

HTV

XII - Std

HALF YEARLY EXAMINATION - 2022

MATHEMATICS

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Time : 3.00 Hrs

Marks : 90

Section - A

I Choose the correct answer :-

20 X 1 = 20

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is.
- a) 17 b) 14 c) 19 d) 21
2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is a adjoint of 3×3 matrix A and $|A| = 4$, then x is.
- a) 15 b) 12 c) 14 d) 11
3. The value of $i^{2018} + i^{2019} + i^{2020} + i^{2021}$ is
- a) 0 b) 1 c) -1 d) i
4. The solution of $|z| - z = 1 + 2i$ is.
- a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} - 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$
5. A zero of $x^3 + 64$ is.
- a) 0 b) 4 c) $4i$ d) -4
6. $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then x is
- a) 4 b) 5 c) 3 d) 2
7. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belong to
- a) $[-1, 1]$ b) $[\sqrt{2}, 2]$ c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ d) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$
8. If $x + y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is
- a) 3 b) -1 c) 9 d) 1
9. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is
- a) 2 b) 4 c) 6 d) 8
10. If $[\vec{a}X\vec{b} \quad \vec{b}X\vec{c} \quad \vec{c}X\vec{a}] = 121$ then $[\vec{a} \quad \vec{b} \quad \vec{c}] =$ is
- a) $\frac{121}{2}$ b) 8 c) 11 d) 121

11. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$, $\vec{c} = \vec{j}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$ then $\lambda + \mu$ is
 a) 0 b) 1 c) 2 d) 3
12. The minimum value of the function $|3 - x| + 9$ is
 a) 0 b) 3 c) 6 d) 9
13. Angle between $y^2 = x$ and $x^2 = y$ at origin is
 a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) can't find
14. $V(x, y) = \log(e^x + e^y)$ then $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} =$
 a) $e^x + e^y$ b) $\frac{1}{e^x + e^y}$ c) 2 d) 1
15. The value of $\int_{-4}^4 |x+3| dx$ is
 a) 50 b) 25 c) 16 d) 8
16. The value of $\int_0^\pi \sin^4 x dx$ is
 a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$
17. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
 a) $xy + k$ b) $y = k \log x$ c) $y = kx$ d) $\log y = kx$
18. The solution of $\frac{dy}{dx} = 2^{y-x}$ is
 a) $2^x + 2^y = c$ b) $2^x - 2^y = c$ c) $\frac{1}{2^x} - \frac{1}{2^y} = c$ d) $x + y = c$
19. If $U = y \sin x$ then $\frac{\partial^2 y}{\partial x \partial y} =$
 a) $\cos x$ b) $\cos y$ c) $\sin x$ d) $\sin y$
20. If $\frac{z-1}{z+1}$ is purely imaginary then $|z|$ is
 a) $\frac{1}{2}$ b) 1 c) 2 d) 3

Section - B

II Answer any 7 questions. Q.No. 30 is compulsory :-

7X2=14

21. If $\text{adj}A = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$ then find A^{-1} .
22. Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.
23. Solve : $x^4 - 14x^2 + 45 = 0$.
24. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$.
25. Obtain the equation of the circle for which $(-4, -2)$ and $(1, 1)$ are the ends of a diameter..
26. Verify that the vectors $\vec{i} + 2\vec{j} - 3\vec{k}$, $2\vec{i} - \vec{j} + 2\vec{k}$, $3\vec{i} + \vec{j} - \vec{k}$ are coplanar.
27. Solve : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
28. Evaluate : $\int_0^1 x^4(1-x)^5 dx$
29. If $\mathbf{V}(x, y, z) = xy + yz + zx$ then find $d\mathbf{v}$.
30. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

Section - C

Answer any 7 questions. Q.No. 40 is compulsory :-

7 X 3 = 21

31. Evaluate. $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$
32. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$ then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
33. Solve : $\frac{dy}{dx} + \frac{y}{x} = \sin x$
34. Show that the shortest distance between the lines
- $$\vec{r} = (6\vec{i} + \vec{j} + 2\vec{k}) + S(\vec{i} + 2\vec{j} - 3\vec{k})$$
- $$\vec{r} = (3\vec{i} + 2\vec{j} - 2\vec{k}) + t(2\vec{i} + 4\vec{j} - 5\vec{k}) \text{ and is } \frac{7\sqrt{5}}{5}.$$
35. If the normal at the point ' t_1 ' on the parabola $y^2 + 4ax$ meets the parabola again at the point ' t_2 ', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.
36. Solve : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, $6x^2 < 1$.
37. If P is real, discuss the nature of the roots of the equation $4x^2 + 4Px + P + 2 = 0$, in terms of P.
38. Show that the equation $z^2 = \bar{z}$ has four solutions.

39. Find the rank of the matrix
- $$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

40. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$

$f(a) + b - n$

Section - IV

$7 \times 5 = 35$

Answer all questions :-

41. a) Investigate for what values of λ, μ the system of linear equations.
 $x+2y+z=7$; $x+y+\lambda z=\mu$; $x+3y+5z=5$ has i) no solution ii) a unique solution iii) an infinite number of solution. (OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

42. a) Solve : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. (OR)

b) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$.

43. a) A bridge has a parabola arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides. (OR)
 b) Prove by vector method $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.

44. a) Sketch the curve $y = f(x) = x^2 - x - 6$. (OR)

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

45. a) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$. (OR)
 b) A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C and another 5 minutes later it has dropped to 65°C . Determine the temperature of the kitchen.

46. a) Find the volume of a right - circular cone of base radius 'r' and height 'h'. (OR)
 b) Find the parametric vector and Cartesian equation of the plane containing the line

$\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ and passing through the point $(-1, 1, -1)$.

47. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then show

that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. (OR)

b) Find the equation of the circle passing through the points $(1, 1)$ $(2, -1)$ and $(3, 2)$.