HALF- YEARLY EXAMINATION -2K22 DEC
SHRI VINAYAGA HR.SEC.SCHOOL-MOREPALAYAM

STD: XII **19.12.2K22**

SUBJECT: MATHEMATICS **ANSWER KEY** **MARKS : 90**

SECTION – I

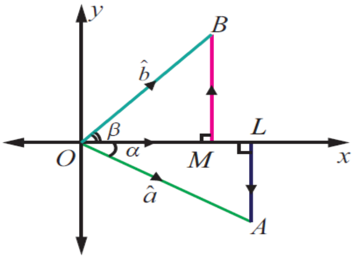
Q.No	ANSWER KEY	MARKS
1.	(c) I_3	1
2.	(a) z	1
3.	(a) $-\frac{q}{r}$	1
4.	(b) $\frac{3\pi}{4}$	1
5.	(b) $2\sqrt{5}$	1
6.	(d) 9	1
7.	(c) $c = \pm\sqrt{3}$	1
8.	(c) $\pm\frac{1}{3}$	1
9.	(d) -1	1
10.	(b) 8	1
11.	(c) $\frac{\sqrt{3}}{12}$	1
12.	(d) 9	1
13.	(b) $-x + \frac{\pi}{2}$	1
14.	(3) $\frac{8}{3}$	1
15.	(b) $\frac{2}{9}$	1
16.	(d) $\cot x$	1
17.	(b) $e^x + e^{-y} = c$	1
18.	(d) 2	1
19.	(b) z	1
20.	(b) 8	1

<table>
<tr><td>21.</td><td>Let A = $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$; $A^T=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$
$AA^T=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$
similarly $A^TA=I_2$
∴ $AA^T=A^TA=I_2$
∴ A is orthogonal</td><td>1
1</td></tr>
<tr><td>22.</td><td>$\sqrt{z}=\sqrt{a+ib}=\pm\left(\sqrt{\frac{|z|+a}{2}}+i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$
$\sqrt{-6+8i}\ =\pm\left(\sqrt{\frac{10-6}{2}}+i\sqrt{\frac{10+6}{2}}\right)\ =\pm\left(\sqrt{2}+i2\sqrt{2}\right)$</td><td>2</td></tr>
<tr><td>23.</td><td>Given $2-\sqrt{3}$ is a root
∴ $2+\sqrt{3}$ is also a root
The equation is x^2 -(S.R)x+ P.R=0
$x^2-4x+1=0$</td><td>1
1</td></tr>
<tr><td>24.</td><td>Given, $\vec{b}=\vec{i}-\vec{j}+\vec{k}$ and $\vec{n}=2\vec{i}-\vec{j}+\vec{k}$
$\sin\theta=\frac{|\hat{b}.\hat{n}|}{|\hat{b}||\hat{n}|}=\frac{4}{3\sqrt{2}}$ $\theta=\text{Sin}^{-1}\left(\frac{4}{3\sqrt{2}}\right)$</td><td>1
1</td></tr>
<tr><td>25.</td><td>$\lim\limits_{x\to 0^+}\left(\frac{\sin x}{x^2}\right)=\infty$
$\lim\limits_{x\to 0^-}\left(\frac{\sin x}{x^2}\right)=-\infty$
limit does not exist.</td><td>1
1</td></tr>
<tr><td>26.</td><td>$f_x=3x^2-6x+5$
$f_x(1,-2)=3-6+5=2$</td><td>1
1</td></tr>
<tr><td>27.</td><td>Let I = $\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$------(1)
I = $\int_2^3\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$ --------(2)
(1)+(2)
$\Rightarrow$ I = $\frac{1}{2}$</td><td>1
1</td></tr>
<tr><td>28.</td><td>$\frac{1}{\sqrt{1-y^2}}dy=\frac{1}{\sqrt{1-x^2}}dx$
Integrating we get,
$\sin^{-1}y=\sin^{-1}x+c$</td><td>1
1</td></tr>
<tr><td>29.</td><td><table>
<tr><th>p</th><th>q</th><th>¬ p</th><th>p → q</th><th>¬ pVq</th></tr>
<tr><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td></tr>
<tr><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td></tr>
<tr><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td></tr>
</table>
Last two columns are identical.
∴ p → q ≡ ¬ pVq</td><td>2</td></tr>
</table>

30.	centre (-g, -f) = $\left(\frac{3}{2},-1\right)$ Radius r = $\sqrt{g^2+f^2-c}$ = $\frac{3}{2}$	1 1
	PART – III	
31.	Given: $x^3 + 27 = 0$. $x = 3(-1)^{\frac{1}{3}} = 3[cos\pi + isin\pi]^{\frac{1}{3}}$ $= 3[\cos(2k+1)\frac{\pi}{3} + isin(2k+1)\frac{\pi}{3}]$, k = 0,1,2 $= 3\ \text{Cis}\frac{\pi}{3}, 3\ \text{Cis}\frac{3\pi}{3}, 3\ \text{Cis}\frac{5\pi}{3},$ The roots are $3\ \text{Cis}\frac{\pi}{3}, -3, 3\ \text{Cis}\frac{5\pi}{3}$	1 1 1
32.	$\tan^{-1}\left[\dfrac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$ $\Rightarrow 2x^2 - 4 = -3$ $\Rightarrow x = \pm\frac{1}{\sqrt{2}}$	1 1 1
33.	Equation of the normal at the point ‘t_1’is $y + xt_1 = at_1^3 + 2at_1$….(1) $(at_2^2, 2at_2)$ $2at_2 + (at_2^2)\, t_1 = at_1^3 + 2at_1$ $2a(t_2 - t_1) = -\, at_1(t_2 - t_1)\,(t_2 + t_1)$ $2 = -\, t_1(t_2 + t_1)$ $t_2 = -\left(\frac{2}{t_1} + t_1\right)$	1 1 1
34.	$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c}$ ------(1) $\vec{b} \times (\vec{c} \times \vec{a}) = (\vec{b}.\vec{a})\,\vec{c} - (\vec{b}.\vec{c})\,\vec{a}$ ------(2) $\vec{c} \times (\vec{a} \times \vec{b}) = (\vec{c}.\vec{b})\,\vec{a} - (\vec{c}.\vec{a})\,\vec{b}$ ------(3) $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$	2 1
35.	$S = x+y = x + \left(\frac{20}{x}\right)$ $S' = 1 - \frac{20}{x^2}$ $S'' = \frac{40}{x^3}$ $S' = 0$, we get $x = 2\sqrt{5}$ $x = 2\sqrt{5}$, $S'' > 0$ $\therefore$ Sum S is minimum at $x = 2\sqrt{5}$. $y = 2\sqrt{5}$ The required two numbers are $2\sqrt{5}, 2\sqrt{5}$	1 1 1
36.	$\frac{1}{T}dT = 0 + \frac{1}{2}\left[\frac{1}{l}dl - 0\right]$ $\frac{dT}{T} = \frac{1}{2l}.dl$	1

	Given : dl= $\frac{2}{100}\, l$ $\therefore \frac{dT}{T} = \frac{1}{100}$ ∴Percentage error = $\frac{dT}{T} \times 100 \quad = \frac{1}{100} \times 100 \quad =1\%$	1 1
37.	tan x = cot x=1⇒ $x = \frac{\pi}{4}$ The required Area A = $\int_0^{\frac{\pi}{4}} \tan x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \, dx$ = log 2 sq. units.	1 1 1
38.	I.F = $e^{\int p dx} = x$ ∴The general solution is y(I.F)=$\int Q(\mathrm{I.F})dx + c$ $yx = \int \log x \; x dx + c$ $4xy = 2x^2 \log x - x^2 + 4c$	1 1 1
39.	$q = \frac{3}{4} \quad p = \frac{1}{4}$ n=8 $P(X=0) = 8C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{8-x}$ x=0,1,2,3,…8	1 2
40.	adjA = $\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$ The solution is X= $A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ x =-11, y = 4	1 1 1
	PART – IV	
41.(a)	$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix} = -22$ $\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = -44$ $\Delta_2 = \begin{vmatrix} 3 & 11 & -11 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix} = -66$ $\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = -88$ By cramer's Rule, we get $x = \frac{\Delta_1}{\Delta} = = 2 \quad y = \frac{\Delta_2}{\Delta} = 3 \quad z = \frac{\Delta_3}{\Delta} = 4$ ∴The solution is (x=2, y=3, z=4)	3 2
41.(b)	Let a = cosα +isinα b= cosβ+isinβ c= cosγ+isinγ a+b+c= (cosα+ cosβ+cosγ)+i(sinα- sinβ+sinγ) = 0 If a+b+c= 0then $a^3+b^3+c^3$= 3abc β $(\cos\alpha+i\sin\alpha)^3+(\cos\beta+i\sin\beta)^3+(\cos\gamma+i\sin\gamma)^3$ =3(cosα+isinα)(cosβ+isinβ)(cosγ+isinγ) (cos3α+isin3α)+(cos3β+isin3β)+(cos3γ+isin3γ) =3cos(α+β+γ)+i3sin(α +β+γ)	1 1 1

	Equating real and imaginary parts we get, (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$and (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$	2
42.(a)	$\frac{2z+1}{iz+1} = \frac{(2x+1)+i2y}{(1-y)+ix}$ $\text{Im}\left(\frac{2z+1}{iz+1}\right) = \frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2}$ $\frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2} = 0$ $2x^2+2y^2+x-2y = 0$	1 2 1 1
42.(b)	$18x^2+12y^2-144x+48y+120 = 0$ $\frac{X^2}{12}+\frac{Y^2}{18} = 1$ (x -4) = X and (y+2) = Y Major axis Y axis, $a^2 = 18 \Rightarrow a = 3\sqrt{2}$ $b^2 = 12 \Rightarrow b = 2\sqrt{3}$ $c^2 = a^2-b^2 = 6$ $c = \sqrt{6}$ $e = \frac{1}{\sqrt{3}}$ $\frac{a}{e} = 3\sqrt{6}$ Center (4,-2) Vertex: $(4,-3\sqrt{2}-2)$, $(4,3\sqrt{2}-2)$ Focus : $(4,-\sqrt{6}-2)$, $(4,-\sqrt{6}-2)$ Equation of diectrices : $y = 3\sqrt{6}-2$; $y = -3\sqrt{6}-2$	1 1 1 1 1
43.(a)	Given $\frac{1}{3}$ is a solution. This is reciprocal equation $\therefore$3 is an other root. $6x^2+15x+6=0$ is another factor. $2x^2+5x+2=0$ $x = \frac{-1}{2}, x = -2$ $\therefore$The solutions of the given equation are $\frac{1}{3}$, 3, $\frac{-1}{2}$, -2	1 1 2 1
43.(b)	$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ $\alpha+\beta+\cos^{-1}z = \pi$ $\alpha+\beta = \pi-\cos^{-1}z$---------1 now, $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\cos(\pi-\cos^{-1}z) = xy-\sqrt{1-x^2}\sqrt{1-y^2}$ $-\cos(\cos^{-1}z) = xy-\sqrt{1-x^2}\sqrt{1-y^2}$ $(z+xy)^2 = (1-x^2)(1-y^2)$ $z^2+x^2y^2+2xyz = 1-y^2-x^2+x^2y^2$ $x^2+y^2+z^2+2xyz = 1$	1 1 2 1

44.(a)	Diagram  $a=\cos\alpha\hat{i}-\sin\alpha\, j$ $\hat{b}=\cos\beta\hat{i}+\sin\beta\, j$ $a.\hat{b}=\lvert a\rvert\lvert\hat{b}\rvert\cos(\alpha+\beta)=\cos(\alpha+\beta) \quad \rightarrow(1)$ $a.\hat{b}=\cos\alpha\cos\alpha-\sin\beta\sin\beta \quad \rightarrow(2)$ $(1)\&(2) \Rightarrow \cos(\alpha+\beta)=\cos\alpha\cos\alpha-\sin\beta\sin\beta$	1 1 1 1 1
44.(b)	**Solution:** Δ=ad-bc Δ_{x^2}=d-b $\quad \Delta_{y^2}$=a-c $\therefore\ x^2=\frac{d-b}{ad-bc}$ $y^2=\frac{a-c}{ad-bc}$ $m_1=-\frac{ax}{by}$ $\qquad m_2=-\frac{cx}{dy}$ $\therefore m_1m_2=-1$ $\left(-\frac{ax}{by}\right)\left(-\frac{cx}{dy}\right)=-1 \qquad \frac{acx^2}{bdy^2}=-1$ $\frac{1}{b}-\frac{1}{d}+\frac{1}{c}-\frac{1}{a}=0$ $\therefore\ \frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$	1 1 1 2
45.(a)	u is not homogeneous function $f(x, y)=\tan u=\frac{x^3+y^3}{x+y}$ $f(\lambda x,\lambda y)=\lambda^2 f(x, y)$ $\therefore$f is homogeneous function with degree 2 By Euler's Tehorem, $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=2f$ $x\frac{\partial(\tan u)}{\partial x}+y\frac{\partial(\tan u)}{\partial y}=\frac{1}{2}\sin u$ $x\sec^2 u\frac{\partial u}{\partial x}+y\sec^2 u\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$ $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\sin 2u$	1 1 1 2
45.(b)	**Diagram** Equation of AB $\quad y=\frac{1}{4}(x+5)$ Equation of BC $\quad y=5-x$ Equation of AC $\quad y=4x+5$ Area of ΔABC $=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\int_{-1}^{3}\frac{1}{4}(x+5)dx$ Required Area of ΔABC $=\frac{15}{2}$ sq. units	1 1 1 2
46.(a)	$\frac{dA}{dt}\propto A\Rightarrow\frac{dA}{dt}=kA \qquad A=Ce^{kt}$	1

$t = 0$ & $A = 3{,}00{,}000$ $[\boldsymbol{C = 3, 00, 000}]$

1

$A = 3{,}00{,}000\, e^{kt}$

1

$t = 40$ & $A = 4{,}00{,}000$ $e^{k} = \left(\frac{4}{3}\right)^{\frac{t}{40}}$

1

$\boldsymbol{A = 3, 00, 000} \left(\frac{4}{3}\right)^{\frac{t}{40}}$

1

$\sum f(x) = 1$ $k = \frac{1}{6}$

1

x	1	2	3	4	5
f(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{6}$	$\frac{3}{6}$

2

$P(2 \le X < 5) = \frac{17}{36}$

1

$P(3 < X) = \frac{5}{6}$

1

47.(a) p→(¬qVr) ≡ ¬ pV(¬qVr)

p	q	r	¬ p	¬ q	(¬qVr)	p→(¬qVr)	¬ pV(¬qVr)
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

2

Last column (2)

Last two columns are identical.

∴ p→(¬qVr) ≡ ¬ pV(¬qVr)

1

47.(b) *Diagram*

1

Volume of cylinder V = $\pi x^2 y = \pi b \left(x^2 - \frac{x^3}{a}\right)$

1

$V' = \pi b \left(2x - \frac{3x^2}{a}\right)$

$V'' = \pi b \left(2 - \frac{6x}{a}\right)$

At $V' = 0$, $x = \frac{2a}{3}$

When $x = \frac{2a}{3}$, $V'' < 0$, volume is maximum.

1

When $x = \frac{2a}{3}$ then $y = \frac{b}{3}$

volume of cylinder V = $\pi \left(\frac{2a}{3}\right)^2 \left(\frac{b}{3}\right) = \frac{4}{9}\left(\frac{1}{3}\pi a^2 b\right)$

volume of cylinder = $\frac{4}{9}$ (Volume of cone)

2

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