COMMON HALF YEARLY EXAMINATION - 2022

Standard XII Reg.No. | | | | | | |

MATHEMATICS

Time: 3.00 hours *Marks: 90*

Part - I

I Choose the correct answer **20 x 1 = 20**

1. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left[\vec{a}, \vec{b}, \vec{a}\times\vec{b}\right] = \frac{1}{4}$ then the angle between $\vec{a}$ and $\vec{b}$ is

   a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

2. If the planes $\vec{r}.\left(2\hat{i} - \lambda\hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(4\hat{i} + \hat{j} - \mu\hat{k}\right) = 5$ are parallel then the value of λ and μ are

   a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$

3. The vertex of the parabola $x^2 - 8y - 1$ is

   a) $\left(-\frac{1}{8}, 0\right)$ b) $\left(\frac{1}{8}, 0\right)$ c) $\left(-6, \frac{9}{2}\right)$ d) $\left(\frac{9}{2}, -6\right)$

4. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   a) $2ab$ b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

5. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if k satisfies

   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

7. The conjugate of a complex number is $\frac{1}{i-2}$, then the complex number is

   a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

8. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{s}\right)$ is

   a) -2 b) -1 c) 1 d) 2

9. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$, then $|\text{adj}\,(AB)| =$

   a) -40 b) -80 c) -60 d) -20

10. Let A be a non-singular matrix then which one of the following is false?

    a) $(\text{adj}\,A)^{-1} = \frac{A}{|A|}$ b) I is an orthogonal matrix

    c) $\text{adj}\,(\text{adj}\,A) = |A|^n A$ d) If A is symmetric then adj A is symmetric

11. Subtraction is not a binary operation in
a) R b) Z c) N d) Q

12. Which one is the inverse of the statement $(p \vee q) \to p \wedge q)$?
a) $(p \wedge q) \to (p \vee q)$ b) $\neg(p \vee q) \to (p \wedge q)$
c) $(\neg p \vee \neg q) \to (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \to (\neg p \vee \neg q)$

13. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$, then standard deviation of X is
a) 6 b) 4 c) 3 d) 2

14. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively
a) 2,3 b) 3,3 c) 2,6 d) 2,4

15. The Integrating factor of $\frac{dy}{dx} + y \cot x = 1$ is
a) $\cot x$ b) $\tan x$ c) $\sin x$ d) $\cos x$

16. The value of $\int_0^1 \log\left(\frac{x}{1-x}\right) dx$
a) 0 b) 2 c) 4 d) 5

17. The value of $\int_0^\infty e^{-3x} x^2 \, dx$ is
a) $7/27$ b) $5/27$ c) $4/27$ d) $2/27$

18. If $u(x.y) = e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is equal to
a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u

19. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is
a) $t = 0$ b) $t = 1/3$ c) $t = 1$ d) $t = 3$

20. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
a) $\tan^{-1} 3/4$ b) $\tan^{-1}(4/3)$ c) $\pi/3$ d) $\pi/4$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **$7 \times 2 = 14$**

21. Prove that $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is orthogonal.

22. Find the value of $\sin^{-1}\left(\sin\left(5\pi/4\right)\right)$

23. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

24. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

25. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

26. Let $f(x) = \sqrt[3]{x}$, find the liniear approximation at $x = 27$ use the linear approximation to approximate $\sqrt[3]{27.2}$

27. Evaluate : $\int_0^{\pi/2} \sin^{10} x \, dx$

28. Write the Maclaurin series expansion of the function e^x

29. Construct the truth table for the statement $\neg(p \wedge \neg q)$

30. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the square root of $6 - 8i$

32. if α and β are the roots of the quadratic equation $2x^2 - 7x - 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

33. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\sqrt{3}/2\right)\right)\right)$

34. Find the equation of the hyperbola with foci $(\pm 3, 5)$ and eccentricity $e = 2$

35. Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$

36. Evaluate the limits if necessary use l'Hôpital Rule $\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

37. Evaluate : $\int_0^1 \frac{2x}{1+x^2} dx$

38. Solve : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

39. The mean and variance of a binomial variate X are respectively 2 and 1.5, find $P(X = 0)$

40. If $A = \begin{bmatrix} -3 & -2 \\ \lambda & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. *(OR)*

b) Solve the following equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

42. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$ *(OR)*

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

43. a) Find the non parametric form of Vector equation and Cartesian equation of the plane passing through the point (2,3,6) and paralle to the straight lines

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$ *(OR)*

b) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2 < X < 6)$ ii) $P(2 \le X < 5)$ iii) $P(X \le 4)$ iv) $P(3 < X)$

44. a) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0.0) and (1.1) *(OR)*

b) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

45. a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ *(OR)*

b) Verify (i) Closure property (ii) Associative property (iii) Existance of identity (iv) Existence of inverse and (v) Commutative property for the operation $+_5$ and Z_5 using table corresponding to addition modulo 5.

46. a) Prove by Vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent. *(OR)*

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$ find the centre vertices foci and the length of latus rectum.

47. a) Evaluate the definite integral $\int_{\pi/8}^{3\pi/8} \frac{1}{1+\sqrt{\tan x}}\,dx$ *(OR)*

b) A hollow cone with base radius 'a' cm and height 'b' cm is placed on a table show that the volume of the largest cylinder that can be hidden underneath is $4/9$ times volume of the cone.

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