

**COMMON HALF YEARLY EXAMINATION - 2022**  
**Standard XI**  
**MATHEMATICS**  
**Part-I**

Reg.No. 

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Marks: 90  
20 x 1 = 20

Time: 3.00 hours

**I Choose the correct answer**

1. The number of relations on a set containing 3 elements is  
 a) 9      b) 81      c) 512      d) 1024
2. If 3 is the logarithm of 343, then the base is  
 a) 5      b) 7      c) 6      d) 9
3.  $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ =$   
 a) 0      b) 1      c) -1      d) 89
4. The number of rectangles that a chess board has  
 a) 81      b) 9<sup>9</sup>      c) 1296      d) 6561
5. The remainder when  $38^{15}$  is divided by 13 is  
 a) 12      b) 1      c) 11      d) 5
6.  $\sin 150^\circ =$   
 a)  $\frac{1}{2}$       b)  $-\frac{1}{2}$       c)  $\frac{\sqrt{3}}{2}$       d)  $-\frac{\sqrt{3}}{2}$
7. The area of the triangle formed by the lines  $x^2 - 4y^2 = 0$  and  $x = a$  is  
 a)  $2a^2$       b)  $\frac{\sqrt{3}}{2}a^2$       c)  $\frac{1}{2}a^2$       d)  $\frac{2}{\sqrt{3}}a^2$
8. If the points  $(x, -2), (5, 2), (8, 8)$  are collinear, then  $x$  is equal to  
 a) -3      b)  $\frac{1}{3}$       c) 1      d) 3
9. If  $|\vec{a}| = 13$ ,  $|\vec{b}| = 5$  and  $\vec{a} \cdot \vec{b} = 60^\circ$  then  $|\vec{a} \times \vec{b}|$  is  
 a) 15      b) 35      c) 45      d) 25
10.  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$   
 a)  $\log ab$       b)  $\log(\frac{a}{b})$       c)  $\log(\frac{b}{a})$       d)  $\frac{a}{b}$
11. If  $y = \frac{1}{a-z}$ , then  $\frac{dy}{dz}$  is  
 a)  $(a-z)^2$       b)  $-(z-a)^2$       c)  $(z+a)^2$       d)  $-(z+a)^2$
12. If A is a square matrix and  $|A| = 2$ , find the value of  $|AA^T|$   
 a) 0      b) 1      c) 2      d) 4
13.  $\int e^{\sqrt{x}} dx$  is  
 a)  $2\sqrt{x}(1 - e^{\sqrt{x}}) + C$       b)  $2\sqrt{x}(e^{\sqrt{x}} - 1) + C$   
 c)  $2e^{\sqrt{x}}(1 - \sqrt{x}) + C$       d)  $2e^{\sqrt{x}}(\sqrt{x} - 1) + C$

14. If two events A and B are independent such that  $P(A) = 0.35$ ,  $P(A \cup B) = 0.6$  then  $P(B)$  is  
 a)  $\frac{5}{13}$       b)  $\frac{1}{13}$       c)  $\frac{4}{13}$       d)  $\frac{7}{13}$
15.  $|x - 9| < 2$  then  
 a)  $7 < x < 11$       b)  $x = 11$       c)  $7 > x > 11$       d)  $x \neq 7$
16. The x-intercept for the equation  $\sqrt{3}x - y + 4 = 0$  is  
 a)  $\frac{2}{\sqrt{3}}$       b)  $\frac{\sqrt{3}}{4}$       c)  $\frac{1}{4}$       d)  $-\frac{4}{\sqrt{3}}$
17. The sum of the squares of the direction cosines of  $\vec{r}$  is  
 a) 0      b) 1      c) -1      d) none of the above
18. Ten coins are tossed. The probability of getting atleast 8 heads is  
 a)  $\frac{7}{64}$       b)  $\frac{7}{32}$       c)  $\frac{7}{16}$       d)  $\frac{7}{128}$
19.  $\int \frac{\sqrt{\tan x}}{\sin 2x} dx$  is  
 a)  $\sqrt{\tan x} + c$       b)  $2\sqrt{\tan x} + c$       c)  $\frac{1}{2}\sqrt{\tan x} + c$       d)  $\frac{1}{4}\sqrt{\tan x} + c$
20. If  $PV = 81$ , then  $\frac{dP}{dV}$  at  $V = 9$  is  
 a) 1      b) -1      c) 2      d) -2
- Part - II**
- II. Answer any 7 questions: (Q.No.30 is compulsory)**  **$7 \times 2 = 14$**
21. If  $f : [-2, 2] \rightarrow B$  is given by  $f(x) = 2x^3$ , then find B so that f is onto.
22. Solve:  $\left| \frac{2}{x-4} \right| > 1, x \neq 4$
23. Express  $\sin 40^\circ \cos 30^\circ$  as a sum.
24. If  $nC_{12} = nC_9$ , find  $21C_n$
25. Write the first 6 terms of the sequence whose  $n^{\text{th}}$  term,  $a_n$  is given below.
- $$a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2, 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$$
26. Define Skew-Symmetric matrix.
27. Show that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$
28. Prove that  $\lim_{x \rightarrow 0} \sin x = 0$
29. Integrate:  $e^{8-7x}$  w.r.t. x
30. Differentiate:  $y = x \log x$  w.r.t x

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Part - III**III. Answer any 7 questions: (Q.No.40 is compulsory)** **$7 \times 3 = 21$** 31. Write the values of  $f$  at  $-3, 5, 2, -1$  if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

32. Simplify by rationalising the denominator  $\frac{7 + \sqrt{6}}{3 - \sqrt{2}}$ 33. If  $\frac{6!}{n!} = 6$ , then find the value of  $n$ .34. Find the last two digits of the number  $7^{400}$ .35. Show the points  $(0, -\frac{3}{2}), (1, -1), (2, -\frac{1}{2})$  are collinear.36. Find the value of the product:  $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ 37. Find the positive integer  $n$  so that  $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 27$ 38. Integrate :  $\frac{x \sin^{-1} x}{\sqrt{1 - x^2}}$  w.r.t  $x$ 39. Prove  $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log 2$ 

40. A die is rolled. If it shows an odd number, find the probability of getting 5.

**Part - IV** **$7 \times 5 = 35$** **IV. Answer all the questions:**41. a) If  $f : R \rightarrow R$  is defined by  $f(x) = 3x - 5$ , prove that  $f$  is a bijection and find its inverse.  
**(OR)**b) Prove that  $32\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} = 3$ 42. a) By the principle of mathematical induction, prove that, for all integers  $n \geq 1$ ,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

**(OR)**

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b) Show that  $\lim_{x \rightarrow 0} x \left[ \left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots + \left\lfloor \frac{15}{x} \right\rfloor \right] = 120$

43. a) Prove that  $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

(OR)

b) Evaluate:  $\int \frac{3x+5}{x^2+4x+7} dx$

44. a) Resolve into partial fractions:  $\frac{x^2+x+1}{x^2-5x+6}$

(OR)

- b) If the equations  $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$  represents a pair of straight lines, find,
- The value of  $\lambda$  and the separate equations of the lines.
  - Point of intersection of the lines
  - Angle between the lines

45. a) Prove that  $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$  is approximately equal to  $\frac{1}{x^2}$  when  $x$  is large.

(OR)

b) If  $y = (\cos^{-1} x)^2$ , prove that  $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$ , hence find  $y_2$  when  $x=0$

46. a) Find the equation of the line through the intersection of the lines  $3x+2y+5=0$  and  $3x-4y+6=0$  and the point  $(1,1)$

(OR)

- b) The chances of A, B and C becoming manager of a certain company are  $5:3:2$ . The probabilities that the office canteen will be improved if A, B and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?

47. a) Prove by vector method, the medians of a triangle are concurrent.

(OR)

b) Prove  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

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