

STD: 12

Model Half Yearly Exam -2022
Businessmaths and Statistics

MARKS: 90
TIME:3hrs

PART-A

I. Choose the best answer from the following options:

(20 x 1 = 20)

1. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ then $\rho(A)$ is (a) 0 (b) 1 (c) 2 (d) n
2. If the number of variables in a non-homogeneous system $AX = B$ is n, then the system possesses a unique solution only when
 - (a) $\rho(A) = \rho(A,B) > n$
 - (b) $\rho(A) = \rho(A,B) = n$
 - (c) $\rho(A) = \rho(A,B) < n$
 - (d) none of these
3. $\int_2^4 \frac{dx}{x}$ is (a) log 4 (b) 0 (c) log 2 (d) log 8
4. The demand and supply function of a commodity are $P(x) = (x - 5)^2$ and $S(x) = x^2 + x + 3$ then the equilibrium quantity x_0 is (a) 5 (b) 2 (c) 3 (d) 1
5. The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is
 - (a) $\frac{d^2y}{dx^2} - y = 0$
 - (b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
 - (c) $\frac{d^2y}{dx^2} = 0$
 - (d) $\frac{d^2y}{dx^2} - x = 0$
6. Area bounded by $y = |x|$ between the limits 0 and 2 is
 - (a) 1sq.units (b) 3 sq.units (c) 2 sq.units (d) 4 sq.units
7. The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ is
 - (a) $e^{\int p dx}$ (b) $\int p dx$ (c) $\int p dy$ (d) $e^{\int p dy}$
8. The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D - a)^2$
 - (a) $\frac{x^2}{2} e^{ax}$ (b) xe^{ax} (c) $\frac{x}{2} e^{ax}$ (d) $x^2 e^{ax}$
9. If c is a constant then $\Delta c =$ (a) c (b) Δ (c) Δ^2 (d) 0
10. If $f(x) = x^2 + 2x + 2$ and the interval of differencing is unity then $\Delta f(x)$
 - (a) $2x - 3$ (b) $2x + 3$ (c) $x + 3$ (d) $x - 3$
11. A discrete probability distribution may be represented by
 - (a) table (b) graph (c) mathematical equation (d) all of these
12. The distribution function $F(x)$ is equal to
 - (a) $P(X = x)$ (b) $P(X \leq x)$ (c) $P(X \geq x)$ (d) all of these
13. In a parametric distribution the mean is equal to variance is
 - (a) binomial (b) normal (d) all the above (c) Poisson
14. If $P(Z > z) = 0.5832$ what is the value of z (z has a standard normal distribution)?
 - (a) -0.48 (b) 0.48 (c) 1.04 (d) -0.21
15. A random sample is a sample selected in such a way that every item in the population has an equal chance of being included
 - (a) Harper (b) Fisher (c) Karl Pearson (d) Dr. Yates
16. Errors in sampling are of
 - (a) Two types (b) three types (c) four types (d) five types
17. The additive model of the time series with the components T, S, C and I is
 - (a) $y = T + S + C \times I$ (b) $y = T + S \times C \times I$ (c) $y = T + S + C + I$ (d) $y = T + S \times C + I$

18. The solution for an assignment problem is optimal if
- each row and each column has no assignment
 - each row and each column has atleast one assignment
 - each row and each column has atmost one assignment
 - each row and each column has exactly one assignment
19. Laspeyre's index = 110, Paasche's index = 108, then Fisher's Ideal index is equal to:
- 110
 - 108
 - 100
 - 109
20. In a degenerate solution number of allocations is
- equal to $m+n-1$
 - not equal to $m+n-1$
 - less than $m+n-1$
 - greater than $m+n-1$

PART -B

II. Two mark Questions :

(7 X 2 = 14)

(Answer any 7, Qno:30 is compulsory)

21. Solve the equations $2x + 3y = 7$; $3x + 5y = 9$ by using Cramer's rule

22. Evaluate $\int \frac{x}{\sqrt{x^2+1}} dx$

23. Solve the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

24. Sketch the graph $y = |x + 3|$ and evaluate $\int_{-6}^0 |x + 3| dx$

25. Let X be a continuous random variable with probability density function.

$$f_x(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{Find the expected value of X.}$$

26. In a Poisson distribution the first probability term is 0.2725.

Find the next Probability term

27. What do you mean by balanced transportation problem?

28. A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Calculate the standard error concerning of good apples.

29. Define seasonal index.

30. Prove that $E = 1 + \Delta$

PART - C.

III . Three mark Questions :

(7 x 3=21)

(Answer any 7, Qno:40 is compulsory)

31. Parithi is either sad (S) or happy (H) each day. If he is happy in one day, he is sad on the next day by four times out of five. If he is sad on one day, he is happy on the next day by two times out of three. Over a long run, what are the chances that Parithi is happy on any given day?

32. If $f'(x) = \frac{1}{x}$ and $f(1) = \frac{\pi}{4}$, then find $f(x)$.

33. The marginal cost function is $MC = 300x^{2/5}$ and fixed cost is zero. Find out the total cost and average cost functions.

34. Find the differential equation corresponding to $y = ae^{4x} + be^{-x}$ where a, b are arbitrary constants,

35. Evaluate $\Delta \left[\frac{5x+12}{x^2+5x+6} \right]$ by taking '1' as the interval of differencing.

36. If x is a binomially distributed random variable with $E(x) = 2$ and $\text{var}(x) = \frac{4}{3}$. Find $P(x=5)$

37. The following information is the probability distribution of successes.

| | | | |
|-------------------------|----------------|----------------|----------------|
| No. of Successes | 0 | 1 | 2 |
| Probability | $\frac{6}{11}$ | $\frac{9}{22}$ | $\frac{1}{22}$ |

Determine the expected number of success.

38. Calculate the cost of living index number for the following data.

| Commodities | Quantity | Price | |
|-------------|----------|-------|------|
| | | 2005 | 2010 |
| A | 10 | 7 | 9 |
| B | 12 | 6 | 8 |
| C | 17 | 10 | 15 |
| D | 19 | 14 | 16 |
| E | 15 | 12 | 17 |

39. A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

| Rainfall | Estimated Conditional Profit(Rs.) | | |
|----------|-----------------------------------|--------|--------|
| | Crop A | Crop B | Crop C |
| High | 8000 | 3500 | 5000 |
| Medium | 4500 | 4500 | 5000 |
| Low | 2000 | 5000 | 4000 |

If the farmer wishes to plant only crop, decide which should be his best crop using (i) Maximin (ii)Minimax

40. State the properties of distribution function.

PART – D

IV. Five mark Questions : (Answer all the Questions)

(7 X 5 = 35)

41.(a) Solve by Cramer's rule $x + y + z = 4$, $2x - y + 3z = 1$, $3x + 2y - z = 1$

[OR]

41.(b) A continuous random variable X has the following probability function Value of

| | | | | | | | | |
|-----------------------|---|---|----|----|----|-------|--------|------------|
| Value of X = x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(x) | 0 | K | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2 + k$ |

(i) Find k (ii) Evaluate $p(x < 6)$, $p(x \geq 6)$ and $p(0 < x < 5)$

(iii) If $p(X \leq x) > \frac{1}{2}$ then find the minimum value of x .

- 42.(a) If $f(x) = \begin{cases} x^2, & -2 \leq x < 1 \\ x, & 1 \leq x < 2 \\ x - 4, & 2 \leq x \leq 4 \end{cases}$ then find the following
- (i) $\int_{-2}^1 f(x) dx$ (ii) $\int_1^2 f(x) dx$ (iii) $\int_2^3 f(x) dx$
 (iv) $\int_{-2}^{1.5} f(x) dx$ (v) $\int_1^3 f(x) dx$

[OR]

42.(b) If the heights of 500 students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches, how many students have height (a) greater than 72 inches (b) less than or equal to 64 inches (c) between 65 and 71 inches.

43.(a) The demand equation for a product is $x = \sqrt{100 - p}$ and the supply equation is $x = \frac{p}{2} - 10$. Determine the consumer's surplus and producer's surplus, under market equilibrium.

[OR]

43.(b) The average score on a nationally administered aptitude test was 76 and the corresponding standard deviation was 8. In order to evaluate a state's education system, the scores of 100 of the state's students were randomly selected. These students had an average score of 72. Test at a significance level of 0.05 if there is a significant difference between the state scores and the national scores.

44.(a) Suppose that the quantity demanded $Q_d = 13 - 6p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2}$ and quantity supplied $Q_s = -3 + 2p$ where p is the price. Find the equilibrium price for market clearance.

[OR]

44.(b) Compute (i) Laspeyre's (ii) Paasche's (iii) Fisher's Index numbers for the 2010 from the following data.

| Commodity | Price | | Quantity | |
|-----------|-------|------|----------|------|
| | 2000 | 2010 | 2000 | 2010 |
| A | 12 | 14 | 18 | 16 |
| B | 15 | 16 | 20 | 15 |
| C | 14 | 15 | 24 | 20 |
| D | 12 | 12 | 29 | 23 |

45.(a) For what values of the parameter λ , will the following equations fail to have unique solution: $3x - y + \lambda z = 1$, $2x + y + z = 2$, $x + 2y - \lambda z = -1$ by rank method.

[OR]

45.(b) Find a polynomial of degree two which takes the values.

| | | | | | | | | |
|---|---|---|---|---|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 1 | 2 | 4 | 7 | 11 | 16 | 22 | 29 |

46.(a) . Evaluate the integral as the limit of a sum: $\int_0^1 x^2 dx$

[OR]

46.(b) Find the optimal solution for the assignment problem with the following cost:

| | | Area | | | |
|-------|---|------|----|----|----|
| | | 1 | 2 | 3 | 4 |
| Sales | P | 11 | 17 | 8 | 16 |
| | Q | 9 | 7 | 12 | 6 |
| | R | 13 | 16 | 15 | 12 |
| | S | 14 | 10 | 12 | 11 |

47.(a) Elasticity of a function $\frac{E_y}{E_x}$ is given by $\frac{E_y}{E_x} = \frac{-7x}{(1-2x)(2+3x)}$.

Find the function When $x = 2$ $y = \frac{3}{8}$.

[OR]

47.(b) Calculate the seasonal indices from the following data using the average from the following data using the average method:

| | I Quarter | II Quarter | III Quarter | IV Quarter |
|------|--------------|---------------|----------------|---------------|
| 2008 | 72 | 68 | 62 | 76 |
| 2009 | 78 | 74 | 78 | 72 |
| 2010 | 74 | 70 | 72 | 76 |
| 2011 | 76 | 74 | 74 | 72 |
| 2012 | 72 | 72 | 76 | 68 |

ALL THE BEST