KALLAKURICHI -DISTRICT

Class : 12

Register Number

COMMON HALFYEARLY EXAMINATION - 2022 - 23

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART – A

$20 X 1 = 20$

I. Answer all the Questions

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$

   (1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

2. If A ,B are orthogonal matrices , then $(AB)^T(AB)$

   (1) A (2) B (3) I (4) A^T

3. The area of the triangle formed by the complex numbers z, iz, and $z+iz$ in the Argand's diagram is

   (1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

4. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

   (1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

5. If α, β, and γ are the roots of x^3+px^2+qx+r, then $\sum\frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

6. The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is

   (1) $[1, 2]$ (2) $[-1, 1]$ (3) $[0, 1]$ (4) $[-1, 0]$

7 If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

   (1) 3 (2) -1 (3) 1 (4) 9

8. The Focus of $(x+3)^2 = -4\left(y-\frac{3}{4}\right)$ is

   (1) (-3 , 4) (2) (3 , $\frac{4}{3}$) (3) (-3 , $\frac{3}{4}$) (4) (-3, -$\frac{1}{4}$)

9. Distance from the origin to the plane $3x-6y+2z+7=0$ is

   (1) 0 (2) 1 (3) 2 (4) 3

10. If $2\hat{i}-\hat{j}+3\hat{k}, 3\hat{i}+2\hat{j}+\hat{k}, \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, then the value of m

   (1) m= -3 (2) m= 3 (3) m= -4 (4) m= 1

11. Angle between $y^2=x$ and $x^2=y$ at the origin is

   (1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

12. The point of inflection of the curve $y=(x-1)^3$ is

   (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

13. Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is

(1) $x+\frac{\pi}{2}$ (2) $-x+\frac{\pi}{2}$ (3) $x-\frac{\pi}{2}$ (4) $-x-\frac{\pi}{2}$

14. If $f(x)=\int_1^x \frac{e^{\sin u}}{u}du, x>1$ and $\int_1^3 \frac{e^{\sin x^2}}{x}dx=\frac{1}{2}[f(a)-f(1)]$, then one of the possible value of a is

(1) 3 (2) 6 (3) 9 (4) 5

15 The value of $\int_0^1 x^3(1-x)^4 dx$

(1) $\frac{1}{460}$ (2) $\frac{1}{360}$ (3) $\frac{1}{720}$ (4) $\frac{1}{280}$

16. The differential equation of the family of curves $y=Ae^x+Be^{-x}$, where A and B are arbitrary constants is

(1) $\frac{d^2y}{dx^2}+y=0$ (2) $\frac{d^2y}{dx^2}-y=0$ (3) $\frac{dy}{dx}+y=0$ (4) $\frac{dy}{dx}-y=0$

17. The degree and order of the differential equation $x^2\frac{d^2y}{dx^2}+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=0$ is $\left[\frac{d^2y}{dx^2}\right]^2$

(1) 4 ,2 (2) 2, 3 (3) ∞ , 2 (4) Nowhere

18. If in 6 trials, X is a binomial variate which follows the relation $9P(X=4)=PX=2)$, then the probability of success is

(1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75

19 The dual of $\neg(p\vee q)\vee[p\vee p(\wedge\neg r)]$ is

(1) $\neg(p\wedge q)\wedge[p\vee(p\wedge\neg r)]$ (2) $(p\wedge q)\wedge[p\wedge(p\vee\neg r)]$

(3) $\neg(p\wedge)\wedge[p\wedge(p\wedge r)]$ (4) $\neg(p\wedge q)\vee[p\wedge(p\vee\neg r)]$

20. Which one of the following is a binary operation on Division ?

(1) $\mathbb{R}$ (2) $\mathbb{Z}$ (3) $\mathbb{N}$ (4) $\mathbb{Q}\backslash\{0\}$

PART -B 7 X 2 = 14

II. Answer any Seven Questions (Question no . 30 is Compulsory)

21. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22. Find the Square root of $4+3i$

23. Find a polynomial equation of minimum degree with rational coefficients, having $3+\sqrt{2}$ as a root.

24. Find the equation of the ellipse for foci $(0,\pm4)$ and end points of major axis are $(0,\pm5)$.

25. Prove that $[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}]=0$

26. Find df for $f(x)=x^2+3x$ and evaluate it for $x=3$ and $dx=0.02$

27. Evaluate: $\int_b^\infty \frac{1}{a^2+x^2}dx, a>0, b\in\mathbb{R}$.

28. A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2\le X<5)$

29. Establish the equivalence property: $p \to q \equiv \neg p \vee q$.

30. Evaluate: $\sin(\sin^{-1}(16))$

PART – C $7 X 3 = 21$

III. Answer any Seven Questions (Question no . 40 is Compulsory)

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

32. State and Prove that Triangle inequality.

33. Find the value of $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

34. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1,-3)$.

35. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

36. Evaluate: $\lim\limits_{x \to 0}\left(\frac{\sin x}{x^2}\right)$

37. If $w(x,y) = x^3 - 3xy + 2y^2$, find the linear approximation for w at $(1,-1)$.

38. Solve $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

39. The probability that Q hits a target at any trial is $\frac{1}{4}$. He tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

40. Evaluate: $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$

PART-D $7 X 5 = 35$

IV. Answer all the Questions

41. Solve $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ by Cramer's rule

(OR)

Solve the equation $z^3 + 8i = 0$, where $z \in R$.

42. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

(OR)

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

43. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

44. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

(OR)

A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between the jeep and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

45. For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

(OR)

If $v(x,y) = \log(x+y)$, Prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$

46. Find the area of the region bounded between the parabola $x^2=y$ and the curve $y=|x|$.

(OR)

Solve: $\left(1+2e^{x/y}\right)dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

47. The probability density function of the random variable X is given by

$$f(x)=\begin{cases}16xe^{-4x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

find the mean and variance of X.

(OR)

(i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $m*n=m+n-mn\,;m,n\in\mathbb{Z}$