STD: XI.

HALF YEARLY EXAMINATION
BUSINESS MATHS
ANSWER KEY

PART-A

1. (d) 2
2. (d) $\frac{16}{5}$
3. (b) $10C_5$
4. (c) $9 \times 9!$
5. (c) 2
6. (b) latus rectum
7. (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
8. (c) 3
9. (b) 1
10. (c) (0, 1)
11. (c) 1
12. (d) -1
13. (c) 13
14. (b) Decreasing function
15. (b) 8.75%
16. (a) Arithmetic Mean
17. (a) 1
18. (b) Positive
19. (d) $r = \pm\sqrt{\frac{1}{b_{xy} \times b_{yx}}}$
20. (c) 0

PART-B

21. Soln,

$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$

To find |AB|:

$AB = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 0+2 \\ 6+1 & 0-2 \end{bmatrix}$

$AB = \begin{bmatrix} 8 & 2 \\ 7 & -2 \end{bmatrix}$

$|AB| = \begin{vmatrix} 8 & -2 \\ 7 & -2 \end{vmatrix}$

$= (-16) - (-14)$

$= -16 + 14$

$= -2$

$\boxed{|AB| = -2}$

22. Soln,

Given: $(98)^3$

$= (100-2)^3$

$= (100)^3 - 3C_1(100)^2(2) + 3C_2(100)^1(2)^2 - (2)^3$

$= 1000000 - 3(10000)(2) + 6(100)(4) - 8$

$= 1000000 - 60000 + 2400 - 8$

$\boxed{(98)^3 = 942392}$

23. Soln:

Given: point (2, 3)

Circle:

$x^2+y^2+8x+4y+8=0$

$\therefore$ Length of the tangent is

$=\sqrt{x_1^2+y_1^2+8x_1+4y_1+8}$

At $(\underset{x_1}{2}, \underset{y_1}{3})$

$=\sqrt{(2)^2+(3)^2+8(2)+4(3)+8}$

$=\sqrt{4+9+16+12+8}$

$=\sqrt{49} = 7$

$\boxed{\text{Length of the tangent} = 7 \text{ units}}$

24. Soln:

Given: $\tan 75^\circ$

$\Rightarrow \tan(45^\circ+30^\circ)$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$

$\tan(45+30) = \dfrac{\tan 45^\circ + \tan 30^\circ}{1-\tan 45^\circ \times \tan 30^\circ}$

$= \dfrac{1+\frac{1}{\sqrt{3}}}{1-(1)\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{3+\sqrt{3}+\sqrt{3}+1}{3-1}$

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$

$\boxed{\tan 75^\circ = 2+\sqrt{3}}$

25. Soln:

Given: $x = 2p^2-5p+1$; $p>3$

$\dfrac{dx}{dp} = 4p-5$

Elasticity of supply :

$\eta_s = \dfrac{p}{x} \cdot \dfrac{dx}{dp}$

$= \dfrac{p}{2p^2-5p+1} \times (4p-5)$

$= \dfrac{4p^2-5p}{2p^2-5p+1}$

$\boxed{\eta_s = \dfrac{4p^2-5p}{2p^2-5p+1}}$

26. Soln:

$a = ₹50$; $i = 5\% = 0.05$

Amount of perpetual annuity:

$p = \dfrac{a}{i}$

$= \dfrac{50}{0.05}$

$= 1000$

$\boxed{P = ₹1000}$

27. Soln,

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$.

(i) Prime number:-

Let A be the event of getting a Prime number

$A = \{1, 3, 5\}$; $n(A) = 3$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$\boxed{P(A) = \frac{1}{2}}$

(ii) a number greater than or equal to 3.

Let B be the event of getting a number greater than or equal to 3.

$B = \{3, 4, 5, 6\}$; $n(B) = 4$

$P(B) = \frac{4}{6} = \frac{2}{3}$

$\boxed{P(B) = \frac{2}{3}}$

28. Soln,

Given: $\Sigma xy = 120$;

$\Sigma x^2 = 90$

$\Sigma y^2 = 640$.

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$

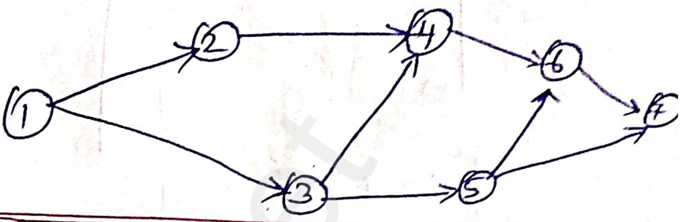
$= \frac{120}{\sqrt{90 \times 640}}$

$= \frac{120}{\sqrt{57600}}$

$= \frac{120}{240} = 0.5$

$\boxed{r = 0.5}$

29. Soln,



30. Soln,

Given xe^x.

$u = x$; $v = e^x$

$u' = 1$; $v' = e^x$.

Diff w.r.t "x"

$d(xe^x) = xe^x + e^x(1)$

$= e^x(x+1)$

$\boxed{\frac{d(xe^x)}{dx} = e^x(x+1)}$

PART-C.

31. Soln,

Given: $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} -\frac{4}{35} & \frac{11}{35} & -\frac{5}{35} \\ -\frac{1}{35} & -\frac{6}{35} & \frac{25}{35} \\ \frac{6}{35} & \frac{1}{35} & -\frac{10}{35} \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} \frac{1}{35}\begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & -10 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 35 & 0 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$BA = \frac{1}{35}\begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & -10 \end{bmatrix}\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 35 & 0 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$AB = BA = I$

32. Soln,

Given: $nP_r = 1680$

$nC_r = 70$

To find ; $n = ?$; $r = ?$

W.K.T ; $nC_r = \frac{nP_r}{r!}$

$70 = \frac{1680}{r!}$

$r! = \frac{1680}{70}$

$r! = 24$

$r! = 4 \times 3 \times 2 \times 1$

$r! = 4!$

$\boxed{r=4}$; $\boxed{r=4}$

33. Soln,

Given:

$3x^2 - 5xy - 2y^2 + 17x + y + 10 = 0$

Here,

$a = 3$; $h = \frac{-5}{2}$; $b = -2$.

$$\theta = \tan^{-1}\left[\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|\right]$$

$$= \tan^{-1}\left[\left|\frac{2\sqrt{\left(\frac{-5}{2}\right)^2 - (3)(-2)}}{3-2}\right|\right]$$

$$= \tan^{-1}\left[\left|\frac{\cancel{2}\left(\frac{7}{\cancel{2}}\right)}{1}\right|\right]$$

$$= \tan^{-1}(7)$$

$$\boxed{\theta = \tan^{-1}(7)}$$

34. Soln,

To prove:

$\sin 600 \cos 390' + \cos 480' \sin 150° = -1$

LHS ⇒

$\sin 600' \cos 390' + \cos 480' \sin 150'$

$\sin 600' = \sin(360+240)$

$= \sin 240'$

$= \sin(180+60)$

$= -\sin 60'$

$= -\frac{\sqrt{3}}{2}$

$\cos 390' = \cos(360+30')$

$= \cos 30' = \frac{\sqrt{3}}{2}$

$\cos 480' = \cos(360+120)$

$= \cos 120'$

$= \cos(180-60)$

$= -\cos 60'$

$= -\frac{1}{2}$

$\sin 150' = \sin(180-30)$

$= \sin 30' = \frac{1}{2}$

$\Rightarrow \sin 600' \cos 390' + \cos 480' \sin 150'$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-3}{4} - \frac{1}{4} = \frac{-4}{4}$

$= -1$

Hence proved

35. Soln,

Given: $\lim\limits_{x\to a} \frac{x^{3/5} - a^{3/5}}{x^{1/5} - a^{1/5}}$

$= \lim\limits_{x\to a} \frac{\frac{x^{3/5} - a^{3/5}}{x-a}}{\frac{x^{1/5} - a^{1/5}}{x-a}}$

$= \frac{\frac{3}{\cancel{5}}(a)^{-2/5}}{\frac{1}{\cancel{5}}(a)^{-4/5}}$

$= 3(a)^{-\frac{2}{5}+\frac{4}{5}}$

$= 3(a)^{2/5}$

$$\boxed{\lim_{x\to a} \frac{x^{3/5} - a^{3/5}}{x^{1/5} - a^{1/5}} = 3a^{2/5}}$$

36. Soln,

Given: $f(x) = x^2+2x-5 \longrightarrow (1)$

Diff $f(x)$ w.r.t "x"

$f'(x) = 2x+2$

At stationary points,

$f'(x) = 0$

$2x+2 = 0$

$\boxed{x = -1}$

When $x = -1$ from (1)

$f(-1) = (-1)^2 + 2(-1) - 5$

$= 1 - 2 - 5 = -6$

Stationary value of $f(x)$ is -6

Hence,

Stationary point is $(-1, -6)$

37. Soln;

Given;

$a = ₹3200$; $n = 12$;

$i = 10\% = 0.1$

$A = \frac{a}{i}\left[(1+i)^n - 1\right]$

$= \frac{3200}{0.1}\left[(1+0.1)^{12} - 1\right]$

$= 32000\left[(1.1)^{12} - 1\right]$

$= 32000\left[3.1384 - 1\right]$

$\boxed{\because (1.1)^{12} = 3.1384}$

$= 32{,}000\left[2.1384\right]$

$= 68426.8$

$\boxed{\text{Amount of an Ordinary Annuity} = ₹68{,}428.80}$

38. Soln;

R_x	R_y	$d = R_x - R_y$	d^2
6	4	2	4
4	1	3	9
3	6	−3	9
1	7	−6	36
2	5	−3	9
7	8	−1	1
9	10	−1	1
8	9	−1	1
10	3	7	49
5	2	3	9
			$\Sigma d^2 = 128$

Here; $N = 10$; $\Sigma d^2 = 128$

Rank Correlation;

$\rho = 1 - \frac{6\Sigma d^2}{N(N^2-1)}$

$= 1 - \frac{6 \times 128}{10(10^2-1)}$

$= 1 - \frac{768}{10(100-1)}$

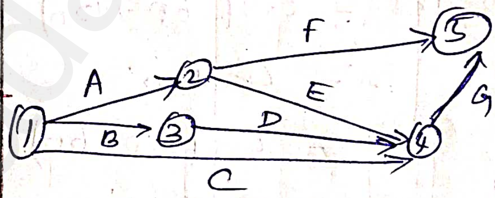
$= 1 - \frac{768}{990}$

$= 1 - 0.7758$

$= 0.2242$

$\boxed{\rho = 0.2242}$

39. Soln;



40. Soln;

Given:

12, 15, 20, 28, 30, 40, 50

To find Q_2, D_5, P_{50}.

Here; $n = 7$

$Q_2 = \text{Size of } 2\left(\frac{n+1}{4}\right)^{th} \text{ value.}$

$= \text{Size of } \left(\frac{n+1}{2}\right)^{th} \text{ value.}$

$= \text{Size of } \left(\frac{7+1}{2}\right)^{th} \text{ value}$

$= \text{Size of } 4^{th} \text{ value}$

$\boxed{Q_1 = 28}$

$D_5 = \text{Size of } 5\left(\frac{n+1}{10}\right)^{th} \text{ value.}$

$= \text{Size of } \left(\frac{7+1}{2}\right)^{th} \text{ value}$

$= \text{Size of } 4^{th} \text{ value.}$

$\boxed{D_5 = 28}$

$P_{50} = \text{Size of } 50\left(\frac{n+1}{100}\right)^{th} \text{ value}$

$= \text{Size of } \left(\frac{7+1}{2}\right)^{th} \text{ value.}$

$= \text{Size of } 4^{th} \text{ Value.}$

$\boxed{P_{50} = 28}$

PART-D

41.(a) Soln,

let x, y and z be the Cost of Onion, wheat and rice per kg respectively.

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$$

$A \quad X = B$

$X = A^{-1}B$

$A^{-1} = \frac{1}{|A|} \text{adj} A$

$$|A| = \begin{array}{ccc} (+) & (-) & (+) \\ \end{array}$$
$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix}$$

$= 4[12-12] - 3[6-36] + 2[4-24]$

$= 0 + 90 - 40 = 50 \neq 0$

$\therefore A^{-1}$ exists.

$$(\text{adj} A) = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}^T$$

$$(\text{adj} A) = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 32 \\ 56 \\ 38 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$$

$\therefore x = 20; \; y = 40; \; z = 60$

Cost of 1Kg onion = ₹20

Cost of 1Kg wheat = ₹40

Cost of 1Kg rice = ₹60

(b) Soln.

LHS ⇒ $\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}+\frac{\sin(A-B)}{\cos A\cos B}$

$=\frac{\cos A[\sin(B-C)]+\cos B[\sin(C-A)]+\cos C[\sin(A-B)]}{\cos A\cos B\cos C}$.

$\cos A[\sin(B-C)]$

$=\cos A[\sin B\cos C-\cos B\sin C]$

$=\cos A\sin B\cos C-\cos A\cos B\sin C \longrightarrow (1)$

$\cos B[\sin(C-A)]$

$\cos B[\sin C\cos A-\cos C\sin A]$

$\cos A\cos B\sin C-\sin A\cos B\cos C \longrightarrow (2)$

$\cos C[\sin(A-B)]$

$\cos C[\sin A\cos B-\cos A\sin B]$

$\sin A\cos B\cos C-\cos A\sin B\cos C \longrightarrow (3)$

Adding (1), (2), (3)

$=\frac{\cancel{\cos A\sin B\cos C}-\cancel{\cos A\cos B\sin C}+\cancel{\cos A\cos B\sin C}-\cancel{\sin A\cos B\cos C}+\cancel{\sin A\cos B\cos C}-\cancel{\cos A\sin B\cos C}}{\cos A\cos B\cos C}$

$=0$ = RHS. LHS = RHS.

Hence Proved

42. (a) Soln.

Here, $a_{11}=30$; $a_{12}=40$

$a_{21}=20$; $a_{22}=10$

$x_1=120$; $x_2=60$.

To find

$b_{11}=\frac{a_{11}}{x_1}=\frac{30}{120}=\frac{1}{4}$

$b_{12}=\frac{a_{12}}{x_2}=\frac{40}{60}=\frac{2}{3}$

$b_{21}=\frac{a_{21}}{x_1}=\frac{20}{120}=\frac{1}{6}$

$b_{22}=\frac{a_{22}}{x_2}=\frac{10}{60}=\frac{1}{6}$

Technology Matrix,

$$B=\begin{bmatrix}\frac{1}{4} & \frac{2}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}$$

$$I-B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}1/4 & 2/3\\ 1/6 & 1/6\end{bmatrix}$$

$$=\begin{bmatrix}3/4 & -2/3\\ -1/6 & 5/6\end{bmatrix}$$

$$|I-B|=\begin{vmatrix}\frac{3}{4} & -2/3\\ -\frac{1}{6} & 5/6\end{vmatrix}$$

$=\frac{15}{24}-\frac{2}{18}$

$=\frac{5}{8}-\frac{1}{9}=\frac{45-8}{72}$

$|I-B|=\frac{37}{72}>0$

∴ The system is Viable.

$$adj(I-B) = \begin{bmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{bmatrix}$$

$$(I-B)^{-1} = \frac{1}{|I-B|} adj(I-B)$$

$$= \frac{1}{37/72}\begin{bmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{bmatrix}$$

$$(I-B)^{-1} = \frac{72}{37}\begin{bmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{bmatrix}$$

$$X = (I-B)^{-1}D$$

$$= \frac{72}{37}\begin{bmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$$= \frac{1}{37}\begin{bmatrix} 60 & 48 \\ 12 & 54 \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$$= \frac{1}{37}\begin{bmatrix} 6720 \\ 3120 \end{bmatrix}$$

$$= \begin{bmatrix} 181.62 \\ 84.32 \end{bmatrix}$$

The output of Industry X should be 181.62 and Y should be 84.32

(b) Soln,

Given: $x^3+y^3 = 3axy$

To find: $\frac{dy}{dx}$.

Diff w.r.t 'x'

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\left[x\frac{dy}{dx}+y\right]$$

$\div$ by 3,

$$x^2 + y^2\frac{dy}{dx} = a\left[x\frac{dy}{dx}+y\right]$$

$$x^2+y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$$

$$y^2\frac{dy}{dx} - ax\frac{dy}{dx} = ay - x^2$$

$$\frac{dy}{dx}\left[y^2-ax\right] = ay-x^2$$

$$\frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}.$$

43. (a) Soln,

Given:

$$\frac{x+2}{(x-1)(x+3)^2}$$

$$\frac{x+2}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$x+2 = A(x+3)^2 + B(x-1)(x+3) + C(x-1) \rightarrow (1)$$

put $x=1$ in (1)

$$1+2 = A(1+3)^2$$

$$3 = 16A \Rightarrow \boxed{A = \frac{3}{16}}$$

Put $x=-3$

$-3+2=C(-3-1)$

$-1=C(-4)$

$C=\frac{1}{4} \rightarrow \boxed{C=\frac{1}{4}}$

Put $x=0$.

$0+2=A(0+3)^2+B(0-1)(0+3)+C(0-1)$

$2=A(3)^2+B(-1)(3)+C(-1)$

$2=9A-3B-C$.

$2=9\left(\frac{3}{16}\right)-3B-\frac{1}{4}$

$2=\frac{27}{16}-3B-\frac{1}{4}$

$3B=\frac{27}{16}-\frac{1}{4}-2$

$3B=\frac{27-4}{16}-2$

$3B=\frac{23-32}{16}$

$3B=\frac{-9}{16} \rightarrow \boxed{B=\frac{-3}{16}}$

$$\boxed{=\frac{3}{16(x-1)}-\frac{3}{16(x+3)}+\frac{1}{4(x+3)^2}}$$

(b) Soln.

Min $Z=5x_1+4x_2$

Subject to,

$4x_1+x_2\geq 40$

$2x_1+3x_2\geq 90$

$x_1, x_2\geq 0$.

① $\rightarrow$ $4x_1+x_2=40$

put $x_1=0 \Rightarrow \boxed{x_2=40}$ $(0,40)$

put $x_2=0 \Rightarrow 4x_1=40$

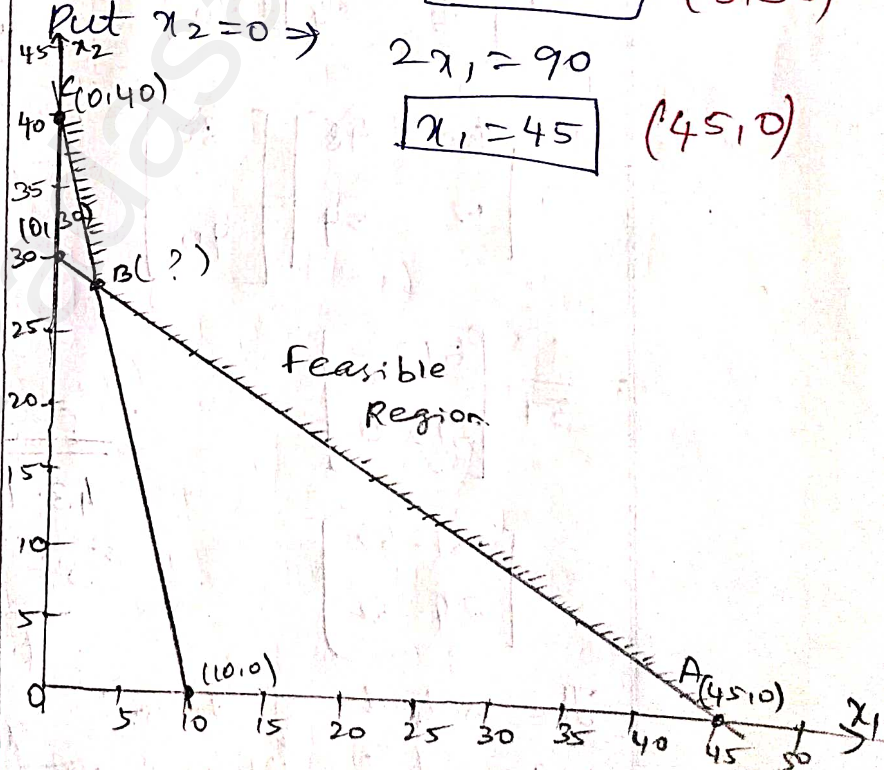
$\boxed{x_1=10}$ $(10,0)$

② $\rightarrow$ put $x_1=0 \Rightarrow 3x_2=90$

$\boxed{x_2=30}$ $(0,30)$

put $x_2=0 \Rightarrow 2x_1=90$

$\boxed{x_1=45}$ $(45,0)$



Now, To find B

$4x_1+x_2=40 \longrightarrow$ ①

$2x_1+3x_2=90 \longrightarrow$ ②

① $\times 3 \Rightarrow 12x_1+\cancel{3x_2}=120$

② $\Rightarrow 2x_1+\cancel{3x_2}=90$

$(-)\quad(-)\quad(-)$

$10x_1=30$

$\boxed{x_1 = 3}$

Sub $x_1 = 3$ in (1)

(1) $\Rightarrow 4x_1 + x_2 = 40$

$4(3) + x_2 = 40$

$12 + x_2 = 40$

$x_2 = 40 - 12$

$\boxed{x_2 = 28}$

$\therefore B(3, 28)$

Corner points	Min $Z = 5x_1 + 4x_2$
A(45, 0)	225
B(3, 28)	127
C(0, 40)	160

Min. value of Z occur at B (3, 28)

$\therefore \boxed{x_1 = 3}$; $\boxed{x_2 = 28}$

$Z_{min} = 127$

44. (a) Soln.,

Let the eqn. of circle be $x^2+y^2+2gx+2fy+c=0$

At (1, 0)

$1^2 + 0^2 + 2g(1) + 2f(0) + c = 0$

$1 + 2g + c = 0$

$2g + c = -1 \longrightarrow (1)$

At (-1, 0)

$(-1)^2 + 0^2 + 2g(-1) + 2f(0) + c = 0$

$1 - 2g + c = 0$

$-2g + c = -1 \longrightarrow (2)$

At (0, 1)

$0^2 + 1^2 + 2g(0) + 2f(1) + c = 0$.

$1 + 2f + c = 0$

$2f + c = -1 \longrightarrow (3)$.

Now,

(1) $\Rightarrow \cancel{2g} + c = -1$

(2) $\Rightarrow \cancel{-2g} + c = -1$

$2c = -2$

$\boxed{c = -1}$

Sub. $c = -1$ in (1)

$2g - 1 = -1$

$2g = -1 + 1$

$2g = 0 \rightarrow \boxed{g = 0}$

Sub. $c = -1$ in (3)

$2f - 1 = -1$

$2f = 0$

$\boxed{f = 0}$

$\therefore$ The eqn of circle is

$x^2 + y^2 + 2(0)x + 2(0)y + (-1) = 0$.

$\boxed{x^2 + y^2 - 1 = 0}$

(b) Soln.

x	y	$dx=x-A$ (60)	$dy=y-A$ (30)	dx^2	dy^2	$dxdy$
35	17	-25	-13	625	169	325
40	28	-20	-2	400	4	40
60	30	0	0	0	0	0
79	32	19	2	361	4	38
83	38	23	8	529	64	184
95	49	35	19	1225	361	665
		$\Sigma dx=32$	$\Sigma dy=14$	Σdx^2 3140	Σdy^2 602	$\Sigma dxdy$ 1252

$N=6.$

$$r=\frac{N\Sigma dxdy-(\Sigma dx)(\Sigma dy)}{\sqrt{N\Sigma dx^2-(\Sigma dx)^2}\sqrt{N\Sigma dy^2-(\Sigma dy)^2}}$$

$$=\frac{6\times1252-(32)(14)}{\sqrt{6\times3140-(32)^2}\sqrt{6\times602-(14)^2}}$$

$$=\frac{7512-448}{\sqrt{18840-1024}\sqrt{3612-196}}$$

$$=\frac{7064}{\sqrt{17816}\sqrt{3416}}$$

$$=\frac{7064}{\sqrt{60859456}}=\frac{7064}{7801.247}$$

$$\boxed{r=0.905}$$

45. (a) Soln.

Given:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

$$\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left[\frac{x+y}{1-xy}\right]$$

$$\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4}$$

$$\frac{\frac{x^2+\cancel{2x}-\cancel{x}-2+x^2+\cancel{x}-\cancel{2x}-2}{x^2-4}}{1-\frac{x^2-1}{x^2-4}}=\tan\frac{\pi}{4}$$

$$\frac{\frac{2x^2-4}{\cancel{x^2-4}}}{\frac{\cancel{x^2}-4-\cancel{x^2}+1}{\cancel{x^2-4}}}=1$$

$$\frac{(2x^2-4)}{-3}=1$$

$$2x^2-4=-3$$

$$2x^2=-3+4$$

$$2x^2=1$$

$$x^2=\frac{1}{2}$$

$$\boxed{x=\pm\frac{1}{\sqrt{2}}}$$

$$\boxed{x=\pm\frac{1}{\sqrt{2}}}$$

(b.) Soln,

Let A be the event of the product of Machine A ; $P(A) = \frac{20}{100}$

Let B be the event of the product of Machine B ; $P(B) = \frac{30}{100}$

Let C be the event of the product of Machine C ; $P(C) = \frac{50}{100}$

Let D be the event of selecting a defective product.

$P(D/A) = \frac{7}{100}$

$P(D/B) = \frac{3}{100}$

$P(D/C) = \frac{5}{100}$

$$P(C/D) = \frac{P(D/C) \cdot P(C)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$= \frac{\frac{5}{100} \times \frac{50}{100}}{\frac{20}{100} \times \frac{7}{100} + \frac{30}{100} \times \frac{3}{100} + \frac{5}{100} \times \frac{50}{100}}$$

$$= \frac{50 \times 5}{(20 \times 7) + (30 \times 3) + (5 \times 50)}$$

$$= \frac{250}{140 + 90 + 250}$$

$$= \frac{250}{480} = \frac{25}{48}$$

$\boxed{P(C/D) = 0.5208}$

46. (a) Soln,

Given: $y = 500 e^{7x} + 600 e^{-7x}$

Diff y w.r.t x,

$y_1 = 500 [7e^{7x}] + 600 [-7e^{-7x}]$

Diff y_1 w.r.t. x,

$y_2 = 500 [7(7) e^{7x}] + 600 [-7 \times -7 e^{-7x}]$

$y_2 = 500 [49 e^{7x}] + 600 [49 e^{-7x}]$

$y_2 = 49 [500 e^{7x} + 600 e^{-7x}]$

$y_2 = 49 y$

$y_2 - 49y = 0$

Henced proved.

(b) Soln:

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
$\Sigma x=28$	$\Sigma y=77$	$\Sigma x^2=140$	$\Sigma y^2=875$	$\Sigma xy=334$

$$\bar{x} = \frac{\Sigma x}{N} = \frac{28}{7} = 4$$

$$\bar{y} = \frac{\Sigma y}{N} = \frac{77}{7} = 11$$

Regression Coefficient of y on x

$$b_{yx} = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7\times 334 - (28)(77)}{7\times 140 - (28)^2}$$

$$= \frac{2338 - 2156}{980 - 784}$$

$$= \frac{182}{196}$$

$$\boxed{b_{yx} = 0.929}$$

Regression equation of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 11 = 0.929(x - 4)$$

$$y - 11 = 0.929x - 3.716$$

$$y = 0.929x - 3.716 + 11$$

$$\boxed{y = 0.929x + 7.284}$$

47. (a) Soln:

Given:

$$q = 13 - 2p_1 - 3p_2^2$$

$$\frac{\partial q}{\partial p_1} = 0 - 2(1) - 0 = -2$$

$$\frac{\partial q}{\partial p_2} = 0 - 0 - 3(2)p_2 = -6p_2$$

$$\boxed{\frac{Eq}{Ep_1} = \frac{-p_1}{q}\frac{\partial q}{\partial p_1}}$$

$$= \frac{-p_1}{13 - 2p_1 - 3p_2^2}(-2)$$

At $p_1 = 2$; $p_2 = 2$.

$$= \frac{-2}{13 - 2(2) - 3(2)^2}(-2)$$

$$= \frac{4}{13 - 4 - 12} = \frac{-4}{3}$$

$$\frac{Eq}{Ep_2} = \frac{-p_2}{q}\cdot\frac{\partial q}{\partial p_2}$$

$$= \frac{-p_2}{13 - 2p_1 - 3p_2^2}(-6p_2)$$

At $p_1 = p_2 = 2$

$$= \frac{-2}{13 - 2(2) - 3(2)^2}(-6\times 2) = \frac{24}{-3} = -8$$

(b) Soln,

(i) Investment = ₹ 96,000

Face = ₹ 100

Market value = ₹ 80

The No. of Shares bought

$$= \frac{\text{Investments}}{\text{M.V. of one share}}$$

$$= \frac{96,000}{80} = 1200$$

No. of Shares bought = 1200 Shares

(ii) Total dividend

= No. of Shares × Rate of dividend × Face value of Share.

$$= 1200 \times \frac{18}{100} \times 100$$

$$= 21600$$

Total dividend = ₹ 21,600.

(iii) Dividend on ₹ 96,000

= ₹ 21600

$$\text{\% of return} = \frac{21600}{96000} \times 100$$

$$= \frac{45}{2}$$

$$= 22.5$$

% of return = 22.5%

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