

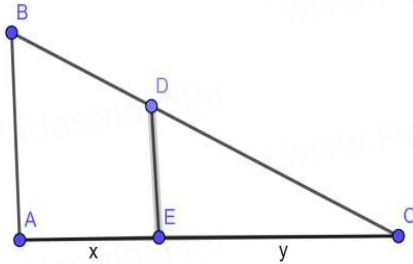
**Marking Scheme**  
**Class XII**  
**Mathematics (Code – 041)**  
**Section : A (Multiple Choice Questions- 1 Mark each)**

Question No	Answer	Hints/Solution
1.	(c)	In a skew-symmetric matrix, the (i, j)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0
2.	(a)	$ AA'  =  A  A'  = (-3)(-3) = 9$
3.	(b)	The area of the parallelogram with adjacent sides AB and AC = $ \vec{AB} \times \vec{AC} $ . Hence, the area of the triangle with vertices A, B, C = $\frac{1}{2}  \vec{AB} \times \vec{AC} $
4.	(c)	The function f is continuous at $x = 0$ if $\lim_{x \rightarrow 0} f(x) = f(0)$ We have $f(0) = k$ and $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1 - \cos}{8x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 2x}{8x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{4x^2}$ $= \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x}\right)^2 = 1$ Hence, $k = 1$
5.	(b)	$\frac{x^2}{2} + \log  x  + C \left( \because f(x) = \int \left(x + \frac{1}{x}\right) dx \right)$
6.	(c)	The given differential equation is $4 \left(\frac{dy}{dx}\right)^3 \frac{d^2y}{dx^2} = 0$ . Here, $m = 2$ and $n = 1$ Hence, $m + n = 3$
7.	(b)	The strict inequality represents an open half plane and it contains the origin as $(0, 0)$ satisfies it.
8.	(a)	Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k}$ $= \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{ \hat{i} + 2\hat{j} - 3\hat{k} } = \frac{7}{\sqrt{14}}$
9.	(c)	$\int_2^3 \frac{x}{x^2+1} = \frac{1}{2} [\log(x^2 + 1)]_2^3 = \frac{1}{2} (\log 10 - \log 5) = \frac{1}{2} \log \left(\frac{10}{5}\right)$ $= \frac{1}{2} \log 2$
10.	(c)	$(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$
11.	(d)	The minimum value of the objective function occurs at two adjacent corner points $(0.6, 1.6)$ and $(3, 0)$ and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region. So, the minimum value occurs at every point of the line-segment joining the two points.
12.	(d)	$2 - 20 = 2x^2 - 24 \Rightarrow 2x^2 = 6 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$
13.	(b)	$ adjA  =  A ^{n-1} \Rightarrow  adjA  = 25$
14.	(c)	$P(A' \cap B') = P(A') \times P(B')$ (As A and B are independent, $A'$ and $B'$ are also independent.) $= 0.7 \times 0.4 = 0.28$
15.	(c)	$ydx - xdy = 0 \Rightarrow ydx - xdy = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$ $\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x} + \log K, K > 0 \Rightarrow \log y  = \log x  + \log K$ $\Rightarrow \log y  = \log x K \Rightarrow  y  =  x K \Rightarrow y = \pm Kx \Rightarrow y = Cx$

16.	(a)	$y = \sin^{-1}x$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$ Again, differentiating both sides w. r. to x, we get $\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \left( \frac{-2x}{2\sqrt{1-x^2}} \right) = 0$ Simplifying, we get $(1-x^2)y_2 = xy_1$
17.	(b)	$ \vec{a} - 2\vec{b} ^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$ $ \vec{a} - 2\vec{b} ^2 = \vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 4\vec{b} \cdot \vec{b}$ $=  \vec{a} ^2 - 4\vec{a} \cdot \vec{b} + 4 \vec{b} ^2$ $= 4 - 16 + 36 = 24$ $ \vec{a} - 2\vec{b} ^2 = 24 \Rightarrow  \vec{a} - 2\vec{b}  = 2\sqrt{6}$
18.	(b)	The line through the points (0, 5, -2) and (3, -1, 2) is $\frac{x}{3-0} = \frac{y-5}{-1-5} = \frac{z+2}{2+2}$ $or, \frac{x}{3} = \frac{y-5}{-6} = \frac{z+2}{4}$ Any point on the line is $(3k, -6k + 5, 4k - 2)$ , where k is an arbitrary scalar. $3k = 6 \Rightarrow k = 2$ The z-coordinate of the point P will be $4 \times 2 - 2 = 6$
19.	(c)	$\sec^{-1}x$ is defined if $x \leq -1$ or $x \geq 1$ . Hence, $\sec^{-1}2x$ will be defined if $x \leq -\frac{1}{2}$ or $x \geq \frac{1}{2}$ . Hence, A is true. The range of the function $\sec^{-1}x$ is $[0, \pi] - \{\frac{\pi}{2}\}$ R is false.
20.	(a)	The equation of the x-axis may be written as $\vec{r} = t\hat{i}$ . Hence, the acute angle $\theta$ between the given line and the x-axis is given by $\cos\theta = \frac{ 1 \times 1 + (-1) \times 0 + 0 \times 0 }{\sqrt{1^2 + (-1)^2 + 0^2} \times \sqrt{1^2 + 0^2 + 0^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

## SECTION B (VSA questions of 2 marks each)

21.	$\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}$ <b>OR</b> Let $y \in N(\text{codomain})$ . Then $\exists 2y \in N(\text{domain})$ such that $f(2y) = \frac{2y}{2} = y$ . Hence, f is surjective. $1, 2 \in N(\text{domain})$ such that $f(1) = 1 = f(2)$ Hence, f is not injective.	.1 1 1 1
22.	Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC be y m. Using similarity of triangles, we have $\frac{4}{1.6} = \frac{x+y}{y} \Rightarrow 3y = 2x$	$\frac{1}{2}$

	 <p>Differentiating both sides w.r.to t, we get <math>3 \frac{dy}{dt} = 2 \frac{dx}{dt}</math></p> $\frac{dy}{dt} = \frac{2}{3} \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$ <p>At any time t seconds, the tip of his shadow is at a distance of <math>(x + y)</math> m from AB.</p> <p>The rate at which the tip of his shadow moving</p> $= \left( \frac{dx}{dt} + \frac{dy}{dt} \right) m/s = 0.5 m/s$ <p>The rate at which his shadow is lengthening</p> $= \frac{dy}{dt} m/s = 0.2 m/s$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1/2</p>
23.	<p><math>\vec{a} = \hat{i} - \hat{j} + 7\hat{k}</math> and <math>\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}</math></p> <p>Hence <math>\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}</math> and <math>\vec{a} - \vec{b} = -4\hat{i} + (7 - \lambda)\hat{k}</math></p> <p><math>\vec{a} + \vec{b}</math> and <math>\vec{a} - \vec{b}</math> will be orthogonal if, <math>(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0</math></p> <p>i.e., if, <math>-24 + (49 - \lambda^2) = 0 \Rightarrow \lambda^2 = 25</math></p> <p>i.e., if, <math>\lambda = \pm 5</math></p> <p style="text-align: center;"><b>OR</b></p> <p>The equations of the line are <math>6x - 12 = 3y + 9 = 2z - 2</math>, which, when written in standard symmetric form, will be</p> $\frac{x-2}{\frac{1}{6}} = \frac{y-(-3)}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$ <p>Since, lines are parallel, we have <math>\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}</math></p> <p>Hence, the required direction ratios are <math>\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)</math> or <math>(1, 2, 3)</math></p> <p>and the required direction cosines are <math>\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>
24.	<p><math>y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1</math></p> <p>Let <math>\sin^{-1}x = A</math> and <math>\sin^{-1}y = B</math>. Then <math>x = \sin A</math> and <math>y = \sin B</math></p> <p><math>y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \Rightarrow \sin B \cos A + \sin A \cos B = 1</math></p> <p><math>\Rightarrow \sin(A + B) = 1 \Rightarrow A + B = \sin^{-1}1 = \frac{\pi}{2}</math></p> <p><math>\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}</math></p> <p>Differentiating w.r.to x, we obtain <math>\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p>
25.	<p>Since <math>\vec{a}</math> is a unit vector, <math>\therefore  \vec{a}  = 1</math></p>	<p><math>\frac{1}{2}</math></p>

	$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12.$ $\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$ $\Rightarrow  \vec{x} ^2 -  \vec{a} ^2 = 12.$ $\Rightarrow  \vec{x} ^2 - 1 = 12$ $\Rightarrow  \vec{x} ^2 = 13 \Rightarrow  \vec{x}  = \sqrt{13}$	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
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## SECTION C

(Short Answer Questions of 3 Marks each)

26.	$\int \frac{dx}{\sqrt{3-2x-x^2}}$ $= \int \frac{dx}{\sqrt{-(x^2+2x-3)}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{2}\right) + C \quad \left[ \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \right]$	 2 1												
27.	<p>P(not obtaining an odd person in a single round) = P(All three of them throw tails or All three of them throw heads)</p> $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{4}$ <p>P(obtaining an odd person in a single round)</p> $= 1 - \text{P(not obtaining an odd person in a single round)} = \frac{3}{4}$ <p>The required probability</p> <p>= P('In first round there is no odd person' and 'In second round there is no odd person' and 'In third round there is an odd person')</p> $= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$ <p style="text-align: center;"><b>OR</b></p> <p>Let X denote the Random Variable defined by the number of defective items.</p> $P(X=0) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$ $P(X=1) = 2 \times \left(\frac{2}{6} \times \frac{4}{5}\right) = \frac{8}{15}$ $P(X=2) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>x_i</math></td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td><math>p_i</math></td> <td><math>\frac{2}{5}</math></td> <td><math>\frac{8}{15}</math></td> <td><math>\frac{1}{15}</math></td> </tr> <tr> <td><math>p_i x_i</math></td> <td>0</td> <td><math>\frac{8}{15}</math></td> <td><math>\frac{2}{15}</math></td> </tr> </table> <p>Mean = <math>\sum p_i x_i = \frac{10}{15} = \frac{2}{3}</math></p>	$x_i$	0	1	2	$p_i$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$	$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$	 1+1/2 $\frac{1}{2}$ 1  2  1/2 1/2
$x_i$	0	1	2											
$p_i$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$											
$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$											
28.	<p>Let <math>I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(i)</math></p>													

	<p>Using <math>\int_a^b f(x) dx = \int_a^b f(a+b-x) dx</math></p> $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}}{\sqrt{\sin(\frac{\pi}{6} + \frac{\pi}{3} - x) + \sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}} dx$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx \text{ ..(ii).}$ <p>Adding (i) and (ii), we get</p> $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx$ $2I = \int_{\pi/6}^{\pi/3} dx$ $= \left[ x \right]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ <p>Hence, <math>I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \frac{\pi}{12}</math></p> <p style="text-align: center;"><b>OR</b></p> $\int_0^4  x-1  dx = \int_0^1 (1-x) dx + \int_1^4 (x-1) dx$ $= \left[ x - \frac{x^2}{2} \right]_0^1 + \left[ \frac{x^2}{2} - x \right]_1^4$ $= \left( 1 - \frac{1}{2} \right) + (8 - 4) - \left( \frac{1}{2} - 1 \right)$ $= 5$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
29.	<p><math>y dx + (x - y^2) dy = 0</math></p> <p>Reducing the given differential equation to the form <math>\frac{dx}{dy} + Px = Q</math></p> <p>we get, <math>\frac{dx}{dy} + \frac{x}{y} = y</math></p> <p>I.F = <math>e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y</math></p> <p>The general solution is given by</p> $x \cdot IF = \int Q \cdot IF dy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C, \text{ which is the required general solution}$ <p style="text-align: center;"><b>OR</b></p> $x dy - y dx = \sqrt{x^2 + y^2} dx$ <p>It is a <b>Homogeneous Equation as</b></p> $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} = \sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x} = f\left(\frac{y}{x}\right).$ <p>Put <math>y = vx</math></p> $\frac{dy}{dx} = v + x \frac{dv}{dx}$	<p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>

	$v + x \frac{dv}{dx} = \sqrt{1 + v^2} + v$ <p>Separating variables, we get</p> $\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ <p>Integrating, we get <math>\log v + \sqrt{1 + v^2}  = \log x  + \log K, K &gt; 0</math></p> $\log y + \sqrt{x^2 + y^2}  = \log x^2 K$ $\Rightarrow y + \sqrt{x^2 + y^2} = \pm Kx^2$ $\Rightarrow y + \sqrt{x^2 + y^2} = Cx^2, \text{ which is the required general solution}$	<p>1/2</p> <p>1/2</p> <p>1+1/2</p>										
30.	<p>We have <math>Z = 400x + 300y</math> subject to  <math>x + y \leq 200, x \leq 40, x \geq 20, y \geq 0</math>                      The corner points of the feasible region are <math>C(20,0), D(40,0), B(40,160), A(20,180)</math></p> <table border="1"> <thead> <tr> <th>Corner Point</th> <th><math>Z = 400x + 300y</math></th> </tr> </thead> <tbody> <tr> <td><math>C(20,0)</math></td> <td>8000</td> </tr> <tr> <td><math>D(40,0)</math></td> <td>16000</td> </tr> <tr> <td><math>B(40,160)</math></td> <td>64000</td> </tr> <tr> <td><math>A(20,180)</math></td> <td>62000</td> </tr> </tbody> </table> <p>Maximum profit occurs at <math>x = 40, y = 160</math>                      and the maximum profit = ₹ 64,000</p>	Corner Point	$Z = 400x + 300y$	$C(20,0)$	8000	$D(40,0)$	16000	$B(40,160)$	64000	$A(20,180)$	62000	<p>1</p> <p>1</p> <p>1</p>
Corner Point	$Z = 400x + 300y$											
$C(20,0)$	8000											
$D(40,0)$	16000											
$B(40,160)$	64000											
$A(20,180)$	62000											
31.	$\int \frac{(x^3 + x + 1)}{(x^2 - 1)} dx = \int \left( x + \frac{2x + 1}{(x - 1)(x + 1)} \right) dx$ <p>Now resolving <math>\frac{2x + 1}{(x - 1)(x + 1)}</math> into partial fractions as</p> $\frac{2x + 1}{(x - 1)(x + 1)} = \frac{A}{x - 1} + \frac{B}{x + 1}$ <p>We get <math>\frac{2x + 1}{(x - 1)(x + 1)} = \frac{3}{2(x - 1)} + \frac{1}{2(x + 1)}</math></p>	<p>1</p> <p>1</p>										

	<p>Hence, <math>\int \frac{(x^3+x+1)}{(x^2-1)} dx = \int \left( x + \frac{2x+1}{(x-1)(x+1)} \right) dx</math></p> <p><math>= \int \left( x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)} \right) dx</math></p> <p><math>= \frac{x^2}{2} + \frac{3}{2} \log x-1  + \frac{1}{2} \log x+1  + C</math></p> <p><math>= \frac{x^2}{2} + \frac{1}{2} (\log (x-1)^3(x+1) ) + C</math></p>	1
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**SECTION D**

*(Long answer type questions (LA) of 5 marks each)*

32.		<p>(Correct Fig: 1 Mark)</p>
	<p>The points of intersection of the parabola <math>y = x^2</math> and the line <math>y = x</math> are <math>(0, 0)</math> and <math>(1, 1)</math>.</p> <p>Required Area = <math>\int_0^1 y_{parabola} dx + \int_1^2 y_{line} dx</math></p> <p>Required Area = <math>\int_0^1 x^2 dx + \int_1^2 x dx</math></p> <p><math>= \left[ \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}</math></p>	<p><math>\frac{1}{2}</math></p> <p>2</p> <p>1+1/2</p>
33.	<p>Let <math>(a, b) \in N \times N</math>. Then we have <math>ab = ba</math> (by commutative property of multiplication of natural numbers)</p> <p><math>\Rightarrow (a, b)R(a, b)</math></p> <p>Hence, R is reflexive.</p> <p>Let <math>(a, b), (c, d) \in N \times N</math> such that <math>(a, b) R (c, d)</math>. Then <math>ad = bc</math></p> <p><math>\Rightarrow cb = da</math> (by commutative property of multiplication of natural numbers)</p> <p><math>\Rightarrow (c, d)R(a, b)</math></p> <p>Hence, R is symmetric.</p> <p>Let <math>(a, b), (c, d), (e, f) \in N \times N</math> such that</p>	<p>1</p> <p>1+1/2</p>

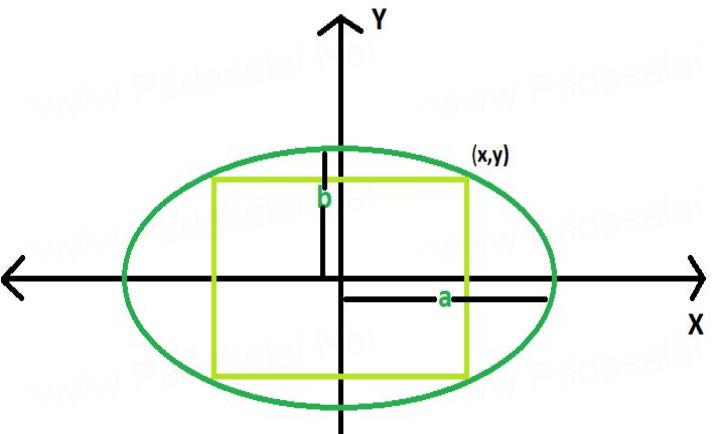
	<p><math>(a, b) R (c, d)</math> and <math>(c, d) R (e, f)</math>.  Then <math>ad = bc, cf = de</math>  <math>\Rightarrow adcf = bcde</math>  <math>\Rightarrow af = be</math>  <math>\Rightarrow (a, b)R(e, f)</math>  Hence, R is transitive.  Since, R is reflexive, symmetric and transitive, R is an equivalence relation on <math>N \times N</math>.</p> <p style="text-align: center;">OR</p> <p>Let <math>A \in P(X)</math>. Then <math>A \subset A</math>  <math>\Rightarrow (A, A) \in R</math>  Hence, R is reflexive.  Let <math>A, B, C \in P(X)</math> such that  <math>(A, B), (B, C) \in R</math>  <math>\Rightarrow A \subset B, B \subset C</math>  <math>\Rightarrow A \subset C</math>  <math>\Rightarrow (A, C) \in R</math>  Hence, R is transitive.  <math>\emptyset, X \in P(X)</math> such that <math>\emptyset \subset X</math>. Hence, <math>(\emptyset, X) \in R</math>. But, <math>X \not\subset \emptyset</math>, which implies that <math>(X, \emptyset) \notin R</math>.  Thus, R is not symmetric.</p>	<p>2</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>2</p> <p>2</p>
34.	<p>The given lines are non-parallel lines. There is a unique line-segment PQ (P lying on one and Q on the other, which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the insects = PQ</p> <p>The position vector of P lying on the line  <math>\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})</math>  is <math>(6 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (2 + 2\lambda)\hat{k}</math> for some <math>\lambda</math></p> <p>The position vector of Q lying on the line  <math>\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})</math>  is <math>(-4 + 3\mu)\hat{i} + (-2\mu)\hat{j} + (-1 - 2\mu)\hat{k}</math> for some <math>\mu</math></p> <p><math>\overline{PQ} = (-10 + 3\mu - \lambda)\hat{i} + (-2\mu - 2 + 2\lambda)\hat{j} + (-3 - 2\mu - 2\lambda)\hat{k}</math></p> <p>Since, PQ is perpendicular to both the lines  <math>(-10 + 3\mu - \lambda) + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)2 = 0,</math></p> <p><i>i. e.</i>, <math>\mu - 3\lambda = 4</math> ... (i)</p> <p>and <math>(-10 + 3\mu - \lambda)3 + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)(-2) = 0,</math></p> <p><i>i. e.</i>, <math>17\mu - 3\lambda = 20</math> ... (ii)</p> <p>solving (i) and (ii) for <math>\lambda</math> and <math>\mu</math>, we get <math>\mu = 1, \lambda = -1</math>.</p> <p>The position vector of the points, at which they should be so that the distance between them is the shortest, are  <math>5\hat{i} + 4\hat{j}</math> and <math>-\hat{i} - 2\hat{j} - 3\hat{k}</math></p> <p><math>\overline{PQ} = -6\hat{i} - 6\hat{j} - 3\hat{k}</math></p> <p>The shortest distance = <math> \overline{PQ}  = \sqrt{6^2 + 6^2 + 3^2} = 9</math></p> <p style="text-align: center;">OR</p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>



	<p>Eliminating t between the equations, we obtain the equation of the path <math>\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}</math>, which are the equations of the line passing through the origin having direction ratios <math>\langle 2, -4, 4 \rangle</math>. This line is the path of the rocket.</p> <p>When <math>t = 10</math> seconds, the rocket will be at the point <math>(20, -40, 40)</math>. Hence, the required distance from the origin at 10 seconds =</p> $\sqrt{20^2 + 40^2 + 40^2} \text{ km} = 20 \times 3 \text{ km} = 60 \text{ km}$ <p>The distance of the point <math>(20, -40, 40)</math> from the given line</p> $= \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{ -30\hat{j} \times (10\hat{i} - 20\hat{j} + 10\hat{k}) }{ 10\hat{i} - 20\hat{j} + 10\hat{k} } \text{ km} = \frac{ -300\hat{i} + 300\hat{k} }{ 10\hat{i} - 20\hat{j} + 10\hat{k} } \text{ km}$ $= \frac{300\sqrt{2}}{10\sqrt{6}} \text{ km} = 10\sqrt{3} \text{ km}$	<p>1</p> <p>1/2</p> <p>1</p> <p>2</p> <p>1/2</p>
35.	$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $ A  = 2(0) + 3(-2) + 5(1) = -1$ $A^{-1} = \frac{\text{adj}A}{ A }$ $\text{adj}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}, A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ $= \frac{1}{(-1)} \begin{bmatrix} 0 + 5 - 6 \\ 22 + 45 - 69 \\ 11 + 25 - 39 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)} \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3.$	<p>1/2</p> <p>3</p> <p>1+1/2</p>

**SECTION E(Case Studies/Passage based questions of 4 Marks each)**

36.	(i) $f(x) = -0.1x^2 + mx + 98.6$ , being a polynomial function, is differentiable everywhere, hence, differentiable in $(0, 12)$	1									
	(ii) $f'(x) = -0.2x + m$ Since, 6 is the critical point, $f'(6) = 0 \Rightarrow m = 1.2$	1									
	(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$ $f'(x) = -0.2x + 1.2 = -0.2(x - 6)$										
	<table border="1"> <thead> <tr> <th>In the Interval</th> <th><math>f'(x)</math></th> <th>Conclusion</th> </tr> </thead> <tbody> <tr> <td>(0, 6)</td> <td>+ve</td> <td>f is strictly increasing in <math>[0, 6]</math></td> </tr> <tr> <td>(6, 12)</td> <td>-ve</td> <td>f is strictly decreasing in <math>[6, 12]</math></td> </tr> </tbody> </table>	In the Interval	$f'(x)$	Conclusion	(0, 6)	+ve	f is strictly increasing in $[0, 6]$	(6, 12)	-ve	f is strictly decreasing in $[6, 12]$	1+1
In the Interval	$f'(x)$	Conclusion									
(0, 6)	+ve	f is strictly increasing in $[0, 6]$									
(6, 12)	-ve	f is strictly decreasing in $[6, 12]$									

	<p style="text-align: center;">OR</p> <p>(iii) <math>f(x) = -0.1x^2 + 1.2x + 98.6</math>,  <math>f'(x) = -0.2x + 1.2, f'(6) = 0</math>,  <math>f''(x) = -0.2</math>  <math>f''(6) = -0.2 &lt; 0</math></p> <p>Hence, by second derivative test 6 is a point of local maximum. The local maximum value = <math>f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2</math>  We have <math>f(0) = 98.6, f(6) = 102.2, f(12) = 98.6</math>  6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.  0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>
37.	<p>(i)</p>  <p>Let <math>(x, y) = \left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)</math> be the upper right vertex of the rectangle.</p> <p>The area function <math>A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}</math>  <math>= \frac{4b}{a}x\sqrt{a^2 - x^2}, x \in (0, a)</math>.</p> <p>(ii) <math>\frac{dA}{dx} = \frac{4b}{a} \left[ x \times \frac{-x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right]</math>  <math>= \frac{4b}{a} \times \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -\frac{4b}{a} \times \frac{2\left(x + \frac{a}{\sqrt{2}}\right)\left(x - \frac{a}{\sqrt{2}}\right)}{\sqrt{a^2 - x^2}}</math>  <math>\frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}</math>  <math>x = \frac{a}{\sqrt{2}}</math> is the critical point.</p> <p>(iii) For the values of <math>x</math> less than <math>\frac{a}{\sqrt{2}}</math> and close to <math>\frac{a}{\sqrt{2}}, \frac{dA}{dx} &gt; 0</math>  and for the values of <math>x</math> greater than <math>\frac{a}{\sqrt{2}}</math> and close to <math>\frac{a}{\sqrt{2}}, \frac{dA}{dx} &lt; 0</math>.</p> <p>Hence, by the first derivative test, there is a local maximum at the critical point <math>x = \frac{a}{\sqrt{2}}</math>. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point <math>x = \frac{a}{\sqrt{2}}</math></p> <p>Thus, for maximum area of the soccer field, its length should be <math>a\sqrt{2}</math> and its width should be <math>b\sqrt{2}</math>.</p> <p style="text-align: center;">OR</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>

	<p>(iii) <math>A = 2x \times 2 \frac{b}{a} \sqrt{a^2 - x^2}, x \in (0, a)</math>.</p> <p>Squaring both sides, we get</p> $Z = A^2 = \frac{16b^2}{a^2} x^2 (a^2 - x^2) = \frac{16b^2}{a^2} (x^2 a^2 - x^4), x \in (0, a)$ <p>A is maximum when Z is maximum.</p> $\frac{dZ}{dx} = \frac{16b^2}{a^2} (2xa^2 - 4x^3) = \frac{32b^2}{a^2} x(a + \sqrt{2}x)(a - \sqrt{2}x)$ $\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$ $\frac{d^2Z}{dx^2} = \frac{32b^2}{a^2} (a^2 - 6x^2)$ $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}} = \frac{32b^2}{a^2} (a^2 - 3a^2) = -64b^2 < 0$ <p>Hence, by the second derivative test, there is a local maximum value of Z at the critical point <math>x = \frac{a}{\sqrt{2}}</math>. Since there is only one critical point, therefore, Z is maximum at <math>x = \frac{a}{\sqrt{2}}</math> hence, A is maximum at <math>x = \frac{a}{\sqrt{2}}</math>.</p> <p>Thus, for maximum area of the soccer field, its length should be <math>a\sqrt{2}</math> and its width should be <math>b\sqrt{2}</math>.</p>	1 1/2 1/2
38.	<p>(i) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:</p> $E_1 = PQ, E_2 = \bar{P}\bar{Q}, E_3 = \bar{P}Q, E_4 = P\bar{Q}$ <p>Let E = The shell fired from exactly one of them hits the plane.</p> $P(E_1) = 0.3 \times 0.2 = 0.06, P(E_2) = 0.7 \times 0.8 = 0.56, P(E_3) = 0.7 \times 0.2 = 0.14, P(E_4) = 0.3 \times 0.8 = 0.24$ $P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$ $P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$ $= 0.14 + 0.24 = 0.38$ <p>(ii) By Bayes' Theorem, <math>P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}</math></p> $= \frac{0.14}{0.38} = \frac{7}{19}$ <p>NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses <math>E_1</math> and <math>E_2</math> are actually eliminated as <math>P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0</math></p> <p><b>Alternative way of writing the solution:</b></p> <p>(i) P(Shell fired from exactly one of them hits the plane)</p> $= P[(\text{Shell from A hits the plane and Shell from B does not hit the plane}) \text{ or } (\text{Shell from A does not hit the plane and Shell from B hits the plane})]$ $= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$ <p>(ii) P(Shell fired from B hit the plane/Exactly one of them hit the plane)</p> $= \frac{P(\text{Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane})}{P(\text{Exactly one of them hit the plane})}$	1 1 2 1 1

$\frac{P(\text{Shell from only B hit the plane})}{P(\text{Exactly one of them hit the plane})}$	1
$\frac{0.14}{0.38} = \frac{7}{19}$	1

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