MATHEMATICS

MODEL PAPER

[DURATION: 3 Hrs. **Max mark: 90]**

I. Choose the appropriate answer **(20x1=20)**

1]. The number of constant functions from a set containing m elements to a set containing n elements.

(A). mn. (B). m. (C). n. (D). $m+n$.

2]. the value of $\log_3 11 . \log_{11} 13 . \log_{13} 15 . \log_{15} 27 . \log_{27} 81$ Is

(A). 1. (B). 2. (C). 3. (D). 4.

3]. the maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$.

(A). $4+\sqrt{2}$. (B). $3+\sqrt{2}$. (C). 9. (D). 4

4]. the product of first n natural numbers equals.

(A). ${}^{2n}C_n \times {}^{n}P_n$.

(B). $(\frac{1}{4})^n \times {}^{2n}C_n \times {}^{2n}P_n$.

(C). $(\frac{1}{2})^n \times {}^{2n}C_n \times {}^{n}P_n$.

(D). ${}^{n}C_n \times {}^{n}P_n$.

5]. the remainder when 38^{15} is divided by 13 is

(A). 1. (B).12 (C). 11. (D). 5.

6]. the equation of the locus of the point whose distance from y- axis is half of the distance from origin is.

(A). $x^2 + 3y^2 = 0$.

(B). $x^2 - 3y^2 = 0$.

(C). $3x^2 + y^2 = 0$.

(D). $3y^2 - y^2$

7]. A bag contains 5 white balls and 3 black balls. Five balls are drawn successively without replacement. The probability that they are alternatively of different colors is

(A). $\frac{68}{105}$. (B). $\frac{71}{105}$. (C). $\frac{64}{105}$. (D). $\frac{73}{105}$.

8]. $\int \frac{\sec x}{\sqrt{\cos 2x}}. dx$

(A). $\tan^{-1}(\sin x) + c$.

(C). $\tan^{-1}(\cos x) + c$.

(B). $2\sin^{-1}(\tan x) + c$. (D). $\sin^{-1}(\tan x) + c$.

9]. It is given that $f'(a)$ exists, then $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$ is

(A). $f(a) - af'(a)$. (C). $-f'(a)$.

(B). $f'(a)$. (D). $f(a) + af'(a)$.

10]. $\lim_{n\to\infty} \frac{\sin\alpha - \cos\alpha}{\alpha - \frac{\pi}{4}}$ is

(A). $\sqrt{2}$. (B). $\frac{1}{\sqrt{2}}$. (C). 1. (D). 2.

11]. If $\vec{a}= \hat{\imath}+\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+x\hat{\jmath}+\hat{k}$, $\vec{c}=\hat{\imath}-\hat{\jmath}+4\hat{k}$ and $\vec{a}.(\vec{b}x\vec{c})$.=70, then x is equal to

(A). 5. (B). 7. (C). 26. (D). 10.

12]. If the square of the matrix $\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ is the unit matrix of order 2, then α, β, γ should satisfy the relation.

(A). $1+\alpha^2+\beta\gamma=0$. (B). $1-\alpha^2-\beta\gamma=0$

(C). $1-\alpha^2+\beta\gamma=0$. (D). $1+\alpha^2-\beta\gamma=0$

13]. Let X={1,2,3,4} and R= ((1,1), (1,2), (1,3), (2,2), (3,3), (2,1), (3,1), (1,4), (4,1)}. Then

(A), Reflexive. (B). Symmetric. (C). Transitive. (D). Equivalence.

14]. When a parabola intersects at x axis at two points and the roots are real and distinct and the discriminant is

(A). positive. (B). Negative. (C). Zero. (D). None of these.

15]. If a vertex of a square I at the origin at its one side lies along the line $4x+3y-20=0$, the area of the square is

(A). 20 sq. units. (B). 16 sq. units. (C). 25 sq. units. (D). 4 sq. units.

16]. The value of $1-\frac{1}{2}\left(\frac{2}{3}\right)+\frac{1}{3}\left(\frac{2}{3}\right)^2-\frac{1}{4}\left(\frac{2}{3}\right)+\ldots$ is

(A). $\log\frac{5}{3}$. (B). $\frac{3}{2}\log\frac{5}{3}$. (C). $\frac{5}{3}\log\frac{5}{3}$. (D). $\frac{2}{3}\log\frac{2}{3}$.

17]. There are three events A,B,C of which one and only one can happen. If the odds are 7 to 4 against A and 5 to 3 against B, then the odds against C is

(A). 23: 65. (B). 65:23. (C). 23:88. (D). 88:23.

18]. If we delete the i^{th} row and j^{th} column from the matrix of $A=[a_{ij}]_{nxm}$ we obtain a determinant order (n-1), which is called

(A). Scalar. (B). Vector. (C). Minor of the element. (D). Major of the element.

19]. What is $A^T = A$ said to be.

(A). Symmetric matrix. (C). Singular Matrix.

(B). Skew-symmetric matrix. (D). Non-singular matrix.

20]. If the function $f:[0,2\pi]\rightarrow[-1,1]$ defined by $f(x)=\sin x$ is

(A). one-to-one. (B). onto. (C). Bijection. (D). all of these.

II. Answer any seven of the following

(Q. No 30 is compulsory) **(7x2=14)**

21]. Factorize: x^2+1. (*Hint*: try complete square method).

22]. If $A+B+C=\pi$, prove that $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$.

23]. How many diagonals are there in a polygon with n sides.

24]. Complete the sum of first n terms of 8+88+888+8888+.....

25]. Find the locus of P,if for all the values of α,m the co-ordinates of a moving point P is ($9\cos\alpha, 9\sin\alpha$)

26]. Differentiate $sin(ax^2+bx+c)$ with respect to $cos(lx^2+mx+n)$.

27]. Integrate $(x+5)^6$

28]. If A and B are two independent events such that $P(A)$=0.4 and $P(A\cup B)$= 0.9. Find $p(B)$.

29]. For any two vectors $\vec{a}$ and $\vec{b}$, prove that

$|\vec{a}x\vec{b}|^2 + (\vec{a}.\vec{b})^2=|\vec{a}|^2 \; |\vec{b}|^2$.

30]. In any triangle ABC, prove that

$a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$.

III. Answer any 7 the following

(Q. No 40 is compulsory) **(7x3=21)**

31]. Prove that $|A|=$

$$\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3.$$

32]. Do the limits of the following functions exist as $x \to 0$? State the reasons for the answer.

(i). $\frac{\sin|x|}{x}$. (ii). $\frac{\sin x}{|x|}$

33]. Evaluate: $\left(\int \tan^{-1}\frac{2x}{1-x^2}\right)dx.$

34]. Find y^n if $x^4+y^4=16$

35]. If $P(A)=0.5$, $P(B)=0.8$ and $P(B/A)=0.8$, find $P(A/B)$ and $P(A\cup B)$.

36]. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

37]. Find the rank of the word PUBLIC.

38]. Express each of the following product as a sum or difference

(i). $\sin 40° \cos 30°$ (ii). $\cos 110° \sin 55°$ (iii). $\sin\frac{x}{2}\ \cos\frac{3x}{2}$.

39]. If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$, the prove that $xyz=1$.

40]. Evaluate the integrate $\frac{1}{7-(4x+1)^2}$.

IV. Answer all the questions

(7x5=25)

41]. Prove that $\cos 20°.\cos 40°.\cos 60°.\cos 80° = \frac{1}{16}$

(OR)

42]. By the principle of mathematical induction, prove that, for $n \in \mathbb{N}$

$\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\ldots+\cos(\alpha+(n-1)\beta)=$

$\cos\left(\alpha+\frac{(n-1)\beta}{2}\right) \times \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$.

43]. Prove that $\sqrt[3]{x^3+7}$ - $\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR)

44]. If the equation αx^2-$10xy$+$12y^2$+$5x$-$16y$-3=0 represents a pair of straight lines, find

(i). The value of α and the separate equation if the lines

(ii). Point of intersection of the lines

(iii). Angle between the lines

45]. When a pair of fair dice is rolled, what are the probabilities of getting the sum

(i). 7. (ii). 7 or 9. (iii). 7 or 12. (iv). 7 or 10. (v). 7 or 9.

(OR)

46].

Evaluate the following integrals

(i). $\int \frac{x+1}{x^2-3x+1}dx$ (ii). $\int \frac{5x-7}{\sqrt{3x-x^2-2}}dx$

47].

(A).Find the second order derivative if x and y are given by x=$a\cos t$;y=$a\sin t$

(B). Find the derivative of x^x with respect to $x\log x$

(OR)

48]. $\lim\limits_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ (a>b).

49]. A triangle is formed by joining the points (1, 0, 0). (0, 1, 0), (0, 0, 1). Find the direction cosines of the medians.

(OR)

50]. If A=$\begin{bmatrix}1/2 & \alpha \\ 0 & 1/2\end{bmatrix}$, prove that $\sum_{k=1}^{n} det(A^k)$=$\frac{1}{3}\left(1-\frac{1}{4^n}\right)$.

51]. For the curve $y = x^3$ draw (i).$y = -x^3$ (ii). $y = x^3$+1

(OR)

52]. Write the values of -3, 5, 2, -1, 0, -11

$$f(x)=\begin{cases} x^2 + x - 5 & if\ x \in (-\infty, 0) \\ x^2 + 3x - 2 & if x \in (3, \infty) \\ x^2 & if\ x \in (0,2) \\ x^2 - 3 & otherwise \end{cases}$$

53]. Resolve into partial fraction

$\frac{7+x}{(1+x)(1+x^2)}$.

(OR)

54]. If one root of $k(x-1)^2 = 5x - 7$ ~~id~~ is double the other root, show that k = 2 or -25.

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