HALF YEARLY EXAM - 2022
ANSWER KEY
11th MATHEMATICS
TIRUVALLUR DISTRICT

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PG Asst. Maths

PART-I

1	c) 512	8	d) 3	15	a) $7<x<11$
2	b) 7	9	d) 25	16	d) $\frac{-4}{\sqrt{3}}$
3	a) 0	10	b) $\log\left(\frac{a}{b}\right)$	17	b) 1
4	c) 1296	11	a) $(a-2)^2$	18	d) $\frac{7}{128}$
5	a) 12	12	d) 4	19	a) $\sqrt{\tan x}+c$
6	a) $\frac{1}{2}$	13	d) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$	20	b) -1
7	c) $\frac{1}{2}a^2$	14	a) $\frac{5}{13}$		

PART-II

21) The minimum value is $f(-2)$ and its maximum value is $f(2)$ which are equal to -16 and 16 respectively.

$\therefore$ B is $\{-16, 16\}$

22)
$$2 > |x-4|$$
$$-2 < x-4 < 2$$
$$2 < x < 6$$
$\therefore$ solution set is
$$(2,4)\cup(4,6)$$

23)
$$2\sin A\cos B = \sin(A-B) + \sin(A-B)$$
$$2\sin 40^\circ \cos 30^\circ = \sin 70^\circ + \sin 10^\circ$$
$$\sin 40^\circ \cos 30^\circ = \frac{1}{2}[\sin 70^\circ + \sin 10^\circ]$$

24)
$$nC_{12} = nC_9$$
$$nC_x = nC_y \Rightarrow x+y=n$$
$$n = 12+9 = 21$$
$$21C_n = 21C_{21}$$
$$= 1.$$

25)
$$a_n = \begin{cases} n & n=1,2,3 \\ a_{n-1}+a_{n-2}+a_{n-3} & , n>3 \end{cases}$$
$n = 1, 2, 3, 4, 5, 6$

$a_1 = 1$, $a_2 = 1$ $a_3 = 1$

$a_4 = a_3+a_2+a_1$
$= 3$

$a_5 = a_4+a_3+a_2$
$= 3+1+1$
$= 5$

$a_6 = a_5+a_4+a_3$
$= 5+3+1$
$= 9$

$\therefore$ The first 6 terms are

1, 1, 1, 3, 5, 9

26. A square matrix A is said to be skew-symmetric if $A^T = -A$.

27. LHS:

$= \vec{a} \times (\vec{b}+\vec{c}) + \vec{b} \times (\vec{c}+\vec{a}) + \vec{c} \times (\vec{a}+\vec{b})$

$= (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{c}) + (\vec{b}\times\vec{a}) + (\vec{c}\times\vec{a}) + \vec{c}\times\vec{b}$

$= \cancel{(\vec{a}\times\vec{b})} + \cancel{(\vec{a}\times\vec{c})} + \cancel{(\vec{b}\times\vec{c})} - \cancel{(\vec{a}\times\vec{b})} - \cancel{(\vec{a}\times\vec{c})} - \cancel{(\vec{b}\times\vec{c})}$

$= 0$

28. $\lim_{x\to 0} \sin x = 0$ (prove that)

Sol:-

$-x \le \sin x \le x \quad \forall x \ge 0$

$\lim_{x\to 0} (-x) = 0$ and

$\lim_{x\to 0} (x) = 0.$

By sandwich theorem

$\lim_{x\to 0} \sin x = 0$

Hence proved.

29. $\int e^{8-7x}\,dx = \frac{e^{8-7x}}{-7} + c$

$= \frac{-e^{8-7x}}{7} + c$

30. $y = x \log x$

Diff w.r to x.

$\frac{dy}{dx} = \cancel{x}\left(\frac{1}{\cancel{x}}\right) + \log x\,(1)$

$= 1 + \log x$

PART - III

31. $x^2 + x - 5 \Rightarrow -1, -2, -3 \ldots$

$x^2 + 3x - 2 \Rightarrow 4, 5, 6 \ldots$

$x^2 \Rightarrow 1$

$x^2 - 3 \Rightarrow$ otherwise

$f(-3) = 9 - 3 - 5$

$= 9 - 8$

$= 1$

$f(5) = 5^2 + 3(5) - 2$

$= 25 + 15 - 2$

$= 40 - 2$

$= 38$

$f(2) = 2^2 - 3$

$= 4 - 3$

$= 1$

$f(-1) = 1 - 1 - 5$

$= -5$

32. $= \frac{7+\sqrt{6}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \frac{21 + 7\sqrt{2} + 3\sqrt{6} + \sqrt{12}}{3^2 - (\sqrt{2})^2}$

$= \frac{21 + 7\sqrt{2} + 3\sqrt{6} + 2\sqrt{3}}{9 - 2}$

$= \frac{21 + 7\sqrt{2} + 3\sqrt{6} + 2\sqrt{3}}{7}$

33. $\frac{6!}{n!} = 6 \Rightarrow 6! = 6n!$

$\cancel{6} \times 5! = n!\ (\cancel{6})$

$\boxed{n = 5}$

34. $7^{400} = (7^2)^{200} = (50-1)^{200}$

$= {}^{200}C_0\, 50^{200} - {}^{200}C_1\, 50^{199} + \dots + {}^{200}C_{200}\,(-1)^{200}$

$= 50^2\left({}^{200}C_0\, 50^{198} - {}^{200}C_1\, 50^{197} + \dots + {}^{200}C_{198}\,(-1)^{198}\right) - 200 \times 50 + 1$

∴ As 50^2 and 200 are divisible by 100, the last two digits of 7^{400} is 01.

35. Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-1 + 3/2}{1 - 0}$

$= \dfrac{-2+3}{2}$

$= 1/2$

Slope of BC $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-1/2 + 1}{2 - 1}$

$= \dfrac{-\frac{1}{2} + \frac{2}{2}}{1}$

$= 1/2$

∴ Slope of AB = slope of BC.

∴ Given points are collinear.

36. $= \begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

$= \begin{vmatrix} \log_3 64 \log_2 3 + \log_4 3 \log_3 4 & \log_3 64 \log_8 3 + \log_4 3 \log_3 4 \\ \log_3 8 \log_2 3 + \log_4 9 \log_3 4 & \log_3 8 \log_8 3 + \log_4 9 \log_3 4 \end{vmatrix}$

$= \begin{vmatrix} \log_2 64 + 1 & \log_8 64 + 1 \\ \log_2 8 + \log_3 9 & 1 + \log_3 9 \end{vmatrix}$

$= \begin{vmatrix} 6+1 & 2+1 \\ 3+2 & 1+2 \end{vmatrix}$

$= \begin{vmatrix} 7 & 3 \\ 5 & 3 \end{vmatrix}$

$= 21 - 15$

$= 6.$

37. $\lim_{x \to 3} \dfrac{x^n - 3^n}{x - 3} = 27$

$\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = n a^{n-1}$

$n(3)^{n-1} = 27$

$= 3 \times 3^2$

$n - 1 = 2$

$n = 2 + 1$

$\boxed{n = 3}$

38 $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$

$t = \sin^{-1} x \Rightarrow x = \sin t$

$dt = \frac{dx}{\sqrt{1-x^2}}$

$I = \int t \sin t \, dt$

$u = t \qquad dv = \sin t \, dt$

$u' = 1 \qquad v = -\cos t$

$\qquad\qquad v_1 = -\sin t$

$I = -t\cos t - (-\sin t) + c$

$= -t\cos t + \sin t + c$

$= -\sin^{-1} x \sqrt{1-x^2} + x + c$

39 LHS

$= \log 75 - \log 16 - 2\log 5 + 2\log 9 + \log 32 - \log 243$

$= \cancel{\log 3} + \cancel{\log 25} - \cancel{\log 16} - \cancel{\log 25} + \cancel{\log 81} + \cancel{\log 16} + \log 2 - \cancel{\log 81} - \cancel{\log 3}$

$= \log 2 = RHS$

40 $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\} \qquad B = \{5\}$

$n(A) = 3 \qquad n(B) = 1$

$A \cap B = \{5\}$

$n(A \cap B) = 1$

$P(B/A) = \frac{P(A\cap B)}{P(A)}$

$= \frac{1}{\cancel{6}} \times \frac{\cancel{6}}{3}$

$= \frac{1}{3}$

41. (a) PART - IV

$f(x) = 3x - 5 \qquad x = \frac{y+5}{3}$

$y = 3x - 5 \qquad g(y) = \frac{y+5}{3}$

$g \circ f(x) = g(f(x))$

$= g(3x-5)$

$= \frac{3x - \cancel{5} + \cancel{5}}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$= x$

$f \circ g(y) = f(g(y))$

$= f\left(\frac{y+5}{3}\right)$

$= \frac{\cancel{3}(y+5)}{\cancel{3}} - 5$

$= y + \cancel{5} - \cancel{5}$

$= y$

$g \circ f = I_x$ and $f \circ g = I_y$

$\therefore f$ is bijection.

Inverse:

$y = 2x - 5$

$x = \frac{y+5}{3} \Rightarrow f^{-1}(x) = \frac{y+5}{3}$

$f^{-1}(y) = \frac{x+5}{3}$

41) b

LHS:

$= 32\sqrt{3} \times \frac{1}{2}\left(2 \sin\frac{\pi}{48} \cos\frac{\pi}{48}\right) \cos\frac{\pi}{24} \cos\frac{\pi}{12} \cos\frac{\pi}{6}$

$= \frac{\overset{8}{\cancel{16}}\sqrt{3}}{\cancel{2}} \sin\frac{\pi}{12} \cos\pi/12 \cos\frac{\pi}{6}$

$= \frac{8\sqrt{3}}{2} \sin\frac{\pi}{6} \cos\frac{\pi}{6}$

$= 4\sqrt{3} \sin\pi/6 \cos\pi/6$

$= \cancel{4}^{2}\sqrt{3} \times \frac{1}{\cancel{2}} \left(2\sin\frac{\pi}{6} \cos\pi/6\right)$

$= 2\sqrt{3} \sin\frac{\pi}{3}$

$= \cancel{2}\sqrt{3} \times \frac{\sqrt{3}}{\cancel{2}}$

$= 3$

42 (a) $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol:

Put $n=1$: $P(1) = 1$

$P(1)$ is true

Put $n=k$:

$P(k) = 1^2+2^2+3^2+\dots+k^2$

$= \frac{k(k+1)(2k+1)}{6}$

$\therefore P(k)$ is true

Put $n=k+1$

$P(k+1) = 1^2+2^2+3^2+\dots k^2+(k+1)^2$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

LHS:

$= P(k) + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$= \frac{(k+1)(2k^2+k+6k+6)}{6}$

$= \frac{(k+1)(2k^2+7k+6)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

$=$ RHS.

By Mathematical Induction

$1^2+2^2+3^2+\dots n^2 = \frac{n(n+1)(2n+1)}{6}$

is true $\forall n \geq 1$.

42 (b) $\frac{1}{x} - 1 \leq \left\lfloor \frac{1}{x} \right\rfloor \leq \frac{1}{x} + 1$

$\frac{2}{x} - 1 \leq \left\lfloor \frac{2}{x} \right\rfloor \leq \frac{2}{x} + 1$

$\vdots \qquad \vdots$

$\frac{15}{x} - 1 \leq \left\lfloor \frac{15}{x} \right\rfloor \leq \frac{15}{x} + 1$

Adding

$\frac{120}{x} - 15 \leq \left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots + \left\lfloor \frac{15}{x} \right\rfloor \leq \frac{120}{x} + 15$

$\lim_{x\to 0^+} (120 - 15x)$

$\leq \lim_{x\to 0^+} x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots \left\lfloor \frac{15}{x} \right\rfloor\right]$

$\leq \lim_{x\to 0^-} (120 + 15x)$

$\lim_{x\to 0^+} x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots + \left\lfloor \frac{15}{x} \right\rfloor\right] = 120$

Hence proved

48 (a)

Let $|A| = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$

put $x = y$

$|A| = \begin{vmatrix} 1 & y^2 & y^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$ $(R_1 \equiv R_2)$

$= 0$

$\therefore (x-y)$ is a factor

III^{ly} $(y-z)$, $(z-x)$ also factor.

$m = 5-3 = 2$

$\therefore$ The factor $k(x^2+y^2+z^2) + l(xy+yz+zx)$

$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = [k(x^2+y^2+z^2) + l(xy+yz+zx)](x-y)(y-z)(z-x) \rightarrow (1)$

put $x=0$, $y=1$ and $z=2$

$5k + 2l = 2 \rightarrow (2)$

put $x=0$, $y=-1$ and $z=1$

$2k - l = -1 \rightarrow (3)$

Solving (2) & (3)

$k = 0$ and $l = 1$

$(1) \Rightarrow \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Hence proved

43 (b)

$I = \int \frac{3x+5}{x^2+4x+7}\,dx$

$3x+5 = A\frac{d}{dx}(x^2+4x+7) + B$

$3x+5 = A(2x+4) + B$

then $A = \frac{3}{2}$ and $B = -1$

$I = \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}\,dx - \int \frac{1}{x^2+4x+7}\,dx$

$= \frac{3}{2}\log|x^2+4x+7| - \int \frac{dx}{(x+2)^2+(\sqrt{3})^2}$

$= \frac{3}{2}\log|x^2+4x+7| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + c.$

44(a)

(i) General equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0 \rightarrow (1)$

comparing:

$a = \lambda$, $b = 12$, $c = -3$, $h = -5$

$g = \frac{5}{2}$ $f = -8$

Condition:

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\lambda(12)(-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - \lambda(-8)^2 - 12(5/2)^2 - (-3)(-5)^2 = 0$

$\lambda = 2$

$2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

$2x^2 - 10xy + 12y^2 \equiv (x-2y)(2x-6y)$

$2x^2 - 10xy + 12y^2 + 5x - 16y - 3$
$= (x - 2y + l)(2x - 6y + m)$

$2l + m = 5$; $3l + m = 8$

$lm = -3$

$l = 3$

$m = -1$

$\therefore$ The seperate equations are $x - 2y + 3 = 0$ and $2x - 6y - 1 = 0$

(ii) point of Intersection

$(x, y) = \left[\dfrac{hf - bg}{ab - h^2}, \dfrac{hg - af}{ab - h^2}\right]$

$= \left[\dfrac{40-30}{-1}, \dfrac{\frac{-25}{2}+16}{-1}\right]$

$= \left[-10, \dfrac{-7}{2}\right]$

(iii) Angle between lines

$\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a+b}\right|$

$= \dfrac{2\sqrt{25-24}}{2+12}$

$= \dfrac{2}{14_{\,7}}$

$= \dfrac{1}{7}$

$\theta = \tan^{-1}\left(\dfrac{1}{7}\right)$

45(a)

$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$

$= \left(x^3\left(1 + \dfrac{7}{x^3}\right)\right)^{1/3} - \left(x^3\left(1 + \dfrac{4}{x^3}\right)\right)^{1/3}$

$= x\left(1 + \dfrac{7}{x^3}\right)^{1/3} - x\left(1 + \dfrac{4}{x}\right)^{1/3}$

$= \left(x + \dfrac{7}{3x^2} - \dfrac{49}{9x^5} + \dots\right) - \left(x + \dfrac{4}{3x^2} - \dfrac{16x}{9x^5} \dots\right)$

$= \cancel{x} + \dfrac{7}{3x^2} - \cancel{x} - \dfrac{4}{3x^2}$

$= \dfrac{3}{3x^2}$

$= \dfrac{1}{x^2}$ (approximately)

45(b)

$y = (\cos^{-1}x)^2$

$y' = \dfrac{-2\cos^{-1}x}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\, y' = -2\cos^{-1}x$

sq. on b/s

$(1-x^2)(y')^2 = 4(\cos^{-1}x)^2$
$= 4y$

Again diff w.r.to x

$(1-x^2)\, 2y'y'' - 2xy'^2 = 4y'$

$\div 2y'$

$(1-x^2)y'' - xy' - 2 = 0$

when $x = 0$

$$y'' - 0 - 2 = 0$$
$$y'' = 2$$
$$\boxed{y_2 = 2}$$

46(a)

$$3x + 2y + 5 = 0$$
$$3x - 4y + 6 = 0$$
point $(1, 1)$

The family of st. lines through the point of intersections of the lines is of the form:

$$(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$$

$$(3x + 2y + 5) + \lambda(3x - 4y + 6) = 0 \longrightarrow (1)$$

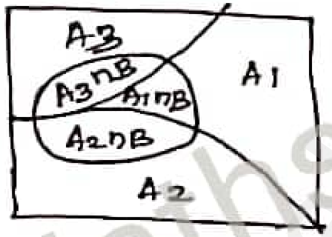
Passes through $(1, 1)$

$$10 + \lambda(9 - 4) = 0$$
$$5\lambda = -10$$

$$\boxed{\lambda = -2}$$

$$\boxed{3x - 10y + 7 = 0}$$

46(b)



$P(A_1) = \frac{5}{10}$ $\quad P(B/A_1) = \frac{4}{10}$

$P(A_2) = \frac{3}{10}$ $\quad P(B/A_2) = \frac{5}{10}$

$P(A_3) = \frac{2}{10}$ $\quad P(B/A_3) = \frac{3}{10}$

By Baye's theorem

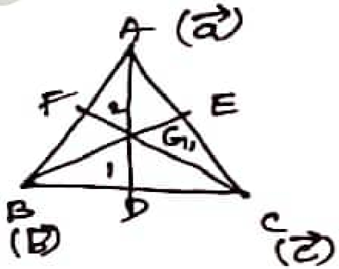
$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$$

$$= \frac{\frac{3}{10} \times \frac{5}{10}}{\frac{5}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{5}{10} + \frac{2}{10} \times \frac{3}{10}}$$

$$= \frac{15}{\cancel{10}} \times \frac{\cancel{10}}{41}$$

$$= \frac{15}{41}$$

47(a)



Let 'O' be the origin

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = \vec{c}$

Also, $\vec{OD} = \frac{\vec{b} + \vec{c}}{2}$, $\vec{OE} = \frac{\vec{c} + \vec{a}}{2}$

$$\vec{OF} = \frac{\vec{a} + \vec{b}}{2}$$

$$m : n = 2 : 1$$

$$\vec{OG_1} = \frac{n\vec{OA} + m\vec{OD}}{n + m}$$

$$= \frac{\vec{a} + \cancel{2}\left(\frac{\vec{b} + \vec{c}}{\cancel{2}}\right)}{3}$$

$$= \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$|||^{ly}$,

$$\vec{OG_2} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \longrightarrow (2)$$

$$\vec{OG_3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \longrightarrow (3)$$

from (1), (2) & (3) we get

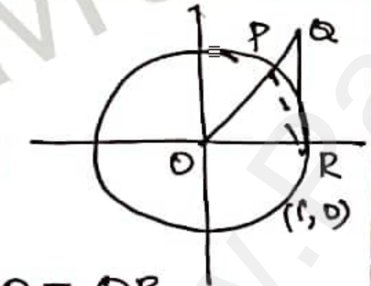
$$\vec{OG_1} = \vec{OG_2} = \vec{OG_3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

$\therefore$ The medians of a triangle are concurrent.

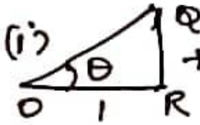
47(b)

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

Consider centre (0,0) and radius 1.



(i)



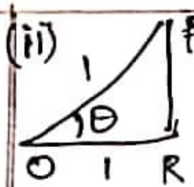
$\tan\theta$

ΔOQR, $\tan\theta = \frac{QR}{1}$

$QR = \tan\theta$

Area of triangle $= \frac{1}{2}(1)\tan\theta = \frac{1}{2}\tan\theta \longrightarrow (1)$

(ii)

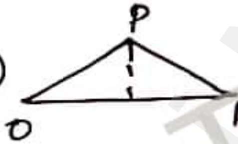


Area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2}(1)^2\theta$

$= \theta/2 \longrightarrow (2)$

(iii)



Area of $\Delta le = \frac{1}{2}(1)\sin\theta$

$= \frac{\sin\theta}{2} \longrightarrow (3)$

By Area property (from (1), (2) & (3))

$$\frac{1}{2}\tan\theta \geq \frac{1}{2}\theta \geq \frac{1}{2}\sin\theta$$

$\times^{ly}$ by $\frac{2}{\sin\theta}$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

Take reciprocal: $\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$

$$\lim_{\theta\to 0}\cos\theta \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$-1 \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq 1$$

By sandwich theorem

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$