

1. Write the use of horizontal line test.

The given curve represents a
one-to-one (or) onto function (or)
not by drawing horizontal line.

2. Is it correct to say $A \times A = \{(a, a) : a \in A\}$?
Justify your answer?

No

$$A \times A = \{(a, b) : a, b \in A\}.$$

3. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ find

$$n[(A \cup B) \times (A \cap B) \times (A \Delta B)].$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(A \cup B) = 6$$

$$A \cap B = \{3, 4\} \Rightarrow n(A \cap B) = 2$$

$$n(A \Delta B) = n(A \cup B) - n(A \cap B) = 4$$

$$n[(A \cup B) \times n(A \cap B) \times n(A \Delta B)] = n(A \cup B) \times n(A \cap B) \times n(A \Delta B)$$

$$= 6 \times 2 \times 4$$

$$= 48.$$

4. Resolve $\frac{3x+1}{(x-2)(x+1)}$ into partial fractions.

$$\frac{3x+1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \quad \text{--- (1)}$$

$$\frac{3x+1}{(x-2)(x+1)} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)}$$

$$3x+1 = A(x+1) + B(x-2)$$

$$\text{put } x = -1 \Rightarrow -3+1 = B(-3)$$

$$-2 = -3B$$

$$B = \frac{2}{3}$$

$$\text{put } x = 2 \Rightarrow 6+1 = A(3)$$

$$7 = 3A$$

$$A = \frac{7}{3}$$

$$\textcircled{1} \Rightarrow \frac{3x+1}{(x-2)(x+1)} = \frac{7}{3(x-2)} + \frac{2}{3(x+1)}$$

⑤ Resolve $\frac{1}{x^2-a^2}$ into partial fractions.

$$\frac{1}{x^2-a^2} = \frac{A}{x+a} + \frac{B}{x-a} \quad \text{--- (1)}$$

$$\frac{1}{x^2-a^2} = \frac{A(x-a) + B(x+a)}{(x+a)(x-a)}$$

$$1 = A(x-a) + B(x+a)$$

$$\text{put } x = -a \Rightarrow 1 = A(-2a)$$

$$A = \frac{-1}{2a}$$

$$\text{put } x = a \Rightarrow 1 = B(2a)$$

$$B = \frac{1}{2a}$$

$$\textcircled{1} \Rightarrow \frac{1}{x^2 - a^2} = \frac{1}{-2a(x+a)} + \frac{1}{2a(x-a)} \quad \text{www.Padasalai.Net}$$

⑥ Find the Complete Set of Values of 'a' for which the quadratic $x^2 - ax + a + 2 = 0$ has equal roots.

$$x^2 - ax + (a+2) = 0$$

Given $b^2 - 4ac = 0$

$$a^2 - 4(a+2) = 0$$

$$a^2 - 4a - 8 = 0$$

$$a = \frac{4 \pm \sqrt{16 + 32}}{2}$$

$$= \frac{4 \pm \sqrt{48}}{2}$$

$$= \frac{4 \pm 4\sqrt{3}}{2}$$

$$a = 2 \pm 2\sqrt{3}$$

⑦ Solve $|2x - 17| = 3$ for x

$$|2x - 17| = 3$$

$$2x - 17 = \pm 3$$

$$2x - 17 = 3$$

$$2x = 3 + 17$$

$$2x = 20$$

$$x = 10$$

$$2x - 17 = -3$$

$$2x = -3 + 17$$

$$2x = 14$$

$$x = 7$$

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8) Solve $|x-9| < 2$ for x .

$$|x-9| < 2$$

$$-2 < x-9 < 2$$

$$-2+9 < x < 2+9$$

$$7 < x < 11$$

9) Express $\cos 6\theta + \cos 2\theta$ as a product.

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos 6\theta + \cos 2\theta = 2 \cos \frac{8\theta}{2} \cos \frac{4\theta}{2}$$

$$= 2 \cos 4\theta \cos 2\theta$$

10) Find the value of $\sin 150^\circ$

$$\sin 150^\circ = \sin (90^\circ + 60^\circ) \quad \boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$= \sin 90^\circ \cos 60^\circ + \cos 90^\circ \sin 60^\circ$$

$$= (1) \left(\frac{1}{2}\right) + 0$$

$$= \frac{1}{2}$$

11) Express $\sin 50^\circ + \sin 20^\circ$ as a product.

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin 50^\circ + \sin 20^\circ = 2 \sin \frac{70^\circ}{2} \cos \frac{30^\circ}{2}$$

$$= 2 \sin 35^\circ \cos 15^\circ$$

(12) Find the value of $\cos 135^\circ$

$$\begin{aligned}\cos(135^\circ) &= \cos(90^\circ + 45^\circ) \quad \boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta} \\ &= \cos 90^\circ \cos 45^\circ - \sin 90^\circ \sin 45^\circ \\ &= 0 - (1) \left(\frac{1}{\sqrt{2}}\right) \\ &= -\frac{1}{\sqrt{2}}.\end{aligned}$$

(13) Find the principal solution of $\cos \theta = -\frac{1}{2}$.

$$\cos \theta = -\frac{1}{2}$$

$$\cos \theta = \cos 2\pi/3$$

$$\theta = 2\pi/3.$$

(14) Prove that $\frac{\cot(180^\circ + \theta) \sin(90^\circ - \theta) \cos(-\theta)}{\sin(270^\circ + \theta) \tan(-\theta) \operatorname{cosec}(360^\circ + \theta)} = \cos^2 \theta \cot \theta$

(5 mark for may 2015)

$$\text{LHS} = \frac{\cot(180^\circ + \theta) \sin(90^\circ - \theta) \cos(-\theta)}{\sin(270^\circ + \theta) \tan(-\theta) \operatorname{cosec}(360^\circ + \theta)}$$

$$= \frac{\cot \theta \cos \theta \cos \theta}{(-\cos \theta) (-\tan \theta) \operatorname{cosec} \theta}$$

$$= \frac{\cot \theta \cos^2 \theta}{\cos \theta \times \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta}}$$

$$= \cot \theta \cos^2 \theta \quad \text{RHS.}$$

15) If $\cos \theta = \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right)$, show that

$$\cos 3\theta = \frac{1}{2} \left(\alpha^3 + \frac{1}{\alpha^3} \right)$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$= 4 \left[\frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \right]^3 - 3 \cdot \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right)$$

$$= 4 \times \frac{1}{8} \left(\alpha + \frac{1}{\alpha} \right)^3 - \frac{3}{2} \left(\alpha + \frac{1}{\alpha} \right)$$

$$= \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \left[\left(\alpha + \frac{1}{\alpha} \right)^2 - 3 \right]$$

$$= \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \left[\alpha^2 + \frac{1}{\alpha^2} + 2 - 3 \right]$$

$$= \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \left[\alpha^2 + \frac{1}{\alpha^2} - 1 \right]$$

$$\cos 3\theta = \frac{1}{2} \left(\alpha^3 + \frac{1}{\alpha^3} \right)$$

16) Write the relationship between Permutation and Combination.

$$nPr = nCr \times r!$$

17) If the letters of the word IITJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE

The lexicographic order E, E, I, I, J, T

$$E \text{ --- } = \frac{5!}{2!} = \frac{120}{2} = 60$$

$$IIE \text{ --- } = 3! = 6$$

$$I I J \text{ --- } = \frac{3!}{2!} = \frac{6}{2} = 3.$$

$$I I T E \text{ --- } = 2! = 2$$

$$I I T J E E = 1.$$

Rank of the word IITJEE is

$$60 + 6 + 3 + 2 + 1 = 72$$

(18) Prove that $\frac{(2n)!}{n!} = 2^n (1 \cdot 3 \cdot 5 \dots (2n-1))$

$$\frac{(2n)!}{n!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \dots (2n-2) (2n-1) 2n}{n!}$$

$$= \frac{(1 \cdot 3 \cdot 5 \dots (2n-1)) (2 \cdot 4 \dots (2n-2) 2n)}{n!}$$

$$= \frac{1 \cdot 3 \cdot 5 \dots (2n-1) \times 2^n (1 \cdot 2 \dots (n-1) n)}{n!}$$

$$= \frac{2^n (1 \cdot 3 \cdot 5 \dots (2n-1)) \times n!}{n!}$$

$$= 2^n (1 \cdot 3 \cdot 5 \dots (2n-1))$$

(19) If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then find the value of

A.

$$\frac{A}{9!} = \frac{1}{7!} + \frac{1}{8!}$$

$$\frac{A}{9 \times 8 \times 7!} = \frac{1}{7!} + \frac{1}{8 \times 7!}$$

$$\frac{A}{9 \times 8 \times 7!} = \frac{1}{7!} \left[1 + \frac{1}{8} \right]$$

$$\frac{A}{8 \times 9} = \frac{9}{8}$$

$$A = 81$$

20) Find the number of ways of arranging the letters of the word "BANANA"

Number of ways of arrangements is $\frac{6!}{3! \times 2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1}$

= 60 ways.

21) Find the number of outcomes when 5 coins are tossed once

Number of outcomes is $2^5 = 32$.

22) Find the value of $\frac{8!}{5! \times 2!}$

$$\frac{8!}{5! \times 2!} = \frac{8 \times 7 \times 6 \times 5!}{5! \times 2 \times 1} = 168$$

23) write the first 4 terms of the sequences whose n^{th} term a_n is given as

$$a_n = \begin{cases} 1 & \text{if } n=1 \\ 2 & \text{if } n=2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

$$a_1 = 1$$

$$a_2 = 2$$

$$a_3 = a_2 + a_1 = 3$$

$$a_4 = a_3 + a_2 = 5$$

24) Write the first 4 terms of the sequences whose n^{th} term a_n is given as

$$a_n = \begin{cases} n & ; \text{ if } n \text{ is } 1, 2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & ; \text{ if } n > 3. \end{cases}$$

$$a_1 = 1$$

$$a_2 = 2$$

$$a_3 = 3$$

$$a_4 = a_3 + a_2 + a_1$$

$$= 3 + 2 + 1$$

$$= 6$$

25) Find the middle term in the expansion of $(x+y)^b$

If n is even then $T_{\frac{n}{2}+1}$ is middle term.

T_{3+1} is middle term.

$$T_{r+1} = n C_r a^{n-r} b^r$$

$$r=3, \quad n=6, \quad a=x \quad b=y.$$

$$\begin{aligned} T_{3+1} &= {}^6C_3 x^3 y^3 \\ &= \frac{6 \times 5 \times 4}{1 \times 2 \times 3} x^3 y^3 \end{aligned}$$

$$= 20 x^3 y^3.$$

(26) Find the coefficient of x^5 in the expansion of $\left(x + \frac{1}{x^3}\right)^{17}$.

$$n=17, \quad a=x \quad b = \frac{1}{x^3} = x^{-3}$$

$$T_{r+1} = n C_r a^{n-r} b^r$$

$$T_{r+1} = 17 C_r x^{17-r} (x^{-3})^r$$

$$= 17 C_r x^{17-r} x^{-3r}$$

$$T_{r+1} = 17 C_r x^{17-4r}$$

Coefficient of x^5 is

$$17-4r = 5$$

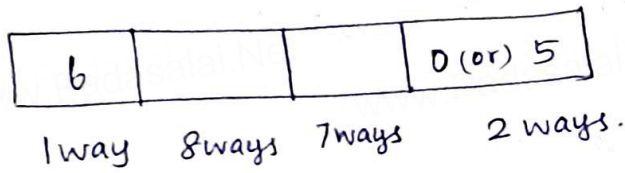
$$-4r = 5 - 17$$

$$-4r = -12$$

$$\begin{aligned}
 T_{3+1} &= {}^{17}C_3 x^{17-12} \\
 &= \frac{17 \times 16 \times 15}{1 \times 2 \times 3} x^5 \\
 &= 680 x^5
 \end{aligned}$$

\therefore Coefficient of x^5 is 680.

- (27) Count the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digits are repeated.



$$\begin{aligned}
 \text{The required no. of numbers} &= 1 \times 8 \times 7 \times 2 \\
 &= 112.
 \end{aligned}$$

- (28) Find the separate equations from a combined equation of straight line

$$2x^2 + xy - 3y^2 = 0$$

$$\begin{aligned}
 2x^2 + xy - 3y^2 &= 2x^2 - 2xy + 3xy - 3y^2 \\
 &= 2x(x-y) + 3y(x-y) \\
 &= (x-y)(2x+3y)
 \end{aligned}$$

Separate equations

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$$\begin{aligned}
 x-y &= 0 \\
 2x+3y &= 0
 \end{aligned}$$

29) Write the equation of the line passing through the point $(1, -1)$ and parallel to the line $x + 3y - 4 = 0$ www.Padasalai.Net

Given line $x + 3y - 4 = 0$

Parallel line $x + 3y + k = 0$ — (1)

At $(1, -1)$

$$1 - 3 + k = 0$$

$$-2 + k = 0$$

$$k = 2$$

$$(1) \Rightarrow x + 3y + 2 = 0.$$

30) Find the equation of the straight line, if the perpendicular from the origin makes an angle of 120° with x axis the length of the perpendicular from the origin is 6 units.

Equation of straight line is

$$x \cos \alpha + y \sin \alpha = p$$

$$\alpha = 120^\circ \quad p = 6.$$

$$x \cos 120^\circ + y \sin 120^\circ = 6$$

$$x \left(-\frac{1}{2}\right) + y \left(\frac{\sqrt{3}}{2}\right) = 6.$$

$$-x + \sqrt{3}y = 12.$$

$$-x + \sqrt{3}y - 12 = 0$$

$$x - \sqrt{3}y + 12 = 0$$

(31) The length of the perpendicular drawn from the origin to the line is 12 and makes an angle 150° with positive direction of the x axis. Find the equation of the line.

Equation of the line is

$$x \cos \alpha + y \sin \alpha = p$$

$$\alpha = 150^\circ \quad p = 12.$$

$$x \cos 150^\circ + y \sin 150^\circ = 12$$

$$x \left(-\frac{\sqrt{3}}{2}\right) + y \left(\frac{1}{2}\right) = 12$$

$$-\sqrt{3}x + y = 24$$

$$-\sqrt{3}x + y - 24 = 0$$

$$\sqrt{3}x - y + 24 = 0.$$

(32) Find the equation of the lines passing through the points $(1, 1)$ and $(-2, 3)$

Equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{3 - 1} = \frac{x - 1}{-2 - 1}$$

$$\frac{y - 1}{2} = \frac{x - 1}{-3}$$

$$-3y + 3 = 2x - 2$$

$$2x + 3y - 2 - 3 = 0$$

$$2x + 3y - 5 = 0.$$

33) Find the equation of the line passing through the point (1,1) with slope 3.

Equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3.$$

$$3x - y - 3 + 1 = 0$$

$$3x - y - 2 = 0.$$

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