

HTV HALF YEARLY EXAMINATION - 2022.

XII - Std

MATHEMATICS

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Time : 3.00 Hrs

Marks : 90

Section - A

I Choose the correct answer :-

20 X 1 = 20

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is.
- a) 17 b) 14 c) 19 d) 21
2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is a adjoint of 3×3 matrix A and $|A| = 4$, then x is.
- a) 15 b) 12 c) 14 d) 11
3. The value of $i^{2018} + i^{2019} + i^{2020} + i^{2021}$ is
- a) 0 b) 1 c) -1 d) i
4. The solution of $|z| - z = 1 + 2i$ is.
- a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} - 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$
5. A zero of $x^3 + 64$ is.
- a) 0 b) 4 c) $4i$ d) -4
6. $\sin^{-1}\left(\frac{x}{5}\right) + \cosec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then x is
- a) 4 b) 5 c) 3 d) 2
7. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belong to
- a) $[-1, 1]$ b) $[\sqrt{2}, 2]$ c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ d) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$
8. If $x + y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is
- a) 3 b) -1 c) 9 d) 1
9. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is
- a) 2 b) 4 c) 6 d) 8
10. If $[\vec{a}X\vec{b} \quad \vec{b}X\vec{c} \quad \vec{c}X\vec{a}] = 121$ then $[\vec{a} \quad \vec{b} \quad \vec{c}] =$ is
- a) $\frac{121}{2}$ b) 8 c) 11 d) 121

11. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$, $\vec{c} = \vec{j}$ and $(\vec{a} X \vec{b}) X \vec{c} = \lambda \vec{a} + \mu \vec{b}$ then $\lambda + \mu$ is
 a) 0 b) 1 c) 2 d) 3

12. The minimum value of the function $|3 - x| + 9$ is
 a) 0 b) 3 c) 6 d) 9

13. Angle between $y^2 = x$ and $x^2 = y$ at origin is
 a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) can't find

14. $V(x, y) = \log(e^x + e^y)$ then $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} =$
 a) $e^x + e^y$ b) $\frac{1}{e^x + e^y}$ c) 2 d) 1

15. The value of $\int_{-4}^4 |x+3| dx$ is
 a) 50 b) 25 c) 16 d) 8

16. The value of $\int_0^\pi \sin^4 x dx$ is
 a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

17. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
 a) $xy + k$ b) $y = k \log x$ c) $y = kx$ d) $\log y = kx$

18. The solution of $\frac{dy}{dx} = 2^{y-x}$ is
 a) $2^x + 2^y = c$ b) $2^x - 2^y = c$ c) $\frac{1}{2^x} - \frac{1}{2^y} = c$ d) $x + y = c$

19. If $U = y \sin x$ then $\frac{\partial^2 y}{\partial x \partial y} =$
 a) $\cos x$ b) $\cos y$ c) $\sin x$ d) $\sin y$

20. If $\frac{z-1}{z+1}$ is purely imaginary then $|z|$ is
 a) $\frac{1}{2}$ b) 1 c) 2 d) 3

Section - B**II Answer any 7 questions. Q.No. 30 is compulsory :-**

7X2=14

21. If $\text{adj}A = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$ then find A^{-1} .
22. Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.
23. Solve : $x^4 - 14x^2 + 45 = 0$.
24. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$.
25. Obtain the equation of the circle for which $(-4, -2)$ and $(1, 1)$ are the ends of a diameter..
26. Verify that the vectors $\vec{i} + 2\vec{j} - 3\vec{k}$, $2\vec{i} - \vec{j} + 2\vec{k}$, $3\vec{i} + \vec{j} - \vec{k}$ are coplanar.
27. Solve : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
28. Evaluate : $\int_0^1 x^4(1-x)^5 dx$
29. If $\nabla(x, y, z) = xy + yz + zx$ then find $d\nabla$.
30. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

Section - C**Answer any 7 questions. Q.No. 40 is compulsory :-**

7 X 3 = 21

31. Evaluate. $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$
32. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$ then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
33. Solve : $\frac{dy}{dx} + \frac{y}{x} = \sin x$
34. Show that the shortest distance between the lines
- $$\vec{r} = (6\vec{i} + \vec{j} + 2\vec{k}) + S(\vec{i} + 2\vec{j} - 3\vec{k})$$
- $$\vec{r} = (3\vec{i} + 2\vec{j} - 2\vec{k}) + t(2\vec{i} + 4\vec{j} - 5\vec{k}) \text{ and is } \frac{7\sqrt{5}}{5}.$$
35. If the normal at the point 't' on the parabola $y^2 + 4ax$ meets the parabola again at the point 't₂' then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.
36. Solve : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, $6x^2 < 1$.
37. If P is real, discuss the nature of the roots of the equation $4x^2 + 4Px + P + 2 = 0$, in terms of P.
38. Show that the equation $z^2 = \bar{z}$ has four solutions.

39. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

40. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$

Section - IV

Answer all questions :-

$7 \times 5 = 35$

41. a) Investigate for what values of λ, μ the system of linear equations.
 $x+2y+z=7$; $x+y+\lambda z=\mu$; $x+3y-5z=5$ has i) no solution ii) a unique solution
 iii) an infinite number of solution. (OR)
- b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.
42. a) Solve : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. (OR)
- b) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$.
43. a) A bridge has a parabola arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides. (OR)
- b) Prove by vector method $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.
44. a) Sketch the curve $y = f(x) = x^2 - x - 6$. (OR)
- b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.
45. a) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$. (OR)
- b) A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C and another 5 minutes later it has dropped to 65°C . Determine the temperature of the kitchen.
46. a) Find the volume of a right-circular cone of base radius 'r' and height 'h'. (OR)
- b) Find the parametric vector and Cartesian equation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ and passing through the point $(-1, 1, -1)$.
47. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. (OR)
- b) Find the equation of the circle passing through the points $(1, 1)$, $(2, -1)$ and $(3, 2)$.

Half yearly Exam - 2022
XII - Maths Answer key / T.V.Malai.Dt

I.		
1) c) 9	11) a) 0	MCQ:
2) d) 11	12) d) 9	
3) a) 0	13) a) $\frac{\pi}{2}$	
4) a) $\frac{3}{2} - 2i$	14) b) 1	
5) d) -4	15) b) 25	
6) c) 3	16) b) $\frac{3\pi}{8}$	
7) d) $[2, -\sqrt{2}] \cup [\sqrt{2}, 2]$	17) c) $y = kx$	B. Jayapradaan
8) c) 9	18) c) $\frac{1}{2^x} - \frac{1}{2^y} = C$	
9) d) 8	19) a) $\cos x$	
10) c) 11	20) b) 1	

II)

$$\text{adj } A = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 5 & -6 \\ 3 & 0 & 6 \end{bmatrix} \quad A^{-1} = \pm \frac{1}{\sqrt{|\text{adj } A|}} \text{ adj } A$$

$$|\text{adj } A| = a(12-0) - b(-2)(3b-18) + c(0+6b) = 6a + 36b = 6$$

$$|\text{adj } A| = 0 + 36b + 0 = 36b$$

$$\sqrt{|\text{adj } A|} = \sqrt{36b} = 6$$

$$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 5 & -6 \\ 3 & 0 & 6 \end{bmatrix}$$

22) ST $(2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10}$ is Purely Imaginary

$$\text{Let } z = (2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10}$$

$$z = (2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10} = (2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10}$$

$$= (2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10} = (2-i\sqrt{3})^{10} (2+i\sqrt{3})^{10}$$

$$= -\{(2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10}\} = -z$$

$$\therefore (2+i\sqrt{3})^{10} (2-i\sqrt{3})^{10} \text{ is purely imaginary}$$

23) Solve $x^4 - 14x^2 + 45 = 0$

$$\text{Let } x^2 = y \rightarrow 0$$

$$x^4 - 14x^2 + 45 = 0 \Rightarrow y^2 - 14y + 45 = 0$$

$$\Rightarrow (y-5)(y-9) = 0$$

$$\Rightarrow y = 5, 9$$

Sub $y = 5, 9$ in ① $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

$$x^2 = 9 \Rightarrow x = \pm 3$$

$$\therefore \text{Thus the roots are } -3, 3, -\sqrt{5}, \sqrt{5}$$

21) $= \cos^{-1}(\frac{1}{2}) + \sin^{-1}(-1)$
 $= \cos^{-1}(\frac{1}{2}) - \sin^{-1}(1)$
 $= \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$

25) End points of diameter $(-\frac{x_1}{2}, -\frac{y_1}{2})$ and $(\frac{x_2}{2}, \frac{y_2}{2})$

The required equation of circle,

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

$$(x+a)(x-d) + (y+2)(y-1) = 0$$

$$\Rightarrow x^2 + y^2 + 3x + y - 6 = 0$$

26) $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a} \vec{b} \vec{c}] = 0$

$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = 0$

∴ The three given vectors are coplanar

27) $\frac{dy}{dx} = \frac{1-y^2}{\sqrt{1-x^2}}$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1}y = \sin^{-1}x + C$

28) $I = \int x^4 (1-x)^5 dx, I = \int x^m (1-x)^n dx = \frac{m! n!}{(m+n)!}$

$= \frac{4! 5!}{(4+5)!} = \frac{4! 5!}{10!} = \frac{4! 5!}{10 \times 9 \times 8 \times 7 \times 6 \times 5!}$

$= \frac{24 \times 120}{10 \times 9 \times 8 \times 7 \times 6} = \frac{24}{120}$

29) $y(x_1, y) = xy + yz + zx$

$\frac{\partial y}{\partial x} = y+z, \frac{\partial y}{\partial y} = x+z, \frac{\partial y}{\partial z} = y+x$

The differential is

$dy = (y+z)dx + (x+z)dy + (y+x)dz$

30) $f(n) = x^2 - 2x - 3$

$f'(x) = 2x-2$, $f'(x)=0 \Rightarrow x=1$

$f'(x) > 0$ at $(2, 0)$, f is strictly increasing

at $(2, 0)$, $f'(x) \geq 0$

Hence, f is strictly increasing at $(2, 0)$

31) $2at_2 + at_2^2 t_1 = at_1^3 + 2at_1$

$\therefore 2at_2 + at_2^2 t_1 = at_1^3 + 2at_1$

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$\therefore 2at_2 + at_2^2 t_1 = at_1^$

41.a/5 Marks

The matrix form of system $AX=B$ is

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & -5 \\ 1 & 3 & 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 14 \\ 5 \end{bmatrix}$$

$$\begin{array}{l} [A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 14 \\ 1 & 3 & 5 & 5 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 14 \\ 1 & 3 & 5 & 5 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 14 \\ 0 & 0 & 10 & -7 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 14 \\ 0 & 1 & 6 & -2 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 6R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 14 \\ 0 & 0 & 1 & -13 \end{array} \right] \end{array}$$

i) $\lambda=7, \mu\neq 9$, $P(A)\neq P(A|B)$, inconsistent, no solution
ii) $\lambda\neq 7$, $P(A)=P(A|B)=3$ = Number of unknowns
Consistent and unique solution.
iii) $\lambda=7, \mu=9$, $P(A)=P(A|B)=2$ < No. of unknowns
Consistent and infinite solutions.

41.b/6 Marks

$$\text{If } z = x+iy, \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{(x^2+y^2-1)+i(2xy)}{(x+1)^2+y^2}$$

$$\text{arg}\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2} \Rightarrow \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = \tan\frac{\pi}{2} \Rightarrow x^2+y^2-1 = 0$$

$$x^2+y^2=1$$

42.a/5 Marks

$$x^4 - 10x^3 + 2bx^2 - 10x + 1 = 0$$

$$(x^2 + \frac{1}{x^2}) - 10(x + \frac{1}{x}) + 2b = 0$$

$$\text{Let } y = x + \frac{1}{x}$$

$$(y^2 - 2) - 10y + 2b = 0 \Rightarrow y^2 - 10y + 2b = 0$$

$$(y-6)(y-4) = 0 \Rightarrow y = 6 \text{ or } y = 4$$

If $y = 6 \Rightarrow x + \frac{1}{x} = 6$
 $x = 3 + 2\sqrt{2}, x = 3 - 2\sqrt{2}$

If $y = 4 \Rightarrow x + \frac{1}{x} = 4$
 $x = 2 + \sqrt{3}, x = 2 - \sqrt{3}$.
Hence the roots are,
 $3 \pm 2\sqrt{2}, 2 \pm \sqrt{3}$.

42.b/5 Marks

domain of $\sin^{-1}x$ is $[-1, 1]$ and $\cos^{-1}x$ is $[0, \pi]$

$$\text{So, } -1 \leq \frac{|x|-2}{3} \leq 1, \quad -1 \leq \frac{|x|}{4} \leq 1$$

$$-3 \leq |x|-2 \leq 3 \quad -4 \leq |x| \leq 4$$

$$-1 \leq |x| \leq 5 \quad -5 \leq |x| \leq 3$$

$$|x| \leq 5 \quad -3 \leq |x| \leq 5$$

$$|x| \leq 5 \quad |x| \leq 5$$

$\therefore x \in [-5, 5]$.

43.a/5 Marks

$x^2 = -4ay \rightarrow \text{①}$

At $(5, -10)$ in ①

 $225 = 40a \quad a = \frac{225}{40}$
 $\alpha = \frac{225}{40} \text{ in ①}$
 $\alpha = -4 \left(\frac{225}{40}\right)y \rightarrow \text{②}$

At $(6, -y)$ in ②

 $(6)^2 = -4 \left(\frac{225}{40}\right)(-y) \Rightarrow y_1 = \frac{36 \times 10}{225}$

The required height $|10-y_1| = |10-1.6| = 8.4 \text{ m}$

43.b/5 Marks

Take two points A and B on the unit circle with centre O.

$|\vec{OA}| = |\vec{OB}| = 1$

$\angle AOX = \alpha, \angle BOX = \beta, \angle AOB = \alpha + \beta$

$\vec{OA} = \vec{OL} + \vec{LA} = \cos\alpha\hat{i} + \sin\alpha\hat{j}$

$\vec{OB} = \vec{OM} + \vec{MB} = \cos\beta\hat{i} - \sin\beta\hat{j}$

$\vec{OB} \times \vec{OA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\alpha & \sin\alpha & 0 \\ \cos\beta & -\sin\beta & 0 \end{vmatrix}$

$\vec{OB} \times \vec{OA} = (\sin\alpha \cos\beta + \cos\alpha \sin\beta)\hat{k} \rightarrow \text{①}$

But $\vec{OB} \times \vec{OA} = |\vec{OB}| |\vec{OA}| \sin(\alpha + \beta) \hat{k}$

$\vec{OB} \times \vec{OA} = \sin(\alpha + \beta) \hat{k} \rightarrow \text{②}$

From ① & ② we get

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta.$$

44.a/5 Marks

$y = f(x) = x^2 - x - b = (x-3)(x+2)$

1) The domain of $f(x)$ is the entire real line

2) Intercepts: put $y=0 \Rightarrow x=3, -2$
 $\therefore x$ intercepts are $(3, 0)$ and $(-2, 0)$
put $x=0 \Rightarrow y=x^2-x-b = -b$
 $\therefore y$ intercepts are $(0, -b)$

3) Critical points:
 $f(x) = x^2 - x - b \Rightarrow f'(x) = 2x - 1$
 $f'(x) = 0 \Rightarrow x = \frac{1}{2}$
The critical point of $f(x)$ occur at $x = \frac{1}{2}$

4) Local extrema:
 $f''(x) = 2 > 0$ At $x = \frac{1}{2}, f''(x) = 2 > 0$
 $f(x)$ has local Minimum at $x = \frac{1}{2}$
Local Minimum value = $f(\frac{1}{2})$

$$= \left(\frac{1}{2}\right)^2 - \frac{1}{2} - b$$

$$= \frac{-25}{4}$$

5) Range: $y \geq \frac{-25}{4}$

6) Concavity:
Since $f''(x) = 2 > 0$, the function is
Concave upward in the entire real line
 \Rightarrow Point of Inflection:

Since $f''(x) = 2 \neq 0$ the curve has no
point of inflection.

7) Asymptote:

The curve has no asymptotes.

44.b/5 Marks

$u = \tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$

$\tan u = \frac{x^3+y^3}{x-y}$ is homogeneous.

$f(x, y) = \frac{x^3(x^3+y^3)}{x-y} = x^2 f(x, y)$

f is homogeneous with degree 2.

By Euler theorem,

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2 f(x, y)$$

45.a/5 Marks

$$x \cdot \frac{\partial}{\partial x}(\tan u) + y \cdot \frac{\partial}{\partial y}(\tan u) = 2 \tan u$$

$$x \cdot \sec^2 u \cdot \frac{\partial u}{\partial x} + y \cdot \sec^2 u \cdot \frac{\partial u}{\partial y} = 2 \tan u$$

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 2 \cdot \frac{\sin u}{\cos^2 u} \cdot \cos^2 u$$

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \sin 2u$$

45.b/5 Marks

Sol:
 $\sin x = \cos x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}$
 $\sin x = \cos x = -\frac{1}{\sqrt{2}} \Rightarrow x = \frac{5\pi}{4}$

From the diagram

$\cos x > \sin x$ for $0 \leq x \leq \frac{\pi}{4}$
 $\sin x > \cos x$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx$

$$= \left[\sin x + \cos x \right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0+1) + (1-0) - \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right)$$

$$= 2\sqrt{2} \text{ sq. units.}$$

45.b/5 Marks

T-be the temperature of boiling water at time t'
S-be the kitchen Temperature

$$\frac{dT}{dt} \propto T-S \Rightarrow \frac{dT}{dE} = k(T-S)$$

$$T-S = e^{kt} \rightarrow \text{①}$$

when $t=0, T=100 \Rightarrow 100-S=c^0$

$$\Rightarrow c=100-S$$

$$T-S=(100-S)e^{kt} \rightarrow \text{②}$$

when $t=5, T=80$:

$$80-S=(100-S)e^{5k} \Rightarrow e^{5k} = \frac{80-S}{100-S}$$

when $t=10, T=65$,

$$65-S=(100-S)e^{10k} \Rightarrow 65-S=(100-S)(e^{5k})^2$$

$$65-S=(100-S)(80-S)(80-S) \over (100-S)(100-S)$$

$$6500 - 165S + S^2 = 6400 - 160S + S^2$$

$$5S = 100 \Rightarrow S = 20^\circ C.$$

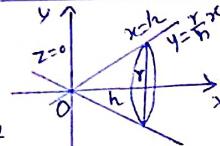
Kitchen Temperature is $20^\circ C$.

46.a) Consider the triangular region in the first quadrant which is bounded by the line $y = \frac{r}{h}x$, x -axis $x=0$ and $x=h$ revolving the region about x -axis base radius r , height $-h$

$$V = \pi \int_0^h y^2 dx = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$= \pi \left(\frac{r}{h}\right)^2 \cdot \int_0^h x^2 dx = \pi \left(\frac{r}{h}\right)^2 \left[\frac{x^3}{3}\right]_0^h$$

$$V = \frac{\pi r^2 h}{3}$$
 cube units.



47.a) Let the two curves intersect at a point (x_0, y_0) . This leads to

$$(a-c)x_0^2 + (b-d)y_0^2 = 0$$

Slope of the curves

$$ax^2 + by^2 = 1 \Rightarrow \frac{dy}{dx} = -\frac{ax}{by}$$

$$cx^2 + dy^2 = 1 \Rightarrow \frac{dy}{dx} = -\frac{cx}{dy}$$

If two curves cut orthogonally, then, $m_1 m_2 = -1$ at (x_0, y_0)

$$\Rightarrow \left(-\frac{ax_0}{by_0}\right) \times \left(-\frac{cx_0}{dy_0}\right) = -1$$

$$acx_0^2 + bd y_0^2 = 0 \quad \rightarrow ①$$

$$\text{Together, } (a-c)x_0^2 + (b-d)y_0^2 = 0 \quad \rightarrow ②$$

From ① & ②

$$\frac{a-c}{ac} = \frac{b-d}{bd}$$

$$\text{That is } \frac{1}{c} - \frac{1}{a} = \frac{1}{d} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$$

47.b) Let the general equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \rightarrow ①$$

It passes through points $(1, 1)$, $(2, -1)$ and $(3, 2)$

$$\therefore 2g + 2f + c = -2, \quad \rightarrow ②$$

$$4g - 2f + c = -5 \quad \rightarrow ③$$

$$9g + 4f + c = -13 \quad \rightarrow ④$$

$$② - ③ \Rightarrow -2g + 4f = 3 \quad \rightarrow ⑤$$

$$④ - ③ \Rightarrow 7g + 6f = -8 \quad \rightarrow ⑥$$

$$⑤ + ⑥ \Rightarrow f = -\frac{1}{2}$$

$$f = -\frac{1}{2} \text{ sub in } ① \quad g = -\frac{5}{2}$$

$$f = -\frac{1}{2} \text{ and } g = -\frac{5}{2} \text{ sub in } ②$$

$$\Rightarrow c = 4$$

$$\therefore \text{The required equation of circle}$$

$$x^2 + y^2 + 2\left(-\frac{5}{2}\right)x + 2\left(-\frac{1}{2}\right)y + 4 = 0$$

$$\Rightarrow x^2 + y^2 - 5x - y + 4 = 0$$

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