

# HALF YEARLY EXAMINATION - 2022

12 - Std

## Mathematics

|  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|

Marks : 90

Time : 3.00 hrs.

### PART - I

**Note :** i) All questions are compulsory. ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

20 X 1 = 20

1. If A is a nonsingular matrix such that  $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$  then  $(A^T)^{-1} =$ 
  - a)  $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$
  - b)  $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$
  - c)  $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$
  - d)  $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
2. The rank of the matrix  $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$  is
  - a) 0
  - b) 1
  - c) 2
  - d) -1
3. If z is a non zero complex number, such that  $2iz^2 = \bar{z}$  then |z| is
  - a)  $\frac{1}{2}$
  - b) 1
  - c) 2
  - d) -1
4. The product of all four values of  $\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{\frac{3}{4}}$  is
  - a) -2
  - b) -1
  - c) 1
  - d) 2
5. If  $\alpha, \beta, \gamma$  are the zeros of  $x^3 + px^2 + qx + r$ , then  $\sum \frac{1}{\alpha}$  is
  - a)  $\frac{-q}{r}$
  - b)  $-\frac{p}{r}$
  - c)  $\frac{q}{r}$
  - d)  $\frac{-q}{r}$
6. The number of real numbers in  $[0, 2\pi]$  satisfying  $\sin^4 x - 2 \sin^2 x + 1$  is
  - a) 2
  - b) 4
  - c) 1
  - d)  $\infty$
7.  $\sin^{-1} \frac{3}{5} - \cos^{-1} \frac{12}{13} + \sec^{-1} \frac{5}{3} - \operatorname{cosec}^{-1} \frac{13}{12}$  is equal
  - a)  $2\pi$
  - b)  $\pi$
  - c) 0
  - d)  $\tan^{-1} \frac{12}{65}$
8. If  $\sin^{-1} x + \cot^{-1} \left(\frac{1}{2}\right) = \frac{\pi}{2}$  then x is equal to
  - a)  $\frac{1}{2}$
  - b)  $\frac{1}{\sqrt{5}}$
  - c)  $\frac{2}{\sqrt{5}}$
  - d)  $\frac{\sqrt{3}}{2}$

12 - Maths Page - 1



9. The radius of the circle  $3x^2 + by^2 + 4bx - 6by + b^2 = 0$  is  
 a) 1                      b) 3                      c)  $\sqrt{10}$                       d)  $\sqrt{11}$
10. The focus of  $y^2 = 4ax$  is  
 a)  $(-a, 0)$                       b)  $(a, 0)$                       c)  $(0, a)$                       d)  $(0, -a)$
11. If the direction cosines of a straight line are  $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$  then  
 a)  $C = \pm 3$                       b)  $C = \pm\sqrt{3}$                       c)  $C > 0$                       d)  $0 < C < 1$
12. If  $[\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}] = 64$ , then  $[\bar{a}, \bar{b}, \bar{c}]$  is equal to  
 a) 64                      b) 8                      c) 6                      d) 4
13. Angle between  $y^2 = x$  and  $x^2 = y$  at the origin is  
 a)  $\tan^{-1}(3/4)$                       b)  $\tan^{-1}(4/3)$                       c)  $\pi/2$                       d)  $\pi/4$
14. The point of inflection of the curve  $y = (x - 1)^3$  is  
 a)  $(0, 0)$                       b)  $(0, 1)$                       c)  $(1, 0)$                       d)  $(1, 1)$
15. The approximate change in the volume  $V$  of a cube of side  $x$  metres caused by increasing the side by 1% is  
 a)  $0.3x \, dx \, m^3$                       b)  $0.03x \, m^3$                       c)  $0.03x^2 \, m^3$                       d)  $0.03 \, x^3 \, m^3$
16. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31  
 a)  $\frac{1}{31}$                       b)  $\frac{1}{5}$                       c) 5                      d) 31
17. The value of  $\int_0^1 x(1-x)^{99} \, dx$  is  
 a)  $\frac{1}{11000}$                       b)  $\frac{1}{10100}$                       c)  $\frac{1}{10010}$                       d)  $\frac{1}{10001}$
18. The order and degree of the differential equation  $\sqrt{\sin x} (dx + dy) = \sqrt{\cos x} (dx - dy)$  are  
 a) 1,2                      b) 2,2                      c) 1,1                      d) 2,1
19. A random variable  $X$  has binomial distribution with  $n = 25$  and  $p = 0.8$  then standard deviation of  $X$  is  
 a) 6                      b) 4                      c) 3                      d) 2
20. In the set  $Q$  define  $a \oplus b = a + b + ab$  for what value of  $y$ ,  $3 \oplus (y \oplus 5) = 7$ ?  
 a)  $y = \frac{2}{3}$                       b)  $y = -\frac{2}{3}$                       c)  $y = -\frac{3}{2}$                       d)  $y = 4$



**PART - II**

**Note : 1) Answer any seven questions.**  
**2) Question number 30 is compulsory.**

7 × 2 = 14

21. If  $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  then find  $A^{-1}$ .
22. Write  $\frac{1}{1+2i}$  in rectangular form.
23. Prove that in an algebraic structure the identity element (if exists) must be unique.
24. Find the value of  $2 \cos^{-1} (1/2) + \sin^{-1} (1/2)$ .
25. Find the centre and radius of the circle  $2x^2 + 2y^2 - 6x + 4y + 2 = 0$ .
26. Find the magnitude of the force about the point  $(2, 0, -1)$  of a force  $2\hat{i} + \hat{j} - \hat{k}$ , whose line of action passes through the origin.
27. Prove that the function  $f(x) = x^2 - 2x - 3$  is strictly increasing in  $(2, \infty)$ .
28. Find  $df$  for  $f(x) = x^2 + 3x$  and evaluate it for  $x = 3$ ,  $dx = 0.02$ .
29. Evaluate :  $\int_0^{\pi/2} \cos^7 x \, dx$ .
30. Find polynomial equation of minimum degree with rational coefficients, having  $2 + \sqrt{3}i$  as a root.

**PART - III**

**Note: 1) Answer any seven questions.**  
**2) Question number 40 is compulsory.**

7 × 3 = 21

31. If  $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ , then verify  $A (\text{adj } A) = (\text{adj } A) A = |A| I_2$ .
32. Show that the square root of  $6 - 8i$  is  $\pm(2\sqrt{2} - i\sqrt{2})$ .
33. Find the lengths of major and minor axes of the ellipse  $9x^2 + 25y^2 = 225$ .
34. With usual notations, in a triangle ABC using vectors prove that  $a^2 = b^2 + c^2 - 2bc \cos A$ .
35. Find the Maclaurin's Series expansion of the function  $e^x$ .
36. Show that the percentage error in the  $n$ th root of a number is approximately  $1/n$  times the percentage error in the number.
37. Establish the equivalence property  $p \rightarrow q \equiv \neg p \vee q$ .
38. Solve :  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

12 - Maths பக்கம் - 3



39. The mean and variance of a binomial variate  $x$  are respectively 2 and 1.5 find  $p(x = 1)$ .
40. Evaluate :  $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$

### PART - IV

**Note : Answer all the questions.**

7 X 5 = 35

41. a) Test for consistency of the following system of linear equations and if possible solve  $x + 2y - z = 3$ ,  $3x - y + 2z = 1$ ,  $x - 2y + 3z = 3$ ,  $x - y + z + 1 = 0$ . **(OR)**  
b) Find the angle between the curves  $y = x^2$  and  $y = (x - 3)^2$ .
42. a) If  $2 \cos \alpha = x + 1/x$  and  $2 \cos \beta = y + 1/y$ , show that  
i)  $x/y + y/x = 2 \cos(\alpha - \beta)$  ii)  $x^m/y^n + y^n/x^m = 2i \sin(m\alpha - n\beta)$ . **(OR)**  
b) Prove that among all the rectangles of the given perimeter, the square has maximum area.
43. a) Solve  $(x-2)(x-7)(x-3)(x+2) + 19 = 0$  **(OR)**  
b) A random variable  $x$  has the following probability mass function
- |        |     |      |      |      |      |       |
|--------|-----|------|------|------|------|-------|
| $x$    | 1   | 2    | 3    | 4    | 5    | 6     |
| $f(x)$ | $k$ | $2k$ | $6k$ | $5k$ | $6k$ | $10k$ |
- Find i)  $P(2 < x < 6)$  ii)  $P(2 \leq x < 5)$  iii)  $P(x \leq 4)$  iv)  $P(3 < x)$
44. a) If  $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ , then show that  $x + y + z = xyz$ . **(OR)**  
b) The maximum and minimum distances of the Earth from the Sun respectively are  $152 \times 10^6$  km  $94.5 \times 10^6$  km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
45. a) By vector method, prove that  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ . **(OR)**  
b)  $u = \cos^{-1} \left( \frac{x-y}{\sqrt{x} + \sqrt{y}} \right)$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} - \frac{1}{2} \cot u = 0$ .
46. a) Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . **(OR)**  
b) Verify (i) closure property ii) Commutative property iii) associative property iv) existence of identity and v) existence of inverse for the operation  $X_{11}$  on a set  $A = \{1, 3, 4, 5, 9\}$  of the set of remainders  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .
47. a) Find the parametric vector non parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points  $(3, 6, -2)$ ,  $(-1, -26)$  and  $(6, 4, -2)$  **(OR)**  
b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours.