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(FIVE MARKS)

Chapter-1

Relations and Functions

1> Let $A=\{x\in N \mid 1<x<4\}$ $B=\{x\in W \mid 0\leq x<2\}$ $C=\{x\in N \mid x<3\}$. Then Verify that

i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Given

$A=\{2,3\}$ $B=\{0,1\}$ $C=\{1,2\}$

i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

Sol:

LHS $A\times(B\cup C)$

$B\cup C=\{0,1\}\cup\{1,2\}$

$B\cup C=\{0,1,2\}$

$A\times(B\cup C)=\{2,3\}\times\{0,1,2\}$

$A\times(B\cup C)=\{(2,0)(2,1)(2,2)(3,0)(3,1)(3,2)\}$ — ①

RHS:

$(A\times B)\cup(A\times C)$

$A\times B=\{2,3\}\times\{0,1\}$

$A\times B=\{(2,0)(2,1)(3,0)(3,1)\}$

$A\times C=\{2,3\}\times\{1,2\}$

$A\times C=\{(2,1)(2,2)(3,1)(3,2)\}$

$(A\times B)\cup(A\times C)=\{(2,0)(2,1)(3,0)(3,1)\}\cup\{(2,1)(2,2)(3,1)(3,2)\}$

$(A\times B)\cup(A\times C)=\{(2,0)(2,1)(3,0)(3,1)(2,2)(3,2)\}$ — ②

① = ②

LHS = RHS

$A\times(B\cup C)=(A\times B)\cup(A\times C)$

Hence proof

ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Sol

LHS $A\times(B\cap C)$

$B\cap C=\{0,1\}\cap\{1,2\}$

$B\cap C=\{1\}$

$A\times(B\cap C)=\{2,3\}\times\{1\}$

$A\times(B\cap C)=\{(2,1)(3,1)\}$ — ①

RHS $(A\times B)\cap(A\times C)$

$A\times B=\{2,3\}\times\{0,1\}$

$A\times B=\{(2,0)(2,1)(3,0)(3,1)\}$

$A\times C=\{2,3\}\times\{1,2\}$

$A\times C=\{(2,1)(2,2)(3,1)(3,2)\}$

$(A\times B)\cap(A\times C)=\{(2,0)(2,1)(3,0)(3,1)\}\cap\{(2,1)(2,2)(3,1)(3,2)\}$

$(A\times B)\cap(A\times C)=\{(2,1)(3,1)\}$ — ②

① = ②

LHS = RHS

$A\times(B\cap C)=(A\times B)\cap(A\times C)$

Hence proof

2> Let A = The set of all natural numbers less than 8, B = The Set of all prime numbers less than 8, C = The Set of even Prime number. Verify that

i) $(A\cap B)\times C=(A\times C)\cap(B\times C)$

ii) $(A\times(B-C)=(A\times B)-(A\times C)$

Given:

$A=\{1,2,3,4,5,6,7\}$

$B=\{2,3,5,7\}$

$C=\{2\}$

Sol:

i) $(A\cap B)\times C = (A\times C)\cap(B\times C)$

LHS

$(A\cap B)\times C$

$A\cap B = \{1,2,3,4,5,6,7\}\cap\{2,3,5,7\}$

$A\cap B = \{2,3,5,7\}$

$(A\cap B)\times C = \{2,3,5,7\}\times\{2\}$

$(A\cap B)\times C = \{(2,2)(3,2)(5,2)(7,2)\}$ — ①

RHS

$(A\times C)\cap(B\times C)$

$A\times C = \{1,2,3,4,5,6,7\}\times\{2\}$

$A\times C = \{(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)(7,2)\}$

$B\times C = \{2,3,5,7\}\times\{2\}$

$B\times C = \{(2,2)(3,2)(5,2)(7,2)\}$

$(A\times C)\cap(B\times C) = \{(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)(7,2)\}\cap\{(2,2)(3,2)(5,2)(7,2)\}$

$(A\times C)\cap(B\times C) = \{(2,2)(3,2)(5,2)(7,2)\}$ — ②

① = ②

LHS = RHS

$(A\cap B)\times C = (A\times C)\cap(B\times C)$

Hence proof

ii) $A\times(B-C) = (A\times B)-(A\times C)$

LHS

$A\times(B-C)$

$B-C = \{2,3,5,7\}-\{2\}$

$B-C = \{3,5,7\}$

$A\times(B-C) = \{1,2,3,4,5,6,7\}\times(3,5,7\}$

$= \{(1,3)(1,5,1,7)(2,3)(2,5)(2,7)(3,3)(3,5)(3,7)(4,3)(4,5)(4,7)(5,3)(5,5)(5,7)(6,3)(6,5)(6,7)(7,3)(7,5)(7,7)\}$ — ①

RHS

$(A\times B)-(A\times C)$

$A\times B = \{1,2,3,4,5,6,7\}\times\{2,3,5,7\}$

$A\times B = \{(1,2)(1,3)(1,5)(1,7)(2,2)(2,3)(2,5)(2,7)(3,2)(3,3)(3,5)(3,7)(4,2)(4,3)(4,5)(4,7)(5,2)(5,3)(5,5)(5,7)(6,2)(6,3)(6,5)(6,7)(7,2)(7,3)(7,5)(7,7)\}$.

$A\times C = \{1,2,3,4,5,6,7\}\times\{2\}$

$A\times C = \{(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)(7,2)\}$

$(A\times B)-(A\times C) = \{(1,3)(1,5)(1,7)(2,3)(2,5)(2,7)(3,3)(3,5)(3,7)(4,3)(4,5)(4,7)(5,3)(5,5)(5,7)(6,3)(6,5)(6,7)(7,3)(7,5)(7,7)\}$ --- ②

① = ②

LHS = RHS

$A\times(B-C) = (A\times B)-(A\times C)$

Hence proof

3, A function f is defined by $f(x) = 2x-3$

i) Find $\frac{f(0)+f(1)}{2}$

ii) Find x such that $f(x)=0$

iii) find x such that $f(x)=x$

iv) Find x such that $f(x)=f(1-x)$

Sol.

i) $\frac{f(0)+f(1)}{2}$

$f(0) = 2(0) - 3$

$\boxed{f(0) = -3}$

$f(1) = 2(1) - 3$

$= 2 - 3$

$\boxed{f(1) = -1}$

$\frac{f(0)+f(1)}{2} = \frac{-3-1}{2}$

$= -4/2 = -2$

ii) $f(x) = 0$

$2x - 3 = 0$

$2x = 3$

$\boxed{x = 2/3}$

iii) $f(x) = x$

$2x - 3 = x$

$2x - x = 3$

$\boxed{x = 3}$

iv) $f(x) = f(1-x)$

$2x - 3 = 2(1-x) - 3$

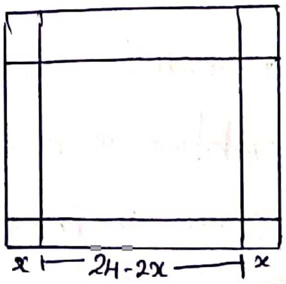
$2x - 3 = 2 - 2x - 3$

$2x + 2x = 2 - 3 + 3$

$4x = 2$

$x = 2/4$ $\boxed{x = 1/2}$

4, An open box is to be made from a Square piece of material, 24cm on a Side, by Cutting equal square from the Corners and turning up the Sides as shown (Fig). Express the Volume V of the box as a function of x.



Sol: Length of the Cuboid (l) = 24 - 2x

Breadth of the Cuboid (b) = 24 - 2x

height of the Cuboid (h) = 2x

Volume of the box } = Volume of the Cuboid

$V = l \times b \times h$

$V = (24-2x)(24-2x)(x)$

$= (24-2x)^2(x)$

$= (576 + 4x^2 - 96x)x$

$V(x) = 576x + 4x^3 - 96x^2$

$\boxed{V(x) = 4x^3 - 96x^2 + 576x}$

5, The data in the adjacent table depicts the length of a person forehand and their Corresponding height. Based on this data, a student find a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constant.

i) check if this relation is a function.

ii) Find a and b.

iii) Find the height of a person whose Forehand length is 40cm.

iv) Find the length of forehand of a Person if the height is 53.3 inches.

Length 'x' of Forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

Sol :-

i) $y = 0.9x + 24.5$

i) Yes the relation is a function.

ii) When Compare $y = ax + b$

$a = 0.9$

$b = 24.5$

iii) Length = 40cm height = ?

$y = 0.9x + 24.5$

$y = 0.9 \times 40 + 24.5$

$= 36 + 24.5$

$\boxed{y = 60.5 \text{ feet}}$

iv) Height = 53.3 inches Length = ?

$y = 0.9x + 24.5$

$53.3 = 0.9x + 24.5$

$53.3 - 24.5 = 0.9x$

$28.8 = 0.9x$

$x = \frac{28.8}{0.9}$

$\boxed{x = 32\text{cm}}$

6, Let f be a function $f: N \to N$ be defined by $f(x) = 3x + 2, x \in N$

i) Find the images of 1, 2, 3

ii) Find the preimages of 29, 53

iii) Identify the type of function.

Sol :-

i) Image of 1, 2, 3

$x = 1 ; f(1) = 3(1) + 2$

$= 3 + 2 = 5$//.

$x = 2 ; f(2) = 3(2) + 2$

$= 6 + 2$

$\boxed{f(2) = 8}$

$x = 3 ; f(3) = 3(3) + 2$

$= 9 + 2$

$\boxed{f(3) = 11}$

Image of 1, 2, 3 is 5, 8, 11

ii) Preimage of 29, 53

$f(x) = 29$	$f(x) = 53$
$3x + 2 = 29$	$3x + 2 = 53$
$3x = 29 - 2$	$3x = 53 - 2$
$3x = 27$	$3x = 51$
$x = 27/3$	$x = 51/3$
$\boxed{x = 9}$	$\boxed{x = 17}$

Preimage of 29, 53 is 9 and 17.

iii) Type of function:-

N have different image in Co-domain

f is one - one function

Range of $f = \{5, 8, 11, 14, 17 \ldots\}$

f is not onto function.

$\therefore$ f is an into function.

$\therefore$ f is one-one and into function.

7, Forensic Scientists can be determine the height (in cm) of a person based on the length of the thigh bone. They usually do so using the function $h(b) = 2.47b + 54.10$ Where b is the length of the thigh bone.

i) Verify the function h is one-one or not.

ii) Also find the height of a person if the length of his thigh bone is 50cm.

iii) Find the length of the thigh bone if the height of a person is 147.96cm.

Sol:

i) h is one-one

$$h(b_1) = h(b_2)$$

$$2.47b_1 + 54.10 = 2.47b_2 + 54.10$$

$$2.47b_1 = 2.47b_2$$

$$b_1 = b_2$$

h is one-one.

ii) Length of thigh bone $b = 50$

$$h(50) = (2.47 \times 50) + 54.10$$

$$\boxed{h(50) = 177.6 \text{ cm}}$$

iii) $h(b) = 147.96$

$$2.47b + 54.10 = 147.96$$

$$2.47b = 147.96 - 54.10$$

$$2.47b = 93.86$$

$$b = \frac{93.86}{2.47}$$

$$\boxed{b = 38 \text{cm}}$$

8, If the function $f: R \to R$ is defined by

$$f(x) = \begin{cases} 2x+7 ; & x < -2 \\ x^2 - 2 ; & -2 \leq x < 3 \\ 3x - 2 ; & x \geq 3 \end{cases}$$

i) $f(4)$ ii) $f(-2)$ iii) $f(4) + 2f(1)$

iv) $\frac{f(1) - 3f(4)}{f(-3)}$

Sol:

i) $f(4) =$

$f(x) = 3x - 2$

$f(4) = 3(4) - 2$

$= 12 - 2$

$\boxed{f(4) = 10}$

ii) $f(-2)$

$f(x) = 3x - 2$

$f(-2) = 3(-2) - 2$

$= -6 - 2$

$\boxed{f(-2) = -8}$

iii) $f(4) + 2f(1)$

$f(4) = 10$

$2f(1)$

$f(1) = x^2 - 2$

$= 1^2 - 2$

$f(1) = -1$

$f(4) + 2f(1) = 10 + 2(-1)$

$= 10 - 2$

$\boxed{f(4)+2f(1) = 8}$

iv) $\frac{f(1) - 3f(4)}{f(-3)}$

$f(1) = -1$

$f(4) = 10$

$f(-3) = 2(-3) + 7$

$= -6 + 7$

$\boxed{f(3) = 1}$

$$\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1 - 3(10)}{1}$$

$$= \frac{-1 - 30}{1}$$

$$\boxed{\frac{f(1) - 3f(4)}{f(-3)} = -31}$$

9, If the function f is defined by

$$f(x) = \begin{cases} x+2 & ; x > 1 \\ 2 & ; -1 \le x \le 1 \\ x-1 & ; -3 < x < -1 \end{cases}$$

i) $f(3)$ ii) $f(0)$ iii) $f(-1.5)$

iv) $f(2)+f(-2)$

Sol :-

i) $f(3)$

$f(3) = 3+2$

$\boxed{f(3) = 5}$

ii) $f(0)$

$\boxed{f(0) = 2}$

iii) $f(-1.5)$

$f(x) = x-1$

$f(-1.5) = -1.5-1$

$\boxed{f(-1.5) = -2.5}$

iv) $f(2)+f(-2)$

$\underline{f(2)}$

$f(x) = x+2$

$f(2) = 2+2$

$\boxed{f(2) = 4}$

$\underline{f(-2)}$

$f(x) = x-1$

$f(-2) = -2-1$

$\boxed{f(-2) = -3}$

$f(2)+f(-2) = 4-3$

$\boxed{f(2)+f(-2) = 1}$

10, A function $f : [-5, 9] \rightarrow R$

$$f(x) = \begin{cases} 6x+1 & ; -5 \le x < 2 \\ 5x^2-1 & ; 2 \le x < 6 \\ 3x-4 & ; 6 \le x \le 9 \end{cases}$$

Find

i) $f(-3)+f(2)$ ii) $f(7)-f(1)$

iii) $2f(4)+f(8)$ iv) $\dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$

Sol :-

i) $f(-3)+f(2)$

$f(x) = 6x+1$

$f(-3) = 6(-3)+1$

$= -18+1$

$\boxed{f(-3) = -17}$

$f(x) = 5x^2-1$

$f(2) = 5(2)^2-1$

$= 5(4)-1$

$= 20-1$

$\boxed{f(2) = 19}$

$f(-3)+f(2) = -17+19$

$\boxed{f(-3)+f(2) = 2}$

ii) $f(7)-f(1)$

$f(x) = 3x-4$

$f(7) = 3(7)-4$

$= 21-4$

$\boxed{f(7) = 17}$

$f(x) = 6x+1$

$f(1) = 6(1)+1$

$= 6+1$

$\boxed{f(1) = 7}$

$f(7)-f(1) = 17-7$

$\boxed{f(7)-f(1) = 10}$

iii) $2f(4)+f(8)$

$f(x) = 5x^2-1$

$f(4) = 5(4)^2-1$

$= 5(16)-1$

$= 80-1$

$\boxed{f(4) = 79}$

$f(x) = 3x-4$

$f(8) = 3(8)-4$

$= 24-4$

$\boxed{f(8) = 20}$

$2f(4)+f(8) = 2(79)+20$

$= 158+20$

$\boxed{2f(4)+f(8) = 178}$

iv) $\dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$

$f(x)=6x+1$

$f(-2)=6(-2)+1$

$=-12+1$

$\boxed{f(-2)=-11}$

$f(x)=3x-4$

$f(6)=3(6)-4$

$=18-4$

$\boxed{f(6)=14}$

$f(x)=5x^2-1$

$f(4)=5(4)^2-1$

$=5(16)-1$

$=80-1$

$\boxed{f(4)=79}$

$f(x)=6x+1$

$$\frac{2f(-2)-f(6)}{f(4)+f(-2)}=\frac{2(-11)-14}{79+(-11)}$$

$$=\frac{-22-14}{79-11}$$

$$=\frac{-36}{68}$$

$$\boxed{\frac{2f(-2)-f(6)}{f(4)+f(-2)}=-\frac{9}{17}}$$

11, The distance S an object travels under the influence of gravity in time t second is given by $s(t)=\frac{1}{2}gt^2+at+b$ where (g is the acceleration due to gravity) a, b are constants. Verify whether the function s(t) is one-one or not.

Sol:

$s(t)=\frac{1}{2}gt^2+at+b$

Let t be 1, 2, 3, ... Seconds

$s(1)=\frac{1}{2}g(1^2)+a(1)+b$

$=\frac{1}{2}g+a+b$

$s(2)=\frac{1}{2}g(2^2)+a(2)+b$

$=2g+2a+b$

There will be different Pre-images for the different Values of the range.

∴ It is one-one function.

12, The function 't' which maps temperature in Celsius (c) into temperature in Fahrenheit (F) is defined by $t(c)=F$ where $F=\frac{9}{5}C+32$. Find i) $t(0)$ ii) $t(28)$ iii) $(t(-10)$ iv) the value of C when $t(c)=212$ v) the temperature when the Celsius value is equal to the Fahrenheit value.

Sol:

i) $t(0)=F$

$F=\frac{9}{5}(c)+32$

$=\frac{9}{5}(0)+32$

$\boxed{t(0)=32^\circ F}$

ii) $t(28)=F$

$=\frac{9}{5}(28)+32$

$=\frac{252}{5}+32$

$= 50.4+32$

$\boxed{t(28) = 82.4^{\circ}F}$

iii) $t(-10) = F$

$F = \frac{9}{5}(-10)+32)$

$= 9\times-2 + 32$

$= -18+32$

$\boxed{t(-10) = 14^{\circ}F}$

iv) $t(c) = 212$

$\frac{9}{5}c + 32 = 212$

$\frac{9}{5}c = 212-32$

$\frac{9}{5}c = 180$

$c = \frac{180\times5}{9}$

$\boxed{c = 100^{\circ}C}$

v) $C = F$

$\frac{9}{5}C+32 = C$

$32 = C - \frac{9}{5}C$

$32 = C(1-\frac{9}{5})$

$32 = C(\frac{-4}{5})$

$C = 32 \times \frac{-5}{4}$

$\boxed{c = -40^{\circ}}$

13, If $f(x) = 2x+3$ $g(x) = 1-2x$ and $h(x) = 3x$. Prove that $fo(goh) = (fog)oh$

Sol:-

LHS $fo(goh)$

$goh = g[h(x)]$

$= g[3x]$

$= 1-2(3x)$

$\boxed{goh = 1-6x}$

$fo(goh) = f[g(h(x))]$

$= f[1-6x]$

$= 2(1-6x)+3$

$= 2-12x+3$

$\boxed{fo(goh) = 5-12x}$ —— ①

RHS

$(fog)oh$

$fog = f[g(x)]$

$= f[1-2x]$

$= 2(1-2x)+3$

$= 2-4x+3$

$\boxed{fog = 5-4x}$

$(fog)oh = fog[h(x)]$

$= fog[3x]$

$= 5-4(3x)$

$\boxed{(fog)oh = 5-12x}$ —— ②

① = ②

LHS = RHS

$fo(goh) = (fog)oh$

Hence proof

14, Find x iff $gff(x) = ffg(x)$ given $f(x) = 3x+1$ and $g(x) = x+3$

Sol:-

LHS $gff(x) = g[f\{f(x)\}]$

$= g[f(3x+1)]$

$= g[3(3x+1)+1]$

$= g[9x+3+1]$

$= g[9x+4]$

$= 9x+4+3$

$\boxed{gff(x) = 9x+7}$ —— ①

RHS

$ffg(x) = f[f\{g(x)\}]$

$= f[f(x+3)]$

$= f[3(x+3)+1]$

$= f[3x+9+1]$

$= f[3x+10]$

$= 3(3x+1)+10$

$= 3x+3+10$

$\boxed{ffg(x) = 3x+13}$ —— ②

① = ②

$9x+7 = 3x+13$

$9x-3x = 13-7$

$6x = 6$

$x = \frac{6}{6}$

$\boxed{x = 1}$

15, If $f(x) = (3x-2)$ $g(x) = 2x+K$ and if $f \circ g = g \circ f$, then find the Value of K.

Sol:-

LHS

$f \circ g = f[g(x)]$

$= f[2x+K]$

$= 3(2x+K)-2$

$\boxed{f \circ g = 6x+3K-2}$ —— ①

RHS

$g \circ f = g[f(x)]$

$= g[3x-2]$

$= 2[3x-2]+K$

$\boxed{g \circ f = 6x-4+K}$ —— ②

① = ②

$6x+3K-2 = 6x-4+K$

$\cancel{6x}+3K-\cancel{6x}-K = -4+2$

$2K = -2$

$K = -\frac{2}{2}$

$\boxed{K = -1}$

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⑨

Chapter - 2

Numbers and Sequences

1) Find the HCF of 396, 504, 636.

Sol:

$$a = bq + r$$

To find 396 and 504

$$\begin{array}{r|l} & 1\,(q) \\ 396\,(b) & 504\,(a) \\ & 396 \\ \hline & 108 \\ & (R) \end{array}$$

$504 = 396 \times 1 + 108$

$396 = 108 \times 3 + 72$

$108 = 72 \times 1 + 36$

$72 = \underline{36} \times 2 + 0$

HCF of 396 and 504 = 36

To find 636 and 36

$636 = 36 \times 17 + 24$

$36 = 24 \times 1 + 12$

$24 = \underline{12} \times 2 + 0$

∴ HCF of (396, 504, 636) = 12

2. Use Euclid's algorithm to find the HCF of 4052 and 12756.

Sol:

$12756 > 4052$

So, $a = b \times q + r$

$12576 = 4052 \times 3 + 420$

$4052 = 420 \times 9 + 272$

$420 = 272 \times 1 + 148$

$272 = 148 \times 1 + 124$

$148 = 124 \times 1 + 24$

$124 = 24 \times 5 + 4$

$24 = \underline{4} \times 6 + 0.$

HCF of (4052, 12576) = 4

3. Prove that $\sqrt{3}$ is irrational.

Sol:

Let us assume that $\sqrt{3}$ is a rational number.

$\sqrt{3} = \frac{a}{b}$ ($b \neq 0$ a, b are Co-prime)

Taking square on both side

$(\sqrt{3})^2 = \frac{a^2}{b^2}$

$3b^2 = a^2$

$b^2 = \frac{a^2}{3}$

3 divides a^2

So, 3 divides a —(1)

Hence

$\frac{a}{3} = c$

$a = 3c$

Now WKT

$3b^2 = a^2$ $\boxed{a = 3c}$

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$b^2 = \frac{9}{3}c^2$

$b^2 = 3c^2$

$\frac{b^2}{3} = c^2$

Hence 3 divides b^2

So, 3 divides b — ②

By ① and ② 3 divides both a and b.

$\sqrt{3}$ is irrational

A.

4, Find the remainder when 2^{81} is divided by 17.

Sol:

$$2^{81} \equiv x \pmod{17}$$

$$2^{40}\times 2^{40}\times 2^{1} \equiv x \pmod{17}$$

$$(2^{4})^{10}\times (2^{4})^{10}\times 2^{1} \equiv x \pmod{17}$$

$$(16)^{10}\times (16)^{10}\times 2 \equiv x \pmod{17}$$

$$(16^{5})^{2}\times (16^{5})^{2}\times 2$$

$$16^{5} \equiv 16 \pmod{17}$$

$$(16^{5})^{2} \equiv 16^{2} \pmod{17}$$

$$(16^{5})^{2} \equiv 256 \pmod{17}$$

$$\equiv 1 \pmod{17} \text{ — ①}$$

(255 is divisible by 17)

$$(16^{5})^{2}\times (16^{5})^{2}\times 2 \equiv 1\times 1\times 2 \pmod{17}$$

$$2^{81} \equiv 2 \pmod{17}$$

$$\boxed{x = 2}$$

2^{81} is divided by 17 is 2.

5, Determine the general term of an A.P whose 7th term is -1 and 16th term is 17.

Sol:

$$\boxed{t_n = a+(n-1)d}$$

$t_7 = -1$

$t_{16} = 17$

$t_7 \Rightarrow a+(7-1)d = -1$

$a+6d = -1$ — ①

$t_{16} \Rightarrow a+(16-1)d = 17$

$a+15d = 17$ — ②

① ⇒ $\cancel{a}+6d = -1$

② ⇒ $\cancel{a}+15d = 17$

$(-)\ (-)\ (-)$

$-9d = -18$

$d = \frac{-18}{-9}$

$$\boxed{d = 2}$$

$d = 2$ Sub in ① eqn

① ⇒ $a+6(2) = -1$

$a+12 = -1$

$a = -1-12$

$$\boxed{a = -13}$$

$a = -13$ $d = 2$

General term $t_n = a+(n-1)d$

$= -13+(n-1)2$

$= -13+2n-2$

$$\boxed{t_n = 2n-15}$$

6, In an A.P Sum of four Consecutive terms is 28 and the sum of their Square is 276. Find the four numbers.

Sol:

Four Consecutive terms

$(a-3d)(a-d)(a+d)(a+3d)$

Sum of the four terms is 28.

$a-3\cancel{d}+a-\cancel{d}+a+\cancel{d}+a+3\cancel{d}=28$

$4a = 28$

$a = 28/4$

$\boxed{a=7}$

Sum of their squares is 276

$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=276$

$(a+b)^2=a^2+b^2+2ab$
$(a-b)^2=a^2+b^2-2ab$

$\left.\begin{matrix}a^2-\cancel{6ad}+9d^2+a^2-\cancel{2ad}+d^2+a^2+\cancel{2ad}\\+d^2+a^2+\cancel{6ad}+9d^2\end{matrix}\right\}=276$

$4a^2+20d^2 = 276$

$a=7 \Rightarrow$

$4(7)^2+20d^2=276$

$4(49)+20d^2=276$

$196+20d^2=276$

$20d^2=276-196$

$20d^2= 80$

$d^2=\frac{80}{20}$

$d^2=4$

$\boxed{d=\pm 2}$

$a=7 \quad d=2$

$\Rightarrow (a-3d), (a-d), (a+d), (a+3d)$

$=(7-3(2)), (7-2), (7+2)(7+3(2))$

$=(7-6), 5, 9, (7+6)$

$= 1, 5, 9, 15$

$a=7 \quad d=-2$

$=(a-3d), (a-d), (a+d), (a+3d)$

$=(7-3(-2)), (7-(-2)), (7-2), (7+3(-2))$

$=(7+6), (7+2), (7-2), (7-6)$

$=15, 9, 5, 1$

∴ The four Consecutive terms of an A.P are 1, 5, 9 and 15.

7) A mother divides Rs. 207 into three parts such that the amount are in A.P. and gives it to her three children. The Product of the two least amount that the children had Rs. 4623. Find the amount received by each child.

Sol: The three terms: $(a-d)(a)(a+d)$

Sum of the amount is Rs.207

$a-\cancel{d}+a+a+\cancel{d}=207$

$3a = 207$

$a = 207/3$

$\boxed{a=69}$

Product of two least number 4623

$(a-d)a=4623$

$a \Rightarrow 69$

$(69-d)69=4623$

$69-d = \frac{4623}{69}$

$69 - d = 67$

$-d = 67 - 69$

$-d = -2$

$\boxed{d = 2}$

The amount is given

$a = 69 \quad d = 2$

$= (a-d), (a), (a+d)$

$= (69-2), 69, (69+2)$

$= 67, 69, 71,$

8) If nine times ninth term is equal to the fifteen times fifteen term, show that six times twenty fourth term is zero.

Sol:

$9t_9 = 15t_{15}$

$6t_{24} = 0.$

$\boxed{t_n = a+(n-1)d}$

$9(a+(9-1)d) = 15(a+(15-1)d)$

$9(a+8d) = 15(a+14)d)$

$9a+72d = 15a+210d$

$9a+72d-15a-210d = 0$

$-6a-138d = 0$

$(-1)\times$

$6a+138d = 0$ — ①

$6t_{24} = 0.$

$6(a+(24-1)d) = 0$

$6(a+23d) = 0$

$6a+138d = 0$ — ②

① = ②

Hence proof

9, The sum of three consecutive terms that are in A.P is 27. and their product is 288. Find the three terms.

Sol:

Three consecutive terms are

$a-d, \; a, \; a+d$

Sum of the terms is 27

$a-\cancel{d}+a+a+\cancel{d} = 27$

$3a = 27$

$a = 27/3$

$\boxed{a = 9}$

Product of the terms is 288.

$(a-d)(a)(a+d) = 288$

$(a^2-d^2)(a) = 288$

$a = 9$

$(9^2-d^2)(9) = 288$

$81-d^2 = 288/9$

$81-d^2 = 32$

$-d^2 = 32-81$

$$-d^2 = -49$$

$$d^2 = 49$$

$$d = \sqrt{49}$$

$$\boxed{d = \pm 7}$$

$a = 9 \quad d = 7$

$$= (a-d), (a), (a+d)$$

$$= (9-7), 9, (9+7)$$

$$= 2, 9, 16$$

$a = 9 \quad d = -7$

$$= (a-d), (a), (a+d)$$

$$= (9-(-7)), 9, (9-7)$$

$$= 9+7, 9, 2$$

$$= 16, 9, 2$$

The three Consecutive terms are 2,9 and 16.

10, The ratio of 6th and 8th term of an A.P is 7:9. Find the ratio of 9th term to 13th term.

Sol:

$$t_9 : t_8 = 7:9$$

$$\frac{t_6}{t_8} = \frac{7}{9}$$

$$\boxed{t_n = a+(n-1)d}$$

$$\frac{t_6}{t_8} \Rightarrow \frac{a+(6-1)d}{a+(8-1)d} = \frac{7}{9}$$

$$\frac{a+5d}{a+7d} = \frac{7}{9}$$

$$9(a+5d) = 7(a+7d)$$

$$9a+45d = 7a+49d$$

$$9a+45d-7a-49d = 0$$

$$2a-4d = 0$$

$$2(a-2d) = 0$$

$$a-2d = 0$$

$$\boxed{a = 2d}$$

$$t_9 : t_{13}$$

$$\frac{t_9}{t_{13}} = \frac{a+(9-1)d}{a+(13-1)d}$$

$$= \frac{a+8d}{a+12d}$$

$a=2d \Rightarrow$

$$= \frac{2d+8d}{2d+12d}$$

$$= \frac{\cancel{10d}^{5}}{\cancel{14d}_{7}}$$

$$= 5/7$$

$$\boxed{t_9 : t_{13} = 5:7}$$

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11) Priya earned Rs.15,000 in the first month. Thereafter her salary increased by Rs.1500 per year. Her expenses are Rs.13,000 during the first month and the expenses increase Rs.900 per year. How long will it take for her to save Rs.20,000 per month.

Sol :-

	yearly Salary	yearly expenses	yearly Savings
1^{st} year	15000	13000	2000
2^{nd} year	16500	13900	2600
3^{rd} year	18000	14800	3200

$d = 2600 - 2000$

$a_1 = 2000 \quad d = 600$

Take for her to Save Rs.20,000 a year

$a_n = 20,000$

$a_n = a+(n-1)d$

$20,000 = 2000+(n-1)600$

$20,000 = 2000+600n-600$

$20,000 = 1400+600n$

$20,000-1400 = 600n$

$18600 = 600n$

$n = \frac{18600}{600}$

$\boxed{n = 31 \text{years}}$

12) The 13^{th} term of an A.P is 3 and the sum of first 13 terms is 234. Find the common difference and the sum of first 21 terms.

Sol :-

$$\boxed{\begin{array}{l} t_n = a+(n-1)d \\ S_n = \frac{n}{2}[2a+(n-1)d] \end{array}}$$

13^{th} term $= 3$

$t_{13} = 3$

$a+(13-1)d = 3$

$a+12d = 3$ —— ①

Sum of first 13 terms $= 234$

$\frac{13}{2}\left[2a+(13-1)d\right] = 234$

$\frac{13}{2}\left[2a+12d\right] = 234$

$2a+12d = 234 \times \frac{2}{13}$

$2a+12d = \frac{468}{13}$

$2a+12d = 36$ —— ②

① => $a+\cancel{12d} = 3$

② => $2a+\cancel{12d} = 36$

$\quad (-) \quad (-) \quad (-)$

$-a = -33$

$\boxed{a = 33}$

$a = 33$ sub in ① eqn.

① => $33+12d = 3$

$12d = 3-33$

$$12d = -30$$

$$d = \frac{-30}{12}$$

$$\boxed{d = -\frac{5}{2}}$$

First term $a = 33$

Common difference $= -\frac{5}{2}$

Sum of first 21 terms.

$$S_{21} = \frac{21}{2}\left[2(33) + (21-1)\left(-\frac{5}{2}\right)\right]$$

$$= \frac{21}{2}\left[66 + (\cancel{20}^{10})\left(-\frac{5}{\cancel{2}_{1}}\right)\right]$$

$$= \frac{21}{2}\left[66 - 50\right]$$

$$= \frac{21}{\cancel{2}_{1}}\left[\cancel{16}^{8}\right]$$

$$\boxed{S_{21} = 168}$$

13, The Sum of first n, 2n and 3n terms of an A.P are S_1, S_2 and S_3 respectively prove that $S_3 = 3(S_2 - S_1)$

Sol:

$$S_1 = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_2 = \frac{2n}{2}\left[2a + (2n-1)d\right]$$

$$S_3 = \frac{3n}{2}\left[2a + (3n-1)d\right]$$

RHS

$$3(S_2 - S_1)$$

$$S_2 - S_1 = \frac{2n}{2}\left[2a+(n-1)d\right] - \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2\{2a+(n-1)d\} - (2a+nd-d)\right]$$

$$= \frac{n}{2}\left[4a+2nd-2d-2a-nd+d\right]$$

$$= \frac{n}{2}\left[2a+nd-d\right]$$

$$S_2 - S_1 = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$3(S_2 - S_1) = \frac{3n}{2}\left[2a+(3n-1)d\right]$$

$$3(S_2 - S_1) = S_3$$

RHS = LHS

Hence proof.

14, The product of three Consecutive terms of a Geometric progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Sol:

Three Consecutive terms in G.P is $\frac{a}{r}$, a, ar

Product of the terms = 343

$$\left(\frac{a}{r}\right)(a)(ar) = 343$$

$$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 343$$

$$a^3 = 343$$

$$a^3 = 7^3$$

$$\boxed{a=7}$$

Sum of the terms $= \frac{91}{3}$

$$\frac{a}{r}+a+ar = \frac{91}{3}$$

$$a\left[\frac{1}{r}+1+r\right] = \frac{91}{3}$$

$$a\left[\frac{1+r+r^2}{r}\right] = \frac{91}{3}$$

$$7\left[\frac{1+r+r^2}{r}\right] = \frac{91}{3}$$

$$\frac{1+r+r^2}{r} = \frac{\overset{13}{\cancel{91}}}{3\times \underset{1}{\cancel{7}}}$$

$$\frac{1+r+r^2}{r} = \frac{13}{3}$$

$$3(1+r+r^2) = 13r$$

$$3+3r+3r^2 = 13r$$

$$3+3r+3r^2-13r = 0$$

$$3r^2-10r+3 = 0$$

9

-1 -9

-10

$$3r^2-r-9r+3 = 0$$

$$r(3r-1)-3(3r-1) = 0$$

$$(3r-1)(r-3) = 0$$

$3r-1=0$ $r-3=0$

$3r=1$ $\boxed{r=3}$

$$\boxed{r=\frac{1}{3}}$$

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$a=7$ $r=3$

$$\frac{a}{r}, a, ar = \frac{7}{3}, 7, 7(3)$$

$$= \frac{7}{3}, 7, 21.$$

$a=7$ $r=\frac{1}{3}$

$$\frac{a}{r}, a, ar = \frac{7}{1/3}, 7, 7\times\left(\frac{1}{3}\right)$$

$$= 7(3), 7, \frac{7}{3}$$

$$= 21, 7, \frac{7}{3}.$$

15, Find the Sum to n terms of the Series 5+55+555+....

Sol:

$= 5+55+555+\dots$ +n terms

$= 5[1+11+111+\dots$ +n terms$]$

Multiply and divided by 9

$= \frac{5}{9}[9+99+999+\dots$ +n terms$]$

$= \frac{5}{9}[(10-1)+(100-1)+(1000-1)+\dots$ +n terms$]$

$= \frac{5}{9}[(10+100+1000+\dots$ +n terms$)-(1+1+1+\dots$ +n terms$)]$

$= \frac{5}{9}[(10+10^2+10^3+\dots$ +n terms$)-n]$

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$= \frac{50(10^n-1)}{9\times 9} - \frac{5n}{9}$$

$$\boxed{S_n = \frac{50(10^n-1)}{81} - \frac{5n}{9}}$$

16, Find the Sum of n terms $3+33+333+\ldots$ to n terms

Sol

$= 3+33+333+\ldots+n$ terms

$= 3[1+11+111+\ldots+n \text{ terms}]$

Multiply and divided by 9

$= \frac{3}{9}[9+99+999+\ldots+n \text{ terms}]$

$= \frac{3}{9}[(10-1)+(100-1)+(1000-1)+\ldots\ldots+n \text{ terms}]$

$= \frac{3}{9}[(10+100+1000+\ldots+n \text{ terms}) - (1+1+1+\ldots\ldots+n \text{ terms}]$

$= \frac{3}{9}[(10+10^2+10^3+\ldots n \text{ terms}) - n]$

$= \frac{3}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$

$$\boxed{S_n = \frac{30(10^n-1)}{81} - \frac{3n}{9}}$$

17, Find the sum of n terms $0.4+0.44+0.444+\ldots$ to n terms.

Sol:

$= 0.4+0.44+0.444+\ldots\ldots+n$ terms

$= \frac{4}{10}+\frac{44}{100}+\frac{444}{1000}+\ldots+n$ terms

$= 4\left[\frac{1}{10}+\frac{11}{100}+\frac{111}{100}+\ldots+n \text{ terms}\right]$

Multiply and divided by 9

$= \frac{4}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots+n \text{ terms}\right]$

$= \frac{4}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\ldots+n \text{ terms}\right]$

$= \frac{4}{9}\left[(1+1+ +\ldots+n \text{ terms}) - \left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots+n \text{ terms}\right)\right]$

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$a = \frac{1}{10}$ $r = \frac{1}{10}$

$= \frac{4}{9}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{4}{9}\left[n - \frac{1}{10}\left(\frac{1-\frac{1}{10^n}}{\frac{10-1}{10}}\right)\right]$

$= \frac{4}{9}\left[n - \frac{1}{\cancel{10}} \times \frac{\cancel{10}}{9}\left(1-\frac{1}{10^n}\right)\right]$

$= \frac{4}{9}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

$$\boxed{S_n = \frac{4n}{9} - \left[\frac{4}{81}\left(1-\frac{1}{10^n}\right)\right]}$$

18, Rekha has 15 Square Colours Papers of Sizes 10cm, 11cm, 12cm, ..., 24cm. How much Can be decorated with these colour Papers?

Sol:

Area of 15 Square Colour papers $= 10^2+11^2+12^2+\ldots+24^2$

$= (1^2+2^2+3^2+\ldots+24^2) - (1^2+2^2+\ldots+9^2)$

$= 24 \times 25$

$$\boxed{\Sigma k^2 = 1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{24(24+1)(2(24)+1)}{6} - \frac{9(9+1)(2(9)+1)}{6}$$

$$= \frac{\overset{4}{\cancel{24}}\times 25\times 49}{\cancel{6}_1} - \frac{\overset{3}{\cancel{9}}\times\overset{5}{\cancel{10}}\times 19}{\cancel{6}_{\cancel{2}_1}}$$

$$= 4\times25\times49 - 15\times19$$

$$= 4900 - 285$$

$$\boxed{\text{Area} = 4615\text{cm}^2}$$

19, Find the Sum of the Series $(2^3-1)+(4^3-3^3)+(6^3-15^3)+\dots$ to i) n terms ii) 8 terms.

Sol:-

i) $\Sigma(2^3+4^3+6^3+\dots+n\text{ terms}) - (1^3+3^3+5^3+\dots+n\text{ terms})$.

$$=\Sigma(2n)^3 - \Sigma(2n-1)^3$$

$$\boxed{a^3-b^3 = (a-b)^3+3ab(a-b)}$$

$$=\Sigma[2n-(2n-1)]^3+3(2n)(2n-1)(2n-(2n-1))$$

$$=\Sigma[\cancel{2n}-\cancel{2n}+1]^3+6n(2n-1)(\cancel{2n}-\cancel{2n}+1)$$

$$=\Sigma(1+12n^2-6n)$$

$$=\Sigma1+12\Sigma n^2-6\Sigma n$$

$$= n+\overset{2}{\cancel{12}}\,\frac{n(n+1)(2n+1)}{\cancel{6}_1} - \frac{\overset{3}{\cancel{6}}(n(n+1))}{\cancel{2}_1}$$

$$= n+2n(2n^2+n+2n+1)-(3n^2+3n)$$

$$= n+4n^3+2n^2+4n^2+2n-3n^2-3n$$

$$= 4n^3+6n^2-3n^2+3n-3n$$

$$\boxed{\text{Sum of n terms} = 4n^3+3n^2}$$

ii) 8 terms

$$\text{Sum of 8 terms} = 4n^3+3n^2$$

$$=4(8)^3+3(8)^2$$

$$=4(512)+3(64)$$

$$=2048+192$$

$$\boxed{\text{Sum of 8 terms} = 2240}$$

Chapter - 3

Algebra

1) The sum of thrice the first number, Second number and twice the third number is 5. If thrice the second number is subtracted from the sum of first number and thrice the third we get 2. If the third number is subtracted from the sum of twice the first, thrice the second, we get 1. Find the numbers.

Sol :-

Let the three numbers x, y, z

$3x+y+2z = 5$ — ①

$x+3z-3y = 2$ — ②

$2x+3y-z=1$ — ③

① ⇒ $\cancel{3x}+y+2z=5$

② × 3 ⇒ $\cancel{3x}-9y+9z=6$

(−) (+) (−) (−)

$10y-7z=-1$ — ④

① × 2 ⇒ $\cancel{6x}+2y+4z=10$

③ × 3 ⇒ $\cancel{6x}+9y-3z=3$

(−) (−) (+) (−)

$-7y+7z=7$ — ⑤

④ ⇒ $10y-\cancel{7z}=-1$

⑤ ⇒ $-7y+\cancel{7z}=7$

$3y=6$

$y=6/3$

$\boxed{y=2}$

y = 2. Sub in ④ eqn.

④ ⇒ $10(2)-7z=-1$

$20-7z=-1$

$-7z=-1-20$

$-7z=-21$

$z=\cancel{-}\frac{\cancel{21}^{3}}{\cancel{-7}}$

$\boxed{z=3}$

y = 2 z = 3 Sub in ③ eqn

③ ⇒ $2x+3(2)-3=1$

$2x+6-3=1$

$2x+3=1$

$2x=1-3$

$2x=-2$

$x=-2/2$

$\boxed{x=-1}$

2, Vani her father and her grand father have an average age of 53. One half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four year ago if Vani's grandfather was four time as old as Vani then how old are they all now?

Sol:

Vani age = x years

Vani father age = y years

Vani grand father = z years

$$\frac{x+y+z}{3}=53$$

$x+y+z=53\times3$

$x+y+z=159$ — ①

$$\frac{1}{2}z+\frac{1}{3}y+\frac{1}{4}x=65$$

$$\frac{6z+4y+3x}{12}=65$$

$3x+4y+6z=65\times12$

$3x+4y+6z=780$ — ②

$z - 4 = 4(x - 4)$

$z - 4 = 4x - 16$

$4x - 16 - z + 4 = 0$

$4x - z - 12 = 0$

$4x - z = 12$ —— ③

① × 4 ⇒ $4x + \cancel{4y} + 4z = 636$

② ⇒ $3x + \cancel{4y} + 6z = 780$

$\quad (-) \quad (-) \quad (-) \quad (-)$

$x - 2z = -144$ — ④

③ × 2 ⇒ $8x - \cancel{2z} = 24$

④ ⇒ $x - \cancel{2z} = -144$

$\quad (-) \quad (+) \quad (+)$

$7x = 168$

$x = \frac{168}{7}$

$\boxed{x = 24}$

$x = 24$ sub in ③ eqn.

③ ⇒ $4(24) - z = 12$

$96 - z = 12$

$-z = 12 - 96$

$\cancel{-}z = \cancel{-}84$

$\boxed{z = 84}$

$x = 24$ $z = 84$ Sub in ① eqn.

① ⇒ $24 + y + 84 = 159$

$y + 108 = 159$

$y = 159 - 108$

$\boxed{y = 51}$

Vani age $x = 24$ years

Vani father age $y = 51$ years

Vani grandfather $z = 84$ years.

3, The sum of the digits of a three digit number is 11. If the digits are reversed the new number is 46 more than five times the old number. If the hundreds digit plus twice the tens digit is equal to the units digit then find the original three digit number?

Sol:

Let Hundred digit $= x$.

Tens digit $= y$

Unit digit $= z$

The number is

$100x + 10y + z$

Digits are reversed

$100z + 10y + x$.

So,

$x + y + z = 11$ —— ①

$100z + 10y + x = 5(100x + 10y + z) + 46$

$100z + 10y + x = 500x + 50y + 5z + 46$

$100z + 10y + x - 500x - 50y - 5z = 46$

$-499x - 40y + 95z = 46$

$499x + 40y - 95z = -46$ — ②

$x + 2y = z$

$x + 2y - z = 0$ —— ③

$① \times 95 \Rightarrow 95x + 95y + 95\cancel{z} = 1045$

$② \Rightarrow 499x + 40y - \cancel{95z} = -46$

$594x + 135y = 999$

$(\div 9)\quad \frac{594x}{9} + \frac{135y}{9} = \frac{999}{9}$

$66x + 15y = 111$

$(\div 3)\quad \frac{66x}{3} + \frac{15y}{3} = \frac{111}{3}$

$22x + 5y = 37$ — ④

$① \Rightarrow x + y + \cancel{z} = 11$

$③ \Rightarrow x + 2y - \cancel{z} = 0$

$2x + 3y = 11$ — ⑤

$④ \Rightarrow \cancel{22x} + 5y = 37$

$⑤ \times 11 \Rightarrow \cancel{22x} + 33y = 121$

$(-) \quad (-) \quad (-)$

$-28y = -84$

$y = \frac{\overset{\overset{\overset{3}{\cancel{21}}}{\cancel{42}}}{\cancel{-84}}}{\underset{\underset{\underset{1}{\cancel{7}}}{\cancel{14}}}{\cancel{-28}}}$

$\boxed{y = 3}$

$y = 3$ Sub in ⑤ eqn.

$⑤ \Rightarrow 2x + 3(3) = 11$

$2x + 9 = 11$

$2x = 11 - 9$

$2x = 2$

$x = 2/2$

$\boxed{x = 1}$

$x = 1 \quad y = 3$ Sub in ① eqn.

$① \Rightarrow 1 + 3 + z = 11$

$4 + z = 11$

$z = 11 - 4$

$\boxed{z = 7}$

Hundred digit $x = 1$

Tens digit $y = 3$

Unit digit $z = 7$

4, There are 12 piece of five, ten and twenty rupee Currencies whose total value is Rs.105. But when first 2 sorts are interchanged in their numbers its value will be increased by 20. Find the number of Currencies in each sort.

Sol:

5 Rs Currencies $= x$

10 Rs Currencies $= y$

20Rs Currencies $= z$

$x + y + z = 12$ — ①

$5x + 10y + 20z = 105$

$\div 5 \quad x + 2y + 4z = 21$ — ②

$10x + 5y + 20z = 105 + 20$

$10x + 5y + 20z = 125$

$\div 5 \quad 2x + y + 4z = 25$ — ③

$① \times 4 \Rightarrow 4x + 4y + 4\cancel{z} = 48$

$② \Rightarrow x + 2y + \cancel{4z} = 21$

$(-) \quad (-) \quad (-) \quad (-)$

$3x + 2y = 27$ — ④

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② => $x + 2y + \cancel{4z} = 21$

③ => $\overset{(-)}{2x} + \overset{(-)}{y} + \overset{(-)}{\cancel{4z}} = \overset{(-)}{25}$

$-x + y = -4$

$\times(-1)$

$x - y = 4$ —— ⑤

④ => $3x + \cancel{2y} = 27$

⑤×2 => $\overset{(+)}{2x} \overset{(+)}{- \cancel{2y}} = \overset{(+)}{8}$

$5x = 35$

$x = 35/5$

$\boxed{x = 7}$

$x = 7$ Sub in ⑤ eqn

⑤ => $x - y = 4$

$7 - y = 4$

$-y = 4 - 7$

$\cancel{-}y = \cancel{-}3$

$\boxed{y = 3}$

$x = 7$ $y = 3$ Sub in ① eqn.

① => $7 + 3 + z = 12$

$10 + z = 12$

$z = 12 - 10$

$\boxed{z = 2}$

5Rs Currencies $x = 7$

10Rs Currencies $y = 3$

20Rs Currencies $z = 2$

5, If $x = \frac{a^2+3a-4}{3a^2-3}$ and $y = \frac{a^2+2a-8}{2a^2-2a-4}$ find the Value of x^2y^{-2}.

Sol

$$x = \frac{a^2+3a-4}{3a^2-3}$$

-4
-1 4
3

$$= \frac{(a-1)(a+4)}{3(a^2-1)}$$

$$= \frac{\cancel{(a-1)}(a+4)}{3(a+1)\cancel{(a-1)}}$$

$$\boxed{x = \frac{a+4}{3(a+1)}}$$

$$y = \frac{a^2+2a-8}{2a^2-2a-4}$$

$a^2+2a-8 = (a-2)(a+4)$

-8
4 -2
2

$2a^2-2a-4 = 2a^2+2a-4a-4$

-8
-4 2
-2

$= 2a(a+1) - 4(a+1)$

$2a^2-2a-4 = (a+1)(2a-4)$

$$y = \frac{(a-2)(a+4)}{(2a+4)(a+1)}$$

$$= \frac{\cancel{(a-2)}(a+4)}{2\cancel{(a+2)}(a+1)}$$

$$\boxed{y = \frac{a+4}{2(a+1)}}$$

$$x^2y^{-2} = \frac{x^2}{y^2} = \left(\frac{x}{y}\right)^2$$

$$= \left[\frac{\frac{a+4}{3(a+1)}}{\frac{a+4}{2(a+1)}}\right]^2$$

$$= \left[\frac{\cancel{a+4}}{3\cancel{(a+1)}} \times \frac{2\cancel{(a+1)}}{\cancel{a+4}}\right]^2$$

$$= \left[\frac{2}{3}\right]^2$$

$$\boxed{x^2 y^{-2} = \frac{4}{9}}$$

6, If a polynomial $p(x)=x^2-5x-14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$ find $q(x)$.

Sol:

$$P(x) = x^2-5x-14$$

-14
-7 2
-5

$$\boxed{P(x) = (x-7)\ (x+2)}$$

$$\frac{P(x)}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2}$$

$$(x-7)(x+2)(x+2) = (x-7)\,q(x)$$

$$\frac{\cancel{(x-7)}\,(x+2)^2}{\cancel{(x-7)}} = q(x)$$

$$q(x) = (x+2)^2$$

$$= x^2+4+2x$$

$$\boxed{q(x) = x^2+2x+4}$$

7, Simplify

$$\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$$

Sol:

$$x^2-5x+6 = (x-2)(x-3)$$

6
-3 -2
-5

$$x^2-3x+2 = (x-1)\ (x-2)$$

2
-1 -2
-3

$$x^2-8x+15 = (x-5)(x-3)$$

15
-5 -3
-8

$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-1)(x-2)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-5x-x+5+x^2-3x-5x+15-(x^2-x-2x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{2x^2-6x-8x+20-x^2+3x-2}{(x-1)\ (x-2)(x-3)(x-5)}$$

$$= \frac{x^2-14x+3x+18}{(x-1)\ (x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)\ (x-2)(x-3)(x-5)}$$

18
-9 -2
-11

$$= \frac{(x-9)\ \cancel{(x-2)}}{(x-1)\ \cancel{(x-2)}(x-3)(x-5)}$$

$$\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15} = \frac{(x-9)}{(x-1)(x-3)(x-5)}$$

8, If $A = \frac{2x+1}{2x-1}$ $B = \frac{2x-1}{2x+1}$ find

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2}$$

Sol:

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2} = \frac{1}{A-B} - \frac{2B}{(A+B)(A-B)}$$

$$= \frac{A+B-2B}{(A+B)(A-B)}$$

$$= \frac{\cancel{A-B}}{(A+B)\cancel{(A-B)}}$$

$$= \frac{1}{A+B}$$

$$= \frac{1}{\frac{2x+1}{2x-1}+\frac{2x-1}{2x+1}}$$

$$= \frac{1}{\frac{(2x+1)(2x+1)+(2x-1)(2x-1)}{(2x-1)(2x+1)}}$$

$$= \frac{(2x-1)(2x+1)}{(2x+1)^2+(2x-1)^2}$$

$$= \frac{4x^2+\cancel{2x}-\cancel{2x}-1}{4x^2+1+\cancel{4x}+4x^2+1-\cancel{4x}}$$

$$= \frac{4x^2-1}{8x^2+2}$$

$$\boxed{\frac{1}{A-B} - \frac{2B}{A^2-B^2} = \frac{4x^2-1}{2(4x^2+1)}}$$

9, If $A = \frac{x}{x+1}$ $B = \frac{1}{x+1}$ prove that

$$\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}$$

Sol:

$$\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{A^2+B^2+\cancel{2AB}+A^2+B^2-\cancel{2AB}}{A \div B}$$

$$= \frac{2A^2+2B^2}{A \div B}$$

$$= \frac{2(A^2+B^2)}{A \div B}$$

$$A^2+B^2 = \frac{x^2}{(x+1)^2} + \frac{1}{(x+1)^2}$$

$$\boxed{A^2+B^2 = \frac{x^2+1}{(x+1)^2}}$$

$$\frac{A}{B} = \frac{x/\cancel{x+1}}{1/\cancel{x+1}}$$

$$= \frac{x}{\cancel{x+1}} \times \frac{\cancel{x+1}}{1}$$

$$\boxed{A/B = x}$$

$$\frac{2(A^2+B^2)}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}$$

$$\boxed{\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}}$$

10, Iniya bought 50kg of fruits Consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought Rs 1800 worth of apples and Rs. 600 worth bananas then how many kgs of each fruit did she buy ?

Sol:

The Weight of apple = a

The Weight of banana = b

$a + b = 50$

$ax = 1800$ —— ①

$by = 600$ —— ②

$x = 2y$ —— ③

③ in ① eqn.

$a(2y) = 1800$

$2ay = 1800$

$y = \frac{\cancel{1800}^{900}}{\cancel{2}_{1}a}$

$y = \frac{900}{a}$ —— ④

④ in ② eqn

$b\left(\frac{900}{a}\right) = 600$

$b = \frac{\cancel{600}^{2}a}{\cancel{900}_{3}}$

$b = \frac{2a}{3}$ —— ⑤

$3b = 2a$

$b = \frac{2a}{3}$ sub in ①

$a + \frac{2a}{3} = 50$

$\frac{3a+2a}{3} = 50$

$\frac{5a}{3} = 50$

$5a = 150$

$a = \frac{150}{5}$

$\boxed{a = 30}$

a = 30 Sub in ① eqn.

① => $30 + b = 50$

$b = 50 - 30$

$\boxed{b = 20}$

Iniya bought 30kg apples and 20kg bananas.

11, Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

Sol

$$\begin{array}{r|l}
 & 8x^2 - x + 1 \\
\hline
8x^2 & 64x^4 - 16x^3 + 17x^2 - 2x + 1 \\
 & \cancel{64x^4} \\
 & (-) \\
\hline
16x^2 - x & \cancel{-16x^3} + 17x^2 \\
 & \cancel{-16x^3} + x^2 \\
 & (+) \quad (-) \\
\hline
16x^2 - 2x + 1 & 16x^2 - 2x + 1 \\
 & 16x^2 - 2x + 1 \\
\hline
 & 0
\end{array}$$

$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$.

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12, If $9x^4+12x^3+28x^2+ax+b$ is a Perfect square, find the value of a and b.

Sol :-

$$\begin{array}{r|l}
 & 3x^2+2x+4 \\
\hline
3x^2 & 9x^4+12x^3+28x^2+ax+b \\
 & 9x^4 \\
 & (-) \\
\hline
6x^2+2x & 12x^3+28x^2 \\
 & 12x^3+4x^2 \\
 & (-)\quad(-) \\
\hline
6x^2+4x+4 & 24x^2+ax+b \\
 & 24x^2+16x+16 \\
\hline
 & 0
\end{array}$$

$a-16=0$ $b-16=0$

$\boxed{a=16}$ $\boxed{b=16}$

13, Find the Square root $121x^4-198x^3-183x^2+216x+144$

Sol :-

$$\begin{array}{r|l}
 & 11x^2-9x-12 \\
\hline
11x^2 & 121x^4-198x^3-183x^2+216x+144 \\
 & 121x^4 \\
 & (-) \\
\hline
22x^2-9x & -198x^3-183x^2 \\
 & -198x^3+81x^2 \\
 & (+)\quad(-) \\
\hline
22x^2-18x-12 & -264x^2+216x+144 \\
 & -264x^2+216x+144 \\
 & (+)\quad(-)\quad(-) \\
\hline
 & 0
\end{array}$$

$\sqrt{121x^4-198x^3-183x^2+216x+144} = |11x^2-9x-12|$

14, Find the square root $ax^4+bx^3+361x^2+220x+100$

Sol

$$\begin{array}{r|l}
 & 10+11x+12x^2 \\
\hline
10 & 100+220x+361x^2+bx^3+ax^4 \\
 & 100 \\
 & (-) \\
\hline
20+11x & 220x+361x^2 \\
 & 220x+121x^2 \\
 & (-)\quad(-) \\
\hline
20+22x+12x^2 & 240x^2+bx^3+ax^4 \\
 & 240x^2+264x^3+144x^4 \\
 & (-)\quad(-)\quad(-) \\
\hline
 & 0
\end{array}$$

$b-264=0$ $a-144=0$

$\boxed{b=264}$ $\boxed{a=144}$

15, Find the square root $x^4-8x^3+mx^2+nx+16$

Sol

$$\begin{array}{r|l}
 & x^2-4x+4 \\
\hline
x^2 & x^4-8x^3+mx^2+nx+16 \\
 & x^4 \\
 & (-) \\
\hline
2x^2-4x & -8x^3+mx^2 \\
 & -8x^3+16x^2 \\
 & (+)\quad(-) \\
\hline
2x^2-8x+4 & (m-16)x^2+nx+16 \\
 & 8x^2-32x+16 \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 0
\end{array}$$

$m-16-8=0$ $n+32=0$

$m-24=0$

$\boxed{m=24}$ $\boxed{n=-32}$

16, Find the square root of the Polynomial $16x^4+8x^2+1$

Sol

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$4x^2+1$

$4x^2 \;|\; 16x^4+8x^2+0x+1$

$16x^4$

$8x^2+1 \;|\; +8x^2+0x+1$

$\cancel{8x^2} + \cancel{0} + \cancel{1}$

$(-)\quad(-)\quad(-)$

0

$$\sqrt{16x^4+8x^2+1} = |4x^2+1|$$

17, Solve $\frac{1}{3}(x+y-5) = y-z = 2x-11 = 9-(x+2z)$.

Sol :

$\frac{1}{3}(x+y-5) = y-z$

$x+y-5 = 3(y-z)$

$x+y-5 = 3y-3z$

$x+y-5-3y+3z = 0$

$x-2y+3z = 5$ —— ①

$y-z = 2x-11$

$y-z-2x = -11$

$2x-y+z = 11$ —— ②

$2x-11 = 9-(x+2z)$

$2x-11 = 9-x-2z$

$2x-11+x+2z = 9$

$3x+2z = 9+11$

$3x+2z = 20$ —— ③

① $\Rightarrow$ $x-\cancel{2y}+3z = 5$

② $\cancel{\Rightarrow}\times 2 \Rightarrow 4x-\cancel{2y}+2z = 22$

$(-)\quad(+)\quad(-)\quad(-)$

$-3x+z = -17$ —— ④

③ $\Rightarrow$ $\cancel{3x}+2z = 20$

④ $\Rightarrow$ $-\cancel{3x}+z = -17$

$3z = 3$

$\boxed{z = 1}$

$z=1$ sub in ③ eqn

③ $\Rightarrow$ $3x+2(1) = 20$

$3x = 20-2$

$3x = 18$

$x = 18/3$

$\boxed{x = 6}$

$x=6$ $z=1$ sub in ① eqn.

① $\Rightarrow$ $6-2y+3(1) = 5$.

$9-2y = 5$

$-2y = 5-9$

$-2y = -4$

$y = \frac{\cancel{+4}^{2}}{\cancel{+2}_{1}}$

$\boxed{y = 2}$

$$\boxed{\begin{array}{l} x = 6 \\ y = 2 \\ z = 1 \end{array}}$$

18, Find the least Common multiple of $xy(k^2+1)+k(x^2+y^2)$ and $xy(k^2-1)+k(x^2-y^2)$.

Sol:-

$xy(k^2+1)+k(x^2+y^2)$

$= xyk^2+xy+kx^2+ky^2$ — ①

$xy(k^2-1)+k(x^2-y^2) =$

$xyk^2-xy+kx^2-ky^2$ — ②

① => $yk(xk+y)+x(xk+y)$

$= (xk+y)(yk+x)$.

② => $yk(xk-y)+x(xk-y)$

$= (xk-y)(yk+x)$.

LCM $= (xk+y)(yk+x)(xk-y)$

$$\boxed{\text{LCM} = (yk+x)(x^2k^2-y^2)}$$

19, Find square root of $289x^4-612x^3+970x^2-684x+361$

Sol:-

$17x^2-18x+19$

$17x^2 \;|\; 289x^4-612x^3+970x^2-684x+361$

$289x^4$

$(-)$

$34x^2-18x \;|\; -612x^3+970x^2$

$-612x^3+324x^2$

$(+) \quad (-)$

$34x^2-18x+19 \;|\; 646x^2-684x+361$

$646x^2-684x+361$

0

$$\sqrt{289x^4-612x^3+970x^2-684x+361} = |17x^2-18x+19|$$

20, Solve $pqx^2-(p+q)^2x+(p+q)^2=0$

Sol

$a = pq \quad b = -(p+q)^2 \quad c = (p+q)^2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-[-(p+q)^2] \pm \sqrt{[-(p+q)^2]^2-4(pq)(p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^4-4pq(p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2[(p+q)^2-4pq]}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2[p^2+q^2+2pq-4pq]}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2[p^2+q^2-2pq]}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2(p-q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{[(p+q)(p-q)]^{\cancel{2}}}}{2pq}$$

$$= \frac{(p+q)^2 \pm (p+q)(p-q)}{2pq}$$

$$= \frac{(p+q)\{(p+q) \pm (p-q)\}}{2pq}$$

$$x = \frac{(p+q)\{p+\cancel{q}+p-\cancel{q}\}}{2pq} \; ; \; \frac{(p+q)\{\cancel{p}+q-\cancel{p}+q\}}{2pq}$$

$$x = \frac{(p+q)(\cancel{2}\cancel{p})}{\cancel{2}\cancel{p}q} ; \frac{(p+q)(\cancel{2}\cancel{q})}{\cancel{2}p\cancel{q}}$$

$$\boxed{x = \frac{p+q}{q} , \frac{p+q}{p}}$$

21, A passenger train take 1 hr more than an express train to travel a distance of 240 km from chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

Sol:

Average speed of Passenger train } = x km/hr

Average Speed of express train } = (x+20) km/hr

Passenger train to Cover distance 240km } = $\frac{240}{x}$ hr

Time taken by express train Cover distance 240km } = $\frac{240}{x+20}$ hr

$$\frac{240}{x} = \frac{240}{x+20} + 1$$

$$\frac{240}{x} - \frac{240}{x+20} = 1$$

$$240\left[\frac{1}{x} - \frac{1}{x+20}\right] = 1$$

$$240\left[\frac{\cancel{x}+20-\cancel{x}}{x(x+20)}\right] = 1$$

$$240\left[\frac{20}{x^2+20x}\right] = 1$$

$$\frac{4800}{x^2+20x} = 1$$

$$4800 = x^2 + 20x$$

$$x^2+20x-4800 = 0$$

$$(x+80)(x-60) = 0$$

-4800

80 -60

$\boxed{x = -80}$ $\boxed{x = 60}$

(-ve value is rejected)

So,

Average speed of Passenger train } = 60 km/hr.

Average speed of express train } = x + 20 km/hr

= 60+20 km/hr

= 80 km/hr //.

22, A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more it would have taken 30 minutes less for the journey. Find the Original speed of the bus.

Sol:

Original speed of bus } = x km/hr

Time taken to cover 90km } = $\frac{90}{x}$

After increasing the speed by 15 km/hr.

Time taken to cover 90km } = $\frac{90}{x+15}$

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$$\frac{90(x+15) - 90x}{x(x+15)} = \frac{1}{2}$$

$$\frac{\cancel{90x}+1350-\cancel{90x}}{x^2+15x} = \frac{1}{2}$$

$$1350 \times 2 = x^2+15x$$

$$2700 = x^2+15x$$

$$x^2+15x-2700 = 0$$

$$(x+60)\ (x-45) = 0$$

-2700

60 -45

15

$\boxed{x=-60}$ $\boxed{x=45}$

(-ve value is rejected)

$\boxed{\text{Original speed of the bus } \} = 45 \text{ km/hr}}$

23, A girl is twice as old as her sister. Five year hence, the product of their ages (in years) will be 375. Find their present ages.

Sol:

Let age of the sister = x.

age of the girl = 2x.

age of the sister = x+5

age of the girl = 2x+5

$$(x+5)(2x+5) = 375$$

$$2x^2+5x+10x+25 = 375$$

$$2x^2+15x+25-375 = 0$$

$$2x^2+15x-350 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=2 \quad b=15 \quad c=-350$

$$= \frac{-15 \pm \sqrt{15^2-4(2)(-350)}}{2(2)}$$

$$= \frac{-15 \pm \sqrt{225+2800}}{4}$$

$$= \frac{-15 \pm \sqrt{3025}}{4}$$

$$= \frac{-15 \pm \sqrt{55^{\cancel{2}}}}{4}$$

$$= \frac{-15 \pm 55}{4}$$

$$= \frac{-15+55}{4} \ ; \ \frac{-15-55}{4}$$

$$= \frac{\cancel{40}^{10}}{\cancel{4}_1} \ ; \ \frac{-70}{4}$$ (-ve value is rejected).

$$\boxed{x = 10}$$

Age of the girl = 10 years

Age of the Sister = $2x$

$= 2\times 10$

$= 20$ years,,

24, From a group of $2x^2$ black bees. Square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there is total?

Sol:

Total no of black bees $= 2x^2$

Half of the group $= \frac{1}{2}\times 2x^2$

$= x^2$

Square root of half of the group $\Big\} = \sqrt{x^2}$

$= x$

Eight-ninth of the bees $\Big\} = \frac{8}{9}\times 2x^2$

$= \frac{16x^2}{9}$

No of bees in the lotus $\Big\} = 2$

By the given Condition

$$x + \frac{16x^2}{9} + 2 = 2x^2$$

$$\frac{9x+16x^2+18}{9} = 2x^2$$

$$9x+16x^2+18 = 2\times 9x^2$$

$$9x-16x^2+18 = 18x^2$$

$$9x+16x^2+18-18x^2 = 0.$$

$$-2x^2+9x+18 = 0.$$

$$(-1)\times \quad 2x^2-9x-18 = 0$$

-36

$-12 \quad 3$

-9

$$2x^2-12x+3x-18 = 0$$

$$2x(x-6)+3(x-6) = 0$$

$$(x-6)(2x+3) = 0$$

$x-6=0 \qquad 2x+3=0$

$\boxed{x=6} \qquad 2x=-3$

$\boxed{x=-3/2}$ -ve Value is rejected

Total no of black bees $\Big\} = 2x^2$

$= 2(6)^2$

$= 2(36)$

$\boxed{\text{Total no of black bees} = 72}$

25, If a, b are real then show that the roots of the equation $(a-b)x^2-6(a+b)x-9(a-b)=0$ are real and unequal.

Sol:

$a = a-b \quad b = -6(a+b) \quad c = -9(a-b)$

$$\Delta = b^2 - 4ac$$

$$= [-6(a+b)]^2 - 4(a-b)(-9)(a-b)$$

$$= 36(a+b)^2 + 36(a-b)^2$$

$$= 36[a^2+b^2+2ab] + 36[a^2+b^2-2ab]$$

$$= 36a^2 + 36b^2 + \cancel{72ab} + 36a^2 + 36b^2 - \cancel{72ab}$$

$$= 72a^2 + 72b^2$$

$$\Delta = 72(a^2+b^2) > 0.$$

The root are real and Unequal.

26, If the roots of the equation $(c^2-ab)x^2 - 2(a^2-bc)x + b^2-ac = 0$ are real and equal. prove that either $a=0$ (or) $a^3+b^3+c^3 = 3abc$.

Sol:

$a = (c^2-ab) \quad b = -2(a^2-bc) \quad c = b^2-ac$

$$\boxed{\Delta = b^2 - 4ac}$$

$$b^2 - 4ac = 0$$

$$[-2(a^2-bc)]^2 - 4(c^2-ab)(b^2-ac) = 0$$

$$4(a^2-bc)^2 - 4(b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$$

$$\left.\begin{matrix} 4(a^4+b^2c^2-2a^2bc) - 4b^2c^2 + 4ac^3 + 4ab^3 \\ -4a^2bc \end{matrix}\right\} = 0$$

$$\left.\begin{matrix} 4a^4 + \cancel{4b^2c^2} - 8a^2bc - \cancel{4b^2c^2} + 4ac^3 + 4ab^3 \\ -4a^2bc \end{matrix}\right\} = 0$$

$$4a^4 - 12a^2bc + 4ac^3 + 4ab^3 = 0$$

$$4a(a^3 - 3abc + c^3 + b^3) = 0$$

$4a = 0 \qquad a^3+b^3+c^3-3abc = 0$

$\boxed{a=0} \qquad \boxed{a^3+b^3+c^3 = 3abc}$

Hence proof

27, The root of the equation $2x^2-7x+5=0$ are α and β. without Solving for the roots. Find i) $\frac{1}{\alpha}+\frac{1}{\beta}$ ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ iii) $\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}$

Sol:

$2x^2 - 7x + 5 = 0$, $a = 2$, $b = -7$, $c = 5$

$$\alpha+\beta = \frac{-b}{a}$$

$$\boxed{\alpha+\beta = \frac{7}{2}}$$

$$\alpha\beta = \frac{c}{a}$$

$$\boxed{\alpha\beta = 5/2}$$

i) $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{(\alpha)(\beta)} = \frac{7/2}{5/2} = 7/\cancel{2} \times \cancel{2}/5$$

$$\boxed{\frac{1}{\alpha} + \frac{1}{\beta} = 7/5}$$

ii) $$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\left(7/2\right)^2 - \cancel{2}\left(5/\cancel{2}\right)}{5/2}$$

$$= \frac{\frac{49}{4} - 5}{5/2}$$

$$= \frac{\frac{49-20}{4}}{5/2}$$

$$= \frac{29}{\cancel{4}2} \times \frac{\cancel{2}^{1}}{5}$$

$$\boxed{\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{29}{10}}$$

iii) $$\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2} = \frac{(\alpha+2)^2+(\beta+2)^2}{(\alpha+2)(\beta+2)}$$

$$= \frac{\alpha^2+4+4\alpha+\beta^2+4+4\beta}{\alpha\beta+2\alpha+2\beta+4}$$

$$= \frac{\alpha^2+\beta^2+4\alpha+4\beta+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$\boxed{\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{\left(7/2\right)^2 - 2\left(5/2\right)+4\left(7/2\right)+8}{\left(5/2\right)+2\left(7/2\right)+4}$$

$$= \frac{\frac{49}{4}-\frac{10}{2}+\frac{28}{2}+8}{\frac{5}{2}+\frac{14}{2}+4}$$

$$= \frac{\frac{49-20+56+32}{4}}{\frac{5+14+8}{2}}$$

$$= \frac{\frac{29+88}{4}}{27/2}$$

$$= \frac{117}{\cancel{4}2} \times \frac{\cancel{2}^{1}}{27}$$

$$\boxed{\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2} = \frac{117}{54}}$$

28, If α, β are the roots of $7x^2+ax+2=0$ and if $\beta-\alpha = \frac{-13}{7}$. Find the values of a.

Sol:-

$$7x^2+ax+2=0.$$

$$\alpha+\beta = -b/a \qquad \alpha\beta = c/a$$

$$\boxed{\alpha+\beta = -\frac{a}{7}} \qquad \boxed{\alpha\beta = 2/7}$$

$$\beta-\alpha = -\frac{13}{7}$$

$$(-1)\times \quad \alpha-\beta = 13/7$$

Squaring on both side

$$(\alpha-\beta)^2 = \left(\frac{13}{7}\right)^2$$

$$\alpha^2+\beta^2-2\alpha\beta = \frac{169}{49}$$

$$(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta = \frac{169}{49}$$

$$(\alpha+\beta)^2-4\alpha\beta = \frac{169}{49}$$

$$\left(\frac{-a}{7}\right)^2-4\left(2/7\right) = \frac{169}{49}$$

$$\frac{a^2}{49} - \frac{8}{7} = \frac{169}{49}$$

$$\frac{a^2 - 56}{49} = \frac{169}{49}$$

$$a^2 - 56 = \frac{169 \times \cancel{49}}{\cancel{49}}$$

$$a^2 - 56 = 169$$

$$a^2 = 169 + 56$$

$$a^2 = 225$$

$$a = \sqrt{225}$$

$$\boxed{a = \pm 15}$$

29, If one root of the equation $3x^2 + Kx + 81 = 0$ (having real roots) is the square of the other than, find K.

Sol :-

Let α and α^2 are the roots

$$3x^2 + Kx + 81 = 0$$

$a = 3$ $b = K$ $c = 81$

Sum of the root $= \frac{-b}{a}$

$$\alpha + \alpha^2 = \frac{-K}{3}$$

$$3(\alpha + \alpha^2) = -K$$

$$3\alpha + 3\alpha^2 = -K \text{ — } ①$$

Product of the root $= c/a$

$$\alpha \times \alpha^2 = \frac{\cancel{81}^{27}}{\cancel{3}_1}$$

$$\alpha^3 = 27$$

$$\alpha^3 = 3^3$$

$$\boxed{\alpha = 3}$$

$\alpha = 3$ sub in ① eqn.

$$① \Rightarrow 3\alpha + 3\alpha^2 = -K$$

$$3(3) + 3(3)^2 = -K$$

$$9 + 3(9) = -K$$

$$9 + 27 = -K$$

$$36 = -K$$

$$\boxed{K = -36}$$

30, If $A = \begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}$

$C = \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$ Compute the following

i) $3A + 2B - C$

ii) $\frac{1}{2}A - \frac{3}{2}B$

Sol :-

i) $3A + 2B - C = 3\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} + 2\begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{bmatrix} + \begin{bmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+16-5 & 24-12-3 & 9-8-0 \\ 9+4+1 & 15+22+7 & 0-6-2 \\ 24+0-1 & 21+2-4 & 18+10-3 \end{bmatrix}$$

$$3A+2B-C = \begin{bmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{bmatrix} 1.$$

ii) $\frac{1}{2}A - \frac{3}{2}B = \frac{1}{2}(A-3B)$

$$= \frac{1}{2}\left[\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} - 3\begin{bmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{bmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{bmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{bmatrix} + \begin{bmatrix} -24 & 18 & 12 \\ -6 & -33 & 9 \\ 0 & -3 & -15 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 1-24 & 8+18 & 3+12 \\ 3-6 & 5-33 & 0+9 \\ 8+0 & 7-3 & 6-15 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -23 & 26 & 15 \\ -3 & -28 & 9 \\ 8 & 4 & -9 \end{bmatrix}$$

$$\frac{1}{2}A - \frac{3}{2}B = \begin{bmatrix} -23/2 & 13 & 15/2 \\ -3/2 & -14 & 9/2 \\ 4 & 2 & -9/2 \end{bmatrix} 1.$$

31, Find X and Y if $X+Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ and $X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

Sol

$$X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix} \text{ — } ①$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \text{ — } ②$$

$$① \Rightarrow X + \cancel{Y} = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$$

$$② \Rightarrow X \cancel{-} Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}$$

$$\boxed{X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}}$$

$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}$ Sub in ② eqn.

$$\frac{1}{2}\begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix} - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$-Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}$$

$$-Y = -\frac{1}{2}\begin{bmatrix} 13 & 0 \\ 3 & 13 \end{bmatrix}$$

$$\boxed{Y = \frac{1}{2}\begin{bmatrix} 13 & 0 \\ 3 & 13 \end{bmatrix}}$$

(or)

$$X = \begin{bmatrix} 5 & 0 \\ 3/2 & 9/2 \end{bmatrix} \quad Y = \begin{bmatrix} 2 & 0 \\ 3/2 & 1/2 \end{bmatrix}$$

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32, Find x and y if

$$x\begin{bmatrix}4\\-3\end{bmatrix}+y\begin{bmatrix}-2\\3\end{bmatrix}=\begin{bmatrix}4\\6\end{bmatrix}$$

Sol:-

$4x-2y=4$ — ①

$-3x+3y=6$ — ②

① × 3 => $12x-\cancel{6y}=12$

② × 2 => $-6x+\cancel{6y}=12$

$6x=24$

$x=24/6$

$\boxed{x=4}$

$x=4$ Sub in ① eqn

① => $4(4)-2y=4$

$16-2y=4$

$-2y=4-16$

$-2y=-12$

$y=\frac{\cancel{+}12}{\cancel{+}2}$

$\boxed{y=6}$

$\boxed{x=4 \quad y=6}$

33, If $A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}$ $B=\begin{bmatrix}1 & -1\\2 & 1\\1 & 3\end{bmatrix}$ and $C=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$ show that $(AB)C=A(BC)$

Sol:-

LHS $(AB)C$

$AB=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\begin{bmatrix}1 & -1\\2 & +1\\1 & 3\end{bmatrix}$

$=\begin{pmatrix}1-\cancel{2}+\cancel{2} & -1-1+6\end{pmatrix}$

$=\begin{pmatrix}1 & -2+6\end{pmatrix}$

$(AB)=\begin{pmatrix}1 & 4\end{pmatrix}$

$(AB)C=\begin{pmatrix}1 & 4\end{pmatrix}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$

$=\begin{pmatrix}1+8 & 2-4\end{pmatrix}$

$(AB)C=\begin{pmatrix}9 & -2\end{pmatrix}$ — ①

RHS

$A(BC)$

$BC=\begin{bmatrix}1 & -1\\2 & 1\\1 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$

$=\begin{bmatrix}1-2 & 2+1\\2+2 & 4-1\\1+6 & 2-3\end{bmatrix}$

$BC=\begin{bmatrix}-1 & 3\\4 & 3\\7 & -1\end{bmatrix}$

$A(BC)=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\begin{bmatrix}-1 & 3\\4 & 3\\7 & -1\end{bmatrix}$

$=\begin{pmatrix}-1-4+14 & 3-3-2\end{pmatrix}$

$=\begin{pmatrix}-5+14 & 3-5\end{pmatrix}$

$A(BC)=\begin{pmatrix}9 & -2\end{pmatrix}$ — ②

① = ②

LHS = RHS

$(AB)C=A(BC)$

Hence Proof

34, If $A=\begin{bmatrix}1&2&1\\2&-1&1\end{bmatrix}$ and $B=\begin{bmatrix}2&-1\\-1&4\\0&2\end{bmatrix}$

show that $(AB)^T=B^TA^T$.

Sol:

LHS $(AB)^T$

$$AB=\begin{bmatrix}1&2&1\\2&-1&1\end{bmatrix}\begin{bmatrix}2&-1\\-1&4\\0&2\end{bmatrix}$$

$$=\begin{bmatrix}2-2+0&-1+8+2\\4+1+0&-2-4+2\end{bmatrix}$$

$$(AB)=\begin{bmatrix}0&9\\5&-4\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}0&9\\5&-4\end{bmatrix}^T$$

$$(AB)^T=\begin{bmatrix}0&5\\9&-4\end{bmatrix} \text{ —— ①}$$

RHS:

B^TA^T

$$B=\begin{bmatrix}2&-1\\-1&4\\0&2\end{bmatrix}$$

$$B^T=\begin{bmatrix}2&-1\\-1&4\\0&2\end{bmatrix}^T$$

$$B^T=\begin{bmatrix}2&-1&0\\-1&4&2\end{bmatrix}$$

$$A=\begin{bmatrix}1&2&1\\2&-1&1\end{bmatrix}$$

$$A^T=\begin{bmatrix}1&2\\2&-1\\1&1\end{bmatrix}$$

$$B^TA^T=\begin{bmatrix}2&-1&0\\-1&4&2\end{bmatrix}\begin{bmatrix}1&2\\2&-1\\1&1\end{bmatrix}$$

$$=\begin{bmatrix}2-2+0&4+1+0\\-1+8+2&-2-4+2\end{bmatrix}$$

$$B^TA^T=\begin{bmatrix}0&5\\9&-4\end{bmatrix} \text{ —— ②}$$

① = ②

LHS = RHS

$(AB)^T=B^TA^T$

Hence proof

35, If $A=\begin{bmatrix}5&2&9\\1&2&8\end{bmatrix}$ $B=\begin{bmatrix}1&7\\1&2\\5&-1\end{bmatrix}$

Verify that $(AB)^T=B^TA^T$

Sol LHS $(AB)^T$

$$AB=\begin{bmatrix}5&2&9\\1&2&8\end{bmatrix}\begin{bmatrix}1&7\\1&2\\5&-1\end{bmatrix}$$

$$=\begin{bmatrix}5+2+45&35+4-9\\1+2+40&7+4-8\end{bmatrix}$$

$$AB=\begin{bmatrix}52&30\\43&3\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}52&43\\30&3\end{bmatrix} \text{ —— ①}$$

RHS

B^TA^T

$B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}$

$A: \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}$ $A^T: \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$

$B^TA^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$

$= \begin{bmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{bmatrix}$

$B^TA^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix}$ — ②

① = ②

LHS = RHS

$(AB)^T = B^TA^T$

Hence proof.

36, If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2-5A+7I_2=0$

Sol

$A^2 = A \times A$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2-5A+7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2-5A+7I_2 = 0$

Hence proof

37, If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Show that $A^2-(a+d)A = (bc-ad)I_2$

Sol:: LHS $A^2-(a+d)A$

$A^2 = A \times A$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$

$(a+d)A = (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} (a+d)a & (a+d)b \\ (a+d)c & (a+d)d \end{bmatrix}$

$$(a+d)A = \begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix}$$

$$A^2-(a+d)A = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} - \begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{a^2}+bc-\cancel{a^2}+ad & \cancel{ab}+\cancel{bd}-\cancel{ab}-\cancel{bd} \\ \cancel{ac}+\cancel{cd}-\cancel{ac}-\cancel{cd} & bc+\cancel{d^2}-ad-\cancel{d^2} \end{bmatrix}$$

$$A^2-(a+d)A = \begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} \quad — ①$$

RHS

$$(bc-ad)\, I_2 = (bc-ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(bc-ad)\, I_2 = \begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} \quad — ②$$

① = ②

$$A^2-(a+d)A = (bc-ad)I_2$$

Hence Proof

38, Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$

$C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ Show that

$(A-B)C = AC-BC$

Sol LHS $(A-B)\,C$

$$A-B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-4 & 2-0 \\ 1-1 & 3-5 \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$(A-B)\,C = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+2 & 0+4 \\ 0-2 & 0-4 \end{bmatrix}$$

$$(A-B)\,C = \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix} \quad — ①$$

RHS $AC-BC$

$$AC = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 0+4 \\ 2+3 & 0+6 \end{bmatrix}$$

$$AC = \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{bmatrix}$$

$$BC = \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$$

$$AC-BC = \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$$

$$AC-BC = \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix} \quad — ②$$

① = ②

LHS = RHS

$(A-B)\,C = AC-BC.$

chapter - 4

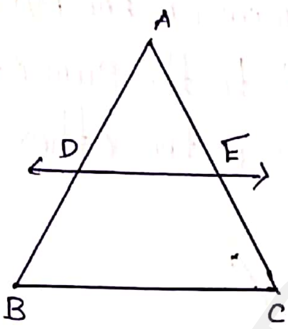
Geometry

1, state and prove Basic Proportionality theorem (or) BPT (or) Thales theorem.

Sol:

statement:

A straight line drawn Parallel to a side of triangle intersecting the other two Sides, divides the side in the Same ratio.



Proof:

Given: In ΔABC, D is a point on AB and E is a point on AC.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line DE∥BC

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statement	Reason
1. $\angle ABC = \angle ADF = \angle 1$	Corresponding angles are equal because DE∥BC
2. $\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because DE∥BC
3. $\angle DAF = \angle BAC = \angle 3$	Both triangle have a common angle
$\Delta ABC \sim \Delta ADE$	By AAA Similarity
$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding Sides are Proportional.
$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E.
$1+\frac{DB}{AD} = 1+\frac{EC}{AE}$	on simplification
$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both Sides.
$\frac{AD}{DB} = \frac{AE}{EC}$	Taking Reciprocals.

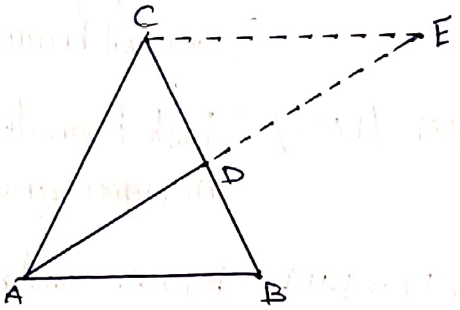
Hence proved.

2, State and prove Angle bisector theorem or ABT.

Sol:

Statement:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.



Proof:

Given: In ΔABC, AD is the internal bisector.

To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

Construction: Draw a line through C parallel to AB. Extend AD to meet line through C at E.

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles are equal.
2.	ΔACE is isosceles $AC = CE$ — ①	In ΔACE, $\angle CAE = \angle CEA$
3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From ① AC=CE

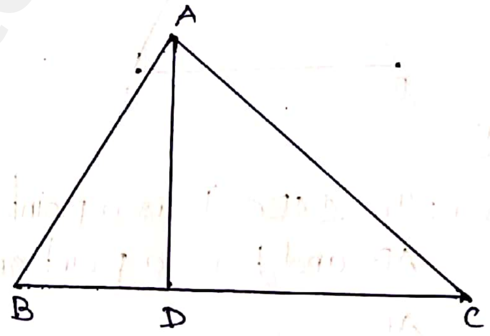
Hence proved.

3, State and prove pythagoras theorem.

Sol:

Statement:

In a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Proof:

Given : In ΔABC, $\angle A = 90^0$

To prove : $AB^2 + AC^2 = BC^2$

Construction: Draw $AD \perp BC$

No	statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is Common $\angle BAC = \angle BDA = 90^\circ$	Given $\angle BAC = 90^\circ$ and by Construction $\angle BDA = 90^\circ$
	Therefore, $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ — ①	By AA Similarity
2.	Compare ΔABC and ΔDAC $\angle C$ is Common $\angle BAC = \angle ABC = 90^\circ$	Given $\angle BAC = 90^\circ$ and by Construction $\angle ADC = 90^\circ$
	Therefore, $\Delta ABC \sim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ — ②	By AA Similarity

Adding ① and ②

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$

$$= BC(BD + DC)$$

$$= BC \times BC$$

$$AB^2 + AC^2 = BC^2$$

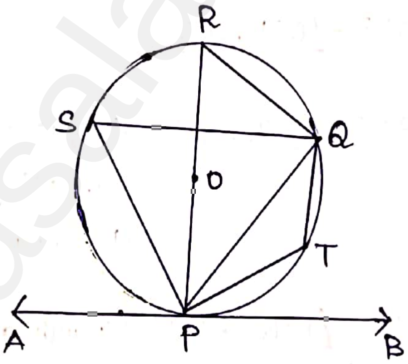
Hence proved

4) State and prove Alternate segment theorem.

Sol:

Statement:

If a line touches a circle and from the point of Contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the Corresponding alternate Segments.



Proof:

Given: A Circle with centre at O, tangent AB. touches the Circle at P and PQ is a Chord. S and T are two Points on the Circle in the opposite sides of chord PQ.

To prove: i) $\angle QPB = \angle PSQ$

ii) $\angle QPA = \angle PTQ$

Construction: Draw the diameter PQR. Draw QR, QS and PS.

No.	Statement	Reason
1.	$\angle RPB = 90°$ Now, $\angle RPQ + \angle QPB = 90°$ – ①	Diameter RP is perpendicular to tangent AB
2.	In ΔRPQ $\angle PQR = 90°$ — ②	Angle in a semicircle is 90°.
3.	$\angle QRP + \angle RPQ = 90°$ – ③	In a right angled triangle sum of the two acute angle is 90°.
4.	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\angle QPB = \angle QRP$ – ④	From ① and ③
5.	$\angle QRP = \angle PSQ$ – ⑤	Angles in the same segment are equal.
6.	$\angle QPB = \angle PSQ$ – ⑥	From ④ and ⑤ Hence i) is proved.
7.	$\angle QPB + \angle QPB = 180°$ – ⑦	Linear pair of angles.
8.	$\angle PSQ + \angle PTQ = 180°$ – ⑧	Sum of opposite angles of a cyclic quadrilateral is 180°.
9.	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From ⑦ and ⑧
10.	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$ from ⑥ Hence (ii) is proved
11.	$\angle QPA = \angle PTQ$	

Hence proof

Chapter - 8

Statistics and Probability

1. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is Selected at random. Find the probability that.

i) The student opted for NCC but not NSS.

ii) The student opted for NSS but not NCC.

iii) The student opted for exactly one of them.

Sol: $n(S) = 50$

$n(A) = 28$

$n(B) = 30$

$n(A \cap B) = 18$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(B) = \frac{n(B)}{n(S)}$

$\boxed{P(A) = \frac{28}{50}}$ $\quad$ $\boxed{P(B) = \frac{30}{50}}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$\boxed{P(A \cap B) = \frac{18}{50}}$

i) The student opted for NCC but not NSS.

$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$= \frac{20}{50} - \frac{18}{50}$

$= \frac{20-18}{50}$

$\boxed{P(A\cap\bar{B}) = \frac{1}{5}}$

ii) The student opted for NSS but not NCC.

$P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$= \frac{30}{50} - \frac{18}{50}$

$= \frac{30-18}{50}$

$\boxed{P(\bar{A}\cap B) = \frac{6}{25}}$

iii) The student opted for exactly one of them.

$= P(A\cap\bar{B}) \cup (\bar{A}\cap B)$

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= \frac{1}{5} + \frac{6}{25}$

$= \frac{5+6}{25}$

$\boxed{P(A\cap\bar{B})\cup(\bar{A}\cap B) = \frac{11}{25}}$

2, A Card is drawn from a Pack of 52 Cards. Find the probability of getting a King or a heart or a red Card.

Sol:

Total no. of Cards = 52

$n(S) = 52$

Let A be the event getting a King Card.

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)}$

$\boxed{P(A) = \frac{4}{52}}$

Let B be the event getting a heart Card.

$n(B) = 13$

$P(B) = \frac{n(B)}{n(S)}$

$\boxed{P(B) = \frac{13}{52}}$

Let C be the event getting a Red Card.

$n(C) = 26$

$P(C) = \frac{n(C)}{n(S)}$

$\boxed{P(C) = \frac{26}{52}}$

45

P(getting heart King) = P(A∩B)

$$\boxed{P(A\cap B) = \frac{1}{52}}$$

P(B∩C) = P(getting King and red)

$$\boxed{P(B\cap C) = \frac{13}{52}}$$

P(A∩C) = P(getting red King)

$$\boxed{P(A\cap C) = \frac{2}{52}}$$

P(A∩B∩C) = P(getting heart, King and red).

$$\boxed{P(A\cap B\cap C) = \frac{1}{52}}$$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$= \frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52}$$

$$= \frac{4+13+26-1-13-2+1}{52}$$

$$= \frac{28}{52}$$

$$\boxed{P(A\cup B\cup C) = \frac{7}{13}}$$

3. Two Unbiased dice are rolled once. Find the probability of getting (i) a doublet (equal numbers on both side) ii) the Product as a prime number. iii) the sum as a prime number iv) the sum as 1.

Sol:

Sample Space = {(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)
(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)
(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)
(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)}

$n(S) = 6\times 6$

$$\boxed{n(S) = 36}$$

i) a doublet (equal numbers on both side).

A = {(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)}

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\boxed{P(A) = \frac{6}{36}}$$

ii) the product as a prime number:

B = {(1,2)(1,3)(1,5)(2,1)(3,1)(5,1)}

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\boxed{P(B) = \frac{6}{36}}$$

iii) the sum as a prime number.

$C = \{(1,1)(1,2)(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4)(4,1)(4,3)(5,2)(5,6)(6,1)(6,5)\}$

$n(c) = 15$

$$P(c) = \frac{n(c)}{n(S)}$$

$$\boxed{P(c) = \frac{15}{36}}$$

iv) the sum as 1.

$n(D) = 0$

$$P(D) = \frac{n(D)}{n(S)}$$

$$\boxed{P(D) = 0}$$

4) Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13.

Sol:

$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$
$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$
$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$
$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$
$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$
$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

$n(S) = 36$

i) Let A be the event of getting the sum of outcomes values equal to 4.

$A = \{(1,3)(2,2)(3,1)\}$

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$$

$$\boxed{P(A) = \frac{1}{12}}$$

ii) Let B be the event of getting the sum of outcome values greater than 10.

$B = \{(5,6)(6,5)(6,6)\}$

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$\boxed{P(B) = \frac{1}{12}}$$

iii) Let c be the event of getting the sum of outcomes less than 13. $C = S$

$n(c) = 36$

$$P(c) = \frac{n(c)}{n(S)} = \frac{36}{36}$$

$$\boxed{P(c) = 1}$$

5, The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23?

Sol :-

$n = 15 \quad \bar{x} = 10 \quad \sigma = 5$

$\bar{x} = \frac{\sum x}{n}$

$\sum x = \bar{x} \times n$

$= 15 \times 10$

$\boxed{\sum x = 150}$

Wrong observation value = 8

Correct observation value = 23

Correct total = 150 − 8 + 23

= 165

Correct mean $\bar{x} = \frac{165}{15}$

$\boxed{\text{Correct mean } \bar{x} = 11}$

Standard deviation $\Big\}\ \sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

Incorrect value of $\sigma = 5$

$5 = \sqrt{\frac{\sum x^2}{15} - (10)^2}$

$25 = \frac{\sum x^2}{15} - 100$

$25 + 100 = \frac{\sum x^2}{15}$

$125 = \frac{\sum x^2}{15}$

$\sum x^2 = 125 \times 15$

$\boxed{\text{Incorrect value of } \sum x^2 = 1875}$

Correct value of $\sum x^2$ $= 1875 - 8^2 + 23^2$

$= 1875 - 64 + 529$

$\boxed{\text{Correct value of } \sum x^2 = 2340}$

Correct standard deviation $\Big\}\ \sigma = \sqrt{\frac{2340}{15} - (11)^2}$

$\sigma = \sqrt{156 - 121}$

$\sigma = \sqrt{35}$

$\boxed{\sigma \simeq 5.9}$

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6) The marks Scored by the Students in a slip test are given below. Find the standard deviation of their marks.

x	4	6	8	10	12
f	7	3	5	9	5

Sol)

x	f	$d=x-A$	fd	fd^2
4	7	-4	-28	112
6	3	-2	-6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	N=29		$\Sigma fd = 4$	$\Sigma fd^2 = 240$

Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2}$$

$$= \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$= \sqrt{\frac{6944}{29 \times 29}}$$

$$\boxed{\sigma \simeq 2.87}$$

7. The following table gives the Value of mean and Variance of heights and weights of the 10th standard students of a School.

	Height	Weight
Mean	155cm	46.50kg
Variance	72.25cm^2	28.09kg

Which is more Varying than the other?

Sol:- Mean Variance

$\bar{x}_1 = 155\text{cm}$ $\sigma_1^2 = 72.25\text{cm}^2$

Standard deviation $\sigma_1 = 8.5$

Coefficient of Variation $C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\%$

$C.V_1 = \frac{8.5}{155} \times 100\%$

$\boxed{C.V_1 = 5.48\%}$ (For heights)

Mean Variance

$\bar{x}_2 = 46.50\text{kg}$ $\sigma_2^2 = 28.09\text{kg}^2$

Standard deviation $\sigma_2 = 5.3\text{kg}$

Coefficient of Variation $C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\%$

$= \frac{5.3}{46.50} \times 100\%$

$\boxed{C.V_2 = 11.40\%}$ (For Weight)

Height is more consistent.

8, For a group of 100 Candidates, the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on, it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the Correct mean and standard deviation.

Sol:-

$n = 100$

Mean $(\bar{x}) = 60$

S.D $(\sigma) = 15$

Mean $(\bar{x}) = \frac{\Sigma x}{n}$

$60 = \frac{\Sigma x}{100}$

$\Sigma x = 60 \times 100$

$\boxed{\Sigma x = 6000}$

Correct total $\Big\} = 6000 + (45-40) + (72-27)$

$= 6000 + 5 + 45$

$\boxed{\text{Correct total} = 6050}$

Correct Mean $(\bar{x}) = \frac{\text{Correct total}}{n}$

$= \frac{6050}{100}$

$\boxed{\text{Correct Mean } (\bar{x}) = 60.5}$

Standard deviation $= 15$

Standard deviation $\Big\} \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$15 = \sqrt{\frac{\Sigma x^2}{100} - (60)^2}$

Squaring on both side we get.

$225 = \frac{\Sigma x^2}{100} - (60)^2$

$225 = \frac{\Sigma x^2}{100} - 3600$

$225 + 3600 = \frac{\Sigma x^2}{100}$

$3825 = \frac{\Sigma x^2}{100}$

$\Sigma x^2 = 3825 \times 100$

$\boxed{\Sigma x^2 = 382500}$

Correct Value of $\Sigma x^2 \Big\} = 382500 + 45^2 + 72^2 - 40^2 - \cancel{7}27^2$

$= 382500 + 2025 + 5184 - 1600 - 729$

$= 382500 + 7209 - 2329$

$= 389709 - 2329$

$\boxed{\text{Correct Value of } \Sigma x^2 = 387380}$

Correct Standard deviation } $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{387380}{100} - (60.5)^2}$

$= \sqrt{3873.8 - 3660.25}$

$= \sqrt{213.55}$

$= 14.613$

$\boxed{\sigma = 14.61}$

Correct mean = 60.5

Correct standard deviation } $\sigma = 14.61$

9, 48 Students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Sol :

x	f	xf	$d = x - \bar{x}$	d^2	fd^2
6	3	18	−3	9	27
7	6	42	−2	4	24
8	9	72	−1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	N=48	Σxf = 432	Σd=0		$\Sigma fd^2 = 124$

Mean

$\bar{x} = \frac{\Sigma xf}{N} = \frac{432}{49}$

$\boxed{\bar{x} = 9}$

standard deviation :-

$\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$

$= \sqrt{\frac{124}{48}}$

$\sigma = \sqrt{2.58}$

$\boxed{\sigma \simeq 1.6}$

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(TWO MARKS) Chapter-1

Relations and Functions

1, If $A = \{1,3,5\}$ $B = \{2,3\}$ then (i) find $A \times B$ and $B \times A$

Sol:

$A \times B = \{1,3,5\} \times \{2,3\}$

$A \times B = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3)\}$

$B \times A = \{2,3\} \times \{1,3,5\}$

$= \{(2,1)(2,3)(2,5)(3,1)(3,3)(3,5)\}$

2, If $A \times B = \{(3,2)(3,4)(5,2)(5,4)\}$ then find A and B.

Sol:

$A = \{3,5\}$ $B = \{2,4\}$

3, Let $A = \{1,2,3,4,\dots 45\}$ and R be the relation defined as "is Square of a number" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R.

Sol:

$A^2 = A \times A$

$= \{1,2,3,4 \dots 45\} \times \{1,2,3,4,\dots 45\}$

$= \{(1,1)(2,2)(3,3) \dots (45,45)\}$

R - is Square of

$R = \{(1,1)(2,4)(3,9)(4,16)(5,25)(6,36)\}$

Domain $= \{1,2,3,4,5,6\}$

Range $= \{1,4,9,16,25,36\}$

4, A relation R is given by the Set $\{(x,y) / y = x+3,\ x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range.

Sol

$x = 0,1,2,3,4,5$

$y = x+3$

$x=0 \Rightarrow y = 0+3 = 3$

$x=1 \Rightarrow y = 1+3 = 4$

$x=2 \Rightarrow y = 2+3 = 5$

$x=3 \Rightarrow y = 3+3 = 6$

$x=4 \Rightarrow y = 4+3 = 7$

$x=5 \Rightarrow y = 5+3 = 8$

$R = \{(0,3)(1,4)(2,5)(3,6)(4,7)(5,8)\}$

Domain $= \{0,1,2,3,4,5\}$

Range $= \{3,4,5,6,7,8\}$

5, Given the function $f: x^2 - 5x + 6$ Evaluate i) $f(-1)$ ii) $f(2a)$ iii) $f(2)$ iv) $f(x-1)$

Sol:

$f(x) = x^2 - 5x + 6$

i) $f(-1) = (-1)^2 - 5(-1) + 6$

$= 1 + 5 + 6$

$\boxed{f(-1) = 12}$

ii) $f(2a) = (2a)^2 - 5(2a) + 6$

$\boxed{f(2a) = 4a^2 - 10a + 6}$

iii) $f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$\boxed{f(2) = 0}$

iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 + 1 - 2x - 5x + 5 + 6$

$\boxed{f(x-1) = x^2 - 7x + 12}$

6, Let $f(x) = 2x+5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$

Sol:

$f(x+2) = 2(x+2)+5$

$= 2x+4+5$

$\boxed{f(x+2) = 2x+9}$

$f(2) = 2(2)+5$

$= 4+5$

$\boxed{f(2) = 9}$

$\frac{f(x+2)-f(2)}{x} = \frac{2x+\cancel{9}-\cancel{9}}{x}$

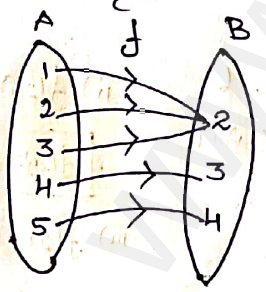
$= \frac{2\cancel{x}}{\cancel{x}}$

$\boxed{\frac{f(x+2)-f(2)}{x} = 2}$

7, Represent the function $f=\{(1,2)(2,2)(3,2)(4,3)(5,4)\}$ through (i) an arrow diagram ii) a table form iii) a graph.

Sol

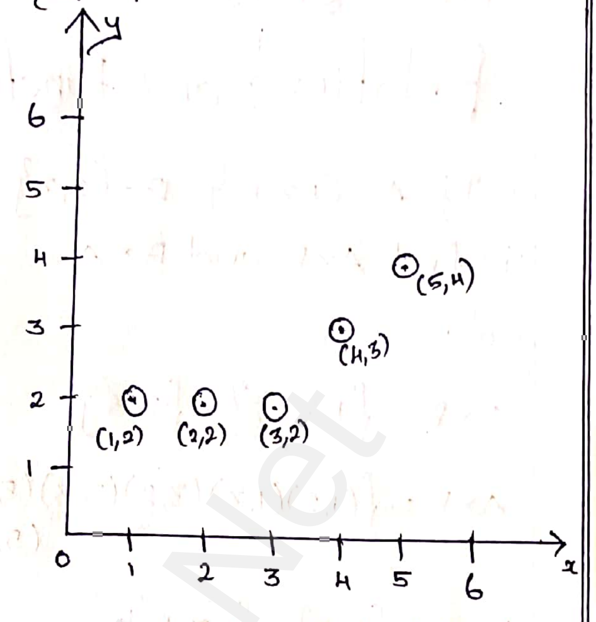
i) An arrow diagram.



ii) a table form.

x	1	2	3	4	5
f	2	2	2	3	4

iii) a graph:



8, show that the function $f:N \rightarrow N$ defined by $f(x)=2x-1$ is one-one but not onto.

Sol:

$x=\{1,2,3,4,5,\dots\}$

$x=1 \Rightarrow f(1) = 2(1)-1 = 2-1$

$= 1$

$x=2 \Rightarrow f(2) = 2(2)-1 = 4-1$

$= 3$

$x=3 \Rightarrow f(3) = 2(3)-1 = 6-1$

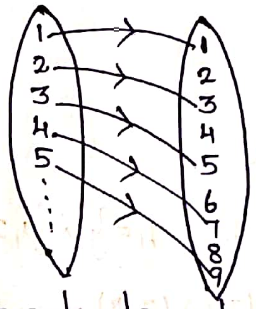
$= 5$

$x=4 \Rightarrow f(4) = 2(4)-1 = 8-1$

$= 7$

$x=5 \Rightarrow f(5) = 2(5)-1 = 10-1$

$= 9$



i) Different element has different image.

ii) Range is not equal to domain.

The given fn. is one-one but not onto

9, The distance S an object travels under the influence of gravity in time t second is given by $s(t)=\frac{1}{2}gt^2+at+b$ where (g is the acceleration due to gravity), a,b are constant. Verify wheather the function s(t) is one-one or not.

Sol:

$$s(t)=\frac{1}{2}gt^2+at+b$$

Let t =1, 2,3..... Seconds

$$s(1)=\frac{1}{2}g(1)^2+a(1)+b$$

$$s(1)=\frac{1}{2}g+a+b$$

$$s(2)=\frac{1}{2}g(2)^2+a(2)+b$$

$$=\frac{1}{2}g(4)^2+2a+b$$

$$s(2)=2g+2a+b$$

Different preimages for the different values of the range.

∴ It is one-one function.

10, Find K if $f\circ f(k)=5$ where $f(k)=2k-1$

Sol

$$f\circ f(k)=f(f(k))$$

$$=f(2k-1)$$

$$=2(2k-1)-1$$

$$=4k-2-1$$

$$f\circ f(k)=4k-3$$

Here $f\circ f(k)=5$

$$4k-3=5$$

$$4k=5+3$$

$$4k=8$$

$$\boxed{k=2}$$

11, Using the function f and g find fog and gof. Check whether fog = gof i) $f(x)=\frac{x+6}{3}$ $g(x)=3-x$

Sol

LHS

$$f\circ g=f[g(x)]$$

$$=f[3-x]$$

$$=\frac{3-x+6}{3}$$

$$\boxed{f\circ g=\frac{9-x}{3}}\text{ — ①}$$

RHS

$$g\circ f=g[f(x)]$$

$$=g\left[\frac{x+6}{3}\right]$$

$$=3-\left(\frac{x+6}{3}\right)$$

$$=3-\frac{(x+6)}{3}$$

$$=\frac{9-x-6}{3}$$

$$\boxed{g\circ f=\frac{3-x}{3}}\text{ — ②}$$

LHS ≠ RHS

$$f\circ g\neq g\circ f$$

12. In electrical circuit theory, a circuit c(t) is called a linear circuit if it satisfies the super-position principle given by $c(at_1+bt_2)=ac(t_1)+bc(t_2)$ where a,b are constant. Show that the circuit $c(t)=3t$ is linear.

Sol:

$$c(at_1)=3at_1$$

$$c(bt_2)=3bt_2$$

$c(at_1+bt_2) = 3at_1+3bt_2$

$= a(3t_1)+b(3t_2)$

$C(at_1+bt_2) = aC(t_1)+bC(t_2)$

Superposition principle is Satisfied.

$c(t)=3t$ is linear function.

Chapter - 2

Numbers and Sequence

1, Show that the square of an odd integer is of the form 4q+1 for some integer q.

Sol:-

Let x be any odd integer.

$x = 2K+1$; integer K

Square on both side

$x^2 = (2K+1)^2$

$= 4K^2+1+4K$

$= 4K(K+1)+1$

$\boxed{x^2 = 4q+1}$ $q = K(K+1)$

2, Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Sol:-

HCF of the number 445-4=441

572-5=567

567 = 441×1+126

441 = 126×3+63

126 = 63×2+0

HCF of 441,567 = 63.

3, If the highest Common factor of 210 and 55 is expressible in the form 55x-325, find x.

Sol:-

210 = 55×3+45

55 = 45×1+10

45 = 10×4+5

10 = 5×2+0

HCF of (210,55) = 5

$55x-325 = 5$

$55x = 5+325$

$55x = 330$

$x = \frac{330}{55}$

$\boxed{x = 6}$

4, Prove that the product of two consecutive positive integers is divisible by 2.

Sol:-

Let n and (n-1) are two Consecutive positive integer.

$n(n-1) = n^2 - n$

Any positive integer form 2q or 2q+1 for same integer q.

Case(i)

When $n=2q$

$n^2-n = (2q)^2-2q$

$= 4q^2-2q$

$= 2q(2q-1)$

$n^2-n = 2r$ Where $r=q(2q-1)$

Hence n^2-n divisble by 2.

Case(ii)

When $n=2q+1$

$n^2-n = (2q+1)^2-(2q+1)$

$=(2q+1)[2q+1-1]$

$=(2q+1)2q$

$n^2-n = 2r$ Where $r=q(2q+1)$

n^2-n is divisble by 2

5, If d is the Highest Common factor of 32 and 60, find x and y Satisfying $d=32x+60y$.

Sol:

$60 = 32\times1+28$

$32 = 28\times1+4$

$28 = 4\times7+0$

HCF of (32,60) is 4

$32 = 28\times1+4$

$32-28\times1 = 4$

$4 = 32-(60-32\times1)\times1$

$4 = 32-60+32$

$4 = 32\times2+(-1)*60$

$\boxed{x=2}$ $\boxed{y=-1}$

6, 'a' and 'b' are two positive integers Such that $a^b\times b^a=800$. Find 'a' and 'b'.

Sol

2 | 800
2 | 400
2 | 200
2 | 100
2 | 50
5 | 25
5

$a^b\times b^a = 2^5\times5^2$

$a=2$ $b=5$ (or) $a=5$ $b=2$.

7, Find the least number that is divisible by the first 10 natural numbers.

Sol

$1=1\times1$ $5=1\times5$, $9=3^2$

$2=1\times2$ $6=2\times3$ $10=2\times5$

$3=1\times3$ $7=1\times7$

$4=4^2$ $8=2^3$

LCM $= 2^3*3^2*5*7$

$= 8\times9\times5\times7$

$\boxed{\text{LCM} = 2520}$

⑤

8, Prove that two consecutive positive integers are always Coprime.

Sol:

Let the number be $l, l+1$

Co-prime if only +ve integer that divides both is 1.

l is given to be +ve integer.

$l = 1, 2, 3, \ldots$

One is odd and another one is even.

HCF of two Consecutive number is 1.

9, If $13824 = 2^a \times 3^b$ then find a and b?

Sol:

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$13824 = 2^9 \times 3^3$

$\boxed{a = 9 \quad b = 3}$

10, Kala and Vani are friends. Kala says, "Today is my birthday" and she ask Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 day ago". Find the day when Vani Celebrated her birthday?

Sol

$-74 \pmod{7} \equiv -4 \pmod{7}$

$\equiv 7-4 \pmod{7}$

$-74 \pmod{7} \equiv 3 \pmod{7}$

$-74 - 3 = -77$ is divisible by 7

$1 - 75 \equiv 3 \pmod{7}$

Vani's birthday must be on Wednesday.

11, Find the first five terms of the following Sequence.

$a_1 = 1 \quad a_2 = 1 \quad a_n = \frac{a_{n-1}}{a_{n-2}+3} \quad n \geq 3, n \in N$

Sol:

$a_1 = 1$

$a_2 = 1$

$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3}$

$\boxed{a_3 = \frac{1}{4}}$

$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{1/4}{1+3}$

$= \frac{1}{4} \times \frac{1}{4}$

$\boxed{a_4 = \frac{1}{16}}$

$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{1/16}{1/4+3}$

$= \frac{1/16}{\frac{1+12}{4}}$

$= \frac{1}{\cancel{16}\,4} \times \frac{\cancel{4}^{1}}{13}$

$\boxed{a_5 = \frac{1}{52}}$

12, Find the first four terms of the Sequence.

i) $a_n = (-1)^{n+1} n(n+1)$ ii) $a_n = 2n^2 - 6$

Sol:

$n = 1, 2, 3, 4$

i) $a_n = (-1)^{n+1} n(n+1)$

$a_1 = (-1)^{1+1}(1)(1+1)$

$= (1)(1)(2)$

$\boxed{a_1 = 2}$

$a_2 = (-1)^{2+1}(2)(2+1)$

$= (-1)(2)(3)$

$\boxed{a_2 = -6}$

$a_3 = (-1)^{3+1} 3(3+1)$

$= (1)(3)(4)$

$\boxed{a_3 = 12}$

$a_4 = (-1)^{4+1} 4(4+1)$

$= (-1)(4)(5)$

$\boxed{a_4 = -20}$

The first four terms are 2, -6, 12, -20

ii) $a_n = 2n^2 - 6$

$a_1 = 2(1)^2 - 6$

$= 2 - 6$

$\boxed{a_1 = -4}$

$a_2 = 2(2)^2 - 6$

$= 2(4) - 6$

$= 8 - 6$

$\boxed{a_2 = 2}$

$a_3 = 2(3)^2 - 6$

$= 2(9) - 6$

$= 18 - 6$

$\boxed{a_3 = 12}$

$a_4 = 2(4)^2 - 6$

$= 2(16) - 6$

$= 32 - 6$

$\boxed{a_4 = 26}$

The first four terms are -4, 2, 12, 26

13, Find the indicated terms of the Sequence whose n th term

i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}

ii) $a_n = -(n^2-4)$; a_4 and a_{11}

Sol:

i) $a_n = \frac{5n}{n+2}$

$a_6 = \frac{5(6)}{6+2}$

$= \frac{\cancel{30}\,15}{\cancel{8}\,4}$

$\boxed{a_6 = \frac{15}{4}}$

ii) $a_{13} = \frac{5(13)}{13+2}$

$= \frac{\cancel{65}\,13}{\cancel{15}\,3}$

$\boxed{a_{13} = \frac{13}{3}}$

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14, Find the number of the terms in the A.P 3, 6, 9, 12, ... 111.

Sol:

$a = 3 \quad d = 6-3 \quad l = 111$

$= 3$

$$n = \left(\frac{l-a}{d}\right)+1$$

$$= \left(\frac{111-3}{3}\right)+1$$

$$\boxed{n = 37}$$

15, Which term of an A.P 16, 11, 6, 1, ... is -54?

Sol:

$t_n = -54$

$a = 16 \quad d = -5$

$$t_n = a+(n-1)d$$

$$-54 = 16+(n-1)(-5)$$

$$-54 = 16-5n+5$$

$$-54 = 21-5n$$

$$-54-21 = -5n$$

$$-75 = -5n$$

$$n = \frac{\cancel{-75}^{15}}{\cancel{-5}_{1}}$$

$$\boxed{n = 15}$$

16, Find the middle terms of an A.P 9, 15, 21, 27, ... 183.

Sol

$a = 9 \quad l = 183$

$d = 15-9$

$= 6$

$$n = \frac{l-a}{d}+1$$

$$= \frac{183-9}{6}+1$$

$$= \frac{174}{6}+1$$

$$= 29+1$$

$$\boxed{n = 30}$$

Middle term = 15th term of 16th term

$$t_n = a+(n-1)d$$

$$t_{15} = 9+(15-1)6$$

$$= 9+14(6)$$

$$= 9+84$$

$$\boxed{t_{15} = 93}$$

$$t_{16} = 9+(16-1)6$$

$$= 9+(15)6$$

$$= 9+90$$

$$\boxed{t_{16} = 99}$$

17, Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Sol:

$301, 308, 315, \ldots, 595$

$a = 301 \quad d = 308 - 301$

$d = 7$

$l = 595$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \frac{595 - 301}{7} + 1$$

$$= \frac{294}{7} + 1$$

$$\boxed{n = 43}$$

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{43}{2}[301 + 595]$$

$$\boxed{S_{43} = 19264}$$

18, Find the 8th term of the G.P 9, 3, 1, . . .

Sol

$$t_n = ar^{n-1}$$

$$a = 9 \quad r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$$

$$t_8 = (9)\left(\frac{1}{3}\right)^{8-1}$$

$$= (9)\left(\frac{1}{3}\right)^{7}$$

$$\boxed{t_8 = \frac{1}{243}}$$

19, Find the least positive integer n such that $1 + 6 + 6^2 + \cdots + 6^n > 5000$.

Sol

$$S_n > 5000$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(6^n - 1)}{6 - 1}$$

$$= \frac{6^n - 1}{5}$$

$$S_n > 5000$$

$$\frac{6^n - 1}{5} > 5000$$

$$6^n - 1 > 25000$$

$$6^n > 25001$$

$$6^5 = 7776$$

$$6^6 = 46656$$

Least positive value of n is 6

20, Find the value of $1 + 2 + 3 + \cdots + 50$

Sol

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 2 + 3 + \cdots + 50 = \frac{50(50+1)}{2}$$

$$= \frac{50(51)}{2}$$

$$= 1275.$$

21, Find the sum of $1^2+2^2+\cdots+19^2$

Sol

$$1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2+2^2+\cdots+19^2 = \frac{19(19+1)(2(19)+1)}{6}$$

$$= \frac{19\times20\times39}{6}$$

$$1^2+2^2+\cdots+19^2 = 2470 //.$$

22, Rekha has 15 square colour Papers of size 10cm, 11cm, 12cm, ... 24cm. How much area can be decorated with these colour papers?

Sol:

$$\left.\begin{array}{l}\text{Area of 15 square}\\ \text{Colour papers}\end{array}\right\} = 10^2+11^2+12^2+\cdots+24^2$$

$$= (1^2+2^2+\cdots+24^2)-(1^2+2^2+\cdots+9^2)$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{24(24+1)(2(24)+1)}{6} - \frac{9(9+1)(2(9)+1)}{6}$$

$$= \frac{24(25)(49)}{6} - \frac{9(10)(19)}{6}$$

$$= 4\times25\times49 - 3\times5\times19$$

$$= 4900 - 285$$

$$= 4615 \text{ cm}^2 //.$$

chapter-3

Algebra

1, The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the Present ages (in years) of the Son and father.

Sol:

Present age of father x.

Present age of son y.

$$x = 6y \quad — ①$$

$$x+6 = 4(y+6) \quad — ②$$

① in ②

$$6y+6 = 4(y+6)$$

$$6y+6 = 4y+24$$

$$6y-4y = 24-6$$

$$2y = 18$$

$$y = 18/2$$

$$\boxed{y = 9}$$

$y=9$ sub in ① eqn.

$$① \Rightarrow \quad x = 6(9)$$

$$\boxed{x = 54}$$

Father's age = 54 years

Son's age = 9 years

2, Find the GCD of the polynomial

i) $x^4-1,\ x^3-11x^2+x-11$

ii) $3x^4+6x^3-12x^2-24x,\ 4x^4+14x^3+8x^2-8x$

Sol :-

i)

$$\begin{array}{r|l} & x+11 \\ \hline x^3-11x^2+x-11 & \cancel{x^4}+0x^3+0x^2+0x-1 \\ & \cancel{x^4}-11x^3+x^2-11x \\ & (-)\ (+)\ (-)\ (+) \\ \hline & 11x^3-x^2+11x-1 \\ & 11x^3-121x^2+11x-121 \\ & (-)\ (+)\ (-)\ (+) \\ \hline & 120x^2+0x+120 \end{array}$$

$$120x^2+120 = 120(x^2+1).$$

$$\begin{array}{r|l} & x-11 \\ \hline x^2+1 & x^3-11x^2+x-11 \\ & x^3-0x^2+x \\ & (-)\ \ (-) \\ \hline & -11x^2+0x-11 \\ & -11x^2+0-11 \\ \hline & 0 \end{array}$$

$$G.C.D = x^2+1$$

ii) $3x^4+6x^3-12x^2-24x = 3x(x^3+2x^2-4x-8)$

$4x^4+14x^3+8x^2-8x = \underset{2}{4}x(2x^3+$

$= 2x(2x^3+7x^2+4x^2-4)$

$$\begin{array}{r|l} & 2 \\ \hline x^3+2x^2-4x-8 & 2x^3+7x^2+4x-4 \\ & 2x^3+4x^2-8x-16 \\ & (-)\ (-)\ (+)\ (+) \\ \hline & 3x^2+12x+12 \end{array}$$

$$3x^2+12x+12 = 3(x^2+4x+4)$$

$$\begin{array}{r|l} & x-2 \\ \hline x^2+4x+4 & x^3+2x^2-4x-8 \\ & x^2+4x^2+4x \\ & (-)\ (-)\ (-) \\ \hline & -2x^2-8x-8 \\ & -2x^2-8x-8 \\ \hline & 0 \end{array}$$

$$GCD = x(x^2+4x+4).$$

3, Find the LCM of the polynomial.

i) $p^2-3p+2,\ p^2-4$

ii) $(2x^2-3xy)^2,\ (4x-6y)^3,\ 8x^3-27y^3.$

Sol)

i) $p^2-3p+2 = (p-1)(p-2)$

2

-1 -2

-3.

$p^2-4 = p^2-2^2$

$p^2-4 = (p+2)(p-2)$

$$LCM = (p-2)(p+2)(p-1)$$

ii) $(2x^2-3xy)^2 = x^2(2x-3y)^2$

$(4x-6y)^3 = 2^3(2x-3y)^2$

$= 8(2x-3y)^2$

$8x^3-27y^3 = (2x)^3-(3y)^3$

$=(2x-3y)\left[(2x)^2+2x\times 3y+(3y)^2\right]$

$=(2x-3y)\left[4x^2+6xy+9y^2\right].$

$$LCM = (2x-3y)^3\, 8x^2\,(4x^2+6xy+9y^2).$$

4) Reduce the rational expressions to its lowest form.

i) $\frac{x^2-11x+18}{x^2-4x+4}$ ii) $\frac{p^2-3p-40}{2p^3-24p^2+64p}$

Sol :-

i) $\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}$

18
-9 -2
-11

$\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)}{(x-2)}$ //.

4
-2 -2
-4

ii) $\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{(p+8)(p+5)}{2p(p^2-12p+32)}$

-40
-8 +5
-3

$= \frac{\cancel{(p+8)}(p+5)}{2p\cancel{(p-8)}(p-4)}$

32
-8 -4
-12

$\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{(p+5)}{2p(p-4)}$ //.

5, Find the excluded value if any expressions:-

i) $\frac{t}{t^2-5t+6}$ ii) $\frac{x^3-27}{x^3+x^2-6x}$

Sol :-

i) $\frac{t}{t^2-5t+6}$

6
-3 -2
-5

$t^2-5t+6 = 0$

$(t-3)(t-2) = 0$

$\boxed{t=3}$ $\boxed{t=2}$

ii) $\frac{x^3-27}{x^3+x^2-6x}$

$x^3-27 = x^3-3^3$

$= (x-3)(x^2+x+3)$

$x^3+x^2-6x = x(x^2+x-6)$

$= x\left((x+3)(x-2)\right)$

-6
3 -2
1

$\frac{x^3-27}{x^3+x^2-6x} = \frac{(x-3)(x^2+x+3)}{x(x+3)(x-2)}$

$x(x+3)(x-2) = 0$

$\boxed{x=0}$ $\boxed{x=-3}$ $\boxed{x=2}$

6, If a polynomial $p(x) = x^2-5x-14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$ find $q(x)$.

Sol :-

$P(x) = x^2-5x-14$

-14
-7 +2
-5

$= \cancel{(x+7)}(x$

$= (x-7)(x+2)$

$\frac{P(x)}{q(x)} = \frac{x-7}{x+2}$

$\frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2}$

$(x-7)(x+2)(x+2) = (x-7)\,q(x)$

$q(x) = \frac{\cancel{(x-7)}(x+2)(x+2)}{\cancel{(x-7)}}$

$q(x) = (x+2)^2$

$\boxed{q(x) = x^2+4x+4}$

7, Simplify $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

Sol

$$9x^2-16y^2 = (3x)^2-(4y)^2$$

$$9x^2-16y^2 = (3x+4y)(3x-4y)$$

$$2x^2+3x-20 = 2x^2+8x-5x-20$$

-40

$8 \quad -5$

3

$$= 2x(x+4)-5(x+4)$$

$$2x^2+3x-20 = (x+4)(2x-5)$$

$$\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20} = \frac{\cancel{x+4}}{\cancel{3x+4y}} \times \frac{\cancel{(3x+4y)}(3x-4y)}{\cancel{(x+4)}(2x-5)}$$

$$= \frac{3x-4y}{2x-5} \text{ //.}$$

8, Multiply $\frac{x^3}{9y^2}$ by $\frac{27y}{x^5}$

Sol

$$\frac{\cancel{x^3}}{\underset{1}{\cancel{9}}y^{\cancel{2}1}} \times \frac{\overset{3}{\cancel{27}}y}{x^{\cancel{5}2}} = \frac{3}{x^2y} \text{ //.}$$

9, Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Sol:

$$\frac{2x^3+x^2+3}{(x^2+2)^2} - \frac{1}{x^2+2} = \frac{2x^3+\cancel{x^2}+3-\cancel{x^2}-2}{(x^2+2)^2}$$

$$= \frac{2x^3+1}{(x^2+2)^2} \text{ //.}$$

10, Pari needs 4 hours to complete a work. His friend yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?

Sol:

Pari = 4hrs

yuvan = 6hrs

Work done by pari in 1hr = $\frac{1}{4}$

Work done by yuvan in 1hr = $\frac{1}{6}$

Time required to complete } = x work.

Time required to complete 1 hrs } = $\frac{1}{x}$

$$\frac{1}{4}+\frac{1}{6} = \frac{1}{x}$$

$$\frac{6+4}{24} = \frac{1}{x}$$

$$\frac{10}{24} = \frac{1}{x}$$

$$\frac{5}{12} = \frac{1}{x}$$

$$x = \frac{12}{5}$$

$$\begin{array}{r|l} & 2.4 \\ 5 & 12 \\ & 10 \\ \hline & 20 \\ & 20 \\ \hline & 0 \end{array}$$

$x = 2.4$ hrs

To complete the work together } = 2hrs 24mins

hrs	min
1	60
0.4	t

$t = 60 \times 0.4$

$$\boxed{t = 24 \text{ min}}$$

11) Iniya bought 50 kg of fruits Consisting of apples and banana. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought Rs.1800 Worth of apples and Rs.600 Worth bananas, then how many kgs of each fruit did she buy?

Sol:

Let

Weight of applies = a kg

Weight of bananas = b kg

$a + b = 50\,kg$ —— Ⓦ

$ax = 1800$ —— ①

$by = 600$ —— ②

$x = 2y$ —— ③

③ in ① =>

① => $a(2y) = 1800$

$$y = \frac{1800}{2a}$$

$$\boxed{y = \frac{900}{a}}$$

$y = \frac{900}{a}$ sub in ② eqn.

② => $b\left(\frac{900}{a}\right) = 600$

$$b = \frac{\overset{2}{\cancel{600}} \times a}{\underset{3}{\cancel{900}}}$$

$$\boxed{b = \frac{2a}{3}}$$

$3b = 2a$

$b = \frac{2a}{3}$ sub in Ⓦ eqn.

$$a + \frac{2a}{3} = 50$$

$$\frac{3a+2a}{3} = 50$$

$$5a = 150$$

$$a = \frac{150}{5}$$

$$\boxed{a = 30}$$

$a = 30$ sub in Ⓦ eqn

$30 + b = 50$

$b = 50 - 30$

$$\boxed{b = 20}$$

Apples = 30 kg

bananas = 20 kg

12) Find the square root $16x^2+9y^2-24xy+24x-18y+9$

Sol:

$$= \sqrt{16x^2+9y^2-24xy+24x-18y+9}$$

$$= \sqrt{(4x)^2+(-3y)^2+(3)^2 - 2(\overset{x}{4})(-3y) + 2(-3y)(3) + 2(4x)3}$$

$$= \sqrt{(4x-3y+3)^{\cancel{2}}}$$

$$= |4x-3y+3|$$

13, Find the Sum and product of the roots of the quadratic equation.

i) $x^2+3x-28=0$ ii) $3+\frac{1}{a}=\frac{10}{a^2}$

iii) $3y^2-y-4=0$.

Sol:

i) $x^2+3x-28=0$

$a=1 \quad b=3 \quad c=-28$

Sum of the root $(\alpha+\beta) = -b/a$

$= -3/1$

$\boxed{\alpha+\beta=-3}$

Product of the root $\alpha\beta = c/a$

$= -28/1$

$\boxed{\alpha\beta=-28}$

ii) $3+\frac{1}{a}=\frac{10}{a^2}$

Multiply by a^2 on both side.

$3a^2+\frac{a^{\cancel{2}}}{\cancel{a}}=\frac{10\cancel{a^2}}{\cancel{a^2}}$

$3a^2+a=10$

$3a^2+a-10=0$

$a=3 \quad b=1 \quad c=-10$

Sum of the root $\alpha+\beta = -b/a$

$\boxed{\alpha+\beta=\frac{-1}{3}}$

Product of the root $\alpha\beta = c/a$

$\boxed{\alpha\beta=\frac{-10}{3}}$

14, The number of Volleyball games that must be Scheduled in a league with n teams is given by $G(n)=\frac{n^2-n}{2}$ where each team plays with every other team exactly once. A league Schedule 15 games. How many teams are in the league?

Sol:

$G(n)=15$

$G(n)=\frac{n^2-n}{2}$

$15=\frac{n^2-n}{2}$

$30=n^2-n$

$n^2-n-30=0$

-30

$-6 \quad 5$

-1

$(n-6)(n+5)=0$

$\boxed{n=6} \quad \boxed{n=-5}$

There are 6 teams in the league.

15, A ball rolls down a slope and travels a distance $d=t^2-0.75t$ feet in t-seconds. Find the time when the distance travelled by the ball is 11.25 feet.

Sol:

$d=11.25$

$d=t^2-0.75t$

$11.25=t^2-0.75t$

Multiply by 100

$1125=100t^2-75t$

$100t^2-75t-1125=0$

÷25

$4t^2-3t-45=0$

$a=4 \quad b=-3 \quad c=-45$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{3\pm\sqrt{9-4(4)(-45)}}{2(4)}$$

$$= \frac{3\pm\sqrt{9+720}}{8}$$

$$= \frac{3\pm\sqrt{729}}{8}$$

$$= \frac{3\pm27}{8}$$

$$= \frac{3+27}{8}, \frac{3-27}{8}$$

$$= \frac{30}{8}, \frac{-24}{8}$$

$= \frac{15}{4}, -3$ (-ve Value is rejected)

Time Required = $\frac{15}{4}$ seconds.

= $3\frac{3}{4}$ seconds (or)

= 3.75 Seconds.

16) The product of Kumaran's age (in years) two year ago and his age four years from now is one more than twice his Present age. What is his Present age?

Sol:.

Present age of Kumaran } = x years

Two years ago = (x-2) years

Four years from now } = (x+4) years

$(x-2)(x+4) = 1+2x$

$x^2+2x-8 = 1+2x$

$x^2+\cancel{2x}-8-1-\cancel{2x}=0$

$x^2-9=0$

$x^2=9$

$\boxed{x=\pm3}$

(age cannot be negative)

So,

Kumaran present age } = 3 years

17) If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

Sol

Let the number be 'x' and its reciprocal is $\frac{1}{x}$

$$x-\frac{1}{x} = \frac{24}{5}$$

$$\frac{x^2-1}{x} = \frac{24}{5}$$

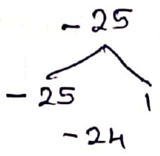
$5(x^2-1) = 24x$

$5x^2-5 = 24x$

$5x^2-5-24x=0$

$5x^2-24x-5=0$

$5x^2-25x+1x-5=0$



$5x(x-5)+1(x-5)=0$

$(x-5)(5x+1)=0$.

$x-5=0$ $\quad$ $5x+1=0$

$\boxed{x=5}$ $\quad$ $5x=-1$.

$\boxed{x=-1/5}$

18, The hypotenuse of a right angled triangle is 25cm and its perimeter 56cm. Find the length of the smallest side.

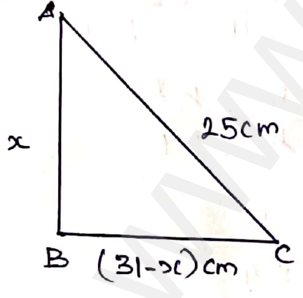
Sol

Perimeter of right angle triangle } = 56cm

Sum of the two sides } + hypotenuse = 56

Sum of to sides = 56 − 25
= 31cm.

One side of the triangle 'x'



By pythagoras thm:-

$AB^2+BC^2=AC^2$

$x^2+(31-x)^2=25^2$

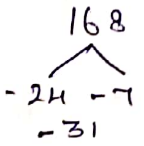
$x^2+961+x^2-62x=625$

$2x^2-62x+961-625=0$

$2x^2-62x+336=0$

$\div 2$

$x^2-31x+168=0$.



$(x-24)(x-7)=0$

$x-24=0$ $\quad$ $x-7=0$

$\boxed{x=24}$ $\quad$ $\boxed{x=7}$

Length of Smallest side } = 7cm.

19, Find the value of 'k' ~~foor~~ which the roots of the equation are real and equal. i) $kx^2+(6k+2)x+16=0$.

Sol:-

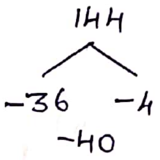
$a=k$ $\quad$ $b=6k+2$ $\quad$ $c=16$

$b^2-4ac=0$

$(6k+2)^2-4(k)(16)=0$

$36k^2+4+24k-64k=0$

$36k^2-40k+4=0$



$36k^2-36k-4k+4=0$

$36k(k-1)-4(k-1)=0$

$(k-1)(36k-4)=0$

$k-1=0$ $\quad$ $36k=4$

$\boxed{k=1}$ $\quad$ $k=\frac{4}{36}$

$k=\frac{\cancel{2}^{1}}{\cancel{18}_{9}}$

$\boxed{k=\frac{1}{9}}$

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20, The root of the equation $2x^2-7x+5=0$ are α and β. Without Solving for the roots.

i) $\frac{1}{\alpha}+\frac{1}{\beta}$ ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ iii) $\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}$

Sol

$$2x^2-7x+5=0$$

$a=2 \quad b=-7 \quad c=5$

$\alpha+\beta=-b/a \qquad \alpha\beta=c/a$

$\boxed{\alpha+\beta=7/2} \qquad \boxed{\alpha\beta=5/2}$

i) $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$$

$$=\frac{7/2}{5/2}$$

$$=\frac{7}{\cancel{2}}\times\frac{\cancel{2}}{5}$$

$$\boxed{\frac{1}{\alpha}+\frac{1}{\beta}=7/5}$$

ii) $$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{(7/2)^2-2(5/2)}{5/2}$$

$$=\frac{\frac{49}{4}-\frac{10}{2}}{5/2}$$

$$=\frac{\frac{49-20}{4}}{5/2}$$

$$=\frac{29}{\cancel{4}2}\times\frac{\cancel{2}1}{5}$$

$$\boxed{\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{29}{10}}$$

iii) $$\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}=\frac{(\alpha+2)^2+(\beta+2)^2}{(\alpha+2)(\beta+2)}$$

$$=\frac{\alpha^2+4+4\alpha+\beta^2+4+4\beta}{\alpha\beta+2\alpha+2\beta+4}$$

$$=\frac{\alpha^2+\beta^2+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$=\frac{(7/2)^2-2(5/2)+4(7/2)+8}{5/2+2(7/2)+4}$$

$$=\frac{\frac{49}{4}-\frac{10}{2}+\frac{28}{2}+8}{5/2+\frac{14}{2}+4}$$

$$=\frac{\frac{49-20+56+32}{4}}{\frac{5+14+8}{2}}$$

$$=\frac{137-20}{\cancel{4}2}\times\frac{\cancel{2}1}{27}$$

$$=\frac{117}{27\times2}$$

$$=\frac{\cancel{117}13}{\cancel{54}9}$$

$$\boxed{\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}=\frac{13}{9}}$$

21, Construct a 3×3 matrix Whose element are $a_{ij} = i^2 j^2$.

Sol

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{11} = 1^2 \times 1^2 = 1 \times 1 = 1$

$a_{12} = 1^2 \times 2^2$
$= 1 \times 4 = 4$

$a_{13} = 1^2 \times 3^2$
$= 1 \times 9 = 9$

$a_{21} = 2^2 \times 1^2 = 4 \times 1 = 4$

$a_{22} = 2^2 \times 2^2$
$= 4 \times 4 = 16$

$a_{23} = 2^2 \times 3^2$
$= 4 \times 9 = 36$

$a_{31} = 3^2 \times 1^2 = 9 \times 1 = 9$

$a_{32} = 3^2 \times 2^2$
$= 9 \times 4 = 36$

$a_{33} = 3^2 \times 3^2$
$= 9 \times 9 = 81$

$$A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{bmatrix}.$$

22, If a matrix has 18 elements, What are the possible orders it can have? What if it has 6 elements?

Sol :-

Possible order 18 element		Possible order 6 element	
1×18	18×1	1×6	6×1
2×9	9×2	2×3	3×2
3×6	6×3		

23, Find the values of x, y and z from the equation.

i) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Sol :-

$x + y = 6$ — ①

$5 + z = 5$ — ②

$xy = 8$

$\boxed{x = 8/y}$

$x + y = 6$

$\frac{8}{y} + y = 6$

$\frac{8 + y^2}{y} = 6$

$8 + y^2 = 6y$

$y^2 - 6y + 8 = 0$

$(y-4)(y-2) = 0$

$\boxed{y = 4}$ $\boxed{y = 2}$

$5 + z = 5$

$z = 5 - 5$

$\boxed{z = 0}$

$y = 4$ $y = 2$

$x = 8/4$ $x = 8/2$

$\boxed{x = 2}$ $\boxed{x = 4}$

23, Solve $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Sol :

$$\begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$2x + y = 4$ — ①

$x + 2y = 5$ — ②

① × 2 ⇒ $4x + \cancel{2y} = 8$

② ⇒ $x + \cancel{2y} = 5$

$(-)$ $(-)$ $(-)$

$3x = 3$

$\boxed{x = 1}$

$x = 1$ Sub in ① eqn

① ⇒ $2(1) + y = 4$

$2 + y = 4$

$y = 4 - 2$

$\boxed{y = 2}$

$x = 1$ $y = 2$

24, If A is of order p×q and B is of order q×r what is the order of AB and BA?

Sol:

$$A = p\times q \quad B = q\times r$$

$$AB = \overset{A}{p\times \boxed{q \quad \overset{B}{q}}\times r}$$

Order of AB = p×q

$$BA = \overset{B}{q\times r} \quad \overset{A}{p\times q}$$

BA is not defined.

25, Verify that $A^2 = I$ when

$$A = \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$$

Sol:

LHS $\quad A^2 = A\times A$

$$= \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}\begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 25-24 & -20+20 \\ 30-30 & -24+25 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^2 = I$

LHS = RHS

Hence proof

Chapter-4

Geometry

1, State Basic proportionality theorem.

Ans

A straight line drawn Parallel to a side of triangle intersecting the other two sides, divides the side in the Same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

2, State Angle bisecotor theorem.

Ans:

The internal bisector of an angle of a triangle divide the opposite side internally in the ratio of the Corresponding sides Containing the angle.

$$\frac{AB}{AC} = \frac{BD}{CD}$$

3, State pythagoras theorem

Ans

In a right angle triangle, the square on the hypotenuse is equal to the sum of the Squares on the other two Sides.

$$AB^2 + AC^2 = BC^2$$

4, : Define alternate Segment theorem.

Ans If a line touches a Circle and from the point of Contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the Corresponding alternate Segments.

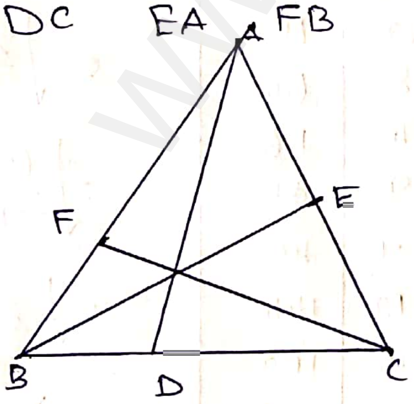
i) $\angle QPB = \angle PSQ$

ii) $\angle QPA = \angle PTQ$

5, Define Ceva's theorem.

Let ABC be a triangle and let D, E, F be points on line BC, CA, AB respectively. Then the Cevians AD, BE, CF are concurrent if and only if

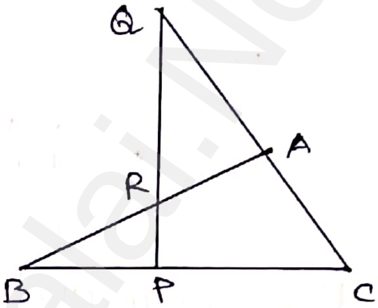
$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1.$$



6, Define Menelaus Theorem

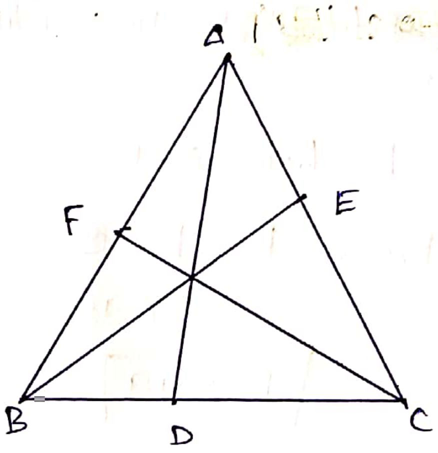
Ans.. A necessary and Sufficient Condition for points P, Q, R on the respective Side BC, CA, AB of a triangle ABC to be Collinear is that

$$\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1.$$



7, Show that in a triangle, the median are concurrent.

Sol



Since D is a midpoint of BC

$BD = DC$ So $\frac{BD}{DC} = 1$ — ①

Since E is a midpoint of CA

$CE = EA$ So $\frac{CE}{EA} = 1$ — ②

Since F is a midpoint of AB

$AF = FB$ So $\frac{AF}{FB} = 1$ — ③

$① \times ② \times ③$

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1\times x 1\times 1$$

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

Ceva's theorem satisfied

Hence, Median are Concurrent

Chapter - 8

statistics and probability

1, Find the range and Coefficient of range of the following data.

25, 67, 48, 53, 18, 39, 44

Sol:

$L = 67 \quad S = 18$

Range $R = L - S$

$= 67 - 18$

$\boxed{R = 49}$

$$\text{Coefficient of Range} = \frac{L-S}{L+S}$$

$$= \frac{67-18}{67+18}$$

$$= 49/85$$

$\boxed{\text{Coefficient of Range} = 0.576}$

2, The range of a set of data is 13.67 and the largest value is 70.08. Find the Smallest Value.

Sol:

Range $R = 13.67$

Largest Value $L = 70.08$

$S = ?$

$R = L - S$

$S = L - R$

$= 70.08 - 13.67$

$\boxed{S = 56.41}$

3, The number of televisions Sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its Standard deviation.

Sol:

x_i	x^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\Sigma x = 63$	$\Sigma x^2 = 623$

$n = 7$

standard deviation

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{81 - 81}$$

$$= \sqrt{8}$$

$\boxed{\sigma \simeq 2.83}$

4, The mark Scored by 10 student in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Sol:

A = 35 n = 10

x	d = x − A = x − 35	d^2
25	−10	100
29	−6	36
30	−5	25
33	−2	04
35	0	0
37	2	04
38	3	09
40	5	25
44	9	81
48	13	169
	Σd = 9	$\Sigma d^2 = 453$

standard deviation

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$$

$$= \sqrt{45.3 - 0.81}$$

$$= \sqrt{44.49}$$

$$\boxed{\sigma \simeq 6.67}$$

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5, Marks of the student in a particular subject of a class are given below. Find its standard deviation.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No of Student	8	12	17	14	9	7	4

Sol: A = 35 c = 10

Marks	Mid Value	f	d = x − A	$d = \frac{x-A}{c}$	d	fd	fd^2
0-10	5	8	−30	−3	9	−24	72
10-20	15	12	−20	−2	4	−24	48
20-30	25	17	−10	−1	1	−17	17
30-40	35	14	0	0	0	0	0
40-50	45	9	10	1	1	9	9
50-60	55	7	20	2	4	14	28
60-70	65	4	30	3	9	12	36
		N = 71				Σfd = −30	$\Sigma fd^2 = 210$

standard deviation $\sigma = c \times \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$$= 10 \times \sqrt{\frac{210}{71} - \left(\frac{-30}{71}\right)^2}$$

$$= 10 \times \sqrt{\frac{210}{71} - \frac{900}{504}}$$

$$= 10 \times \sqrt{2.779}$$

$$\simeq 16.67$$

6, The mean of a data is 25.6 and its Coefficient of Variation is 18.75. Find the standard deviation.

Sol:

$\bar{x} = 25.6$ $C.V = 18.75$

$C.V = \frac{\sigma}{x} \times 100\%$

$18.75 = \frac{\sigma}{25.6} \times 100$

$\frac{18.75 \times 25.6}{100} = \sigma$

$\boxed{\sigma = 4.8}$

7, If $n=5$ $\bar{x}=6$ $\Sigma x^2 = 765$ then Calculate the Coefficient of Variation.

Sol:

S.D $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{765}{5} - (\bar{x})^2}$

$= \sqrt{\frac{765}{5} - 6^2}$

$= \sqrt{153 - 36}$

$= \sqrt{117}$

$\boxed{\sigma = 10.816}$

$C.V = \frac{\sigma}{x} \times 100$

$= \frac{10.816}{6} \times 100$

$= 108.266\%$

$\boxed{C.V = 108.27\%}$

8, A bag contain 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Sol:

$n(s) = 5+4$

$\boxed{n(s) = 9}$

i) Let A be the event getting blue ball.

$n(A) = 5$

$P(A) = \frac{n(A)}{n(s)}$

$\boxed{P(A) = \frac{5}{9}}$

ii) Let ~~B~~ $\bar{A}$ be the even getting not blue ball.

$P(\bar{A}) = 1 - P(A)$

$= 1 - \frac{5}{9}$

$= \frac{9-5}{9}$

$\boxed{P(\bar{A}) = 4/9}$

9, Find the standard deviation of first 21 natural numbers.

Sol:

$n = 21$

$S.D = \sqrt{\frac{n^2-1}{12}}$

$= \sqrt{\frac{21^2-1}{12}}$

$= \sqrt{\frac{441-1}{12}}$

$= \sqrt{\frac{440}{12}}$

$= \sqrt{36.666}$

$\boxed{S.D = 6.06}$

10, Two Coins are tossed together. What is the Propability of getting different face on the coins?

Sol:

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

Let A be the event getting different face on the Coins.

$A = \{HT, TH\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{2}{4}$

$\boxed{P(A) = \frac{1}{2}}$

11, What is the probability that a leap year stelected at random will Contain 53 Saturday.

Sol:

A leap year = 366 days.

It has 52 Weeks and 2 days

So, 52 Weeks have 52 Saturdays.

S = {Sun-mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun}

$n(S) = 7$

Let A be the event getting 53^{rd} Saturday.

A = {Fri-sat, Sat-Sun}

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)}$

$\boxed{P(A) = \frac{2}{7}}$

12, A die is rolled and a Coin tossed Simultaneously. Find the probability that the die shows an odd numbers and the coin shows a head,

Sol:

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$n(S) = 12.$

Let A be a getting odd number and a head.

$A = \{1H, 3H, 5H\}$ $n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12}$ $\boxed{P(A) = \frac{1}{4}}$