HALF YEARLY EXAMINATION - 2019

12 - STD **MATHS**

Time Allowed : 3.00 Hours Maximum Marks : 90

PART -I

Note : i) All questions are compulsory. ii) Choose the most suitable answer from the given four correct alternatives and write the option code and the corresponding answer. 20 X 1 = 20

1. If A is a non - singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

   a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

   a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

   a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

4. If $x^2 + y^2 = 1$, then the value of $\frac{1 + x + iy}{1 + x - iy}$ is

   a) $x - iy$ b) $2x$ c) $-2iy$ d) $x + iy$

5. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4}$ is

   a) -2 b) -1 c) 1 d) 2

6. The polynomial $x^3 + 2x + 3$ has

   a) one negative and two imaginary zeros b) one positive and two imaginary zeros
   c) three real zeros d) no zeros

7. The value of $\sin^{-1}(\cos x)$, $0 \leq x \leq \pi$ is

   a) $\pi - x$ b) $x - \frac{\pi}{2}$ c) $\frac{\pi}{2} - x$ d) $x - \pi$

8. An ellipse has OB as semi minor axis, F and F^I its foci and the angle FBF^I is a right angle. Then the eccentricity of the ellipse is

   a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{\sqrt{3}}$

9. The vertex of the parabola $x^2 = 8y - 1$ is

   a) $\left(-\frac{1}{8}, 0\right)$ b) $\left(\frac{1}{8}, 0\right)$ c) $\left(0, \frac{1}{8}\right)$ d) $\left(0, -\frac{1}{8}\right)$

10. The value of $\left[\hat{i}+\hat{j},\ \hat{j}+\hat{k},\ \hat{k}+\hat{i}\right]$ is equal to

a) 0 b) 1 c) 2 d) 4

11. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

12. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height when t =

a) 2 b) 2.5 c) 3 d) 3.5

13. The value of the limit $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$ is a) 1 b) 1 c) 2 d) ∞

14. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is

a) xye^{xy} b) $(1 + xy)e^{xy}$ c) $(1 + y)e^{xy}$ d) $(1 + x)e^{xy}$

15. The value of $\int_{-\pi/4}^{\pi/4}\left(\frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x}\right)dx$ is

a) 4 b) 3 c) 2 d) 0

16. The value of $\int_0^{\pi/6}\cos^3 3x\,dx$ is a) $\frac{2}{3}$ b) $\frac{2}{9}$ c) $\frac{1}{9}$ d) $\frac{1}{3}$

17. The value of $\int_0^{\pi}\frac{dx}{1+5^{\cos x}}$ is a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) 2π

18. The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

a) $y + \sin^{-1}x = c$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = c$ d) $x^2 + 2\sin^{-1}y = 0$

19. If sinx is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, then P is

a) log sinx b) cos x c) tan x d) cot x

20. The order and degree of the differential equation $[x + y']^2 = (y')^2 + 3x$ are

a) 1, 2 b) 1, 1 c) 2, 1 d) 2, 2

PART - II

Note : Answer any seven questions. Question No. 30 is compulsory. 7X2=14

21. Simplify : $i^{59}+\frac{1}{i^{59}}$.

22. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are 2α, 2β and 2γ.

23. Find the principal value of $\operatorname{cosec}^{-1}(-1)$.
24. Find the equation of the parabola if the curve is opened upward, vertex is (–1, –2) and the length of the latus rectum is 4.
25. If $2\hat{i}-\hat{j}+3\hat{k},\ 3\hat{i}+2\hat{j}+\hat{k},\ \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.
26. Determine whether the pair of straight lines $\vec{r}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)+t\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$, $\vec{r}=\left(2\hat{j}-3\hat{k}\right)+s\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ are parallel.
27. Using Rolle's theorem find the point on the curve $y=x^2+1,\ -2\le x\le 2$ where the tangent is parallel to x - axis.
28. Find differential for by $y=\dfrac{(1-2x)^3}{3-4x}$.
29. Evaluate : $\displaystyle\int_{-\pi/2}^{\pi/2} x\cos x\,dx$.
30. Write the Maclaurin series expansion of e^{-x}.

PART - III

Note : Answer any seven questions. Questions No. 40 is compulsory.

7 X 3 = 21

31. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=f(-\alpha)$.
32. Find the values of the real numbers x and y, if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.
33. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.
34. Solve : $\tan^{-1}\left(\dfrac{x-1}{x-2}\right)+\tan^{-1}\left(\dfrac{x+1}{x+2}\right)=\dfrac{\pi}{4}$.
35. The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.
36. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$.
37. Evaluate : $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.
38. Evaluate : $\displaystyle\int_{-1}^{1}\log\left(\frac{3-x}{3+x}\right)dx$.
39. Form the differential equations by eliminating arbitrary constants given in bracket $Y=e^{3x}(C\cos 2x+D\sin 2x)$ {C, D}.
40. Draw the Geometrical diagram for the sum of two complex numbers Z_1 and Z_2 and verify the result.

PART - IV

Note : Answer all the questions. $7 \times 5 = 35$

41. a) Solve, by Cramer's rule, the system of equations. $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$. ***(OR)*** b) Test for consistency and if possible, solve the equations by rank method $2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$.

42. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ show that $x^2 + y^2 = 1$. ***(OR)***

    b) Sketch the graph of sinx in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin^{-1}x$ $[-1, 1]$.

43. a) Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is $(2, 3)$ and a directrix is $x = 7$. Also find the length of the major and minor axes of the ellipse.

    (OR)

    b) A tunnel through a mountain for a lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m hight to clear it the highest point of the opening is to be 5m approximately. How wide must the opening be?

44. a) By vector method, prove that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. ***(OR)***

    b) Find the vector and Cartesian eqauation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$.

45. a) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m / min, how fast is the depth of the water increases when the water is 8 metres deep? ***(OR)***

    b) Prove that among all the rectangles of the given area, square has the least perimeter.

46. a) If $U(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. ***(OR)***

    b) Solve : $\frac{dy}{dx} + 2y\cot x = 3x^2 \operatorname{cosec}^2 x$.

47. a) The rate at which the population of city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990, what population may be anticipated in 2050.

    (approximate to thousands)? $\left[\left(\frac{16}{13}\right)^3 \simeq 1.86\right]$ ***(OR)***

    b) Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.