COMMON HALF YEARLY EXAMINATION – 2022

Standard XI

Reg No. :

MATHEMATICS

Time: 3.00 hrs. **Marks: 90**

Part - I

I. Choose the correct answer: **20 x 1 = 20**

1. If two sets A and B have 17 elements in common, then the number of elements common to the set A x B and B x A is
   a) 2^{17} b) 17^2 c) 34 d) insufficient data
2. If 3 is the logaritham of 343, then the base is
   a) 5 b) 7 c) 6 d) 9
3. The number of roots of $(x+3)^4 + (x+5)^4 = 16$ is
   a) 4 b) 2 c) 3 d) 0
4. $\cos 1° + \cos 2° + \cos 3° + \ldots\ldots + \cos 179° =$
   a) 0 b) 1 c) –1 d) 89
5. Area of triangle ABC is
   a) $\frac{1}{2}ab\cos C$ b) $\frac{1}{2}ab\sin C$ c) $\frac{1}{2}bc\cos C$ d) $\frac{1}{2}bc\sin C$
6. In a plane there are 10 points are there out of which 4 points are collinear, then the number of triangles formed is
   a) 110 b) $10C_3$ c) 120 d) 116
7. $1 + 3 + 5 + 7 + \ldots\ldots + 17$ is equal to
   a) 101 b) 81 c) 71 d) 60
8. The value of $\frac{1}{2} + \frac{7}{4} + \frac{13}{8} + \frac{19}{16} + \ldots\ldots$ is
   a) 14 b) 7 c) 4 d) 6
9. If the pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$ are perpendicular then
   a) $a + b = 0$ b) $a - b = 0$ c) $h^2 - ab = 0$ d) $h^2 + ab = 0$
10. The length of from the origin to the line $\frac{x}{3} - \frac{y}{4} = 1$ is
    a) $\frac{11}{5}$ b) $\frac{5}{12}$ c) $\frac{12}{5}$ d) $\frac{5}{7}$
11. $A = \begin{pmatrix} \lambda & 1 \\ -1 & -\lambda \end{pmatrix}$, then for what value of λ, $A^2 = 0$?
    a) 0 b) ± 1 c) –1 d) 1
12. The order of matrix B = [1 2 5 7] is
    a) 1 x 4 b) 4 x 1 c) 2 x 1 d) 1 x 1
13. $\lambda\vec{i} + 2\lambda\vec{j} + 2\lambda\vec{k}$ is a unit vector, then the value of λ is
    a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$

14. $\lim_{\theta \to 0} \left(\frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}} \right)$

a) 1 b) −1 c) 0 d) 2

15. The value of $\lim_{x \to 0} \left(\frac{\sin x}{\sqrt{x^2}} \right)$

a) 1 b) −1 c) 0 d) ∞

16. If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is

a) 1 b) 2 c) 3 d) −3

17. The derivative of $f(x) = x|x|$ at $x = -3$ is

a) 6 b) −6 c) does not exist d) 0

18. $\frac{\sqrt{\tan x}}{\sin 2x} dx =$

a) $\sqrt{\tan x} + c$ b) $2\sqrt{\tan x} + c$ c) $\frac{1}{2}\sqrt{\tan x} + c$ d) $\frac{1}{4}\sqrt{\tan x} + c$

19. $\int \frac{1}{e^x} dx =$

a) $\log e^x + c$ b) $\frac{-1}{e^x} + c$ c) $\frac{1}{e^x} + c$ d) $x + c$

20. Ten coins are tossed. The probability of getting at least 8 heads is

a) $\frac{7}{64}$ b) $\frac{7}{32}$ c) $\frac{7}{16}$ d) $\frac{7}{128}$

Part - II

II. Answer any 7 questions: (Q.No.30 is compulsory) **7 x 2 = 14**

21. Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \leq n \leq 5, n \in N\}$.
22. Compute $\log_9 27 - \log_{27} 9$
23. Find the value of $\cos 15°$
24. A polygon has 90 diagonals. Find the number of its sides?
25. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x axis. Find the equation of the line.
26. If G is the centroid of a triangle ABC, prove that $\overline{GA} + \overline{GB} + \overline{GC} = 0$
27. Compute $\lim_{x \to 1} \left(\frac{x^3 - 1}{x - 1} \right)$
28. Find $\frac{dy}{dx}$ if $x = a(t - \sin t)$, $y = (1 - \cos t)$.
29. Integrate : $\int (\tan x + \cot x)^2 dx$
30. Let f and g be the two functions from R to R defined by $f(x) = 3x - 4$ and $g(x) = x^2 + 3$. Find gof and fog.

Part - III

III. Answer any 7 questions: (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$

32. Find the postive number smaller than $\frac{1}{2^{1000}}$. Justify

33. A straight tunnel to be made through a mountain. A surveyor observes the two extremities A and B of the tunnel to be built from a point P in front of the mountain. If AP = 3 km, BP =5 km and $\angle APB = 120^{\circ}$, then find the length of the tunnel to be built.

34. Prove that $\frac{(2n)!}{n!} = 2^n (1.3.5.....(2n-1))$

35. Find the constant term of $\left(2x^3 - \frac{1}{3x^2}\right)$

36. If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, find $A^2 - 7A - 21$

37. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

28. If $y = e^{\tan^{-x}}$, show that $(1 + x^2)\, y'' + (2x—1)\, y' = 0$.

39. Evaluate: $\int x(1-x)^{17} dx$

40. Prove that the points whose position vectors $2\vec{i} + 4\vec{j} + 3\vec{k}$, $4\vec{i} + \vec{j} + 9\vec{k}$ and $10\vec{i} - \vec{j} + 6\vec{k}$ from a right angled triangle.

Part - IV

IV. Answer all the questions: **7 x 5 = 35**

41. a) If $f : R \to R$ is defined by $f(x) = 2x - 3$, prove that f is a bijection and find its inverse.

(OR)

b) Resolve into partial fractions:

$$\frac{x^2 + x + 1}{x^2 - 5x + 6}$$

42. a) $A + B + C = \pi$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$.

(OR)

b) State and prove Nepier's formula

43. a) Use induction to prove that $n^3 - 7n + 3$, is divisible by 3, for all natural numbers n.

(OR)

b) Prove that $\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

44. a) For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight line.

(OR)

b) Solve : $2X + Y + \begin{bmatrix} -2 & 1 & 3 \\ 5 & -7 & 3 \\ 4 & 5 & 4 \end{bmatrix} = 0$ $X - Y = \begin{bmatrix} 4 & 7 & 0 \\ -1 & 2 & -6 \\ -2 & 8 & -5 \end{bmatrix}$

45. a) Prove that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

(OR)

b) Evaluate : $\lim_{x \to 0} \left(\frac{3^x - 1}{\sqrt{x+1}-1} \right)$

46. a) Show that the vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-\vec{j} + 2\vec{k}$ are coplanar.

(OR)

b) i) If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y

ii) Differentiate : $y = \frac{\log x}{e^x}$

47. a) Integrate the following with respect to x :

i) $\int e^{3x} \cos 2x \, dx$

ii) $\int \cos 5x \sin 3x \, dx$

(OR)

b) The chances of X, Y and Z becoming managers of a certain company are 4:2:3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

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