23.12.2022 **Standard 12**

Time Allowed: 3.00 Hours **MATHEMATICS** Maximum Marks: 90

PART - I

All questions are compulsory: **20×1=20**

1) If $A^T A^{-1}$ us symmetric, then A^2 =
   1) A^{-1} 2) $(A^T)^2$ 3) A^T 4) $(A^{-1})^2$

2) $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is
   1) 0 2) 1 3) −1 4) i

3) If α, β and γ are the zeros of x^3+px^2+qx+r, then $\sum\frac{1}{\alpha}$ is
   1) $-\frac{q}{r}$ 2) $-\frac{p}{r}$ 3) $\frac{q}{r}$ 4) $-\frac{q}{p}$

4) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
   1) [1, −2] 2) [−1, 1] 3) [0, 1] 4) [−1, 0]

5) The area of quadrilateral formed with foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is
   1) $4(a^2+b^2)$ 2) $2(a^2+b^2)$ 3) a^2+b^2 4) $\frac{1}{2}(a^2+b^2)$

6) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to
   1) 81 2) 9 3) 27 4) 18

7) If $|\text{adj}(\text{adj } A)| = |A|^{16}$, then the order of the square matrix A is
   1) 3 2) 4 3) 2 4) 5

8) The maximum value of the function x^2e^{-2x}, $x>0$ is
   1) $\frac{1}{e}$ 2) $\frac{1}{2e}$ 3) $\frac{1}{e^2}$ 4) $\frac{4}{e^4}$

9) If $v(x, y) = \log(e^x+e^y)$ then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}$ is equal to
   1) e^x+e^y 2) $\frac{1}{e^x+e^y}$ 3) 2 4) 1

10) The value of $\int_0^{\pi}\sin^4 x\,dx$ is
   1) $\frac{3\pi}{10}$ 2) $\frac{3\pi}{8}$ 3) $\frac{3\pi}{4}$ 4) $\frac{3\pi}{2}$

11) The solution of $\frac{dy}{dx}+p(x)y=0$ is
   1) $y = ce^{\int pdx}$ 2) $y = ce^{-\int pdx}$ 3) $x = ce^{-\int pdy}$ 4) $x = ce^{\int pdy}$

12) If $P(X=0)=1-P(X-1)$. If $E(X)=3Var(X)$, then $P(X=0)$ is

1) $\frac{2}{3}$ 2) $\frac{2}{5}$ 3) $\frac{1}{5}$ 4) $\frac{1}{3}$

13) Which one of the following statements has truth value F?

1) Chennai is in India or $\sqrt{2}$ is an integer.

2) Chennai is in India or $\sqrt{2}$ is an irrational number.

3) Chennai is in China or $\sqrt{2}$ is an integer.

4) Chennai is in China or $\sqrt{2}$ is an irrational number.

14) If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are (11, 2) the coordinates of the other end are

1) (−5, 2) 2) (−3, 2) 3) (5, −2) 4) (−2, 5)

15) Simplify $i^{-1924}+i^{2018}$

1) 0 2) i 3) −i 4) 1

16) The volume of the parallelopiped with its edges represented by the vectors $\vec{i}+\vec{j}$, $\vec{i}+2\vec{j}$, $\vec{i}+\vec{j}+\pi\vec{k}$ is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) π 4) $\frac{\pi}{4}$

17) A stone is thrown up vertically. The height is reaches at time t seconds is given by $x = 80t-16t^2$. The stone reaches the maximum height in time t seconds is given by

1) 2 2) 2.5 3) 3 4) 3.5

18) If f(x) is an odd function, then $\int_{-a}^{a} f(x)\,dx =$

1) 0 2) $2\int_{0}^{a} f(x)\,dx$ 3) 2 4) $\int_{0}^{a} f(x)\,dx$

19) The operation * defined by $a*b = \frac{ab}{7}$ is not a binary operation on

1) Q^+ 2) Z 3) R 4) C

20) Find the principal value of $\sin^{-1}(2)$, if it exists.

1) Does not exists 2) 2 3) 0 4) −1

PART - II

(i) Answer any 7 questions. (ii) Question No. 30 is compulsory. **7×2=14**

21) If $adjA = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22) Find the value of $\left[\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right]^{10}$.

23) If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

24) Find the value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$.

25) If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ find $\vec{a}.(\vec{b}\times\vec{c})$.

26) A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100\times(1-0.1t)^2$, $0\le t\le 10$. What is the rate at which the person forgets the words 2 days after learning?

27) Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

28) Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$

29) Let X be a random variable denoting the life time of an electrical equipment having probability density function.

$$f(x) = \begin{cases} ke^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$ find the k value.

30) In the set Q define $a\odot b = a+b+ab$. For what value of y, $3\odot(y\odot 5) = 7$?

PART - III

(i) Answer any 7 questions. (ii) Question No. 40 is compulsory. **7×3=21**

31) Find the inverse of the non-singular matrix $A = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ by Gauss - Jordan method.

32) Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real.

33) If the equation $3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

34) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

35) Show that the points (2, 3, 4), (−1, 4, 5) and (8, 1, 2) are collinear.

36) If $X\sim B(n, p)$ such that $4P(X=4)=P(X=2)$ and $n=6$. Find the distribution, mean and standard deviation of X.

37) Solve: $\frac{dy}{dx} + 2y = e^{-x}$

38) Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

39) Find the values in the interval (1, 2) of the mean value theorem satisfied by the function $f(x) = x-x^2$ for $1\le x\le 2$.

40) Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

PART - IV

Answer all the questions: **7×5=35**

41) a) Determine the values of λ for which the following system of equations: $x+y+3z = 0$, $4x+3y+\lambda z = 0$, $2x+y+2z = 0$ has (i) a unique solution (ii) a non-trivial solution. **(OR)**

b) Solve the equation $6x^4-5x^3-38x^2-5x+6 = 0$ if it is known that 1/3 is a solution.

42) a) If $z = x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2+y^2 = 1$.

(OR)

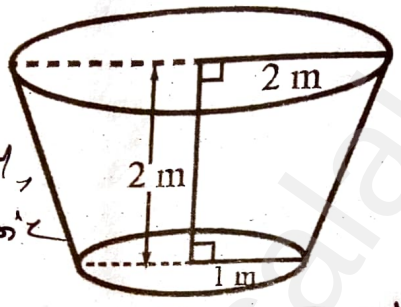
b) Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

43) a) Prove by vector method that $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

(OR)

b) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is 4/9 times volume of the cone.

44) a) Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the figure.



(OR)

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

45) a) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0, 0) and (1, 1).

(OR)

b) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$.

46) a) Find the population of a city at any time t, given that the rate of incerase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

(OR)

b) Show that the line $x-y+4 = 0$ is a tangent to the ellipse $x^2+3y^2 = 12$. Also find the coordinate of the point of contact.

47) a) A random variable X has the following probability mass function.

x	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

(1) Find k. (2) Evaluate $P(X<4)$, $P(X\geq5)$ and $P(3<X\leq6)$

(OR)

b) Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identy and (v) existence of inverse for the operation $+_5$ and Z_5 using table corresponding to addition modulo 5.
