

HALF YEARLY EXAM – DECEMBER-2022

11 - STD

MATHS

Time : 3 Hours

Maximum Marks : 90

Part - I

Note : i) All questions are compulsory

20 × 1 = 20

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer

1. The number of relations on a set containing 3 elements is
 a) 9 b) 81 c) 512 d) 1024
2. The number of constant functions from a set containing m elements to a set containing n elements is
 a) mn b) m c) n d) $m+n$
3. The number of solutions of $x^2 + |x - 1| = 1$ is
 a) 1 b) 0 c) 2 d) 3
4. If $\log_a 243 = 5$, then the value of a is
 a) 1 b) 3 c) 2 d) 0
5. Which of the following is not true?
 a) $\sin \theta = -\frac{3}{4}$ b) $\cos \theta = -1$ c) $\tan \theta = 25$ d) $\sec \theta = \frac{1}{4}$
6. Number of sides of a polygon having 44 diagonals is ...
 a) 4 b) 4! c) 11 d) 22
7. The HM of two positive numbers whose AM and GM are 16, 8 respectively is
 a) 10 b) 6 c) 5 d) 4
8. The value of $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$ is
 a) $\frac{e+e^{-1}}{2}$ b) $\frac{e^2+e^{-1}}{2}$ c) $\frac{e+e^{-1}}{e}$ d) $\frac{e^2-1}{2e}$
9. Which of the following equation is the locus of $(at^2, 2at)$
 a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ c) $x^2 + y^2 = a^2$ d) $y^2 = 4ax$
10. The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$ is
 a) $2a^2$ b) $\frac{\sqrt{3}}{2} a^2$ c) $\frac{1}{2} a^2$ d) $\frac{2}{\sqrt{3}} a^2$
11. If A is a square matrix, then which of the following is not symmetric?
 a) $A + A^T$ b) AA^T c) $A^T A$ d) $A - A^T$

12. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is
 a) $-2abc$ b) abc c) 0 d) $a^2 + b^2 + c^2$
13. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is
 a) 42 b) 12 c) 22 d) 32
14. Find the value λ for the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular
 a) $\frac{5}{2}$ b) $-\frac{5}{2}$ c) $\frac{2}{5}$ d) -2
15. $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$
 a) $\log ab$ b) $\log\left(\frac{a}{b}\right)$ c) $\log\left(\frac{b}{a}\right)$ d) $\frac{a}{b}$
16. The derivative of $f(x) = x|x|$ at $x = -3$ is
 a) 6 b) -6 c) does not exist d) 0
17. Evaluate the integrals: x^{-1}
 a) $\log x$ b) 0 c) 1 d) $\frac{1}{x}$
18. $\int \tan^{-1} \left[\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right] dx =$
 a) $x^2 + c$ b) $2x^2 + c$ c) $\frac{x^2}{2} + c$ d) $-\frac{x^2}{2} + c$
19. A number is selected from the set $\{1, 2, 3, \dots, 20\}$. The probability that the selected number is divisible by 3 or 4 is
 a) $\frac{2}{5}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$
20. If two events A and B are independent such that $P(A) = 0.35$ and $P(A \cup B) = 0.6$, then $P(B)$ is
 a) $\frac{5}{13}$ b) $\frac{1}{13}$ c) $\frac{4}{13}$ d) $\frac{7}{13}$

Part - II

Note : i) Answer any **Seven** questions

$$7 \times 2 = 14$$

ii) Question number **30** is compulsory

21. If $n(A) = 10$ and $n(A \cap B) = 3$, find $n((A \cap B) \cap A)$.
22. Solve : $|2x - 3| = |x - 5|$
23. Find the value of $\sin 150^\circ$
24. Using binomial theorem, Find the value of 9^4
25. Find the equation of a straight line cutting an intercept of 5 from the negative direction of the y -axis and is inclined at an angle 150° to the x -axis.

26. Determine the value of $x + y$ if $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 13 \\ y & x + 6 \end{bmatrix}$.
27. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a .
28. Differentiate : $y = e^{\sin x}$
29. Integrate with respect to x : \sqrt{x}
30. Given that $P(A)=0.52$, $P(B)=0.43$ and $P(A \cap B)=0.24$, find $P(A \cap \bar{B})$.

Part - III

Note : i) Answer any **Seven** questions

$7 \times 3 = 21$

ii) Question number **40** is compulsory

31. In the set Z of integers, define mRn if $m-n$ is divisible by 7. Prove that R is an equivalence relation.
32. Rationalize the denominator of $\frac{\sqrt{5}}{(\sqrt{6}+\sqrt{2})}$.
33. If the roots of the equation $(q-r)x^2 + (r-p)x + p-q = 0$ are equal, then show that p, q , and r are in AP.
34. If $((n+2)P_4) = 42 \times nP_2$, prove that $n = 5$.
35. Find the distance i) between two parallel lines $3x + 4y = 12$ and $6x + 8y + 1 = 0$.
36. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$.
37. If $\vec{a} = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$, verify
i) \vec{a} and $\vec{a} \times \vec{b}$ are perpendicular to each other.
38. Prove that $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$.
39. Integrate with respect to x : $\int \frac{x+1}{3x^2+6x+2} dx$
40. An integer is chosen at random from the first ten positive integers. Find the Probability that it is i) an even number ii) multiple of three.

Part - IV

Note : i) Answer all the questions

$7 \times 5 = 35$

41. a) If $f: R \rightarrow R$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse. (OR)
- b) Evaluate the integrals : $\int \frac{3x+5}{x^2+4x+7} dx$
42. a) Resolve into partial fractions : $\frac{x+1}{x^2(x-1)}$. (OR)

b) A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

43. a) Express the following matrices as the sum of a symmetric matrix and a

skew-symmetric matrix : $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(OR)

b) If $A + B + C = \frac{\pi}{2}$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C.$$

44. a) Using Factor theorem, Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2 (x+2a).$

(OR)

b) If $y = e^{\tan^{-1} x}$, show that $(1+x^2)y'' + (2x-1)y' = 0$.

45. a) Prove that the points whose position vectors $2\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 9\hat{k}$, and $10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle.

(OR)

b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

46. a) The medians of a triangle are concurrent.

(OR)

b) Find the value of k , if the following equation represents a pair of straight lines.

Further find whether these lines are parallel or intersecting,

$$12x^2 + 7xy - 12y^2 - x + 7y + k = 0$$

47. a) By the principle of Mathematical induction, prove that for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(OR)

b) Evaluate : $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x-2} \right)^x$.