hat{a}\cdot\hat{b}=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.......①

By definition,

$\hat{a}\cdot\hat{b}=|\vec{a}||\vec{b}|\cos(\alpha+\beta)=\cos(\alpha+\beta)$......②

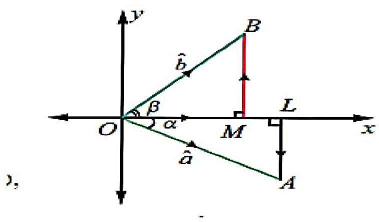
From ① & ② *we get,*

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

19. Prove by vector method that

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

Solution:



$\hat{a}=\cos\alpha\,\hat{\imath}-\sin\alpha\,\hat{\jmath}$

$\hat{b}=\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath}$

$\hat{a}\times\hat{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \cos\beta & \sin\beta & 0\\ \cos\alpha & -\sin\alpha & 0\end{vmatrix}$

$\hat{a}\times\hat{b}=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\hat{k}$........①

$\hat{a}\times\hat{b}=|\vec{a}||\vec{b}|\sin(\alpha+\beta)\hat{k}$

$\hat{a}\times\hat{b}=\sin(\alpha+\beta)\hat{k}$.......②

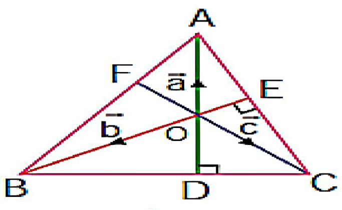
From ① & ② *we get,*

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

20. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are

concurrent.

Solution:



AD ⊥ BC $\Rightarrow \overrightarrow{OA} \,.\, \overrightarrow{BC} = 0$

$\Rightarrow \overrightarrow{OA} \,.\, (\overrightarrow{OC} - \overrightarrow{OB}) = 0$

$\Rightarrow \vec{a} \,.\, (\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a} \,.\, \vec{c} - \vec{a} \,.\vec{b} = 0$..........(1)

BE ⊥ CA $\Rightarrow \overrightarrow{OB} \,.\, \overrightarrow{CA} = 0$

$\Rightarrow \overrightarrow{OB} \,.\, (\overrightarrow{OA} - \overrightarrow{OC}) = 0$

$\Rightarrow \vec{b} \,.\, (\vec{a} - \vec{c}) = 0$

$\Rightarrow \vec{b} \,.\, \vec{a} - \vec{b} \,.\, \vec{c} = 0$...........(2)

(1) + (2) $\Rightarrow \vec{a} \,.\, \vec{c} - \vec{a} \,.\, \vec{b} + \vec{b} \,.\, \vec{a} - \vec{b} \,.\, \vec{c} = 0$

$\Rightarrow (\vec{a} - \vec{b}) \,.\, \vec{c} = 0$

$\Rightarrow \overrightarrow{BA} \,.\, \overrightarrow{OC} = 0$

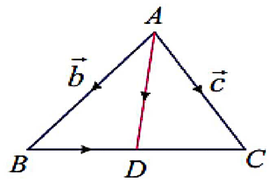
The altitudes are concurrent.

21. **Apollonius's theorem**:

If D is the midpoint of the side BC of a triangle ABC, show by vector method that

$$|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2\left(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2\right).$$

Solution:



A−Origin

D − Midpoint of AB

$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2}$

$\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \frac{\vec{b}+\vec{c}}{2} - \vec{b} = \frac{\vec{c}-\vec{b}}{2}$

$|\overrightarrow{AD}|^2 = \overrightarrow{AD} \cdot \overrightarrow{AD} = (\frac{\vec{b}+\vec{c}}{2}) \cdot (\frac{\vec{b}+\vec{c}}{2})$

$= \frac{1}{4}\left(|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\,.\,\vec{c}\right)$--------(1)

$|\overrightarrow{BD}|^2 = \overrightarrow{BD} \cdot \overrightarrow{BD} = (\frac{\vec{c}-\vec{b}}{2}) \cdot (\frac{\vec{c}-\vec{b}}{2})$

$= \frac{1}{4}\left(|\vec{c}|^2 + |\vec{b}|^2 - 2\vec{c} \cdot \vec{b}\right)$ .... (2)

(1) + (2) $\Rightarrow$

$|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2 = \frac{1}{2}\left(|\vec{b}|^2 + |\vec{c}|^2\right)$

$|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2 = \frac{1}{2}\left(|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2\right)$

$2\left(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2\right) = |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2$

22. If $\vec{a} = \hat{\imath} - \hat{\jmath}$, $\vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}$, $\vec{c} = 3\hat{\jmath} - \hat{k}$ and $\vec{d} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$, verify that

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

Solution:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 0 \\ 1 & -1 & 4 \end{vmatrix} = 4\hat{\imath} + 4\hat{\jmath}$$

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 5 & 1 \end{vmatrix} = 8\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$$

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 4 & 0 \\ 8 & -2 & -6 \end{vmatrix}$$

$= -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k}$.......(1)

$[\vec{a}, \vec{b}, \vec{d}] = 28$, $\quad [\vec{a}, \vec{b}, \vec{c}] = 12$

$[\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d} = -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k}$...(2)

From (1) & (2) we get,

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

23. If $\vec{a} = \hat{\imath} - \hat{\jmath}$, $\vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}$, $\vec{c} = 3\hat{\jmath} - \hat{k}$ and $\vec{d} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$, verify that

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

Solution:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 0 \\ 1 & -1 & 4 \end{vmatrix} = 4\hat{\imath} + 4\hat{\jmath}$$

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 5 & 1 \end{vmatrix} = 8\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$$

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 4 & 0 \\ 8 & -2 & -6 \end{vmatrix}$$

$= -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k}$.......①

$[\vec{a},\vec{c},\vec{d}] = 10$, $[\vec{b},\vec{c},\vec{d}] = 34$

$[\vec{a},\vec{c},\vec{d}]\vec{b} - [\vec{b},\vec{c},\vec{d}]\vec{a} = -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k}$...②

From ① & ② *we get,*

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{c},\vec{d}]\vec{b} - [\vec{b},\vec{c},\vec{d}]\vec{a}$$

24. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

Solution:

Given $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

Let $f(x,y) = \sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$

f is a homogeneous function of degree $n = \frac{1}{2}$

and $f = \sin u$

By Euler's theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial(\sin u)}{\partial x} + y\frac{\partial(\sin u)}{\partial y} = \frac{1}{2}\sin u$$

$$\cos u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \frac{1}{2}\sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

25 . If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.

Solution:

Given $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$

Let $f(x,y) = e^v = \dfrac{x^2+y^2}{x+y}$

f is a homogeneous function of degree $n = 1$

and $f = e^v$

By Euler's theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial(e^v)}{\partial x} + y\frac{\partial(e^v)}{\partial y} = 1\, e^v$$

$$e^v\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = e^v$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1.$$

26. $w(x,y,z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$

find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$

Solution:

Given w= $\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$

Let $f = e^v = \dfrac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}$

f is a homogeneous function of degree $n = 5$

and $f = e^v$

By Euler's theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial(e^v)}{\partial x} + y\frac{\partial(e^v)}{\partial y} = 5\, e^v$$

$$e^v\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = 5e^v$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 5$$

27. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

Solution:

Given $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$

u is a homogeneous function of degree $n = \frac{3}{2}$

By Euler's theorem,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$$

28. Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

Solution: $\mathbb{Z}_5 = \{[0], [1], [2], [3], [4]\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(i) Each box has an unique element of $\mathbb{Z}_5$

(ii) The entries are symmetrical about the main diagonal.

(iii) As usual, the associative property can be seen to be true.

(iv) The identity element of $\mathbb{Z}_5$ is 0.

(v) The inverse of 0 is 0

The inverse of 1 is 4

The inverse of 2 is 3

The inverse of 3 is 2

The inverse of 4 is 1

29. Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.

Solution: $A = \{1, 3, 4, 5, 9\}$

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

(i) Each box has an unique element of A

(ii) The entries are symmetrical about the main diagonal.

(iii) As usual, the associative property can be seen to be true.

(iv) The identity element of A is 1.

(v) The inverse of 1 is 1

The inverse of 3 is 4

The inverse of 4 is 3

The inverse of 5 is 9

The inverse of 9 is 5

30. Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the following operation on the given set $m * n = m + n - mn, \forall\, m, n \in \mathbb{Z}$.

Solution:

Let $m, n, p \in \mathbb{Z}$

(i) Closure property:

Clearly $m + n - mn = m * n \in \mathbb{Z}$

(ii) Commutative property:

$$m * n = m + n - mn$$
$$n * m = n + m - nm$$
$$\therefore\ m * n = n * m$$

(iii) Associative property:

$(m * n) * p = (m + n - mn) * p$

$= m + n + p - mn - mp - np + mnp$...①

$m * (n * p) = m * (n + p - np)$

$= m + n + p - mn - mp - np + mnp$..②

From & we get,

$$(m * n) * p = m * (n * p)$$

(iv) Identity property:

$$m * e = e * m = m$$

$m * e = m \Rightarrow\ m + e - me = m$

$$e(1 - m) = 0$$
$$\Rightarrow e = 0 \in \mathbb{Z}$$

(v) Inverse property:

$$m * m' = m' * m = e$$

$m * m' = 0 \Rightarrow m + m' - mm' = 0$

$$m'(1 - m) = -m$$
$$m' = \frac{-m}{1 - m} = \frac{m}{m - 1}$$

When $m = 1$, m' is not defined.

When $m = 3$, $m' = \frac{3}{2} \notin \mathbb{Z}$

Inverse does not exist in $\mathbb{Z}$.

31. Let A be $\mathbb{Q}\backslash\{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so,(i) examine the commutative and associative properties satisfied by $*$ on A. (ii) examine the existence of identity and existence of inverse properties for the operation $*$ on A.

Solution:

Let $x, y, z \in \mathbb{Q}\backslash\{1\}$

(i) Closure property:

Let $x, y \in \mathbb{Q}\backslash\{1\}$, then $x \neq 1$ and $y \neq 1$

$x - 1 \neq 0$ and $y - 1 \neq 0$

$(x - 1)(y - 1) \neq 0$

$xy - x - y + 1 \neq 0$

$1 \neq x + y - xy$

$1 \neq x * y$

$x * y \in \mathbb{Q}\backslash\{1\}$

(ii) Commutative property:

$$x * y = x + y - xy$$
$$y * x = y + x - yx$$
$$\therefore\ x * y = y * x$$

(iii) Associative property:

$(x * y) * z = (x + y - xy) * z$

$= x + y + z - xy - xz - yz + xyz$.......①

$x * (y * z) = x * (y + z - yz)$

$= x + y + z - xy - xz - yz + xyz$ .....②

From & we get,

$$(x * y) * z = x * (y * z)$$

(iv) Identity property:

$$x * e = e * x = x$$

$x * e = x \Rightarrow\ x + e - xe = x$

$$e(1 - x) = 0$$
$$\Rightarrow e = 0 \in \mathbb{Q}\backslash\{1\}$$

Queen of science is mathematics

(v) Inverse property:

$$x * x' = x' * x = e$$

$x * x' = 0 \Rightarrow x + x' - xx' = 0$

$$x'(1-x) = -x$$

$$x' = \frac{-x}{1-x} = \frac{x}{x-1} \in \mathbb{Q}\backslash\{1\}$$

32. Define on operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right); a, b \in \mathbb{Q}$. (i) Examine the closure, commutative and associative properties satisfied by $*$ on $\mathbb{Q}$.
(ii) Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.

Solution: Let $a, b, c \in \mathbb{Q}$

(i) Closure property:

$$a * b = \frac{a+b}{2} \in \mathbb{Q}$$

(ii) Commutative property:

$$a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a$$

(iii) Associative property:

$$(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{a+b+2c}{4}$$

$$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{2a+b=c}{4}$$

$$(a * b) * c \neq a * (b * c)$$

(iv) Identity property:

$$a * e = e * a = a$$

$a * e = a \Rightarrow \frac{a+e}{2} = a$

$$\Rightarrow e = a \in \mathbb{Q}$$

Here $e = a$ is not unique.

Identity property is not satisfied.

(v) Inverse property:

Identity property is not satisfied, we cannot find inverse.

33. Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, (i) examine the commutative and associative properties satisfied by $*$ on M. (ii) examine the existence of identity and existence of inverse properties for the operation $*$ on M.

Solution:

(i) Closure property:

Let $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}, B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in M$

$AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$

(ii) Commutative property:

$$AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$$

$$BA = \begin{pmatrix} y & y \\ y & y \end{pmatrix}\begin{pmatrix} x & x \\ x & x \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$$

$$\therefore AB = BA$$

(iii) Associative property:

Matrix multiplication is always associative.

(iv) Identity property:

$$A * E = E * A = A$$

$AE = A \Rightarrow \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\Rightarrow 2xe = x$$

$$e = \frac{1}{2}$$

$$E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in M$$

(v) Inverse property:

$$A * A' = A' * A = A$$

$AA' = A \Rightarrow \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} x' & x' \\ x' & x' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$$\begin{pmatrix} 2xx' & 2xx' \\ 2xx' & 2xx' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\Rightarrow 2xx' = \frac{1}{2}$$

$$x' = \frac{1}{4x}$$

$$A' = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \in M$$

34. Find theparametric form, non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$	Point: $\vec{a}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ Vectors: $\vec{b}=2\hat{\imath}+3\hat{\jmath}+1\hat{k}$ $\vec{c}=2\hat{\imath}-5\hat{\jmath}-3\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s\,\vec{b}+t\,\vec{c}$ $\vec{r}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}+$ $s\,(2\hat{\imath}+3\hat{\jmath}+1\hat{k})+$ $t\,(2\hat{\imath}-5\hat{\jmath}-3\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-3 & z-6 \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix}=0$ $(x-2)(-9+5)-(y-3)(-6-2)$ $+(z-6)(-10-6)=0$ $\Rightarrow x-2y+4z-20=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a}).(\vec{b}\times\vec{c})=0$ $\vec{r}.(\hat{\imath}-2\hat{\jmath}+4\hat{k})=20$
35. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+s(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=(\hat{\imath}-3\hat{\jmath}+5\hat{k})+t(\hat{\imath}+\hat{\jmath}-\hat{k})$.	Point: $\vec{a}=\hat{\jmath}-5\hat{k}$ Vectors: $\vec{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ $\vec{c}=\hat{\imath}+\hat{\jmath}-\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s\,\vec{b}+t\,\vec{c}$ $\vec{r}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}+$ $s\,(2\hat{\imath}+3\hat{\jmath}+1\hat{k})+$ $t\,(2\hat{\imath}-5\hat{\jmath}-3\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x & y-1 & z+5 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}=0$ $x(-3-6)-(y-1)(-2-6)$ $+(z+5)(2-3)=0$ $\Rightarrow -9x+8y-z-13=0$ $\Rightarrow 9x-8y+z+13=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(-9\,\hat{\imath}+8\hat{\jmath}-\hat{k})=13$
36. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=(6\hat{\imath}-\hat{\jmath}+\hat{k})+s(-\hat{\imath}+2\hat{\jmath}+\hat{k})+t(-5\hat{\imath}-4\hat{\jmath}-5\hat{k})$.	Point $\vec{a}=6\hat{\imath}-\hat{\jmath}+\hat{k}$ Vectors: $\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ $\vec{c}=-5\hat{\imath}-4\hat{\jmath}-5\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s\,\vec{b}+t\,\vec{c}$ $\vec{r}=6\hat{\imath}-\hat{\jmath}+\hat{k}+$ $s\,(-\hat{\imath}+2\hat{\jmath}+\hat{k})+$ $t\,(-5\hat{\imath}-4\hat{\jmath}-5\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-6 & y+1 & z-1 \\ -1 & 2 & 1 \\ -5 & -4 & -5 \end{vmatrix}=0$ $(x-6)(-10+4)-(y+1)(5+5)$ $+(z-1)(4+10)=0$ $\Rightarrow 3x+5y-7z-6=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(3\hat{\imath}+5\hat{\jmath}-7\hat{k})=6$

your positive action combined with positive thinking result in success

37. Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.	Point: $\vec{a}=\hat{\imath}-2\hat{\jmath}+4\hat{k}$ Vectors: $\vec{b}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ $\vec{c}=3\hat{\imath}-\hat{\jmath}+\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$ $\vec{r}=\hat{\imath}-2\hat{\jmath}+4\hat{k}+s(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(3\hat{\imath}-\hat{\jmath}+\hat{k})$	Cartesian Equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix}=0$ $(x-1)(2-3)-(y+2)(1+9)$ $+(z-4)(-1-6)=0$ $\Rightarrow x+10y+7z-9=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(\hat{\imath}+10\hat{\jmath}+7\hat{k})=9$
38. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+t(2\hat{\imath}-\hat{\jmath}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+\hat{k})=8$.	Point: $\vec{a}=\hat{\imath}-\hat{\jmath}+3\hat{k}$ Vectors: $\vec{b}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$ $\vec{c}=\hat{\imath}+2\hat{\jmath}+\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$ $\vec{r}=\hat{\imath}-\hat{\jmath}+3\hat{k}+s(2\hat{\imath}-\hat{\jmath}+4\hat{k})+t(\hat{\imath}+2\hat{\jmath}+\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$ $(x-1)(-1-8)-(y+1)(2-4)$ $+(z-3)(4+1)=0$ $\Rightarrow 9x-2y-5z+4=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(9\hat{\imath}-2\hat{\jmath}-5\hat{k})=-4$ $\vec{r}\cdot(-9\hat{\imath}+2\hat{\jmath}+5\hat{k})=4$
39. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0)$, $(2,2,-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.	Points: $\vec{a}=-\hat{\imath}+2\hat{\jmath}$ $\vec{b}=2\hat{\imath}+2\hat{\jmath}-\hat{k}$ $\vec{b}-\vec{a}=3\hat{\imath}-\hat{k}$ Vector: $\vec{c}=\hat{\imath}+\hat{\jmath}-\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\vec{c}$ $\vec{r}=-\hat{\imath}+2\hat{\jmath}+s(3\hat{\imath}-\hat{k})+t(\hat{\imath}+\hat{\jmath}-\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x+1 & y-2 & z \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix}=0$ $(x+1)(0+1)-(y-2)(-3+1)$ $+z(3-0)=0$ $\Rightarrow x+2y+3z-3=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+3\hat{k})=3$

An equation means nothing to us unless express a thought of god.

40. Find the parametric form of vector equation, non parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1)$, $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.	Points: $\vec{a}=2\hat{\imath}+2\hat{\jmath}+\hat{k}$ $\vec{b}=9\hat{\imath}+3\hat{\jmath}+6\hat{k}$ $\vec{b}-\vec{a}=7\hat{\imath}+\hat{\jmath}+5\hat{k}$ Vector: $\vec{c}=2\hat{\imath}+6\hat{\jmath}+6\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\,\vec{c}$ $\vec{r}=(2\hat{\imath}+2\hat{\jmath}+\hat{k})+$ $s\,(7\hat{\imath}+\hat{\jmath}+5\hat{k})+$ $t\,(2\hat{\imath}+6\hat{\jmath}+6\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix}=0$ $(x-2)(6-30)-(y-2)(42-10)$ $+(z-1)(42-2)=0$ $\Rightarrow 3x+4y-5z-9=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times\vec{c})=0$ $\vec{r}\cdot(3\hat{\imath}+4\hat{\jmath}-5\hat{k})=9$
41. Find parametric form of vector equation, non parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1)$, $(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.	Points: $\vec{a}=2\hat{\imath}+2\hat{\jmath}+\hat{k}$ $\vec{b}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$ $\vec{b}-\vec{a}=-\hat{\imath}-4\hat{\jmath}+2\hat{k}$ Vector: $\vec{c}=(-1-2)\hat{\imath}+(5-1)\hat{\jmath}+(-8+3)\hat{k}$ $\vec{c}=-3\hat{\imath}+4\hat{\jmath}-5\hat{k}$	Parametric form of vector equation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\,\vec{c}$ $\vec{r}=(2\hat{\imath}+2\hat{\jmath}+1\hat{k})+$ $s\,(-\hat{\imath}-4\hat{\jmath}+2\hat{k})+$ $t\,(-3\hat{\imath}+4\hat{\jmath}-5\hat{k})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-2 & z-1 \\ -1 & -4 & 2 \\ -3 & 4 & -5 \end{vmatrix}=0$ $(x-2)(20-8)-(y-2)(5+6)$ $+(z-1)(-4-12)=0$ $\Rightarrow 12x-11y-16z+14=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times\vec{c})=0$ $\vec{r}\cdot(12\hat{\imath}-11\hat{\jmath}-16\hat{k})=-14$ $\vec{r}\cdot(-12\hat{\imath}+11\hat{\jmath}+16\hat{k})=14$
42. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the non-collinear points $(3,6,-2)$, $(-1,-2,6)$ and $(6,4,-2)$.	Points $\vec{a}=3\hat{\imath}+6\hat{\jmath}-2\hat{k}$ $\vec{b}=-\hat{\imath}-2\hat{\jmath}+6\hat{k}$ $\vec{c}=6\hat{\imath}+4\hat{\jmath}-2\hat{k}$ $\vec{b}-\vec{a}=-4\hat{\imath}-8\hat{\jmath}+8\hat{k}$ $\vec{c}-\vec{a}=3\hat{\imath}-2\hat{\jmath}$	Parametric form of vector equation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\,(\vec{c}-\vec{a})$ $\vec{r}=(3\hat{\imath}+6\hat{\jmath}-2\hat{k})+$ $s\,(-4\hat{\imath}-8\hat{\jmath}+8\hat{k})+$ $t\,(3\hat{\imath}-2\hat{\jmath})$	Cartesian equation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$ $\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$ $(x-3)(0+16)-(y-6)(0-24)$ $+(z+2)(8-24)=0$ $\Rightarrow 2x+3y+4z-16=0$	Non -Parametric form of vector equation $(\vec{r}-\vec{a})\cdot$ $((\vec{b}-\vec{a})\times(\vec{c}-\vec{a}))=0$ $\vec{r}\cdot(2\hat{\imath}+3\hat{\jmath}+4\hat{k})=16$

To achieve your target plan well