

COMMON HALF YEARLY EXAMINATION - 2022

Standard XII
MATHEMATICS

Reg.No.

--	--	--	--	--	--

Time: 3.00 hours

Marks: 90

Part - I

I Choose the correct answer

20 x 1 = 20

- If \bar{a} and \bar{b} are unit vectors such that $[\bar{a}, \bar{b}, \bar{a} \times \bar{b}] = \frac{1}{4}$ then the angle between \bar{a} and \bar{b} is
 - $\frac{\pi}{6}$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{3}$
 - $\frac{\pi}{2}$
- If the planes $\bar{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\bar{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel then the value of λ and μ are
 - $\frac{1}{2}, -2$
 - $-\frac{1}{2}, 2$
 - $-\frac{1}{2}, -2$
 - $\frac{1}{2}, 2$
- The vertex of the parabola $x^2 - 8y - 1$ is
 - $(-\frac{1}{8}, 0)$
 - $(\frac{1}{8}, 0)$
 - $(-6, \frac{9}{2})$
 - $(\frac{9}{2}, -6)$
- Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
 - $2ab$
 - ab
 - \sqrt{ab}
 - $\frac{a}{b}$
- $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9})$ is equal to
 - $\frac{1}{2} \cos^{-1}(\frac{3}{5})$
 - $\frac{1}{2} \sin^{-1}(\frac{3}{5})$
 - $\frac{1}{2} \tan^{-1}(\frac{3}{5})$
 - $\tan^{-1}(\frac{1}{2})$
- The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if k satisfies
 - $|k| \leq 6$
 - $k = 0$
 - $|k| > 6$
 - $|k| \geq 6$
- The conjugate of a complex number is $\frac{1}{i-2}$, then the complex number is
 - $\frac{1}{i+2}$
 - $\frac{-1}{i+2}$
 - $\frac{-1}{i-2}$
 - $\frac{1}{i-2}$
- The product of all four values of $(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ is
 - 2
 - 1
 - 1
 - 2
- If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$, then $|\text{adj}(AB)| =$
 - 40
 - 80
 - 60
 - 20
- Let A be a non-singular matrix then which one of the following is false?
 - $(\text{adj} A)^{-1} = \frac{A}{|A|}$
 - I is an orthogonal matrix
 - $\text{adj}(\text{adj} A) = |A|^n A$
 - If A is symmetric then $\text{adj} A$ is symmetric

(2)

XII Mathematics

11. Subtraction is not a binary operation in
 a) R b) Z c) N d) Q
12. Which one is the inverse of the statement $(p \vee q) \rightarrow p \wedge q$?
 a) $(p \wedge q) \rightarrow (p \vee q)$ b) $\neg(p \vee q) \rightarrow (p \wedge q)$
 c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$
13. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$, then standard deviation of X is
 a) 6 b) 4 c) 3 d) 2
14. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively
 a) 2,3 b) 3,3 c) 2,6 d) 2,4
15. The Integrating factor of $\frac{dy}{dx} + y \cot x = 1$ is
 a) $\cot x$ b) $\tan x$ c) $\sin x$ d) $\cos x$
16. The value of $\int_0^1 \log\left(\frac{x}{1-x}\right) dx$
 a) 0 b) 2 c) 4 d) 5
17. The value of $\int_0^\infty e^{-3x} x^2 dx$ is
 a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$
18. If $u(x,y) = e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is equal to
 a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u
19. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is
 a) $t = 0$ b) $t = \frac{1}{3}$ c) $t = 1$ d) $t = 3$
20. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
 a) $\tan^{-1} \frac{3}{4}$ b) $\tan^{-1} \left(\frac{4}{3}\right)$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

Part - II**II. Answer any 7 questions. (Q.No.30 is compulsory)****7 x 2 = 14**

21. Prove that $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$ is orthogonal.

22. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

(3)

XII Mathematics

23. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.
24. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$
25. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a} \ \vec{b} \ \vec{c}]$
26. Let $f(x) = \sqrt[3]{x}$, find the linear approximation at $x = 27$ use the linear approximation to approximate $\sqrt[3]{27.2}$

27. Evaluate: $\int_0^{\pi/2} \sin^{10} x \, dx$

28. Write the Maclaurin series expansion of the function e^x .
29. Construct the truth table for the statement $\neg(p \wedge \neg q)$
30. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory)

7 x 3 = 21

31. Find the square root of $6 - 8i$
32. if α and β are the roots of the quadratic equation $2x^2 - 7x - 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2 .
33. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$
34. Find the equation of the hyperbola with foci $(\pm 3, 5)$ and eccentricity $e = 2$
35. Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$
36. Evaluate the limits if necessary use l'Hôpital Rule $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$
37. Evaluate: $\int_0^1 \frac{2x}{1+x^2} \, dx$
38. Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
39. The mean and variance of a binomial variate X are respectively 2 and 1.5, find $P(X = 0)$
40. If $A = \begin{bmatrix} -3 & -2 \\ \lambda & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$

Part - IV

IV. Answer all the questions.

7 x 5 = 35

41. a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (OR)

- (4)
- b) Solve the following equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$
42. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$ (OR)
- b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.
43. a) Find the non parametric form of Vector equation and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines
- $$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1} \text{ and } \frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3} \quad (\text{OR})$$
- b) A random variable X has the following probability mass function.
- | | | | | | | |
|------|---|----|----|----|----|-----|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | k | 2k | 6k | 5k | 6k | 10k |
- Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$ (iii) $P(X \leq 4)$ (iv) $P(3 < X)$
44. a) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0) and (1,1) (OR)
- b) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left[\frac{x + y + z - xyz}{1 - xy - yz - zx} \right]$
45. a) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ (OR)
- b) Verify (i) Closure property (ii) Associative property (iii) Existence of identity (iv) Existence of inverse and (v) Commutative property for the operation $+_5$ and Z_5 using table corresponding to addition modulo 5.
46. a) Prove by Vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent. (OR)
- b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$ find the centre vertices foci and the length of latus rectum.
47. a) Evaluate the definite integral $\int_{\pi/8}^{3\pi/8} \frac{1}{1 + \sqrt{\tan x}} dx$ (OR)
- b) A hollow cone with base radius 'a' cm and height 'b' cm is placed on a table show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

HALF YEARLY EXAM - 2022தொகுப்பு - 2022

பகுதி: XII

பொருள் - Mathematics

A. K. RAJADHURAI - TIRUVALLUR DIST

1) a) $\frac{\pi}{6}$ 2) c) $-\frac{1}{2}, -2$ 3) a) $(0, -\frac{1}{8})$ 4) a) $2ab$

5) d) $\tan^{-1}(\frac{1}{2})$ 6) $|k| \geq 6$ 7) b) $\frac{-1}{i+2}$ 8) c) 1

9) b) -80 10) c) $\text{adj}(\text{adj} A) = |A|^n A$ 11) c) $|A|$

12) d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$ 13) d) 2

14) a) 2, 3 15) c) $\sin x$ 16) a) 0 17) d) $\frac{2}{27}$

18) b) $2au$ 19) b) $t = \frac{1}{3}$ 20) $\frac{\pi}{2}$

பகுதி - 25II
21)

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$AA^T = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$AA^T = I$$

 \therefore A ஒரு ஒர்த்தோக்டிரல் மாட். \therefore A is orthogonal.

22)

$$\begin{aligned} \sin^{-1}(\sin(\frac{5\pi}{4})) &= \sin^{-1}(\sin(\pi + \frac{\pi}{4})) \\ &= \sin^{-1}(-\sin \frac{\pi}{4}) \\ &= \sin^{-1}(\sin(-\frac{\pi}{4})) \\ &= -\frac{\pi}{4} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{aligned}$$

$$23) \frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1-1+2i}{1+1} = i$$

$$\frac{1-i}{1+i} = \frac{1}{i} = -i$$

$$\begin{aligned} \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 &= i^3 - (-i)^3 = i^3 + i^3 \\ &= 2i^3 = 2(-i) = -2i \end{aligned}$$

$$24) P(x) = 9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2$$

①) வாய்மைச் சார்பின் தனிமங்களின் = 4.

②) தவறான வகை தனிம தனிமங்களின் தனிமங்களின் = 4.

$$P(-x) = -9x^9 - 4x^8 - 4x^7 - 3x^6 - 2x^5 - x^3 + 7x^2 - 7x + 2$$

③) வாய்மைச் சார்பின் தனிமங்களின் = 3

④) தவறான வகை தனிம தனிமங்களின் தனிமங்களின் = 3

25)

$$\begin{aligned} [\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] &= \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}] \\ &= [\vec{a}, \vec{b}, \vec{c}] \end{aligned}$$

26)

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$$

$$f(x_0) = f(27) = (27)^{\frac{1}{3}} = 3$$

$$f'(x_0) = f'(27) = \frac{1}{3} (27)^{-\frac{2}{3}} = \frac{1}{27}$$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$L(x) = 3 + \frac{1}{27}(x - 27) = \frac{x}{27} + 2$$

$$\sqrt[3]{27.2} \approx L(27.2)$$

$$\approx \frac{27.2}{27} + 2 \approx 1.0074 + 2$$

$$\sqrt[3]{27.2} \approx 3.0074.$$

27)

$$\int_0^{\pi/2} \sin^{10} x \, dx = \frac{9}{10} \cdot \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{945\pi}{7680} = \frac{189\pi}{1536} = \frac{63\pi}{512}$$

28)

$$f(x) = e^x \quad f(0) = 1$$

$$f'(x) = e^x \quad f'(0) = 1$$

$$f''(x) = e^x \quad f''(0) = 1$$

தொடர்வாக்கம் என்பது:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

29)

P	Q	$\neg Q$	$P \wedge \neg Q$	$\neg(P \wedge \neg Q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

30)

$$xy = ae^x + be^{-x} + x^2 \quad \text{--- (1)}$$

Diff, 2nd time,

$$xy' + y = ae^x - be^{-x} + 2x$$

Again 2nd time, Again diff,

$$xy'' + y' + y' = ae^x + be^{-x} + 2$$

① Along, From ①

$$xy'' + 2y' = xy - x^2 + 2$$

$$xy'' + 2y' - xy + x^2 - 2 = 0.$$

31)

$\sqrt{6-8i}$ or $\sqrt{6-8i}$, Square root of $6-8i$

$$|6-8i| = \sqrt{6^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10.$$

$$\sqrt{a+ib} = \pm \left(\sqrt{\frac{|z|+a}{2}} + i \frac{b}{|b|} \sqrt{\frac{|z|-a}{2}} \right)$$

$$\sqrt{6-8i} = \pm \left(\sqrt{\frac{10+6}{2}} + i \frac{-8}{8} \sqrt{\frac{10-6}{2}} \right)$$

$$= \pm (\sqrt{8} - i\sqrt{2}) = \pm (2\sqrt{2} - i\sqrt{2})$$

அரசு உயர்கல்வியுறைவுத் துறை
 குடிநிலைப்பராமல்-631 208,
 திருவள்ளூர் மாவட்டம்.

32) α & β are the roots of $2x^2 - 7x - 13 = 0$.
 $2x^2 - 7x - 13 = 0$ ஓர் சமன்பாட்டின் α, β .

$$\alpha + \beta = \frac{7}{2} \quad \alpha\beta = \frac{-13}{2}$$

$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= \left(\frac{7}{2}\right)^2 - 2\left(\frac{-13}{2}\right) \\ &= \frac{49}{4} + 13 = \frac{101}{4} \end{aligned}$$

$$\alpha^2\beta^2 = \frac{169}{4}$$

பின்னர் α^2 & β^2 ஓர் சமன்பாட்டின், Quadratic equation

$$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$$

$$x^2 - \frac{101}{4}x + \frac{169}{4} = 0$$

$$4x^2 - 101x + 169 = 0$$

33)

$$\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$$

$$= \sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

34) The equation of the hyperbola
 அகலம் 2000 மீட்டர் டீர் (0, 5)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$2ae = 6$$

$$ae = 3$$

$$a(2) = 3$$

$$\boxed{a = \frac{3}{2}}$$

Centre or vertex $(h, k) = (0, 5)$

$$\frac{x^2}{a^2} - \frac{(y-5)^2}{b^2} = 1$$

$$b^2 = a^2 (e^2 - 1)$$

$$= \frac{9}{4} (2^2 - 1)$$

$$= \frac{9}{4} (3)$$

$$\boxed{b^2 = \frac{27}{4}}$$

அகலம் 2000 மீட்டர் டீர்

$$\frac{4x^2}{9} - \frac{4(y-5)^2}{27} = 1$$

35)

$$2x = 3y = -z$$

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$$

$$(b_1, b_2, b_3) = \left(\frac{1}{2}, \frac{1}{3}, -1\right)$$

$$6x = -y = -4z$$

$$\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}}$$

$$(d_1, d_2, d_3) = \left(\frac{1}{6}, -1, -\frac{1}{4}\right)$$

பெரிய கோணம் சமவெட்டு

$$\theta = \cos^{-1} \left(\frac{\vec{b} \cdot \vec{d}}{|\vec{b}| |\vec{d}|} \right)$$

$$= \cos^{-1} \left(\frac{\frac{1}{12} - \frac{1}{3} + \frac{1}{4}}{\sqrt{\frac{1}{4} + \frac{1}{9} + 1} \sqrt{\frac{1}{36} + 1 + \frac{1}{16}}} \right)$$

$$= \cos^{-1} \left(\frac{0}{\sqrt{\frac{1}{4} + \frac{1}{9} + 1} \sqrt{\frac{1}{36} + 1 + \frac{1}{16}}} \right)$$

$$= \cos^{-1}(0) = \frac{\pi}{2}$$

$$36) \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \infty - \infty$$

தேயமானம் வந்தது

$$= \lim_{x \rightarrow 0} \left(\frac{x - \sin x}{x \sin x} \right) = \frac{0}{0}$$

லாக்ரேஞ்சின் விதியை பயன்படுத்தி;

$$= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cos x + \sin x} = \frac{0}{0}$$

தேயமானம் வந்தது

லாக்ரேஞ்சின் விதியை மீண்டும் பயன்படுத்தி;

$$\lim_{x \rightarrow 0} \frac{\sin x}{x(-\sin x) + \cos x + \cos x} = \frac{0}{2} = 0 //$$

$$37) \int_0^1 \frac{2x}{1+x^2} dx$$

$$= \left[\log(1+x^2) \right]_0^1$$

$$= \log 2 - \log 1$$

$$= \log 2 //$$

$$38) \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}(y) = \sin^{-1}(x) + C$$

39) சராசரி $np = 2$

புறக்கூறு $npq = 1.5$

$$\frac{npq}{np} = \frac{1.5}{2} = \frac{3}{4}$$

$$q = \frac{3}{4}$$

$$p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$p = \frac{1}{4}$$

$$h\left(\frac{1}{4}\right) = 2$$

$$h = 8$$

$$P(X=x) = \binom{8}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{8-x}$$

$$P(X=0) = \binom{8}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^8 = \left(\frac{3}{4}\right)^8$$

40)

$$A^2 = A \cdot A = \begin{pmatrix} -3 & -2 \\ \lambda & -2 \end{pmatrix} \begin{pmatrix} -3 & -2 \\ \lambda & -2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 9 - 2\lambda & 10 \\ -5\lambda & -2\lambda + 4 \end{pmatrix}$$

$$A^2 = \lambda A - 2I$$

$$\begin{pmatrix} 9 - 2\lambda & 10 \\ -5\lambda & -2\lambda + 4 \end{pmatrix} = \begin{pmatrix} -3\lambda & -2\lambda \\ \lambda^2 & -2\lambda \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 9 - 2\lambda & 10 \\ -5\lambda & -2\lambda + 4 \end{pmatrix} = \begin{pmatrix} -3\lambda - 2 & -2\lambda \\ \lambda^2 & -2\lambda - 2 \end{pmatrix}$$

$$-2\lambda = 10 \Rightarrow \lambda = -5$$

$$\underline{UBS - \Gamma.}$$

41) a)

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{array} \right]$$

$$R_3 \rightarrow R_3 - R_1$$

$$R_2 \rightarrow 2R_2 - 7R_1$$

$$\longrightarrow$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array} \right]$$

By row analysis it is in echelon form

Case (i) $\lambda = 5, \mu \neq 9$

$$P(A) = 2 \quad P(A|B) = 3$$

$$P(A) \neq P(A|B)$$

അല്ലാത്ത എണ്ണം സമീകരണം. No solution.

Case (ii) $\lambda \neq 5, \mu \in \mathbb{R}$

$$P(A) = 3 \quad P(A|B) = 3$$

$$P(A) = P(A|B) = 3 = 3 \quad (\text{b. o. o.})$$

ഒരേ എണ്ണം സമീകരണം.

a unique solution.

Case (iii) $\lambda = 5, \mu = 9$

$$P(A) = P(A|B) = 2 < 3 \quad (\text{b. o. o.})$$

\(\therefore\) അനന്തമായി എണ്ണം സമീകരണം.

An infinite number of solutions.

41) b) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

$$x^2 \left[\left(x^2 + \frac{1}{x^2} \right) - 10 \left(x + \frac{1}{x} \right) + 26 \right] = 0$$

$$\left(x^2 + \frac{1}{x^2} \right) - 10 \left(x + \frac{1}{x} \right) + 26 = 0$$

Let $y = x + \frac{1}{x}$ then $\Rightarrow y^2 - 2 = x^2 + \frac{1}{x^2}$

$$(y^2 - 2) - 10y + 26 = 0$$

$$y^2 - 10y + 24 = 0$$

$$(y-6)(y-4) = 0$$

$$y = 6, \quad y = 4$$

Case (i)

If $y = 6$ then $x + \frac{1}{x} = 6$

$$\Rightarrow x = 3 + 2\sqrt{2}$$

$$x = 3 - 2\sqrt{2}$$

Case (ii)

If $y = 4$ then $x + \frac{1}{x} = 4$

$$\Rightarrow x = 2 + \sqrt{3}$$

$$x = 2 - \sqrt{3}$$

Solution:

$$3 \pm 2\sqrt{2}, \quad 2 \pm \sqrt{3}$$

42) a)

$$\frac{z-i}{z+2} = \frac{x+iy-i}{x+iy+2} = \frac{x+i(y-1)}{(x+2)+iy}$$

$$= \frac{x+i(y-1)}{(x+2)+iy} \times \frac{(x+2)-iy}{(x+2)-iy}$$

$$\frac{z-i}{z+2} = \frac{x(x+2) - ixy + i(y-1)(x+2) + y(y-1)}{(x+2)^2 + y^2}$$

$$\frac{z-i}{z+2} = \frac{x(x+2) + y(y-1) + i[(y-1)(x+2) - xy]}{(x+2)^2 + y^2}$$

$$\arg\left(\frac{z-i}{z+2}\right) = \tan^{-1} \frac{(y-1)(x+2) - xy}{x(x+2) + y(y-1)} = \frac{\pi}{4}$$

$$\frac{xy + 2y - x + 2 - xy}{x^2 + 2x + y^2 - y} = \tan \frac{\pi}{4}$$

$$+2y - x + 2 = x^2 + 2x + y^2 - y$$

$$x^2 + y^2 + 3x - 3y + 2 = 0$$

A2)

b)

சமன்பாட்டின் மூலம்
Equation of Parabola

$$x^2 = -4ay \quad \text{--- (1)}$$

It passes through (6, -4)

$$36 = -4a(-4) \Rightarrow 4a = 9$$

$$\text{(1)} \Rightarrow x^2 = -9y \quad \text{--- (2)}$$

(2) ன்வு சமன்பாடு, Diff wrt x,

$$2x = -9 \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{2x}{9}$$

$$m = \left(\frac{dy}{dx}\right)_{(6, -4)} = \frac{12}{9} = \frac{4}{3}$$

But $m = \tan \theta$

$$\tan \theta = \frac{4}{3}$$

அல்லது $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

The angle of Projection $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

43)

a)

$$(x_1, y_1, z_1) = (2, 3, 6)$$

$$(l_1, m_1, n_1) = (2, 3, 1)$$

$$(l_2, m_2, n_2) = (2, -5, -3)$$

Cartesian equation of plane is
கார்டீசியன் சமன்பாடு

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-6 \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix} = 0$$

$$(x-2)(-4) - (y-3)(-8) + (z-6)(-16) = 0$$

$$\boxed{x - 2y + 4z - 20 = 0}$$

$$\vec{r} \cdot (\vec{i} - 2\vec{j} + 4\vec{k}) = 20 \text{ சமன்பாடு}$$

இது ஒரு நேரிய சமன்பாடு ஆகும்.

It is the non-parametric vector equation.

43/b)

$$\sum_n f(n) = 1$$

$$\therefore k + 2k + 6k + 5k + 6k + 10k = 1$$

$$30k = 1$$

$$\boxed{k = \frac{1}{30}}$$

$$x : 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$$f(n) : \frac{1}{30} \quad \frac{2}{30} \quad \frac{6}{30} \quad \frac{5}{30} \quad \frac{6}{30} \quad \frac{10}{30}$$

$$(i) P(2 < x < 6) = f(3) + f(4) + f(5) \\ = \frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$$

$$(ii) P(2 \leq x < 5) = f(2) + f(3) + f(4) \\ = \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{13}{30}$$

$$(iii) P(x \leq 4) = f(1) + f(2) + f(3) + f(4) \\ = \frac{1}{30} + \frac{2}{30} + \frac{6}{30} + \frac{5}{30} \\ = \frac{14}{30}$$

$$(iv) P(3 < x) = f(4) + f(5) + f(6) \\ = \frac{5}{30} + \frac{6}{30} + \frac{10}{30} = \frac{21}{30}$$

44) a) The equation of the curves
 சமன்பாடுகள் தரப்பட்டுள்ளன

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$x = y^2$$

$$1 = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

In the point (0,0).

$$\tan \theta_1 = \left| \frac{2x - \frac{1}{2y}}{1 + (2x)\left(\frac{1}{2y}\right)} \right|$$

$$\tan \theta_1 = \lim_{(x,y) \rightarrow (0,0)} \left| \frac{2x - \frac{1}{2y}}{1 + (2x)\left(\frac{1}{2y}\right)} \right|$$

$$\tan \theta_1 = \lim_{(x,y) \rightarrow (0,0)} \left| \frac{4xy-1}{2(y+2)} \right|$$

$$\tan \theta_1 = \infty$$

$$\theta_1 = \frac{\pi}{2}$$

In the part (1,1)

$$m_1 = \left(\frac{dy}{dx} \right)_{(1,1)} = 2(1) = 2$$

$$m_2 = \left(\frac{dy}{dx} \right)_{(1,1)} = \frac{1}{2(1)} = \frac{1}{2}$$

$$\tan \theta_2 = \left| \frac{2 - \frac{1}{2}}{1 + 2\left(\frac{1}{2}\right)} \right| = \frac{3}{4}$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right)$$

44) b) ^{CHS} $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}(z)$$

$$= \tan^{-1}\left[\frac{\frac{x+y}{1-xy} + z}{1 - z\left(\frac{x+y}{1-xy}\right)}\right]$$

$$= \tan^{-1}\left(\frac{x+y+z - \frac{xyz}{1-xy}}{1-xy - \frac{z(x+y)}{1-xy}}\right)$$

$$= \tan^{-1}\left(\frac{x+y+z - xyz}{1-xy-yz-zx}\right) = \text{RHS}$$

45) a)

$$u = \sin^{-1} \left(\frac{x+y}{\sqrt{x+y}} \right)$$

$$f(x, y) = \sin u = \frac{x+y}{\sqrt{x+y}}$$

$$f(tx, ty) = \frac{tx+ty}{\sqrt{tx+ty}} = t^{\frac{1}{2}} \cdot \frac{x+y}{\sqrt{x+y}}$$

$$f(tx, ty) = t^{\frac{1}{2}} \cdot f(x, y)$$

f homogeneous of order $\frac{1}{2}$ \therefore Euler's theorem applies.

பொருள்: \therefore By Euler's theorem.

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{2} \cdot f$$

$$x \frac{\partial}{\partial x} (\sin u) + y \frac{\partial}{\partial y} (\sin u) = \frac{1}{2} \sin u$$

$$x \cos u \frac{\partial u}{\partial x} + y \cos u \frac{\partial u}{\partial y} = \frac{1}{2} \sin u$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$$

45)

b)

$$Z_5 = \{ [0] [1] [2] [3] [4] \}$$

பொருள்: $\{0, 1, 2, 3, 4\}$

$\{ [0] [1] [2] [3] [4] \}$ சார்பு முறை

பொருள்: $\{0, 1, 2, 3, 4\}$.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(i) திறமையுடையது:

அகலகணக்கின் அடிப்படையில் Z_5 -ன் திறமையுடையது

Z_5 -ன் திறமையுடையது

$\therefore +_5$ திறமையுடைய Z_5 -ன் திறமையுடையது.

(ii) பரிமாற்றத்துடன் கூடியது:

அகலகணக்கின் திறமையுடையது

பரிமாற்றத்துடன் கூடியது என்பதற்கான சான்றிதழை Z_5 -ன் திறமையுடையது

$+_5$ திறமையுடையது பரிமாற்றத்துடன் கூடியது

(iii) பிணைப்பு திறமையுடையது

$$2, 3, 4 \in Z_5$$

$$(2 +_5 3) +_5 4 = 0 +_5 4 = 4 \text{ (6C6A5)}$$

$$2 +_5 (3 +_5 4) = 2 +_5 2 = 4 \text{ (6C6A5)}$$

$$(2 +_5 3) +_5 4 = 2 +_5 (3 +_5 4)$$

பிணைப்பு திறமையுடையது என்பதற்கான சான்றிதழை Z_5 -ன் திறமையுடையது என்பதற்கான சான்றிதழை.

(iv) சென்ட்ரல் அங்குலம் வாய்

அளவீடுகள்

சென்ட்ரல் அங்குலம் $0 \in \mathbb{Z}_5$

0 ஓர் அங்குலம் $0 \in \mathbb{Z}_5$

1 ஓர் அங்குலம் $4 \in \mathbb{Z}_5$

2 ஓர் அங்குலம் $3 \in \mathbb{Z}_5$

3 ஓர் அங்குலம் $2 \in \mathbb{Z}_5$

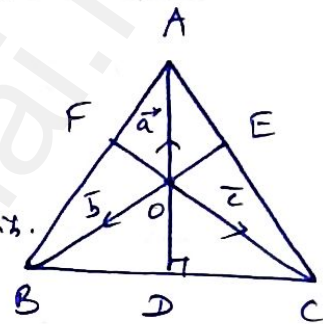
4 ஓர் அங்குலம் $1 \in \mathbb{Z}_5$

4b/a)

$\triangle ABC$,
 \vec{AD}, \vec{BE} சென்ட்ரல் அங்குலம்
 சென்ட்ரல் அங்குலம் O ஆகும்.

சென்ட்ரல் அங்குலம் O ஆகும்.

சென்ட்ரல் அங்குலம் O ஆகும்.



$$\vec{OA} = \vec{a}$$

$$\vec{OB} = \vec{b}$$

$$\vec{OC} = \vec{c}$$

$$\vec{AD} \perp BC \Rightarrow \vec{OA} \perp BC$$

$$\vec{OA} \cdot BC = 0$$

$$\vec{a} \cdot (\vec{c} - \vec{b}) = 0$$

$$\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} = 0 \quad \text{--- (1)}$$

$$\vec{BE} \perp CA \Rightarrow \vec{OB} \perp CA$$

$$\vec{OB} \cdot CA = 0$$

$$\vec{b} \cdot (\vec{a} - \vec{c}) = 0$$

$$\vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{c} = 0 \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow$$

$$\vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} = 0$$

$$\vec{c} \cdot (\vec{a} - \vec{b}) = 0$$

$$\vec{OC} \cdot \vec{BA} = 0$$

$$\Rightarrow \vec{OC} \perp \vec{BA}$$

$$\Rightarrow \vec{CF} \perp \vec{BA}$$

\therefore \vec{OC} \perp \vec{BA} \Rightarrow $\vec{CF} \perp \vec{BA}$
 \therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}
 \therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}
 \therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}

b) \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}

$$4x^2 + 24x + y^2 - 2y + 21 = 0$$

$$4(x^2 + 6x + 9 - 9) + (y^2 - 2y + 1 - 1) + 21 = 0$$

$$4(x+3)^2 - 36 + (y-1)^2 - 1 + 21 = 0$$

$$4(x+3)^2 + (y-1)^2 = 16$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

Centre $(-3, 1)$ $a = 4$, $b = 2$

\therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}

$$c^2 = 16 - 4 = 12$$

$$c = \pm 2\sqrt{3}$$

\therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}

\therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}

$$\therefore \vec{CF} \perp \vec{BA} \Rightarrow \vec{CF} \perp \vec{BA}$$

47) a)

$$I = \int_{\pi/8}^{3\pi/8} \frac{1}{1 + \sqrt{\tan x}} dx$$

$$I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{--- (1)}$$

$$I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cos(\pi/8 + 3\pi/8 - x)}}{\sqrt{\cos(\pi/8 + 3\pi/8 - x)} + \sqrt{\sin(\pi/8 + 3\pi/8 - x)}} dx$$

$$I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{--- (2)}$$

$$\text{(1) + (2)} \Rightarrow$$

$$2I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_{\pi/8}^{3\pi/8} dx = \left[x \right]_{\pi/8}^{3\pi/8} = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{\pi}{4}$$

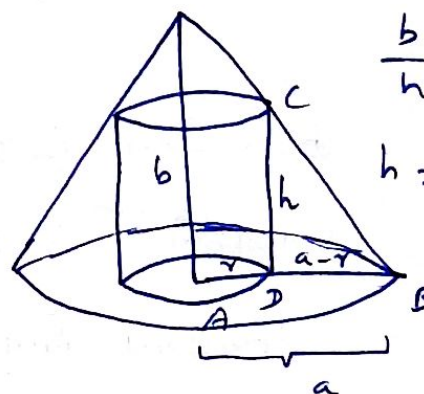
$$I = \frac{\pi}{8}$$

b)

Volume of cylinder
2 @mmndir kmndy

$$V = \pi r^2 h$$

$$= \pi r^2 \frac{b}{a} (a-r)$$



$$\frac{b}{h} = \frac{a}{a-r}$$

$$h = \frac{b}{a} (a-r)$$

$$V = \frac{\pi b}{a} (ar^2 - r^3) \quad \text{--- (1)}$$

$$V' = \frac{\pi b}{a} (2ar - 3r^2)$$

$$V'' = \frac{\pi b}{a} (2a - 6r)$$

Put $V' = 0$

$$\frac{\pi b}{a} (2ar - 3r^2) = 0$$

$$r(2a - 3r) = 0$$

$$\begin{array}{l|l} r = 0 & 2a - 3r = 0 \\ \hline \underline{\underline{r \neq 0}} & \boxed{r = \frac{2a}{3}} \end{array}$$

At $r = \frac{2a}{3}$, $V'' = \frac{\pi b}{a} \left[2a - 6 \left(\frac{2a}{3} \right) \right]$

$$V'' = \frac{\pi b}{a} (-2a) < 0$$

V attains maximum at $r = \frac{2a}{3}$

$$\text{(1)} \Rightarrow V = \frac{\pi b}{a} \left[a \frac{4a^2}{9} - \frac{8a^3}{27} \right]$$

$$V = \frac{\pi b}{a} \left(\frac{4a^3}{27} \right) = \frac{4}{9} \times \frac{1}{3} \pi a^2 b$$

Volume of cylinder = $\frac{4}{9}$ x Volume of cone

By

A K RAJADHURAI

MATHS TEACHER

GGHSS - PODATURPET