Ts11M

Tenkasi District Common Examinations
Common Half Yearly Examination - December 2022

23-12-2022

Standard 11

MATHEMATICS

Time Allowed: 3.00 Hours   Maximum Marks: 90

PART - I

Answer all the questions: **20×1=20**

1) For any non-empty sets A and B, if $A \subset B$, then $(A\times B) \cap (B\times A)$ is equal to
a) $A\cap B$ b) $A\times A$ c) $B\times A$ d) None of these

2) Let $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d\}$ and $f = \{(1, a), (4, b), (2, c), (3, d), (2, d)\}$, then f is
a) an one to one function b) an onto function
c) a function which is not one to one d) not a function

3) The value of $\log_{\sqrt{2}} 512$ is
a) 16 b) 18 c) 9 d) 12

4) If $|x+2| \le 9$, then x belongs to
a) $(-\infty, -7)$ b) $[-11, 7]$ c) $(-\infty, -7) \cup [11, \infty)$ d) $(-11, 7)$

5) $\cos 1^\circ + \cos 2^\circ + \dots\dots\dots + \cos 179^\circ =$
a) 0 b) 1 c) -1 d) 89

6) The value of $\sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right]$
a) $\frac{12}{13}$ b) $\frac{5}{13}$ c) $\frac{5}{12}$ d) 1

7) The remainder when 38^{15} is divided by 13 is
a) 12 b) 1 c) 11 d) 5

8) The sum upto 'n' terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\dots\dots\dots$ is
a) $\frac{n(n+1)}{2}$ b) $2n(n+1)$ c) $\frac{n(n+1)}{\sqrt{2}}$ d) 1

9) If 'p' is the distance of the perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then which of the following is incorrect?
a) $p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$ b) $p=\frac{ab}{\sqrt{a^2+b^2}}$
c) $\frac{1}{b^2}=\frac{1}{a^2}+\frac{1}{p^2}$ d) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$

10) A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is
a) 6 b) 3 c) 0 d) -6

11) If $m(2\hat{i}+\hat{j}-\hat{k})$ is a unit vector, then the values of m ______.
a) $\pm\frac{1}{\sqrt{3}}$ b) $\pm\frac{1}{\sqrt{5}}$ c) $\pm\frac{1}{\sqrt{6}}$ d) $\pm\frac{1}{\sqrt{2}}$

12) If α, β, γ are the direction angles of the vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then which one of the following is incorrect?

a) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma \neq 1$

c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

d) the direction cosines of $\vec{r}$ are $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$

13) $\lim_{x\to 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$

a) 2 log2 b) 2 $(\log 2)^2$

c) log2 d) 3 log2

14) $\lim_{x\to 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$

a) 1 b) e c) 1/2 d) 0

15) If $f(x) = x.\tan^{-1}x$, then $f'(1)$ is

a) $1+\frac{\pi}{4}$ b) $\frac{1}{2}+\frac{\pi}{4}$ c) $\frac{1}{2}-\frac{\pi}{4}$ d) 2

16) It is given that $f'(a)$ exists, then $\lim_{x\to a} \frac{x.f(a) - a.f(x)}{x-a}$ is

a) $f(a)-a.f'(a)$ b) $f'(a)$

c) $-f'(a)$ d) $f(a)+a.f'(a)$

17) $\int \tan^{-1}\left[\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right] dx$ is

a) x^2+c b) $2x^2+c$ c) $\frac{x^2}{2}+c$ d) $\frac{-x^2}{2}+c$

18) $\int \frac{x+2}{\sqrt{x^2-1}} dx$ is

a) $\sqrt{x^2-1} - 2\log\left|x+\sqrt{x^2-1}\right| + c$ b) $\sin^{-1}x - 2\log\left|x+\sqrt{x^2-1}\right| + c$

c) $2.\log\left|x+\sqrt{x^2-1}\right| - \sin^{-1}x + c$ d) $\sqrt{x^2-1} + 2.\log\left|x+\sqrt{x^2-1}\right| + c$

19) If A and B are any two events, then the probability that exactly one of them occur is

a) $P(A\cup\bar{B}) + P(\bar{A}\cup B)$ b) $P(A\cap\bar{B}) + P(\bar{A}\cap B)$

c) $P(A) + P(B) - P(A\cap B)$ d) $P(A)+P(B)+2.P(A\cap B)$

20) It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then P(B) is

a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

PART - II

Answer any 7 questions: (Qn.No. 30 is compulsory) **7x2=14**

21) If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.

22) Solve $2|x+1|-6 \leq 7$ and graph the solution set in a number line.

23) Find the general solution of $\sin\theta = \frac{-\sqrt{3}}{2}$.

24) Determine the number of permutations of the letters of the word 'SIMPLE' if all are taken at a time?

25) Expand $(1+x)^{2/3}$ upto four terms for $|x| < 1$.

26) Transform the equation $3x+4y+12 = 0$ into normal form.

27) For what value of x, the matrix A, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.

28) For λ, when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

29) Given that $P(A) = 0.52$, $P(B) = 0.43$ and $P(A \cap B) = 0.24$, find $P(\bar{A} \cup \bar{B})$.

30) Integrate with respect to x : $(1+x^2)^{-1}$

PART - III

Answer any 7 questions. Qn.No. 40 is compulsory: **7x3=21**

31) From the curve $y = \sin x$, draw $y = \sin|x|$ (Hint : $\sin(-x) = -\sin x$).

32) Resolve into partial fractions: $\frac{3x+1}{(x-2)(x+1)}$

33) Show that $32\sqrt{3}.\sin\frac{\pi}{48}.\cos\frac{\pi}{48}.\cos\frac{\pi}{24}.\cos\frac{\pi}{12}.\cos\frac{\pi}{6} = 3$.

34) If $nP_r = 11880$ and $nC_r = 495$, find 'n' and 'r'.

35) If a, b, c are in Geometric progression and if $a^{1/x} = b^{1/y} = c^{1/z}$, then prove that x, y, z are in Arithmetic progression.

36) Find the image of the point $(-2, 3)$ about the line $x+2y-9 = 0$.

37) Prove that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

38) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} - 4\hat{j} - 4\hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ form a right angled triangle.

39) Evaluate the following integral: $\frac{12}{(4x-5)^3} + \frac{6}{3x+2} + 16e^{4x+3}$

40) If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots$, then prove that $\frac{dy}{dx} = y$.

PART - IV

Answer all the questions: **7×5=35**

41) a) Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

(OR)

b) Solve: $\log_{5-x}(x^2-6x+65) = 2$

42) a) If $A+B+C = \pi$, prove that $\cos^2 A+\cos^2 B+\cos^2 C = 1-2.\cos A.\cos B.\cos C$.

(OR)

b) Prove that the sum of the first 'n' non-zero even numbers is n^2+n.

43) a) Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR)

b) Find the value of k, if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting,

$12x^2+7xy-12y^2-x+7y+k=0$

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44) a) Using factor theorem, show that

$$\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$$

(OR)

b) Prove that the medians of a triangle are concurrent.

45) a) Show that the vectors $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$ are coplanar.

(OR)

b) Show that the function $f(x) = \begin{cases} \frac{x^3-1}{x-1}, & \text{if } x \neq 1 \\ 3, & \text{if } x = 1 \end{cases}$ is continuous on $(-\infty, \infty)$.

46) a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2-3xy_1-y = 0$.

(OR)

b) Evaluate: $\int \frac{3x+5}{x^2+4x+7}\,dx$

47) a) The chances of X, Y and Z becoming managers of a certain company are 4:2:3. The probabilities that scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

(OR)

b) If $f : R \to R$ is defined by $f(x) = 3x-5$. Prove that f is a bijection and find its inverse.
