VINAYAGA MATRIC HIGHER SECONDARY SCHOOL
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HALF YEARLY EXAMINATION DECEMBER - 2022

STD: X KEY ANSWER SUB: MATHEMATICS

I - ONE MARK :

1. c) 2^{mn}-1
2. a) 1
3. d) 2520
4. b) 2n
5. b) 5
6. b) 4
7. c) $\angle B = \angle D$
8. b) Two parallel and two non-parallel sides
9. a) $\frac{1}{2}$
10. b) $\frac{1}{25}$
11. d) Frustum of a cone and a hemisphere
12. d) 3 : 1 : 2
13. d) 225
14. b) $\frac{7}{12}$

2. $f(x)+f(1-x)=2$

$f\left(\frac{1}{2}\right)+f(1-\frac{1}{2})=2$

$f\left(\frac{1}{2}\right)+f\left(\frac{2-1}{2}\right)=2$

$f\left(\frac{1}{2}\right)+f\left(\frac{1}{2}\right)=2$

$2f\left(\frac{1}{2}\right)=2$

$f\left(\frac{1}{2}\right)=\frac{2}{2}$

$f\left(\frac{1}{2}\right)=1$

4. 2+ 2+2+2+ up to n terms
Given series A.P and G.P
First term a = 2
Last term $l = 2$
Common difference d = 0

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{n}{2}(2+2)$

$S_n = \frac{4n}{2}$

$S_n = 2n$

10. $5x = \sec\theta$ $\quad \frac{5}{x} = \sec\theta$

$x = \frac{\sec\theta}{5}$ $\quad \frac{1}{x} = \tan\theta$

Square on both sides

$x^2 = \frac{\sec^2\theta}{25}$ $\quad \frac{1}{x^2} = \tan^2\theta$

$x^2 - \frac{1}{x^2} = \frac{\sec^2\theta}{25} - \frac{\tan^2\theta}{25}$

$x^2 - \frac{1}{x^2} = \frac{\sec^2\theta - \tan^2\theta}{25}$

$\sec^2\theta - \tan^2\theta = 1$

$x^2 - \frac{1}{x^2} = \frac{1}{25}$

14. 31 Days in English month Jan-31, Aug – 31
March – 31, Oct – 31
May- 31, Dec – 31
July – 31

P(A) $= \frac{7}{12}$

II. 2 MARKS

15. If A×B = {(3,2), (3,4), (5,2), (5,4)} then find A and B.

Solution $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$

We have A = {set of all first coordinates of elements of $A \times B$}. $\therefore$ $A = \{3,5\}$

B = {set of all second coordinates of elements of $A \times B$}. $\therefore$ $B = \{2,4\}$

Thus $A = \{3,5\}$ and $B = \{2,4\}$.

16. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution The number 800 can be factorized as

$$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$$

Hence, $a^b \times b^a = 2^5 \times 5^2$

This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

17. Find the sum of the series $1 + 4 + 9 + 16 + \cdots + 225$

Solution

iv) 1+4+9+16+. . . +225

$= 1^2+2^2+3^2+4^2+\ldots+15^2$

$= \dfrac{n(n+1)(2n+1)}{6}$ where n = 15

$= \dfrac{15\times16\times31}{6}$

$= 1240$

So 1+4+9+16+. . . +225 = 1240

18. Reduce the rational expressions to its lowest form $\dfrac{x^2-16}{x^2+8x+16}$

Solution

$$\frac{x^2-16}{x^2+8x+16} = \frac{(x+4)(x-4)}{(x+4)^2} = \frac{x-4}{x+4}$$

19. If the difference between a number and its reciprocal is $\dfrac{24}{5}$, find the number.

Solution

Let the number be x

It's reciprocal is $\dfrac{1}{x}$

Given that,

$$x - \frac{1}{x} = \frac{24}{5}$$

$$\Rightarrow \frac{x^2-1}{x} = \frac{24}{5}$$

$$\Rightarrow 5(x^2-1) = 24x$$

$$\Rightarrow 5x^2 - 24x - 5 = 24x$$

$$\Rightarrow (5x+1)(x-5) = 0$$

$$\therefore x = 5 \text{ or } -\frac{1}{5}$$

The number is 5 (or) $-\dfrac{1}{5}$

20. If $A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$ find the value of

i) $B - 5A$ ii) $3A - 9B$

Solution :

i) $B - 5A$ $= \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} - 5\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} + \begin{bmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{bmatrix} = \begin{bmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{bmatrix}$

21. The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

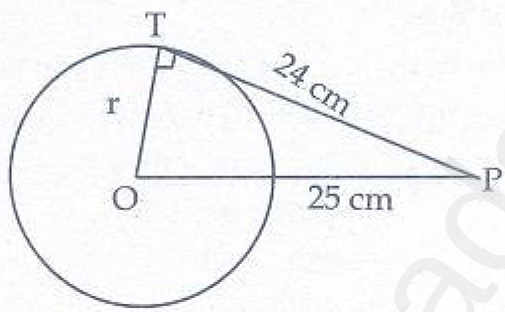
Solution :

radius r $= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

$= \sqrt{49}$

$= 7$ cm



22. Find the value of 'a', if the line through (–2,3) and (8,5) is perpendicular to $y = ax + 2$

Solution:

Here the two points (–2, 3) & (8, 5)

the slope of these two points is

$m_1 = \frac{5-3}{8+2} = \frac{2}{10} = \frac{1}{5}$

Given y = ax+2

$m_2 = a$

Two straight lines are perpendicular

So, $m_1 \times m_2 = -1$

$\frac{1}{5} \times a = -1 \Rightarrow \boxed{a = -5}$

23. Prove the identity $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

Solution

(ii) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \text{Sec}\ \theta + \tan\ \theta = \text{RHS}.$$

24. If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution Given that, radius $r = 7$ cm

Now, total surface area of the cone $= \pi r(l+r)$ sq. units

$$\text{T.S.A.} = 704\ \text{cm}^2$$

$$704 = \frac{22}{7} \times 7(l+7)$$

$$32 = l + 7 \Rightarrow l = 25\ \text{cm}$$

Therefore, slant height of the cone is 25 cm.

25. A hemi-spherical hollow bowl has material of volume $\dfrac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution:

R = 7 cm

Volume of Hollow Hemisphere $= \dfrac{436\pi}{3}$

$$\frac{\cancel{2}}{\cancel{3}}\cancel{\pi}(R^3 - r^3) = \frac{436\cancel{\pi}}{\cancel{3}}$$

$$7^3 - 3^3 = \frac{436}{2} = 218$$

$$343 - r^3 = 218$$

$$r^3 = 343 - 218$$

$$r^3 = 218$$

$$r^3 = 125$$

$$\boxed{r = 5\ \text{cm}}$$

Thickness = R–r

= 7–5 = 2 cm

26. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution:

range = 20

L–S = 20 -----(1)

Coefficient of range = 0.2

$$\frac{L-S}{L+S} = 0.2$$

$$\frac{20}{L+S} = 0.2$$

$$L+S = \frac{20}{0.2}$$

$$= \frac{200}{2} = 100$$

L+S = 100 ----(2)

Solving (1) & (2)

$$L-\cancel{S} = 20$$

$$L+\cancel{S} = 100$$

$$2L = 120$$

$$L = \frac{120}{2} = 60$$

$$S = 100-60 = 40$$

Answer: Largest value = 60

Smallest value = 40

27. Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution When two coins are tossed together, the sample space is

$$S = \{HH, HT, TH, TT\};\ n(S) = 4$$

Let A be the event of getting different faces on the coins.

$$A = \{HT, TH\}; \qquad n(A) = 2$$

Probability of getting different faces on the coins is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

28. Let A = { 1 , 2, 3 } and B = { a , b } . Write any two functions from A to B in the form of Set of ordered pairs.

Solution

Given sets A = { 1 , 2, 3 } and B = { a , b }

Functions: f = { (1, a) , (2,a) ,(3,b) } Many one and onto function

g = { (1, a) , (2,b) ,(3,b) } Many one and onto function

h = { (1, a) , (2,a) ,(3,a) } Constant function

j = { (1, b) , (2,b) ,(3,b) } Constant function

III. FIVE MARKS:

29. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find,

(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$

(iv) the value of C when $t(C) = 212$

(v) the temperature when the Celsius value is equal to the Farenheit value.

Solution:

Given t(c) = F

$$F = \frac{9c}{5} + 32$$

i) $t(0) = \frac{0}{5} + 32$

ii) $t(28) = \frac{9 \times 28}{5} + 32$

$= \frac{252}{5} + 32$

$= 50.4 + 32 = 82.4$

iii) $t(-10) = \frac{9 \times -\cancel{10}^{2}}{\cancel{5}} + 32$

$= -18 + 32$

$= 24$

iv) The value of c when t(c) = 212

t(c) = 212

$\frac{9C}{5} + 32 = 212$

$9C + 160 = 212 \times 5$

$9C = 1060 - 160$

$9C = 900 \Rightarrow C = 100^{\circ}C$

v) the temperature when the celsius value is equal to the Farenheit value

$C = \frac{9C}{5} + 32$

$5C = 9C + 160$

$4C = -160$

$C = -40$

30. Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

Solution $gff(x) = g[f\{f(x)\}]$ (This means "g of f of f of x")

$= g[f(3x+1)] = g[3(3x+1)+1] = g(9x+4)$

$g(9x+4) = [(9x+4)+3] = 9x+7$

$fgg(x) = f[g\{g(x)\}]$ (This means "f of g of g of x")

$= f[g(x+3)] = f[(x+3)+3] = f(x+6)$

$f(x+6) = [3(x+6)+1] = 3x+19$

These two quantities being equal, we get $9x + 7 = 3x + 19$. Solving this equation we obtain $x = 2$.

31. The sum of first n, $2n$ and $3n$ terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution If S_1, S_2 and S_3 are sum of first n, $2n$ and $3n$ terms of an A.P. respectively then

$$S_1 = \frac{n}{2}[2a+(n-1)d], \quad S_2 = \frac{2n}{2}[2a+(2n-1)d], \quad S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

Consider,

$$S_2 - S_1 = \frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[[4a+2(2n-1)d]-[2a+(n-1)d]]$$

$$S_2 - S_1 = \frac{n}{2}\times[2a+(3n-1)d]$$

$$3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)d]$$

$$3(S_2 - S_1) = S_3$$

32. Find the sum to n terms of the series 3 + 33 + 333 + to n terms

Solution

ii) 3 + 33 + 333 +. . . to n terms

Given 3 + 33 + 333 +. . . to n terms

$= 3[1+11+111+ \ldots\ldots$ n terms$]$

$= \frac{3}{9}[9+99+999+ \ldots\ldots$ n terms$]$

$= \frac{1}{3}[(10-1)+(100-1)+(1000-1)\ldots\ldots$ n terms$]$

$= \frac{1}{3}[(10+100+1000+\ldots)-(1+1+1+\ldots$ n terms$)]$

$$=\frac{1}{3}\left[\frac{a(r^n-1)}{r-1}-n\right] \text{ Where } a=10, r=10>1$$

$$=\frac{1}{3}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$=\frac{1}{3}\left[\frac{10(10^n-1)}{9}-n\right]$$

$$=\frac{10(10^n-1)}{27}-\frac{n}{3}$$

33. Find the square root of the following polynomial $37x^2 - 28x^3 + 4x^4 + 42x + 9$

ii)

$$\begin{array}{r|l}
 & 2x^2-7x-3 \\
\hline
2x^2 & 4x^4-28x^3+37x^2+42x+9 \\
 & 4x^4 \qquad (-) \\
\hline
4x^2-7x & -28x^3+37x^2 \\
 & -28x^3+49x^2 \qquad (-) \\
\hline
 & -12x^2+42x+9 \\
4x^2-14x-3 & -12x^2+42x+9 \qquad (-) \\
\hline
 & 0
\end{array}$$

$$\therefore \sqrt{4x^4-28x^3+37x^2+42x+9} = |2x^2-7x-3|$$

34. If α, β are the roots of the equation $3x^2+7x-2=0$, find the values of (i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution $3x^2+7x-2=0$ here, $a=3$, $b=7$, $c=-2$

since, α, β are the roots of the equation

(i) $\alpha+\beta=\frac{-b}{a}=\frac{-7}{3}$; $\alpha\beta=\frac{c}{a}=\frac{-2}{3}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(\frac{-7}{3}\right)^2-2\left(\frac{-2}{3}\right)}{\frac{-2}{3}}=\frac{-61}{6}$$

(ii) $$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^3-3\left(-\frac{2}{3}\right)\left(-\frac{7}{3}\right)}{-\frac{2}{3}}=\frac{469}{18}$$

35. State and prove Thales Theorem

Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem

Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

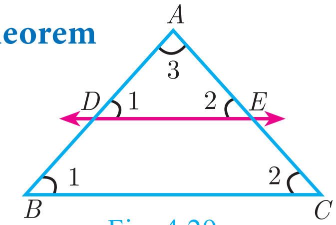


Fig. 4.29

Proof

Given: In ΔABC , D is a point on AB and E is a point on AC.

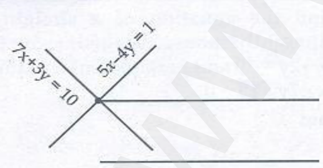
To prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \sim \Delta ADE$	By AAA similarity
	$\dfrac{AB}{AD} = \dfrac{AC}{AE}$	Corresponding sides are proportional
	$\dfrac{AD+DB}{AD} = \dfrac{AE+EC}{AE}$	Split AB and AC using the points D and E.
	$1+\dfrac{DB}{AD} = 1+\dfrac{EC}{AE}$	On simplification
	$\dfrac{DB}{AD} = \dfrac{EC}{AE}$	Cancelling 1 on both sides
	$\dfrac{AD}{DB} = \dfrac{AE}{EC}$	Taking reciprocals
	Hence proved	

36. Find the equation of a straight line through the intersection of lines $7x+3y=10$, $5x-4y=1$ and parallel to the line $13x+5y+12=0$

Solution:



Given two lines

$7x+3y = 10$ -------(1)

$5x-4y = 1$ -------(2)

Solving the equations (1) & (2)

$(1) \times 4 \Rightarrow \quad 28x + 12\cancel{y} = 40$

$(2) \times 3 \Rightarrow \quad 15x - \cancel{12y} = 3$

$43x = 43$

$x = \dfrac{43}{43}$

$\boxed{x = 1}$

$x = 1$ sub. in (1), we get

$7x+3y = 10$

$7(1)+3y = 10$

$3y = 10-7$

$3y = 3$

$\boxed{y = 1}$

Now, the point of intersection of (1) & (2) is (1, 1)

The required line is parallel to $13x+5y+12=0$

Then, the required line is

$13x+5y+k = 0$

Since it passes through (1, 1)

$13(1) + 5(1) + k = 0$

$13+5+k = 10$

$18+k = 0$

$\boxed{k = -18}$

The required straight line is $13x+5y-18=0$

37. $\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{1}{a}$, then prove that $\dfrac{a^2-1}{a^2+1}=\sin\theta$

Solution: $$\frac{\cos\theta}{1+\sin\theta}=\frac{1}{a}$$

$$\frac{\cos^2\theta}{(1+\sin\theta)^2}=\frac{1}{a^2}$$

$$\frac{1-\sin^2\theta}{(1+\sin\theta)^2}=\frac{1}{a^2}$$

$$\frac{\cancel{(1+\sin\theta)}(1-\sin\theta)}{(1+\sin\theta)^2}=\frac{1}{a^2}$$

$$\frac{1-\sin\theta}{1+\sin\theta}=\frac{1}{a^2}$$

$$a^2-a^2\sin\theta = 1+\sin\theta$$

$$a^2-1 = a^2\sin\theta+\sin\theta$$

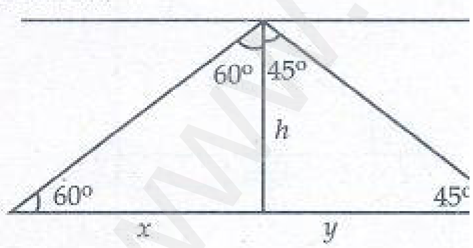
$$a^2-1 = (a^2+1)\sin\theta$$

$$\sin\theta = \frac{a^2-1}{a^2+1}$$

Hence Proved

38. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are $60°$ and $45°$ respectively. If the distance between the ships is $200\left(\dfrac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the lighthouse.

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45° = \frac{h}{y}$$

$$1 = \frac{h}{y}$$

$$\boxed{y=h}$$

$$\tan 60° = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$\boxed{x=\frac{h}{\sqrt{3}}}$$

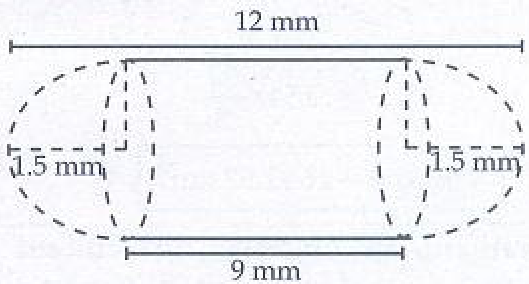
$$x=y=h+\frac{h}{\sqrt{3}}=\frac{\sqrt{3}h+h}{\sqrt{3}}$$

$$=h\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)=200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$

$$\boxed{h=200m}$$

39. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?

Solution:



Cylinder:	Hemisphere:
radius $r = \frac{3}{2}$ mm	radius $r = \frac{3}{2}$ cm
height $h = 9$ mm	

Volume of the Capsule

= Volume of Cylinder + 2 × Volume of Hemisphere

$$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^2 h$$

$$= \pi r^2 \left[h + \frac{4}{3} r \right]$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \left[9 + \frac{\overset{2}{\cancel{4}}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}} \right]$$

$$= \frac{22}{7} \times \frac{9}{4} \times 11$$

$$= 77.715 \text{ cm}^3$$

40. Solution

Data in ascending order 16, 19, 20, 22, 23

Mean $\bar{x} = \frac{\sum x}{n}$

$$\bar{x} = \frac{16 + 19 + 20 + 22 + 23}{5} = \frac{100}{5}$$

$$\bar{x} = 20$$

x	$d = x - \overline{x}$	d^2
16	-4	16
19	-1	1
20	0	0
22	2	4
23	3	9
	$\sum d = 0$	$\sum d^2 = 30$

$$\sigma = \sqrt{\frac{\sum d^2}{n}}$$

$$\sigma = \sqrt{\frac{30}{5}}$$

$$\sigma = \sqrt{6}$$

$$\sigma = 2.45$$

$$C.V = \frac{\sigma}{\overline{x}} \times 100$$

$$C.V = \frac{2.45}{20} \times 100$$

$$C.V = 12.25$$

41. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

Two dice are rolled once.

S = { (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n(S) = 36

Let A be the event of getting even number on first die

A = {(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n(A) = 18

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Let B be the event of getting total of face sum as 8

B = {(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)}

n(B) = 5

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$A \cap B$ = { (2, 6) (4, 4) (6, 2)}

$$n(A \cap B) = 3 \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

Using addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$

$$= \frac{20}{36} = \frac{5}{9}$$

Answer: P(even number on first die 0. a total of face sum 8) = $\frac{5}{9}$

42. Solution

$$\text{Given } \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 2 & -4 & 6 \\ 3 & -6 & 9 \\ 1 & -2 & 3 \end{pmatrix}$$

Possible dimension $3 \times 1 \quad 1 \times 3 = 3 \times 3$

i) Order of matrix A is 1×3

Let matrix A = $(x \quad y \quad z)$

$$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} (x \quad y \quad z) = \begin{pmatrix} 2 & -4 & 6 \\ 3 & -6 & 9 \\ 1 & -2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 2y & 2z \\ 3x & 3y & 3z \\ x & y & z \end{pmatrix} = \begin{pmatrix} 2 & -4 & 6 \\ 3 & -6 & 9 \\ 1 & -2 & 3 \end{pmatrix}$$

Comparing third row

$x = 1$, $y = -2$, $z = 3$

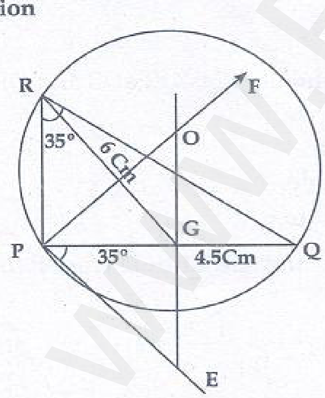
ii) The elements of the matrix A = (1 -2, 3)

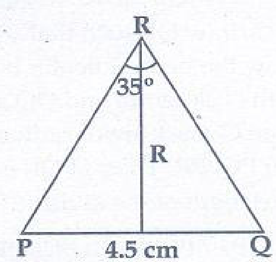
Elements are 1 , -2 , 3

IV. 8 MARKS

43. a) Construct a ΔPQR which the base $PQ = 4.5$ cm, $\angle R = 35$ and the median RG from R to PG is 6 cm.

Solution





Rough Diagram

Construction :

1. Draw a line segment PQ = 4.5 cm
2. At P draw PE such that $\angle QPE = 35^0$
3. At P draw PF such that $\angle EPF = 90^0$
4. Draw the perpendicular bisector to PQ, which intersects PF at O and PQ at G.
5. With O as centre and OP as radius draw a circle.
6. From G mark are of radius 6 cm on the circle.
7. Join PR and RQ. Then ΔPQR is the required triangle.

b) Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Solution Given, diameter $(d) = 6$ cm, we find radius $(r) = \frac{6}{2} = 3$ cm

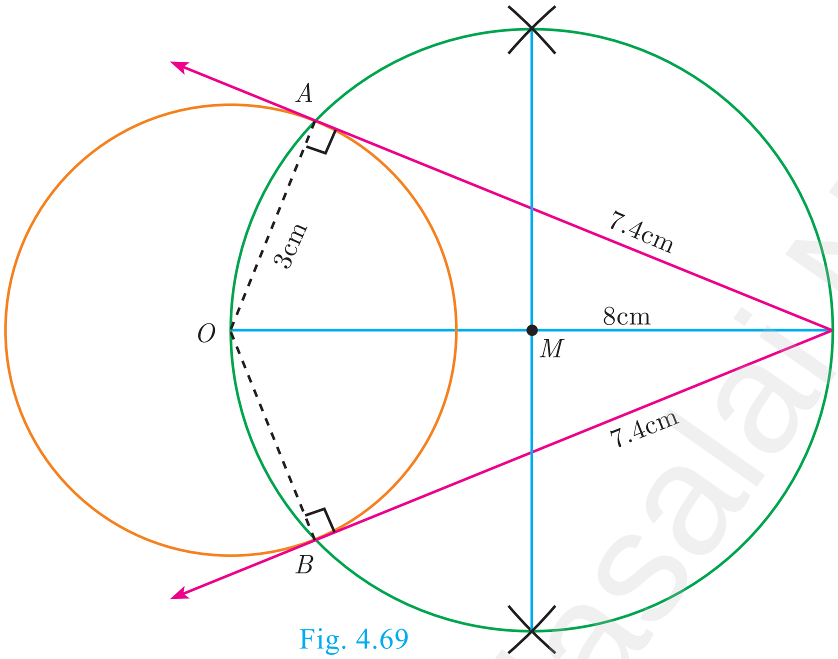
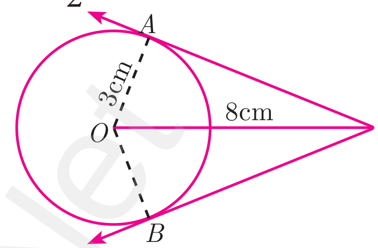


Fig. 4.69



Rough diagram

Construction

Step 1: With centre at O, draw a circle of radius 3 cm.

Step 2: Draw a line OP of length 8 cm.

Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4: With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

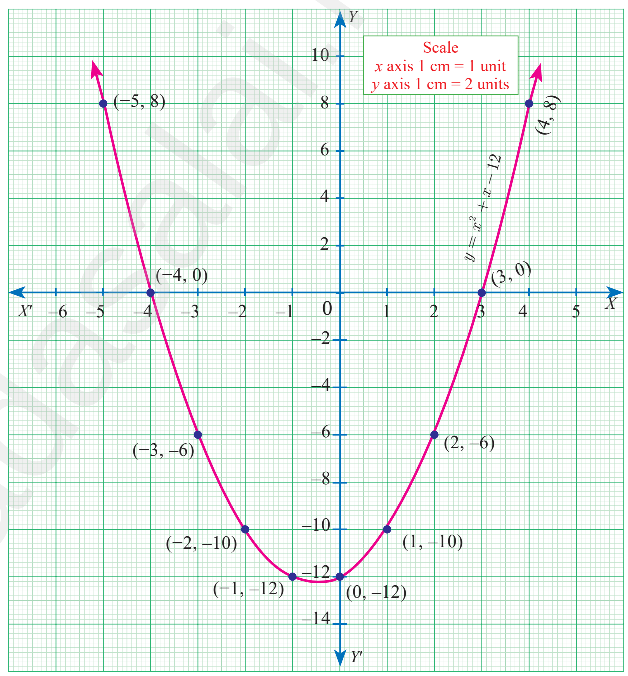
Step5: Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are $PA = PB = 7.4$ cm.

Verification : In the right angle triangle OAP, $PA^2 = OP^2 - OA^2 = 8^2 - 3^2 = 64 - 9 = 55$

$$PA = \sqrt{55} = 7.4 \text{ cm (approximately)}.$$

44. a Prepare the table of values for the equation $y = x^2 + x - 12$.

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	8	0	-6	-10	-12	-12	-10	-6	0	8



P

Since there are two points of intersection with the X axis, the quadratic equation $x^2 + x - 12 = 0$ has real and unequal roots.

b) A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Step 1

Given,

A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as shown below.

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹(y)	180	90	60	45	36

We have to draw the graph and find how much will each participant get if the number of participants are 12

As x increases y decreases, therefore the given variation is an inverse variation.

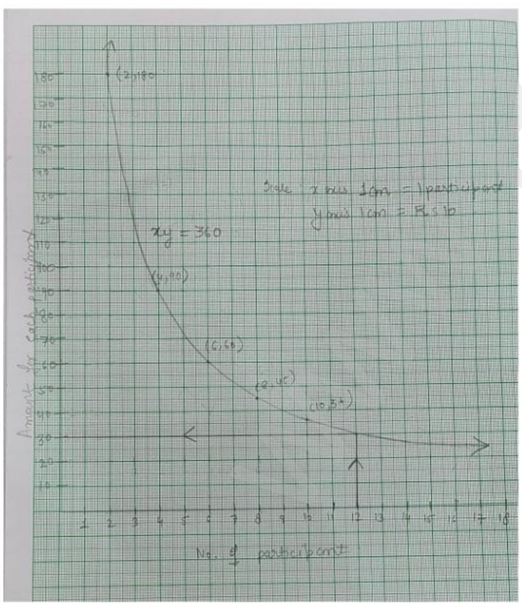
$\therefore$ Equation of curve is $xy = k$ or $y = \frac{k}{x}$

$2 \times 180 = k,\ 4 \times 90 = k,\ 6 \times 60 = k,\ 8 \times 45 = k,\ 10 \times 36 = k$

$\therefore$ Constant of variation is $k = 360$

Plot the points $(2,\ 180),\ (4,\ 90),\ (6,\ 60),\ (8,\ 45),\ (10,\ 36)$ on the graph and join them to get a smooth curve.

Scale: X axis - $1\,\text{cm} = 1\ \text{paticipant}$, Y axis - $1\ \text{cm} = \text{Rs. } 10$



Therefore, From the graph when the number of participants are 12 each will get Rs. 30

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HALF YEARLY EXAMINATION - 2022

X - Std

MATHS

Time : 3.15 Hrs Marks : 100

PART - I

Choose the correct answer from the four alternatives and write the option code and the corresponding answer :- 14 X 1 = 14

1. Let n(A) = m and n(B) = n then the total number of non- empty relations that can be defined from A to B is
 a) m^n b) n^m c) $2^{mn} - 1$ d) 2^{mn}
2. If $f(x) + f(1-x) = 2$ then $f(1/2) = ?$ a) 1 b) -1 c) 5 d) -9
3. Theleast number that is divisible by all the numbers from 1to 10 (both inclusive) is
 a) 2025 b) 5220 c) 5025 d) 2520
4. Sum of the series 2 + 2 + 2 + ... upto n terms is
 a) 2^n b) 2n c) n^2 d) n + 2
5. If $(x-6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of K is
 a) 3 b) 5 c) 6 d) 8
6. If A is 2 X 3 matrix and B is 3 X 4 matrix, how many clumns does AB have
 a) 3 b) 4 c) 2 d) 5
7. If in triangle ABC and EDF, $\frac{AB}{\Delta E} = \frac{BC}{FD}$ then they will be similar, when
 a) b) c) d)
8. When proving that a quadrilateral is a trapezium it is necessary to show.
 a) Two sides are parallel b) Two parallel and two non - parellel sides
 c) Opposite sides areparellel d) All sides are of equal length
9. Find the slope of the line 2y = x + 8 a) 1/2 b) 1 c) 8 d) 2
10. If $5x = \sec\theta$, $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to
 a) 25 b) 1/25 c) 5 d) 1
11. A shuttle cock used for playing badminton has the shape of the combination of
 a) a cylinder and a sphere b) a hemisphere and a cone
 c) a sphere and a cone d) frustum of a cone and a hemisphere
12. The ratio of volumes of a cyclinder, a cone and a sphere, if each has the same diametre and same height is
 a) 1 : 2 : 3 b) 2 : 1 : 3 c) 1 : 3 : 2 d) 3 : 1 : 2
13. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
 a) 3 b) 15 c) 5 d) 225
14. An English month is selected at random in a year. The probability that it contains 31 days is.
 a) $\frac{6}{12}$ b) $\frac{7}{12}$ c) $\frac{5}{12}$ d) $\frac{1}{12}$

PART - II

Answer any 10 questions. Q.No. 28 is compulsory :- 10 X 2 = 20

15. If A X B = {(3, 2), (3, 4), (5,2), (5, 4)} then find A and B.
16. 'a'and 'b'are two positive integers such that $a^b \times b^a = 800$. Find a and b.
17. Find the sum of the series 1 + 4 + 9 + 16 + ...+ 225.
18. Reduce to its lowest form : $\frac{x^2-16}{x^2+8x+16}$
19. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.
20. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ then find the value of B – 5A.
21. The length of the tangent to a circle from a point p, which is 25cm away from the center is 24cm.What is the radius of the circle?
22. Find the value of a if the line through (–2, 3) and (8, 5) is perpendicular to y = ax + 2.
23. Prove the identity $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$.

24. If the total surface area ofacone of radius 7cm is $704 cm^2$, then find its slant height.

25. A hemi- spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14cm. Find its thickness.

26. If the range and coefficient of range of a data are 20 and 0.2 respectively, then find the largest and smallest values of data.

27. Two coins are tossed together, what is the probability of getting different faces on the coins?

28. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$.Write any two functions from A to B in the form of set of ordered pairs.

PART - III

Answer any 10 questions. Q.No. 42 is compulsory:- 10 X 5 = 50

29. The function 't' which maps temperature in celsius (C) into temperature in Fahrenheit is defined by $t(C)= F$, where $F = 9/5\ C + 32$. Find (i) $t(0)$. (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C)= 212$ (v) the temperature when the celsius value is equal to the Fahrenheit value.

30. Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$,and $g(x) = x + 3$.

31. The sum of first n, 2n and 3n termsof an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

32. Find the sum to n terms of the series 3 + 33 + 333 + ... upto n terms.

33. Find the square root of $37x^2 - 28x^3 + 4x^4 + 42x + 9$.

34. If α, β are roots of the equation $3x^2 + 7x - 2 = 0$. find the values of (i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

35. State and prove Thales Theorem.

36. Find the equation of the straight line through the intersection of the lines $7x + 3y = 10$, $5x - 4y = 1$ and parellel to the line $13x + 5y + 12 = 0$.

37. If $\frac{\cos\theta}{1+\sin\theta}=\frac{1}{a}$ then prove that $\frac{a^2-1}{a^2+1}=\sin\theta$.

38. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the light house are 60^o and 45^o respectively. If the distance between the ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the light house.

39. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12mm and the diametre of the capsule is 3mm, how much medicine it can hold?

40. Find the coefficient of variation of 20, 22, 19, 23, 16.

41. Two dices are rolled. Find the probability of getting an even number on the first die or a total of face sum 8.

42. If $\begin{pmatrix}2\\3\\1\end{pmatrix}A=\begin{pmatrix}2&-4&6\\3&-6&9\\1&-2&3\end{pmatrix}$ then find (i) the order of the matrix A. (ii) the elements of the matrix A.

PART - IV

Answer both the questions choosing either of the alternatives:- 2 X 8 = 16

43. a) Construct a ΔPQR with the base PQ = 4.5cm, $\angle R = 35^o$ and the median from R to RG is 6cm. ***(OR)***

b) Draw a circle of diameter 6cm from a point P, which is 8cm away from its centre.Draw the two tangents PA and PB to the circle and measure their lengths.

44. a) Discuss the nature of solutions of the quadratic equation $x^2 + x - 12 = 0$. ***(OR)***

b) A School announces that for a certain competitions, the cash price will be distributed for all the particpants equally as shown below.

No.of Participants (x)	2	4	6	8	10
Amount for each participant (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.