SUN TUITION CENTRE

poon thotta pathai hindu mission hospital opposite
villupuram

+1 MATHEMATICS

Life is a good circle,
you choose the best radius...

PUBLIC PREPARATION- 2022-2023

XI – MATHS - TWO AND THREE MARKS

1. SETS, RELATIONS AND FUNCTIONS

1. Find the number of subsets of A if A = $\{x : x = 4n+1, 2 \le n \le 5, n \in N\}$
2. If n(A) = 10 and $n(A \cap B) = 3$, find $n((A \cap B)' \cap A)$.
3. If $\rho(A)$ denotes the power set of A, then find $n(\rho(\rho(\rho(\phi))))$.
4. If n(ρ(A)) = 1024; n(A ∪ B) = 15 and n(ρ(B)) = 32, then find n(A ∩ B).
5. If n(A ∩ B) = 3 and n(A ∪ B) = 10, then find n(ρ(A Δ B)).
6. In the set Z of integers, define mRn if m- n is a multiple of 12. Prove that R is an equivalence relation.
7. Let P be the set of all triangles in a plane and R be the relation defined on P as aRb if a is similar to b. Prove that R is an equivalence relation.
8. Prove that the relation "friendship" is not an equivalence relation on the set of all people in Chennai.
9. Let A = $\{a, b, c\}$. What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A?
10. In the set Z of integers, define mRn if m - n is divisible by 7. Prove that R is an equivalence relation.
11. Write the values of f at -4, 1,-2, 7, 0 if f(x) = $\begin{cases} -x+4 & if\ -\infty < x \le -3 \\ x+4 & if\ -3 < x < -2 \\ x^2 - x & if\ -2 \le x < 1 \\ x - x^2 & if\ 1 \le x < 7 \\ 0 & otherwise \end{cases}$
12. Write the values of f at -3, 5, 2,-1, 0 if f(x) = $\begin{cases} x^2 + x - 5 & , x \in (-\infty, 0) \\ x^2 + 3x - 2 & , x \in (3, \infty) \\ x^2 & , x \in (0, 2) \\ x^2 - 3 & , otherwise \end{cases}$
13. On the set of natural numbers let R be the relation defined by aRb if a + b ≤ 6. Write down the relation by listing all the pairs. Check whether it is
    (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
14. Discuss the following relations for reflexivity, symmetricity and transitivity:
    "The relation R defined on the set of all positive integers by "mRn if m divides n".

15. On the set of natural numbers let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is

(i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

2. BASIC ALGEBRA

1. Solve $\left|\frac{2}{x-4}\right| > 1, x \neq 4$.

2. Solve $|5x-12| < -2$

3. Solve $3x-5 \leq x+1$ for x.

4. If a and b are the roots of the equation $x^2 - px + q = 0$, find the value of $\frac{1}{a}+\frac{1}{b}$.

5. Construct a quadratic equation with roots 7 and -3.

6. Find the condition that one of the roots of $ax^2 + bx + c$ may be (i) negative of the other, (ii) thrice the other, (iii) reciprocal of the other.

7. Solve the equation $\sqrt{6-4x-x^2} = x+4$

8. Resolve into partial fractions: $\frac{1}{x^2-a^2}$

9. Find the square root of $7 - 4\sqrt{3}$

10. Simplify and hence find the value of $n: \frac{3^{2n}9^2 3^{-n}}{3^{3n}} = 27$

11. Find the radius of the spherical tank whose volume is $\frac{32\pi}{3}$ units.

12. Simplify by rationalising the denominator. $\frac{7+\sqrt{6}}{3-\sqrt{2}}$

13. Prove $\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=0.$

14. Prove $\log_{a^2} a.\log_{b^2} b.\log_{c^2} c = \frac{1}{8}$.

15. Prove $\log a + \log a^2 + \log a^3 + \_\_\_ + \log a^n = \frac{n(n+1)}{2}\log a$

16. If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$ then prove that xyz = 1.

17. If $a^2 + b^2 = 7ab$, show that $\log\frac{a+b}{3}=\frac{1}{2}(\log a+\log b)$

18. Simplify $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

19. Simplify: $\left(3^{-6}\right)^{\frac{1}{3}}$

20. Simplify: $\left(x^{1/2}y^{-3}\right)^{1/2}$; where x, y $\geq$ 0.

3. TRIGONOMETRY

1. Find the value of $\cos(300^0)$

2. Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

3. Show that $tan\left(45^0+A\right)=\frac{1+\tan A}{1-\tan A}$

4. Prove that $cot\left(A+B\right)=\frac{\cot A\cot B-1}{\cot A+\cot B}$

5. If A + B = 45^0, show that (1 + tanA) (1 + tanB) = 2.

6. Find the value of sin 34^0 + cos 64^0 - cos 4^0.

7. Prove that $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\tan\theta$

8. Express the following as a sum or difference 2 sin 10θcos 2θ

9. Show that $\frac{sin8x\ cosx-sin6x\ cos3x}{cos2x\ cos\ x-sin3x\ sin4x}=\tan 2x.$

10. Prove that $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x}=\tan 3x.$

4.Combinatorics and Mathematical Induction

1. If $\frac{1}{7!}+\frac{1}{8!}=\frac{A}{9!}$ then find the value of A.

2. Count the number of three-digit numbers which can be formed from the digits 2,4,6,8 if (i) repetitions of digits is allowed. (ii) repetitions of digits is not allowed

3. If $^{(n+2)}P_4 = 42\times {}^nP_2$, find n.

4. If $^{10}P_r = {}^7P_{r+2}$ find r.

5. Find the number of ways of arranging the letters of the word BANANA.

6. Find the number of ways of arranging the letters of the word RAMANUJAN so that the relative positions of vowels and consonants are not changed.

7. If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in this list before the first word starting with B?

8. Three men have 4 coats, 5 waist coats and 6 caps. In how many ways can they wear them?

9. Find the distinct permutations of the letters of the word MISSISSIPPI?

10. In how many ways can the letters of the word SUCCESS be arranged so that all Ss are together?
11. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY.
12. If ${}^{n}P_{r} = 11880$ and ${}^{n}C_{r} = 495$, Find n and r.
13. A Mathematics club has 15 members. In that 8 are girls. 6 of the members are to be selected for a competition and half of them should be girls. How many ways of these selections are possible?
14. A box of one dozen apple contains a rotten apple. If we are choosing 3 apples simultaneously, in how many ways, one can get only good apples.
15. Prove that ${}^{15}C_3 + 2\times{}^{15}C_4 + {}^{15}C_5 = {}^{17}C_5$.
16. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.
17. Prove that ${}^{35}C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4 = {}^{40}C_5$
18. How many ways can a team of 3 boys, 2 girls and 1 transgender be selected from 5 boys, 4 girls and 2 transgenders?
19. A trust has 25 members.

    (i) How many ways 3 officers can be selected?

    (ii) In how many ways can a President, Vice President and a Secretary be selected?
20. In an examination a student has to answer 5 questions, out of 9 questions in which 2 are compulsory. In how many ways a student can answer the questions?
21. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly three aces in each combination.
22. There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find,

    (i) the number of straight lines that can be obtained from the pairs of these points?

    (ii) the number of triangles that can be formed for which the points are their vertices?
23. A polygon has 90 diagonals. Find the number of its sides?
24. If ${}^{n}C_{12} = {}^{n}C_9$ find ${}^{21}C_n$.
25. There are 5 teachers and 20 students. Out of them a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees

    (i) a particular teacher is included? (ii) a particular student is excluded?

5.Binomial Theorem,Sequences and Series

1. $a_n = \begin{cases} n+1 \ if \ \ n \ is \ odd \\ n if \ \ n \ is \ even \end{cases}$ Write the first 6 terms of the sequences

2. $a_n = \begin{cases} 1 & if \quad n=1 \\ 2 & if \quad n=2 \\ a_{n-1}+a_{n-2} & if \quad n>2 \end{cases}$ Write the first 6 terms of the sequences

3. $\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6},......$ Write the n^{th} term of the sequences

4. If t_k is the k^{th} term of a GP, then show that t_{n-k}, t_k, t_{n+k} also form a GP for any positive integer k.

5. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

6. If a, b, c are respectively the p^{th}, q^{th} and r^{h} terms of a GP, show that $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$.

7. Find $\sum_{k=1}^{n} \frac{1}{k(k+1)}$.

8. Find $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$.

9. Find $\sqrt[3]{65}$

10. Find $\sqrt[3]{1001}$ approximately (two decimal places).

6.Two Dimensional Analytical Geometry

1. A straight rod of length 8 units slides with its ends A and B always on the x and y axes respectively. Find the locus of the mid point of the line segment AB

2. Show the points $\left(0,-\frac{3}{2}\right),(1,-1) \, and \left(2,-\frac{1}{2}\right)$ are collinear.

3. Express the equation $\sqrt{3}x - y + 4 = 0$ in Normal form

4. If p is length of perpendicular from origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

5. Find the equation of the lines passing through the point (1, 1)and (-2, 3)

6. Find the distance

   (i) between two points (5, 4) and (2, 0) (ii) from a point (1, 2) to the line $5x + 12y - 3 = 0$

   (iii) between two parallel lines $3x + 4y = 12$ and $6x + 8y + 1 = 0$

7. Find the nearest point on the line 2x + y = 5 from the origin.

8. Show that the lines are 3x + 2y + 9 = 0 and 12x + 8y - 15 = 0 are parallel lines.

9. Find the distance between the line 4x + 3y + 4 = 0; and a point (i) (-2, 4) (ii) (7,-3)

10. Write the equation of the lines through the point (1,-1)

(i) parallel to x + 3y - 4 = 0 (ii) perpendicular to 3x + 4y = 6

11. Find the equations of two straight lines which are parallel to the line 12x + 5y + 2 = 0 and at a unit distance from the point (1, - 1).

12. Find the distance between the parallel lines

(i) 12x + 5y = 7 and 12x + 5y + 7 = 0(ii) 3x - 4y + 5 = 0 and 6x - 8y - 15 = 0.

13. Find the combined equation of the straight lines whose separate equations are x - 2y -3 = 0 and x + y + 5 = 0.

14. Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

15. Show that $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.

16. Find the separate equation of the following pair of straight lines

(i) $3x^2 + 2xy - y^2 = 0$

(ii) $6(x - 1)^2 + 5(x - 1)(y - 2) - 4(y - 2)^2 = 0$

(iii) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$.

17. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.

18. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

19. Separate the equations $5x^2 + 6xy + y^2 = 0$

20. Find the image of the point (-2, 3) about the line x + 2y - 9 = 0.

7. Matrices and determinants

1. Find the sum *A* + *B* + *C* if *A*, *B*, *C* are given by $A=\begin{bmatrix}\sin^2\theta & 1\\ \cot^2\theta & 0\end{bmatrix}, B=\begin{bmatrix}\cos^2\theta & 0\\ -\operatorname{cosec}^2\theta & 1\end{bmatrix}, c=\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$

2. Simplify : $\sec\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & \sec\theta\end{bmatrix}-\tan\theta\begin{bmatrix}\tan\theta & \sec\theta\\ \sec\theta & \tan\theta\end{bmatrix}$

3. Construct an $m \times n$ matrix is $A=\left[a_{ij}\right]$ where a_{ij} given by $a_{ij}=\frac{(i-2j)^2}{2}$ with m = 2,n = 3

4. Find the values of *p, q, r,* and *s* if $\begin{bmatrix}p^2-1 & 0 & -31-q^3\\ 7 & r+1 & 9\\ -2 & 8 & s-1\end{bmatrix}=\begin{bmatrix}1 & 0 & -4\\ 7 & \frac{3}{2} & 9\\ -2 & 8 & -\pi\end{bmatrix}$

5. Construct the matrix $A=\left[a_{ij}\right]_{3\times3}$ where $a_{ij}=i-j$. State whether A is symmetric or skew-symmetric.

6. Find $|A|$ if $A=\begin{bmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$

7. Compute $|A|$ using Sarrus rule if $A=\begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$.

8. Find the value of x if , $A=\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix}=0$

9. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}=0$

10. Prove that $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix}=0$.

11. If $A=\begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n}\det\left(A^k\right)=\frac{1}{3}\left(1-\frac{1}{4^n}\right)$.

12. If A is a square matrix and | A | = 2, find the value of $|AA^T|$.

13. If A and B are square matrices of order 3 such that $|A|=-1$ and $|B|=3$, find the value of $|3AB|$.

14. Verify that $|AB|=|A||B|$ if $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

15. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2=\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$.

16. Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

17. Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2=\begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$.

18. Find the area of the triangle whose vertices are (0, 0), (1, 2) and (4, 3).

19. Identify the singular and non-singular matrices:

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & a-b & k \\ b-a & 0 & 5 \\ -k & -5 & 0 \end{bmatrix}$.

20. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$.

8.VECTOR ALGEBRA-1

1. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\vec{BE}+\vec{DC}=\frac{3}{2}\vec{BC}$.

2. If $\vec{PO}+\vec{OQ}=\vec{QO}+\vec{OR}$, prove that the points P, Q, R are collinear.

3. If D is the midpoint of the side BC of a triangle ABC, prove that $\vec{AB}+\vec{AC}=2\vec{AD}$

4. If G is the centroid of a triangle ABC, prove that $\vec{GA}+\vec{GB}+\vec{GC}=\vec{0}$

5. Show that the points whose position vectors are $2\hat{\imath}+3\hat{\jmath}-5\hat{k}$, $3\hat{\imath}+\hat{\jmath}-2\hat{k}$,$6\hat{\imath}-5\hat{\jmath}+7\hat{k}$ are collinear.

6. Find the value of λ for which the vectors $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+\lambda\hat{j}+3\hat{k}$ are parallel.

7. Find the unit vector parallel to $3\vec{a}-2\vec{b}+4\vec{c}$ if $\vec{a}=3\hat{i}-\hat{j}-4\hat{k}, \vec{b}=-2\hat{i}+4\hat{j}-3\hat{k}$, and $\vec{c}=\hat{i}+2\hat{j}-\hat{k}$

8. Find the value or values of m for which $m(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

9. Find $(\vec{a}+3\vec{b}).(2\vec{a}-\vec{b})$ if $\vec{a}=\hat{i}+\hat{j}+2\hat{k}, \vec{b}=3\hat{i}+2\hat{j}-\hat{k}$.

10. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ prove that $\vec{a}$ and $\vec{b}$ are perpendicular.

11. For any vector $\vec{r}$ prove that $\vec{r}=(\vec{r}.\hat{i})\hat{i}+(\vec{r}.\hat{j})\hat{j}+(\vec{r}.\hat{k})\hat{k}$.

12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors satisfying $\vec{a}-\sqrt{3}\vec{b}+\vec{c}=\vec{0}$ then find the angle between , $\vec{a}$ and $\vec{c}$.

13. Find the angle between the vectors

(i) $2\hat{i}+3\hat{j}-6\hat{k}, and\, 6\hat{i}-3\hat{j}+2\hat{k}$ (ii) $\hat{i}-\hat{j}\, and\, 6\hat{j}-\hat{k}$.

14. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$.

15. Find λ, when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.

16. Show that the vectors $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}, \vec{b}=6\hat{i}+2\hat{j}-3\hat{k}$,and $\vec{c}=3\hat{i}-6\hat{j}+2\hat{k}$ are mutually orthogonal.

17. Find $\left|\vec{a}\times\vec{b}\right|$, where $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$.

18. Find the vectors of magnitude 6 which are perpendicular to both vectors $\vec{a}=4\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$.

19. Find the area of the parallelogram whose adjacent sides are $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

20. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}\cdot\vec{b}\right)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$.

21. Show that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$.

22. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

23. Prove that $\vec{a}=\pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$.

24. Find the area of the parallelogram whose two adjacent sides are determined by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

25. For any vector $\vec{a}$ prove that $\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=2\left|\vec{a}\right|^2$.

9. DIFFERENTIAL CALCULUS – LIMITS AND CONTINUITY

1. Consider the function $f(x)=\sqrt{x}, x\geq 0$. Does $\lim\limits_{x\to 0} f(x)$ exist?

2. Evaluate $\lim\limits_{x\to 2^-}\lfloor x\rfloor$ and $\lim\limits_{x\to 2^+}\lfloor x\rfloor$

3. Let $f(x)=\begin{cases} x+1, x>0 \\ x-1, x<0 \end{cases}$. Verify the existence of limit as $x\to 0$.

4. Evaluate : $\lim\limits_{x\to 3}\frac{x^2-9}{x-3}$ if it exists by finding $f(3^-)$ and $f(3^+)$.

5. Calculate $\lim\limits_{x\to 3}(x^3-2x+6)$

6. Compute $\lim\limits_{x\to 0}\left[\frac{x^2+x}{x}+4x^3+3\right]$

7. Compute $\lim\limits_{x\to 1}\frac{x^3-1}{x-1}$

8. Find the positive integer n so that $\lim\limits_{x\to 3}\frac{x^n-3^n}{x-3}=27$

9. Show that $\lim_{n\to\infty}\frac{1^2+2^2+3^2+\ldots+(3n)^2}{(1+2+\ldots+5n)(2n+3)}=\frac{9}{25}$

10. Evaluate the limit $\lim_{x\to 3}\frac{x^2-9}{x^2\left(x^2-6x+9\right)}$

11. Evaluate : $\lim_{x\to 0}(1+x)^{\frac{1}{3x}}$

12. Evaluate : $\lim_{x\to 0}\frac{\sin\alpha x}{\sin\beta x}$

13. Evaluate : $\lim_{x\to 0}\frac{\tan 2x}{\sin 5x}$

14. Evaluate : $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

15. Evaluate : $\lim_{x\to 0}\frac{2arc\sin x}{3x}$

16. Prove that $f(x)=2x^2+3x-5$ is continuous at all points in R.

17. For what value of α is this function $f(x)=\begin{cases}\frac{x^4-1}{x-1} & x\neq 1\\ \alpha & x=1\end{cases}$ continuous at x = 1?

10. DIFFERENTIAL CALCULUS.

1. Find the derivatives of the function $f(x) = -4x + 7$ using first principle.

2. Find the derivatives from the left and from the right at $x = 1$ (if they exist) of the function $f(x) = |x - 1|$. Are the functions differentiable at $x = 1$?

3. Differentiate with respect to x : $y=\frac{\log x}{e^x}$

4. Differentiate g(t) = 4 sec t + tan t

5. Differentiate with respect to x. $y=\left(x^2+5\right)\log\left(1+x\right)e^{-3x}$

6. Differentiate with respect to x. $y=\sin x^0$

7. Differentiate with respect to x. $y=\log_{10}x$

8. Differentiate : (i) $y = \sin(x^2)$ (ii) $y = \sin^2 x$

9. Differentiate : $y = e^{\sin x}$.

10. Differentiate y = cos (tanx)

11. Differentiate : $y=e^{\sqrt{x}}$

12. Differentiate : $y=\sqrt{1+2\tan x}$

13. Differentiate : $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$

14. Find $\frac{dy}{dx}$ if $x^2+y^2=1$..

15. Differentiate . $y=x^{\sqrt{x}}$.

16. Find $\frac{dy}{dx}$ if $x=at^2; y=2at, t\neq 0$.

17. Find the derivative of y = x$^{\cos x}$

18. Find the derivative of(cos x) $^{\log x}$

19. Find the derivative of x = a cos^{3}t ; y = a sin^{3}t

20. If y = sin^{-1}x then find y''.

11. INTEGRAL CALCULUS

1. Integrate with respect to x: $\frac{\cos x}{\sin^2 x}$

2. Integrate with respect to x: $\frac{1}{(3x-2)}$

3. Integrate with respect to x: $\sec^2\frac{x}{5}$

4. Integrate with respect to x: $4\cos(5-2x)+9e^{3x-6}+\frac{24}{6-4x}$

5. If $f''(x)=12x-6$ and $f(1)=30, f'(1)=5$, find $f(x)$

6. Integrate with respect to x: $x\log x$

7. Integrate with respect to x: $25xe^{-5x}$

8. Integrate with respect to x: $e^{-x}\cos 2x$

9. Integrate with respect to x: $e^{-3x}\sin 2x$

10. Integrate with respect to x: $e^x\left(\frac{x-1}{2x^2}\right)$

11. Integrate with respect to x: $e^x\sec x\left(1+\tan x\right)$

12. Integrate with respect to x: $\frac{1}{25-4x^2}$

13. Integrate with respect to x: $\frac{1}{\sqrt{(2+x)^2-1}}$

12. INTRODUTION TO PROBABILITY THEORY

1. (i) The odds that the event A occurs is 5 to 7, find P(A).

(ii) Suppose $P(B)=\frac{2}{5}$. Express the odds that the event B occurs.

2. Given that $P(A)=0.52, P(B)=0.43$ and $P(A\cap B)=0.24$, find $P(\overline{A}\cap\overline{B})$

3. If A and B are mutually exclusive events $P(A)=\frac{3}{8}$ and $P(B)=\frac{1}{8}$, then find $P(\overline{A}\cap B)$

4. If A and B are two events associated with a random experiment for which
$P(A)=0.35, P(AorB)=0.85,$ and $P(A\ and\ B)=0.15$ Find $P(\overline{B})$

5. A die is thrown twice. Let A be the event, 'First die shows 5' and B be the event, 'second die shows 5'. Find $P(A\cup B)$

6. A die is rolled. If it shows an odd number, then find the probability of getting 5.

7. If A and B are two independent events such that $P(A)=0.4$ and $P(A\cup B)=0.9$. Find $P(B)$

8. If A and B are two events such that $P(A\cup B)=0.7, P(A\cap B)=0.2$ and $P(B)=0.5$, then show that A and B are independent.

9. If $P(A)=0.5, P(B)=0.8$ and $P(B/A)=0.8$, find $P(A/B)$ and $P(A\cup B)$.

10. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3},\frac{1}{4}$ and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

11. A year is selected at random. What is the probability that (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays

12. Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

13. Urn-I contains 8 red and 4 blue balls and urn-II contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red.

14. A factory has two machines I and II. Machine-I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item.

15. A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

XI-MATHEMATICS

IMPORTANT 5 MARK QUESTIONS

1.SETS,RELATIONS AND FUNCTIONS

Exercise - 1.3

12. If $f : R \to R$ is defined by f(x) = 3x - 5, prove that f is a bijection and find its inverse.
16. A salesperson whose annual earnings can be represented by the function A(x)=30, 000+0.04x, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function S(x) = 25, 000 + 0.05x. Find(A + S)(x) and determine the total family income if they each sell Rupees 1,50, 00, 000 worth of merchandise.
17. The function for exchanging American dollars for Singapore Dollar on a given day is f(x) =1.23x, where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is g(y) = 50.50y, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.
19. The formula for converting from Fahrenheit to Celsius temperatures is y = $\frac{5x}{9} - \frac{160}{9}$.

    Find the inverse of this function and determine whether the inverse is also a function.

Exercise - 1.2

2. Let X = $\{a, b, c, d\}$ and R = {(a, a), (b, b), (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
3. Let A = $\{a, b, c\}$ and R = {(a, a), (b, b), (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
5. On the set of natural numbers let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
7. On the set of natural numbers let R be the relation defined by aRb if a + b ≤ 6. Write down the relation by listing all the pairs. Check whether it is
   (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence
9. In the set Z of integers, define mRn if m - n is divisible by 7. Prove that R is an equivalence relation.

Example 1.11 Let S = $\{1, 2, 3\}$ and ρ = {(1, 1), (1, 2), (2, 2), (1, 3), (3, 1)}

(i) Is ρ reflexive? If not, state the reason and write the minimum set of ordered pairs to be included to ρ so as to make it reflexive.

(ii) Is ρ symmetric? If not, state the reason, write minimum number of ordered pairs to be included to ρ so as to make it symmetric and write minimum number of ordered pairs to be deleted from ρ so as to make it symmetric.

(iii) Is ρ transitive? If not, state the reason, write minimum number of ordered pairs to be included to ρ so as to make it transitive and write minimum number of ordered pairs to be deleted from ρ so as to make it transitive.

(iv) Is ρ an equivalence relation? If not, write the minimum ordered pairs to be included to ρ so as to make it an equivalence relation.

Example 1.12 Let $A=\{0,1,2,3\}$. Construct relations on A of the following types:

(i) not reflexive, not symmetric, not transitive.
(ii) not reflexive, not symmetric, transitive.
(iii) not reflexive, symmetric, not transitive.
(iv) not reflexive, symmetric, transitive.
(v) reflexive, not symmetric, not transitive.
(vi) reflexive, not symmetric, transitive.
(vii) reflexive, symmetric, not transitive.
(viii) reflexive, symmetric, transitive.

Example 1.13 In the set Z of integers, define mRn if m- n is a multiple of 12. Prove that R is an equivalence relation.

Example 1.30 If $f: R\to R$ is defined by f(x) = 2x - 3 prove that f is a bijection and find its inverse.

2. BASIC ALGEBRA

Exercise - 2.9

Resolve the following rational expressions into partial fractions.

3. $\dfrac{x}{(x^2+1)(x-1)(x+2)}$ 4. $\dfrac{x}{(x-1)^3}$

5. $\dfrac{1}{x^4-1}$ 6. $\dfrac{(x-1)^2}{x^3+x}$ 7. $\dfrac{x^2+x+1}{x^2-5x+6}$ 8. $\dfrac{x^3+2x+1}{x^2+5x+6}$

9. $\dfrac{x^1+12}{(x+1)^2(x-2)}$ 10. $\dfrac{6x^2-x+1}{x^3+x^2+x+1}$ 11. $\dfrac{2x^2+5x-11}{x^2+2x-3}$ 12. $\dfrac{7+x}{(1+x)(1+x^2)}$

Exercise - 2.11

5. Find the radius of the spherical tank whose volume is $\dfrac{32\pi}{3}$ units.

7. Solve $(x+1)^{\frac{1}{3}}=\sqrt{x-3}$

8. Simplify by rationalising the denominator. $\dfrac{7+\sqrt{6}}{3-\sqrt{2}}$

Exercise - 2.12

7. Prove that $\log 2+16\log\dfrac{16}{15}+12\log\dfrac{25}{24}+7\log\dfrac{81}{80}=1$

10. If $\dfrac{\log x}{y-z}=\dfrac{\log y}{z-x}=\dfrac{\log z}{x-y}$ then prove that xyz = 1.

Exercise - 2.8

1. Find all values of x for which $\dfrac{x^3(x-1)}{(x-2)}>0$

2. Find all values of x that satisfies the inequality $\dfrac{2x-3}{(x-2)(x-4)}<0$

3. Solve $\frac{x^2-4}{x^2-2x-15} \le 0$

4. Solve $\frac{x-2}{x+4} \ge \frac{5}{x+3}$

Exercise - 2.3

6. A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?
9. A Plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will paid rupees 120 per hour. If he works x hours to complete the job, then for what value of x does the first scheme give better wages?
10. A and B are working on similar jobs but their monthly salaries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?

Exercise - 2.4

7. If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots, then prove that $ae = 2(b + f)$.
4. If one root of $k(x- 1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or $- 25$.

Example 2.26 Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$

Example 2.27 Resolve into partial fractions: $\frac{x+1}{x^2(x-1)}$

Example 2.24 Solve $\frac{x+1}{x+3} < 3$

Example 2.15 Solve the equation

Example 2.13 Solve $3x^2 + 5x -2 \le 0$

Example 2.14 Solve $\sqrt{x+14} < x+2$.

Example 2.12 Find the number of solutions of $x^2+|x-1|=1$

Example 2.36 Prove $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$

Example 2.40 Given that $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.47712$ (approximately), find the number of digits in $2^8.3^{12}$.

3. Trigonometry

Exercise - 3.3

4. Prove that $\frac{\cot(180^0+\theta)\sin(90^0-\theta)\cos(-\theta)}{\sin(270^0+\theta)\tan(-\theta)\operatorname{cosec}(360^0+\theta)} = \cos^2\theta\cot\theta.$

5. Find all the angles between 0^0 and 360^0 which satisfy the equation $\sin^2\theta = \frac{3}{4}$.

6. Show that $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$

Example 3.12 Find the value of (i) $\sin 765^0$ (ii) $\operatorname{cosec}(-1410^0)$ (iii) $\cot\left(\frac{-15\pi}{4}\right)$

Example 3.13 Prove that $\tan(315^0)\cot(-405^0) + \cot(495^0)\tan(-585^0) = 2$

Exercise - 3.4

7. Find a quadratic equation whose roots are sin 15^0 and cos 15^0.

16. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, find the value of xy + yz + zx.

19. If cos(α-β) + cos(β-γ) + cos(γ-α) = $-\frac{3}{2}$, then prove that

cos α + cos β+ cos γ = sin α+ sin β+ sin γ = 0.

25. If θ+∅= α and tanθ = k tan∅, then prove that $\sin(\theta-\phi)=\frac{k-1}{k+1}\sin\alpha$.

Exercise - 3.5

4. Prove that cos5 θ = 16 $\cos^5\theta$ - 20 $\cos^3\theta$ + 5cos θ.

6. If A + B = 45^0, show that (1 + tanA) (1 + tanB) = 2.

9. Show that $\cot\left(7\frac{1^0}{2}\right)=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}.$

10. Prove that (1 + sec2θ)(1 + sec4θ) ………… (1+sec $2^n\theta$) = tan $2^n\theta$ cotθ.

11. Prove that $32(\sqrt{3})\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}=3.$

Exercise - 3.6

3. Show that sin 12^0 sin 48^0 sin 54^0 = $\frac{1}{8}$.

4. Show that $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}=\frac{1}{128}$

Example 3.38 Show that cos 10^0 cos 30^0 cos 50^0 cos 70^0 = $\frac{3}{16}$

Example 3.36 Show that cos 36^0 cos 72^0 cos 108^0 cos 144^0 = $\frac{1}{16}$

Exercise - 3.3

4. Prove that $\frac{\cot(180^0+\theta)\ \sin(90^0-\theta)\ \cos(-\theta)}{\sin(270^0+\theta)\ \ \tan(-\theta)\operatorname{cosec}(360^0+\theta)}=\cos^2\theta\cot\theta.$

6. Show that $\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}=2$

Prove that cos(α + β) = cos α cos β **-** sinα sinβ

Example 3.19 A ripple tank demonstrates the effect of two water waves being added together. The two waves are described by h = 8 cos t and h = 6 sin t, where t∈ [0, 2π) is in seconds and h is the height in millimeters above still water. Find the maximum height of the resultant wave and the value of t at which it occurs

Example 3.29 Find the values of (i) sin 18^0 (ii) cos 18^0 (iii) sin 72^0 (iv) cos 36^0 (v) sin 54^0

Example 3.30 If $\tan\frac{\theta}{2}=\sqrt{\frac{1-a}{1+a}}\tan\frac{\phi}{2}$, then prove that $\cos\phi=\frac{\cos\theta-a}{1-a\cos\theta}$

4.Combinatorics andMathematical Induction

Exercise - 4.2

8. 8 women and 6 men are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position?

(ii) In how many arrangements will all 6 men be standing next to one another?

(iii) In how many arrangements will no two men be standing next to one another?

14. How many strings are there using the letters of the word INTERMEDIATE, if

(i) The vowels and consonants are alternative

(ii) All the vowels are together

(iii) Vowels are never together

(iv) No two vowels are together.

15. Each of the digits 1, 1, 2, 3, 3 and 4 is written on a separate card. The six cards are then laid out in a row to form a 6-digit number.

(i) How many distinct 6-digit numbers are there?

(ii) How many of these 6-digit numbers are even?

(iii) How many of these 6-digit numbers are divisible by 4?

16. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) GARDEN (ii) DANGER.

17. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85^{th} string?

Exercise - 4.3

17. Find the number of ways of forming a committee of 5 members out of 7Indians and 5 Americans, so that always Indians will be the majority in the committee.

18. A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of

(i) exactly 3 women?

(ii) at least 3 women?

(iii) at most 3 women?

19. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife's relatives?

20. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

21. Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION?

24. There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find,

(i) the number of straight lines that can be obtained from the pairs of these points?

(ii) the number of triangles that can be formed for which the points are their vertices?

Exercise - 4.4

1. By the principle of mathematical induction, prove that, for $n \geq 1$,

$$1^3+2^3+3^3+........+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

2. By the principle of mathematical induction, prove that, for n ≥1,

$$1^2+3^2+5^2+........+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$$

4. By the principle of Mathematical induction, prove that, for n ≥1,

$$1.2+2.3+3.4+........+n.(n+1)=\frac{n(n+1)(n+2)}{3}.$$

5. Using the Mathematical induction, show that for any natural number n≥2.

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)...........\left(1-\frac{1}{n^2}\right)=\frac{(n+1)}{2n}$$

6. Using the Mathematical induction, show that for any natural number n≥2,

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+........+\frac{1}{1+2+3+.......+n}=\frac{n-1}{n+1}$$

7. Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+........+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}.$$

8. Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+........+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}.$$

9. Prove by Mathematical Induction that

$1!+(2\times2!)+(3\times3!)+...........+(n\times n!)=(n+1)!-1.$

10. Using the Mathematical induction, show that for any natural number n,
$x^{2n}-y^{2n}$ is divisible by x + y.

11. By the principle of Mathematical induction, prove that, for n≥1,

$$1^2+2^2+3^2+........+n^2>\frac{n^3}{3}$$

13. Use induction to prove that $5^{n+1}+4\times6^n$ when divided by 20 leaves a remainder 9, for all natural numbers n.

14. Use induction to prove that $10^n+3\times4^{n+2}+5$, is divisible by 9, for all natural numbers n.

Example 4.56 An exam paper contains 8 questions, 4 in Part A and 4 in Part B. Examiners are required to answer 5 questions. In how many ways can this be done if (i) There are no restrictions of choosing a number of questions in either parts.
(ii) At least two questions from Part A must be answered.

Example 4.34 A van has 8 seats. It has two seats in the front with two rows of three seats behind. The van belongs to a family, consisting of seven members, F, M, S_1, S_2, S_3, D_1, D_2. How many ways can the family sit in the van if
(i) There are no restriction?
(ii) Either F or M drives the van?
(iii) D_1, D_2 sits next to a window and F is driving?

Example 4.35 If the letters of the word TABLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order),find the ranks of the words (i) TABLE, (ii) BLEAT

Example 4.58 Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION.

Example 4.42 If the letters of the word IITJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE.

Example 4.63 By the principle of mathematical induction, prove that, for all integers n ≥ 1,

$$1^2+2^2+3^2+........+n^2=\frac{n(n+1)(2n+1)}{6}$$

Example 4.64 Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+........+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

Example 4.65 Prove that for any natural number n, $a^n - b^n$ is divisible by a - b, where a > b.

Example 4.66 Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all n ≥ 1.

Example 4.67 Using the Mathematical induction, show that for any integer $n \geq 2, 3n^2 > (n+1)^2$

Example 4.70 Using the Mathematical induction, show that for any natural number n; with the assumption $i^2 = -1, (r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$.

5.Binomial Theorem,Sequences and Series

Exercise - 5.2

4. The product of three increasing numbers in GP is 5832. If we add 6 to the second number and 9 to the third number, then resulting numbers form an AP. Find the numbers in GP.

7. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

10. If a, b, c are respectively the p^{th}, q^{th} and r^{h} terms of a GP, show that $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$.

Exercise - 5.4

2. Find $\sqrt[3]{1001}$ approximately (two decimal places).

3. Prove that $\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

4. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1-x+\frac{x^2}{2}$ when x is very small.

8. If p - q is small compared to either p or q, then show that $\sqrt[n]{\frac{p}{q}} \approx \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$

   Hence find $\sqrt[8]{\frac{15}{16}}$

10. Find the value of $\sum_{n=1}^{\infty}\frac{1}{2n-1}\left(\frac{1}{9^{n-1}}+\frac{1}{9^{2n-1}}\right)$

Example 5.24 Find $\sqrt[3]{65}$

Example 5.25 Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

Example 5.20 Find $\sum_{n=1}^{\infty}\frac{1}{n^2+5n+6}$.

Example 5.25 Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

6.Two Dimensional Analytical Geometry

Exercise - 6.1

9. The coordinates of a moving point P are $\left(\frac{a}{2}(\operatorname{cosec}\theta+\sin\theta),\frac{b}{2}(\operatorname{cosec}\theta-\sin\theta)\right)$, where θ is a variable parameter. Show that the equation of the locus P is $b^2x^2 - a^2y2 = a^2b^2$.

13. If Q is a point on the locus of $x^2 + y^2 + 4x - 3y + 7 = 0$, then find the equation of locus of P which divides segment OQ externally in the ratio 3:4,where O is origin.

15. The sum of the distance of a moving point from the points (4, 0) and (-4, 0) is always 10 units.Find the equation of the locus of the moving point.

Exercise - 6.2

5. The normal boiling point of water is 100°C or 212°F and the freezing point of water is 0°C or 32° F. (i) Find the linear relationship between C and F. Find (ii) the value of C for 98.6°F and (iii) the value of F for 38°C

6. An object was launched from a place P in constant speed to hit a target. At the 15th second it was 1400m away from the target and at the 18th second 800m away. Find (i) the distance between the place and the target (ii) the distance covered by it in 15 seconds. (iii) time taken to hit the target.

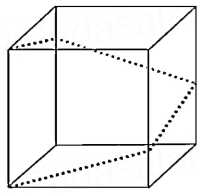
12. A 150m long train is moving with constant velocity of 12.5 m/s. Find (i) the equation of the motion of the train, (ii) time taken to cross a pole. (iii) The time taken to cross the bridge of length 850m is?

13. A spring was hung from a hook in the ceiling. A number of different weights were attached to the spring to make it stretch, and the total length of the spring was measured each time shown in the following table.

Weight,(kg)	2	4	5	8
Length,(cm)	3	4	4.5	6

(i) Draw a graph showing the results.
(ii) Find the equation relating the length of the spring to the weight on it.
(iii) What is the actual length of the spring.
(iv) If the spring has to stretch to 9 cm long, how much weight should be added?
(v) How long will the spring be when 6 kilograms of weight on it?

15. In a shopping mall there is a hall of cuboid shape with dimension 800 x 800 x 720 units, which needs to be added the facility of an escalator in the path as shown by the dotted line in the figure. Find (i)the minimum total length of the escalator.
(ii) the heights at whichthe escalator changes its direction.
(iii) the slopes of the escalator at the turning points.



11. A straight line is passing through the point A(1, 2) with slope $\left(\frac{5}{12}\right)$. Find points on the line which are 13 units away from A.

Exercise - 6.3

11. If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines x sec θ + y cosec θ = 2a and x cos θ - y sin θ = a cos 2θ, then prove that $P_1^2+P_2^2=a^2$.

18. A photocopy store charges Rs 1.50 per copy for the first 10 copies and Rs 1.00 per copy

after the 10th copy. Let x be the number of copies, and let y be the total cost of photocopying. (i) Draw graph of the cost as x goes from 0 to 50 copies. (ii) Find the cost of making 40 copies

Exercise - 6.4

5. Prove that the equation to the straight lines through the origin, each of which makes an angle α with the straight line $y = x$ is $x^2 - 2xy \sec 2\alpha + y^2 = 0$.
8. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.
9. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.
10. A ΔOPQ is formed by the pair of straight lines $x^2 -4xy +y^2 = 0$ and the line PQ. The equation of PQ is $x + y - 2 = 0$. Find the equation of the median of the triangle ΔOPQ drawn from the origin O.
15. Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.
16. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$
18. Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles.
14. Show that the equation $9x^2-24xy+16y^2-12x+16y-12 = 0$ represents a pair of parallel lines. Find the distance between them.

Example 6.6 If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $(a(\theta - \sin \theta), a(1 - \cos \theta))$.

Example 6.10 The Pamban Sea Bridge is a railway bridge of length about 2065 m constructed on the PalkStrait, which connects the Island town of Rameswaram to Mandapam, the main land of India. The Bridge is restricted to a uniform speed of only 12.5 m/s. If a train of length 560m starts at the entry point of the bridge from Mandapam, then

(i) find an equation of the motion of the train.
(ii) when does the engine touch island
(iii) when does the last coach cross the entry point of the bridge
(iv) what is the time taken by a train to cross the bridge.

Example 6.12 The seventh term of an arithmetic progression is 30 and tenth term is 21.

(i) Find the first three terms of an A.P.
(ii) Which term of the A.P. is zero (if exists)
(iii) Find the relation ship between Slope of the straight line and common difference of A.P.

Example 6.13 The quantity demanded of a certain type of Compact Disk is 22,000 units when a unit price is Rs 8. The customer will not buy the disk, at a unit price of Rs 30 or higher. On the other side the manufacturer will not market any disk if the price is Rs 6 or lower. However, if the price Rs 14 the manufacturer can supply 24,000 units. Assume that the quantity demanded and quantity supplied are linearly proportional to the price. Find (i) the demand equation (ii) supply equation (iii) the market equilibrium quantity and price. (iv) The quantity of demand and supply when the price is Rs 10.

Example 6.15 A straight line L with negative slope passes through the point (9, 4) cuts the positive coordinate axes at the points P and Q. As L varies, find the minimum value of $|OP|+|OQ|$, where O is the origin.

Example 6.18 Find the equation of the lines make an angle 60° with positive x-axis and at a distance $5\sqrt{2}$ units measured from the point (4, 7), along the line $x - y + 3 = 0$.

Example 6.19 Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form:

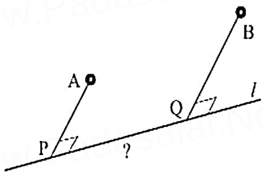
(i) Slope and Intercept form

(ii) Intercept form

(iii) Normal form

Example 6.21 Consider a hollow cylindrical vessel, with circumference 24 cm and height 10 cm. An ant is located on the outside of vessel 4 cm from the bottom. There is a drop of honey at the diagrammatically opposite inside of the vessel, 3 cm from the top. (i) What is the shortest distance the ant would need to crawl to get the honey drop? (ii) Equation of the path traced out by the ant. (iii) Where the ant enter in to the cylinder? Here is a picture that illustrates the position of the ant and the honey.

Example 6.30 Suppose the Government has decided to erect a new Electrical Power Transmission Substation to provide better power supply to two villages namely A and B. The substation has to be on the line *l*. The distances of villages A and B fromthe foot of the perpendiculars P and Q on the line *l* are3 km and 5 km respectively and the distance between P andQ is 6 km. (i) What is the smallest length of cable required to connect the two villages. (ii) Find the equations of the cable lines that connect the power station to two villages. (Using the knowledge in conjunction with the principle of reflection allows for approach to solve this problem.)



Example 6.35 Find the equation of the pair of lines through the origin and perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$.

Example 6.36 Show that the straight lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle.

Example 6.38 If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, find (i) the value of λ and the separate equations of the lines (ii) point of intersection of the lines (iii) angle between the lines.

Example 6.39 A student when walks from his house, at an average speed of 6 kmph, reaches his school ten minutes before the school starts. When his average speed is 4 kmph, he reaches his school five minutes late. If he starts to school every day at 8.00 A.M, then find (i) the distance between his house and the school (ii) the minimum average speed to reach the school on time and time taken to reach the school (iii) the time the school gate closes (iv) the pair of straight lines of his path of walk.

Example 6.40 If one of the straight lines of $ax^2+2hxy+by^2 = 0$ is perpendicular to $px + qy = 0$, then show that $ap^2 + 2hpq + bq^2 = 0$

Example 6.41 Show that the straight lines joining the origin to the points of intersection of $3x - 2y + 2 = 0$ and $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at right angles.

IMPORTANT FIVE MARKS(VOL-II)

7. Matrices and determinants

EXERCISE 7.1

13.Verify the property $A(B+C)=AB+AC$, when the matrices A, B, and C are given by

$$A=\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}, B=\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}, c=\begin{bmatrix}4 & 7\\ 2 & 1\\ 1 & -1\end{bmatrix}$$

18.Find the matrix A such that . $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A^T=\begin{bmatrix}-1 & -8 & -10\\ 1 & 2 & -5\\ 9 & 22 & 15\end{bmatrix}$.

EXERCISE 7.2

(2) Show that $\begin{vmatrix}b+c & bc & b^2c^2\\ c+a & ca & c^2a^2\\ a+b & ab & a^2b^2\end{vmatrix}=0$

(3) Prove that $\begin{vmatrix}a^2 & bc & ac+c^2\\ a^2+ab & b^2 & ac\\ ab & b^2+bc & c^2\end{vmatrix}=4a^2b^2c^2$

(4) Prove that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(8) If $\begin{vmatrix}a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}=0$ prove that a, b, c are in $G.P.$ or α is a root of $ax^2+2bx+c=0$

(11) Show that is $\begin{vmatrix}a^2+x^2 & ab & ac\\ ab & b^2+x^2 & bc\\ ac & bc & c^2+x^2\end{vmatrix}$ divisible by x^4.

(12) If a, b, c are all positive, and are p^{th}, q^{th} and r^{th} terms of a $G.P.$, show that $\begin{vmatrix}\log a & p & 1\\ \log b & q & 1\\ \log c & r & 1\end{vmatrix}=0$

(14) If $A=\begin{bmatrix}\frac{1}{2} & \alpha\\ 0 & \frac{1}{2}\end{bmatrix}$, prove that $\sum_{k=1}^{n}\det\left(A^k\right)=\frac{1}{3}\left(1-\frac{1}{4^n}\right)$.

EXERCISE 7.3

(1) Show that $\begin{vmatrix}x & a & a\\ a & x & a\\ a & a & x\end{vmatrix}=(x-a)^2(x+2a)$.

(2) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.

(4) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

(5) Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$

(6) Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

Example 7.23 Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$

Example 7.24 Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

Example 7.25 Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$.

Example 7.29 Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

Example 7.30 Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$.

Example 7.31 If A_i, B_i, C_i are the cofactors of, a_i, b_i, c_i respectively, $i = 1$ to 3 in ,

$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$,show that $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$.

8.VECTOR ALGEBRA-I

EXERCISE 8.1

(5) Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

(6) Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

(12) If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$.

EXERCISE 8.2

(4) A triangle is formed by joining the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). Find the direction cosines of the medians.

(7) Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, 3\hat{i}-4\hat{j}-4\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ form a right angled triangle.

(9) Show that the following vectors are coplanar

(i) $\hat{i}-2\hat{j}+3\hat{k}, -2\hat{i}+3\hat{j}-4\hat{k}, -\hat{j}+2\hat{k}$.

(ii) $5\hat{i}+6\hat{j}+7\hat{k}, 7\hat{i}-8\hat{j}+9\hat{k}, 3\hat{i}+20\hat{j}+5\hat{k}$.

(10) Show that the points whose position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

(12) The position vectors of the vertices of a triangle are $\hat{i}+2\hat{j}+3\hat{k}, 3\hat{i}-4\hat{j}-5\hat{k}$, and $-2\hat{i}+3\hat{j}-7\hat{k}$. Find the perimeter of the triangle.

(17) Show that the points *A* (1, 1, 1), *B* (1, 2, 3) and *C*(2, - 1, 1) are vertices of an isosceles triangle.

EXERCISE 8.3

(7) Show that the vectors $-\hat{i}-2\hat{j}-6\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$, and $-\hat{i}+3\hat{j}+5\hat{k}$ form a right angled triangle.

(10) If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that ,

(i) $\sin\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}-\vec{b}\right|$ (ii) $\cos\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}+\vec{b}\right|$ (iii) $\tan\frac{\theta}{2}=\frac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}+\vec{b}\right|}$

(14) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\left|\vec{a}\right|=2, \left|\vec{b}\right|=3, \left|\vec{c}\right|=4$, and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$.

Find $4\vec{a}.\vec{b}+3\vec{b}.\vec{c}+3\vec{c}.\vec{a}$.

EXERCISE 8.4

(4) Find the unit vectors perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

(6) Find the area of the triangle whose vertices are A(3, - 1, 2), B(1, - 1, - 3) and C(4, - 3, 1).

(7) If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices *A, B, C* of a triangle *ABC*, show that the area of the triangle *ABC* is $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$. Also deduce the condition for collinearity of the points *A, B,* and *C.*

(8) For any vector $\vec{a}$ prove that $\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=2\left|\vec{a}\right|^2$

(9) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

Prove that $\vec{a}=\pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$.

Example 8:9 Prove that the points whose position vectors $2\hat{\imath}+4\hat{\jmath}+3\hat{k}$, $4\hat{\imath}+\hat{\jmath}+9\hat{k}$ and $10\hat{\imath}-\hat{\jmath}+6\hat{k}$ form a right angled triangle.

Example 8.10 Show that the vectors $5\hat{\imath}+6\hat{\jmath}+7\hat{k}$, $7\hat{\imath}-8\hat{\jmath}+9\hat{k}$, $3\hat{\imath}+20\hat{\jmath}+5\hat{k}$ are coplanar.

Example 8.19 Show that the points (4, - 3, 1), (2, - 4, 5) and (1, - 1, 0) form a right angled triangle.

Example 8.23 Find the cosine and sine angle between the vectors $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$.

Example 8.26 Find the area of a triangle having the points as its vertices. A(1,0,0),B(0,1,0), and C(0,0,1).

Theorem 8.1 (Section Formula - Internal Division)

Let O be the origin. Let A and B be two points. Let P be the point which divides the line segment AB internally in the ratio $m : n$. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, then the position vector $\vec{op}$ of P is given by $\vec{op}=\dfrac{n\vec{a}+m\vec{b}}{n+m}$.

Theorem 8.3 The medians of a triangle are concurrent.

Theorem 8.4 A quadrilateral is a parallelogram if and only if its diagonals bisect each other

9. DIFFERENTIAL CALCULUS – LIMITS AND CONTINUITY

EXERCISE 9.1

Sketch the graph of f, then identify the values of x_0 for which $\lim\limits_{x\to x_0} f(x)$ exists.

(16) $f(x)=\begin{cases} x^2 & x\le 2 \\ 8-2x & 2<x<4 \\ 4 & x\ge 4 \end{cases}$

(17) $f(x)=\begin{cases} \sin x & x<0 \\ 1-\cos x & 0\le x\le \pi \\ \cos x & x>\pi \end{cases}$

(23) Verify the existence of $\lim\limits_{x\to 1} f(x)$ where $f(x)=\begin{cases} \dfrac{|x-1|}{x-1}, & for \quad x\ne 1 \\ 0, & for \quad x=1 \end{cases}$.

EXERCISE 9.2

Evaluate the following limits:

(8) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$ (10) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$ (11) $\lim\limits_{x\to 2}\dfrac{2-\sqrt{x+2}}{\sqrt[3]{2}-\sqrt[3]{4-x}}$

(12) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1}{x}$ (15) $\lim\limits_{x\to a}\dfrac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}(a>b)$

EXERCISE 9.4

Evaluate the following limits:

(22) $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$ (23) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ (24) $\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+1}{x^2-4x+2}\right)^x$

(4) $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+3}{2x^2+5}\right)^{8x^2+3}$ (28) $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$ (16) $\lim\limits_{x\to 0}\dfrac{3^x-1}{\sqrt{x+1}-1}$

(18) $\lim\limits_{x\to\infty} x\left[3^{\frac{1}{x}}+1-\cos\left(\dfrac{1}{x}\right)-e^{\frac{1}{x}}\right]$ (26) $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}$

EXERCISE 9.5

(9) Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

(i) $f(x)=\begin{cases}2x+1 & x\le -1\\ 3x & -1<x<1\\ 2x-1 & x\ge 1\end{cases}$ (ii) $f(x)=\begin{cases}(x-1)^3 & x<0\\ (x+1)^3 & x\ge 0\end{cases}$

(10) A function f is defined as follows:

$f(x)=\begin{cases}0 & x<0\\ x & 0\le x<1\\ -x^2+4x-2 & 1\le x<3\\ 4-x & x\ge 3\end{cases}$ Is the function continuous?

(7) Let $f(x)=\begin{cases}0 & x<0\\ x^2 & 0\le x<2\\ 4 & x\ge 2\end{cases}$. Graph the function. Show that $f(x)$ continuous on (- ∞, ∞).

(3) Find the points of discontinuity of the function f, where

(i) $f(x)=\begin{cases}4x+5 & x\le 3\\ 4x-5 & x>3\end{cases}$ (ii) $f(x)=\begin{cases}x+2 & x\ge 2\\ x^2 & x<2\end{cases}$

(iii) $f(x)=\begin{cases}x^3-3 & x\le 2\\ x^2+1 & x>2\end{cases}$ (iv) $f(x)=\begin{cases}\sin x & 0\le x\le \dfrac{\pi}{4}\\ \cos x & \dfrac{\pi}{4}<x<\dfrac{\pi}{2}\end{cases}$

(5) Show that the function $\begin{cases}\dfrac{x^3-1}{x-1} & x\ne 1\\ 3 & x=1\end{cases}$ is continuous on (- ∞, ∞)

Example 9.6 Test the existence of the limit $\lim\limits_{x\to 1}\dfrac{4|x-1|+x-1}{|x-1|}, x\ne 1.$

Example 9.31 Show that $\lim\limits_{x\to 0^+} x\left[\left\lfloor\dfrac{1}{x}\right\rfloor+\left\lfloor\dfrac{2}{x}\right\rfloor+.....+\left\lfloor\dfrac{15}{x}\right\rfloor\right]=120$

Example 9.34 Evaluate : $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$

Example 9.33 Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x-2}\right)^n$

Example 9.26 According to Einstein's theory of relativity, the mass m of a body moving with velocity v is $m=\dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$, where m_0 is the initial mass and c is the speed of light. What happen to m as $v\to c^-$. Why is a left hand limit necessary?

Example 9.28

Suppose that the diameter of an animal's pupils is given by $f(x)=\dfrac{160x^{-0.4}+90}{4x^{-0.4}+15}$, where x is the intensity of light and $f(x)$ is in mm. Find the diameter of the pupils with (a) minimum light (b) maximum light.

Example 9.35

Do the limits of following functions exist as $x\to 0$? State reasons for your answer.

(i) $\dfrac{\sin|x|}{x}$ (ii) $\dfrac{\sin x}{|x|}$ (iii) $\dfrac{x\lfloor x\rfloor}{\sin|x|}$ (iv) $\dfrac{\sin\left(x-\lfloor x\rfloor\right)}{x-\lfloor x\rfloor}$

Example 9.36 Evaluate : $\lim\limits_{x\to 0}\dfrac{3^x-1}{\sqrt{1+x}-1}$

Example 9.37 Describe the interval(s) on which each function is continuous.

(i) $f(x)=\tan x$ (ii) $g(x)=\begin{cases}\sin\dfrac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$ (iii) $h(x)=\begin{cases}x\sin\dfrac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$

Example 9.7 Calculate $\lim\limits_{x\to 3}(x^3-2x+6)$

10. DIFFERENTIAL CALCULUS.

EXERCISE 10.1

(3) Determine whether the following function is differentiable at the indicated values.

(i) $f(x) = x\,|x|$ at $x = 0$ (ii) $f(x) = |x^2 - 1|$ at $x = 1$

(iii) $f(x) = |x| + |x - 1|$ at $x = 0,1$ (iv) $f(x) = \sin |x|$ at $x = 0$.

(6) If $f(x) = |x + 100| + x^2$, test whether $f'(100)$ exists.

EXERCISE 10.3

(28) $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$ (30) $y=\sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

EXERCISE 10.4

(20) Find the derivative of $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}x$.

(21) If $u=\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$ and $v=\tan^{-1}x$, find $\dfrac{du}{dv}$.

(22) Find the derivative with $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$

(23) If $y = \sin^{-1}x$ then find y''.

(24) If $y=e^{\tan^{-1}x}$ show that $(1+x^2)y''+(2x-1)y'=0$

(25) If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2-3xy_1-y=0$.

(26) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ then prove that at $\theta=\dfrac{\pi}{2}, y''=\dfrac{1}{a}$.

(27) If $\sin y = x\sin(a+y)$ then prove that $\dfrac{dy}{dx}=\dfrac{\sin^2(a+y)}{\sin a}, a\neq n\pi$.

(28) If $y=\left(\cos^{-1}x\right)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$. Hence find y_2 when x=0.

(12) Find the derivative $\cos\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$.

Example 10.36 Find $\frac{d^2y}{dx^2}$ if $x^2+y^2=4$.

Example 10.34 Find y'' if $x^4+y^4=16$.

Example 10.30 Differentiate $\sin\left(ax^2+bx+c\right)$ with respect to $\cos\left(lx^2+mx+n\right)$.

Example 10.29 Find the derivative of $\tan^{-1}(1+x^2)$ with respect to x^2+x+1.

Example 10.34 Find y'' if $x^4+y^4=16$.

11. INTEGRAL CALCULUS

EXERCISE 11.4

(4) A ball is thrown vertically upward from the ground with an initial velocity of 39.2 m/sec. If the only force considered is that attributed to the acceleration due to gravity, find

(i) how long will it take for the ball to strike the ground?

(ii) the speed with which will it strike the ground? and

(iii) how high the ball will rise?

(5) A wound is healing in such a way that *t* days since Sunday the area of the wound has been decreasing at a rate of $\frac{-3}{(t+2)^2}cm^2$ per day. If on Monday the area of the wound was

(i) What was the area of the wound on Sunday?

(ii) What is the anticipated area of the wound on Thursday if it continues to heal at the same rate?

EXERCISE 11.7

Integrate the following with respect to *x*:

(3) (i) $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ (ii) $x^5e^{x^2}$ (iii) $\tan^{-1}\left(\frac{8x}{1-16x^2}\right)$ (iv) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

EXERCISE 11.11

Integrate the following with respect to x :

(1) (i) $\frac{2x-3}{x^2+4x-12}$ (ii) $\frac{5x-2}{2+2x+x^2}$ (iii) $\frac{3x+1}{2x^2-2x+3}$

(2) (i) $\frac{2x+1}{\sqrt{9+4x-x^2}}$ (ii) $\frac{x+2}{\sqrt{x^2-1}}$ (iii) $\frac{2x+3}{\sqrt{x^2+4x+1}}$

Examples 11.40

Evaluate the following integrals

(i) $\int\frac{3x+5}{x^2+4x+7}dx$ (ii) $\int\frac{x+1}{x^2-3x+1}dx$ (iii) $\int\frac{5x-7}{\sqrt{3x-x^2-2}}dx$

Example 11.34 Evaluate: $\int\tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Example 11.32

Integrate the following with respect to x.

(i) $\int \frac{2x+4}{x^2+4x+6}dx$ (iv) $\int \frac{\sin x+\cos x}{\sin x-\cos x}dx$ (v) $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

12. INTRODUTION TO PROBABILITY THEORY

EXERCISE 12.2

(5) A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96.

(i) What is the probability that a fire engine is available when needed?

(ii) What is the probability that neither is available when needed?

(6) The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards.

EXERCISE 12.3

(6) A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3},\frac{1}{4}$ and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

(7) The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15.

(i) If the oil had to be changed, what is the probability that a new oil filter is needed?

(ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

(8) One bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. If one ball is drawn from each bag, find the probability that (i) both are white (ii) both are black (iii) one white and one black(10) Given $P(A)=0.4$ and $P(A\cup B)=0.7$ Find $P(B)$ if

(i) A and B are mutually exclusive (ii) A and B are independent events

(iii) $P(A/B)=0.4$ (iv) $P(B/A)=0.5$

(11) A year is selected at random. What is the probability that (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays

(12) Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

EXERCISE 12.4

(1) A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

(2) There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

(3) A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,

(i) find the probability that the selected pipe is a defective one.

(ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ?

(4) The chances of A, B and C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B, and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?

(5) An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.

Example 12.26

A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

Example 12.27

A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

Example 12.28

The chances of X, Y and Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

Example 12.29

A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?

Example 12.21

An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane in the first, second, third, and fourth shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane.

Example 12.22

X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

Example 12.23

A main road in a City has 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the probability of 0.4 and 0.6 respectively. Determine the probability of

(i) a car crossing the first crossroad without stopping

(ii) a car crossing first two crossroads without stopping

(iii) a car crossing all the crossroads, stopping at third cross.

(iv) a car crossing all the crossroads, stopping at exactly one cross.

Example 12.15

The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get atleast one of the two jobs (ii) she will get only one of the two jobs.