EDUCATION DEPARTMENT
VILLUPURAM DISTRICT

MATHEMATICS

10

Minimum Material for Slow Learners

2022-23

BEST WISHES
Mrs. K.Krishnapriya, B.Sc., M.A., B.Ed.,
Chief Educational Officer, Villupuram District.

At the state level, we secured the 11th place in the Government Public Examination through our hard work and deserve appreciation.

தன்னம்பிக்கை + விடாமுயற்சி + கடின உழைப்பு = வெற்றி
"The Struggle you're in Today will definitely develop the strength you need for Tomorrow.

A. SIVAMOORTHY, Government High School, Perumbakkam, **Villupuram District.**
Mobile: 9750827997

MESSAGE TO TEACHERS

First and Foremost I would like to express by hearty gratitude to all the teachers who are taking much effort to attain the outstanding performance in Tenth Public Examination last year.

Congratulations to all the teachers who are taking atmost care to improve the level of gifted students as well as the slow learners with colourful marks.

Still we are in a position to enhance the percentage of X Standard result in Villupuram District in the State Level.

"In my point of view a dedicated and service - minded teacher is blessed by the God Over"

Hence it is my appeal to all the Tenth handing teachers to devote more time for the welfare and upliftment of the poor, the destitute, the down trodden and the rural pupils fruitfully.

With Best Wishes

Mrs.K.Krishnapriya, B.Sc., M.A., B.Ed.,

Chief Educational Officer, Villupuram District.

Preface

This material has been prepared in accordance with The TamilNadu Government State Board Syllabus. I am very happy to inform you that by practicing all the problems in this material thoroughly will definitely make the students to score more than 90 percentage of marks in Mathematics in the Public Examination. I am in a position to express my hearty gratitude to our respected CEO Madam and DEO Sir for having encouraged my serious attempt to prepare this material for the welfare of the students. Constructive criticisms and valuable suggestions are always welcome.

A. SIVAMOORTHY,
Government High School,
Perumbakkam,
Villupuram District.

Chapter	Title	Marks			
		1	2	5	8
1	Relations and Functions	2	1	1	-
2	Numbers and Sequences	2	1	1	-
3	Algebra	2	1	1	1
4	Geometry	2	1	1	1
5	Coordinate Geometry	2	1	1	-
6	Trigonometry	1	1	-	-
7	Mensuration	2	1	1	-
8	Statistics and Probability	1	2	1	-
	Total Questions	14(12)	14(9)	14(7)	2(2)
	Total Marks	12	18	35	16

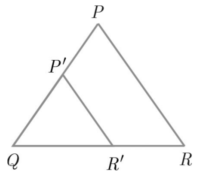
**All the Book Back 1 mark questions well,
we can make the students to get atleast 10 marks.**

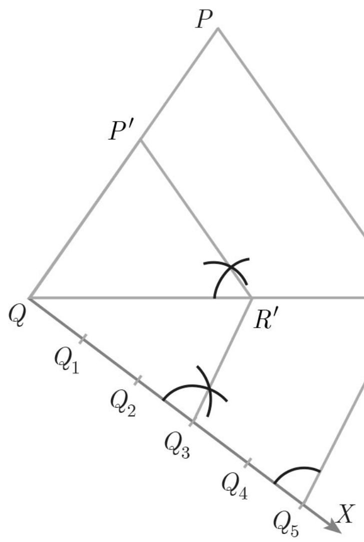
4 GEOMETRY

1. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)**

Solution:

Rough diagram

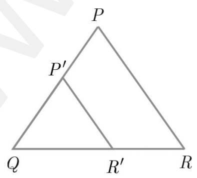


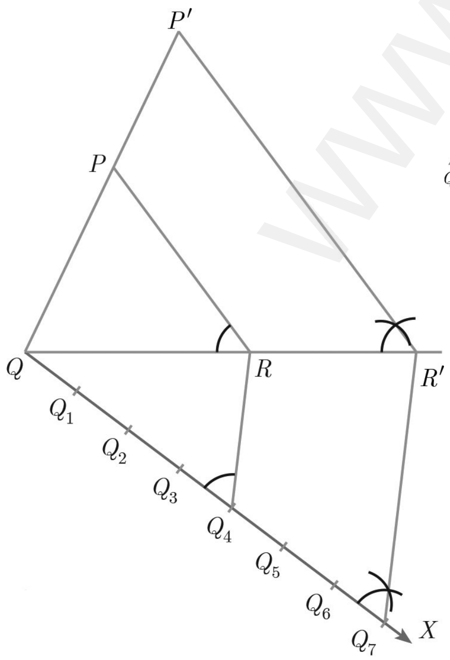


2. **Cons Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)**

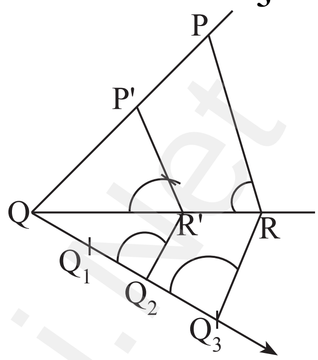
Solution:

Rough diagram

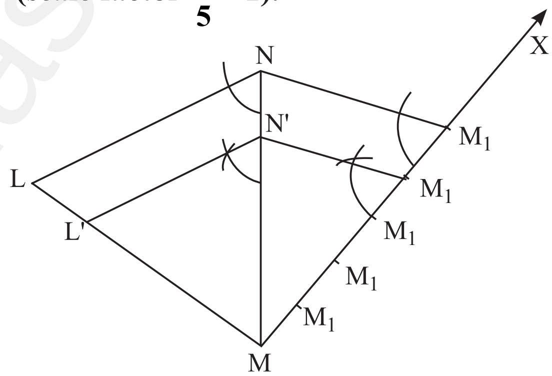




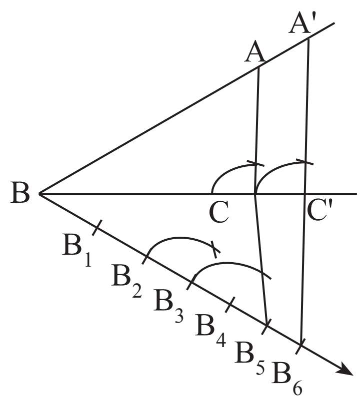
3. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3} < 1$).**



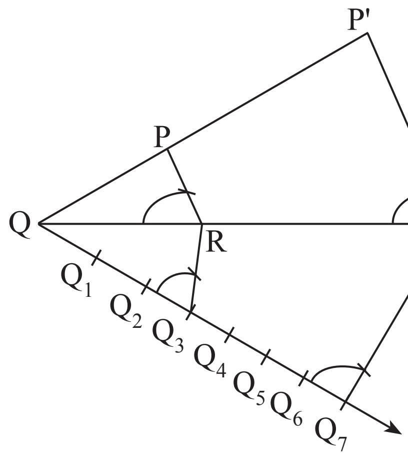
4. **Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).**



5. **Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$).** SEP-20



6. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3} > 1$).

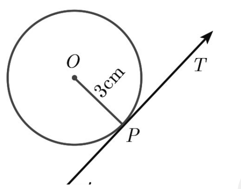


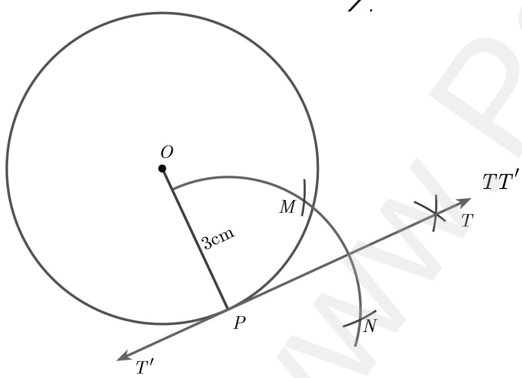
7. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution:

Given, radius r = 3 cm

Rough diagram

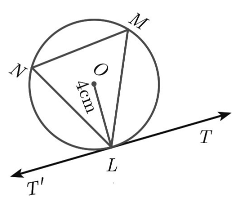


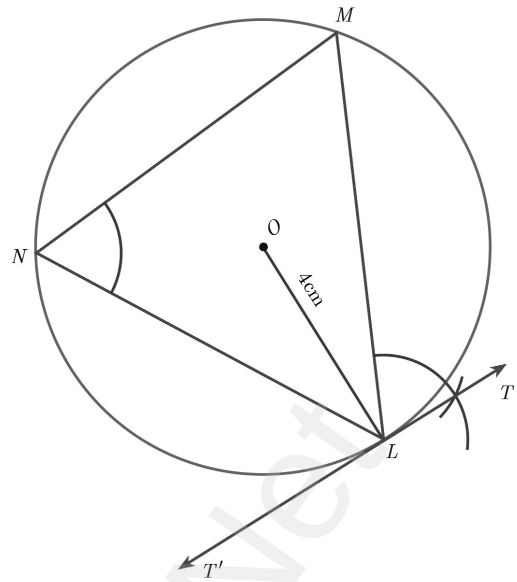


8. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution:

Rough diagram





9. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

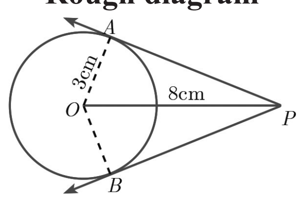
Verification: In the right angle triangle OAP.

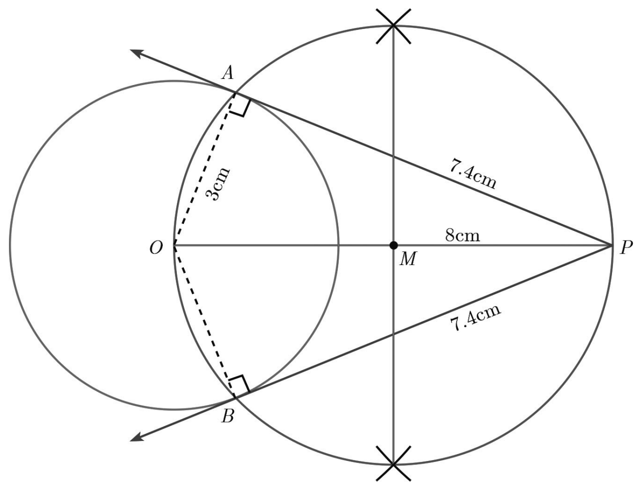
$PA^2 - OA^2 = 64 - 9 = 55$

$PA = \sqrt{55} = 7.4$ cm

Solution:

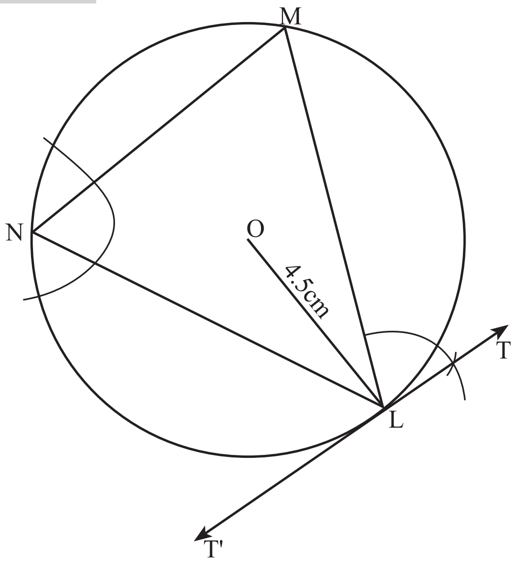
Rough diagram





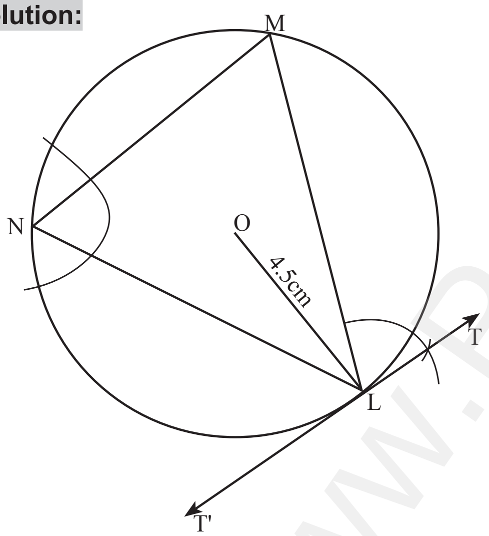
10. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:



11. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

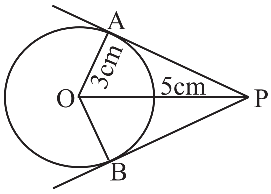
Solution:

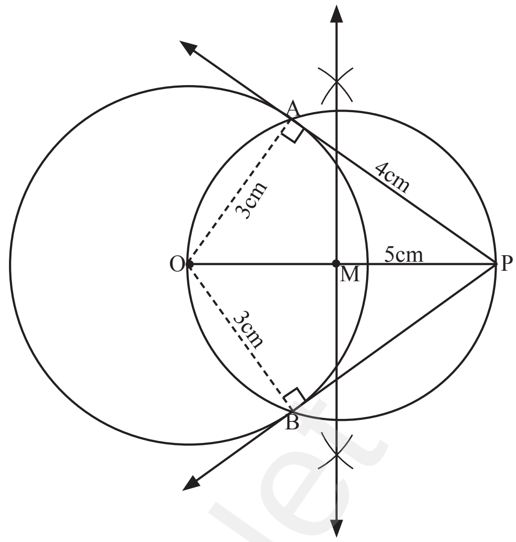


12. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents. **SEP-20**

Solution:

Rough Diagram





Proof:

In ΔOPA

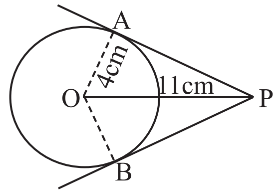
$PA^2 = OP^2 - OA^2$

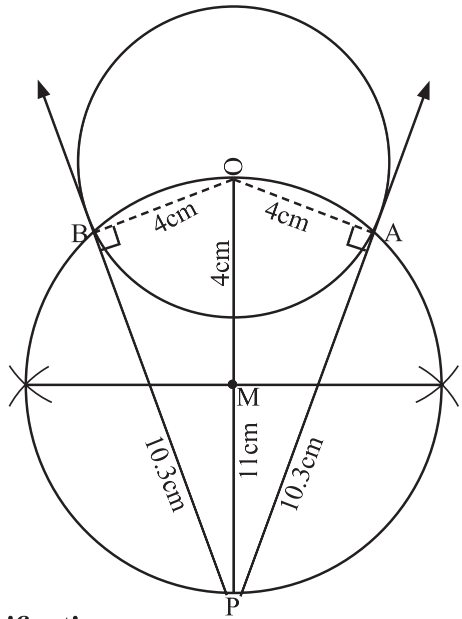
$= 10^2 - 5^2 = 100 - 25 = 75$

$PA = \sqrt{75} = 8.6$ cm (approx)

13. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

Rough Diagram





Verification:

In ΔOPA $AP^2 = OP^2 - OA^2$

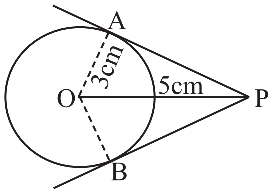
$= 11^2 - 4^2 = 121 - 16 = 105$

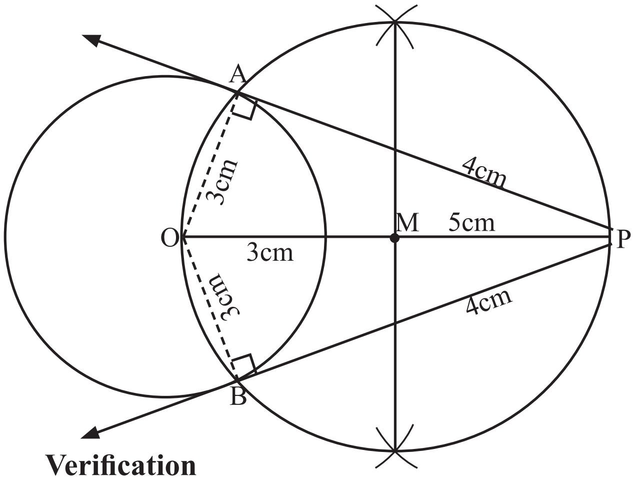
$AP = \sqrt{105} = 10.2$ cm

14. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents. SEP-21

Solution:

Rough Diagram





Verification

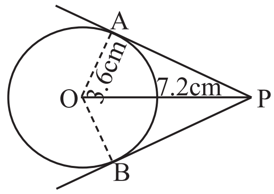
In ΔOPA $AP^2 = OP^2 - OA^2$

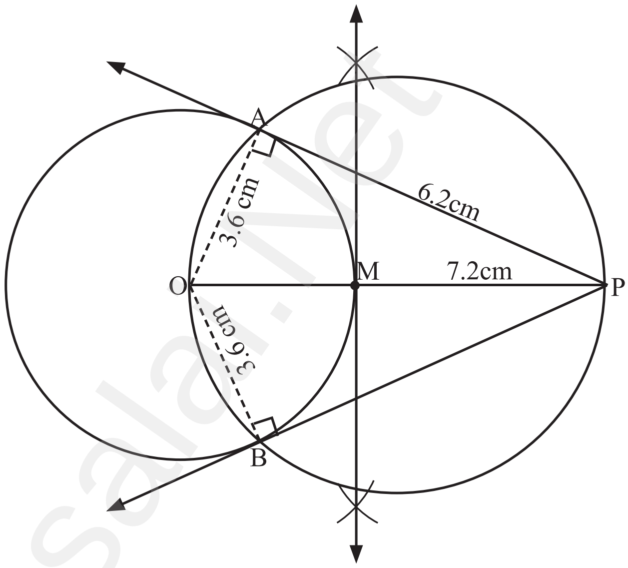
$= 5^2 - 3^2 = 25 - 9 = 16$

$AP = \sqrt{16} = 4$ cm

15. Draw a tangent to the circle from the point P having radius 3.6cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

Solution: **Rough Diagram**





Verification:

In ΔOPA, $PA^2 = OP^2 - OA^2$

$= 7.2^2 - 3.6^2$

$= 51.84 - 12.96$

$= 38.88$

$PA = \sqrt{38.88} = 6.2$ cm (approx)

***

GRAPH

1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

Solution:

I. Table (Given)

Diameter(x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

Let y = kx, where k is a constant of proportionality.

From the given values, we have,

$$k = \frac{y}{x} = \frac{3.1}{1} = \frac{6.2}{2} = \frac{9.3}{3} = \frac{12.4}{4} = ... = 3.1$$

$\therefore y = 3.1x$

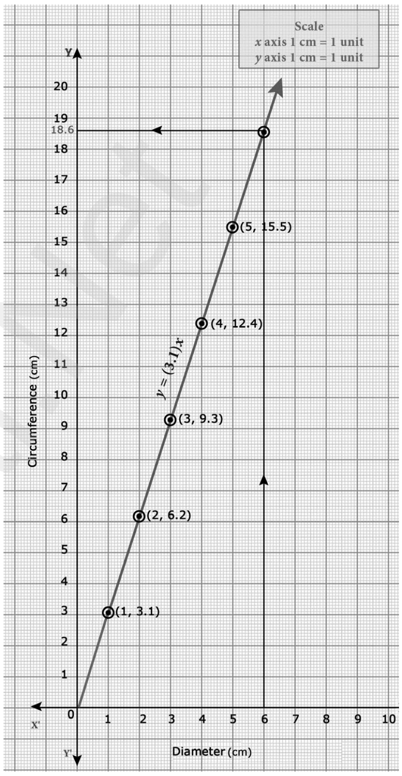
III. Points

(1, 3.1) (2, 6.2) (3, 9.3), (4, 12.4), (5, 15.5)

IV. Solution:

From the graph, when diameter is 6 cm, its circumference is 18.6 cm.

Verify: When $x = 6$, $y = (3.1) \times 6 = 18.6$



2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find (i) the constant of variation (ii) how far will it travel in 1½ hr (iii) the time required to cover a distance of 300 km from the graph.

Solution: I. Table

Time taken x (in minutes)	60	120	180	240	300	360
Distance y (in km)	50	100	150	200	250	300

II. Variation:

When 'x' increases, 'y' also increases.

Thus, the variation is a direct variation.

$$\frac{y}{x} = \frac{50}{60} = \frac{100}{120} = \frac{5}{6} \quad \therefore \text{Equation } y = \frac{5}{6}x$$

III. Points: (60, 50), (120, 100), (180, 150), (240, 200), (300, 250)

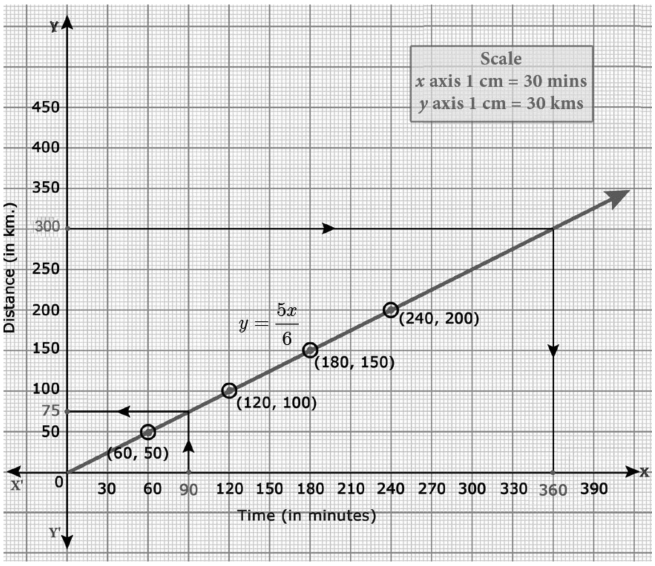
IV. Solution:

(i) the constant of variation $k = \frac{y}{x} = \frac{5}{6}$

(ii) the bus will travel 75 km in 90 mins

(verify : $y = \frac{5}{6} \times 90 = \frac{450}{6} = 75$)

(iii) from the graph, the time required to cover a distance of 300 km is 360 minutes.



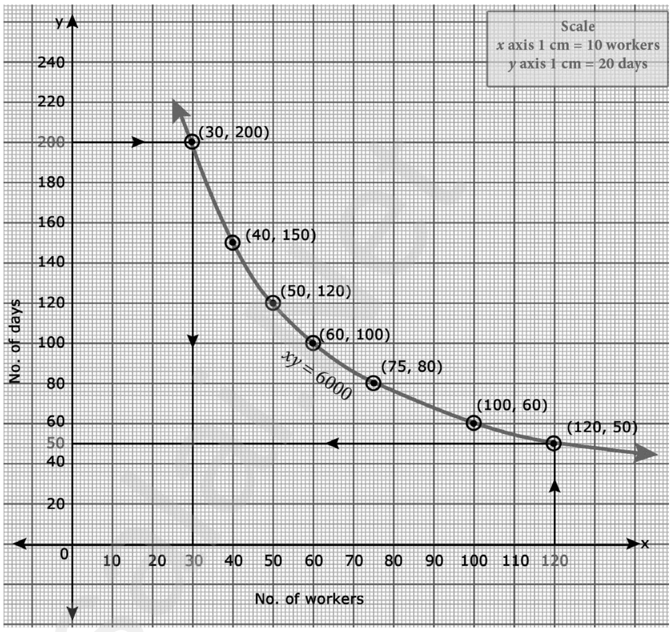
3. A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	**40**	**50**	**60**	**75**
Number of days (y)	**150**	**120**	**100**	**80**

(i) Graph the above data and identify the type of variation. (ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers? (iii) If the work has to be completed by 30 days, how many workers are required?

Solution: **I. Table (Given)**

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80



II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation. i.e. $xy = k$

$xy = 40 \times 150 = 50 \times 120 = \ldots.. 6000$ (k)

$\therefore$ Required Equation $xy = 6000$

III. Points: (40, 150) (50, 120) (60, 100), (75, 80)

IV. Solution:

(i) Inverse Variation

(ii) When $x = 120 \Rightarrow 120 \times y = 6000$

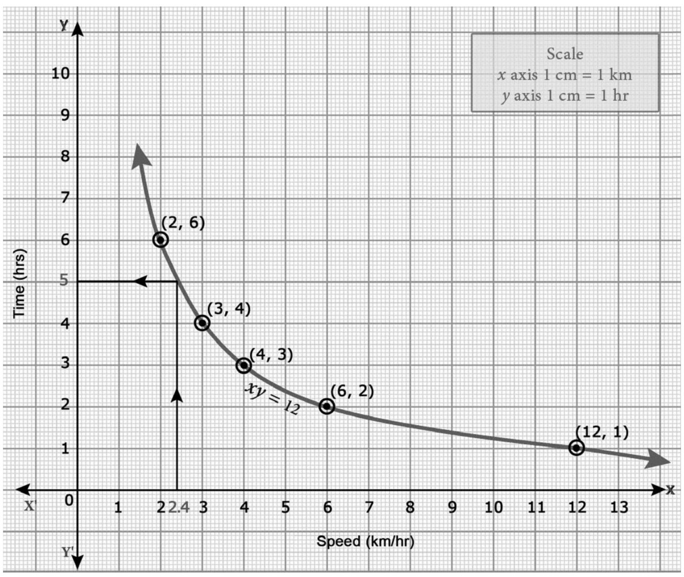
$\Rightarrow y = \frac{6000}{120} = 50$. Also from the graph, the No. of days required to complete the work if the company decides to opt for 120 workers is 50 days.

(iii) When $y = 200 \Rightarrow x \times 200 = 6000 \Rightarrow x = \frac{6000}{200} = 30$. Also from the graph, the No. of workers required to complete in 200 days is 30.

4. Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Ponmozhi, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively. Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

Solution: **I. Table:**

Speed x(km / hr)	12	6	4	3	2
Time y (hours)	1	2	3	4	6



II. Variation:

From the table, we observe that as x decreases, y increases. Hence, the type is inverse variation.

Let y = — $\Rightarrow xy = k, k > 0$ is called the constant of variation.

$k = 12 \times 1 = 6 \times 2 = \ldots = 2 \times 6 = 12$ (k)

Therefore, $xy = 12$.

III. Points: (12, 1), (6, 2), (4, 3), (3, 4), (2, 6)

IV. Solution:

When $x = 2.4 \Rightarrow 2.4 \times y = 12$.

$$y = \frac{12}{2.4} = 5$$

Also, from the graph, the time taken to Kaushik with his speed of 2.4 km / hr is 5 hours .

5. **A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find**
(i) the marked price when a customer gets a discount of ₹ 3250 (from graph)
(ii) the discount when the marked price is ₹ 2500

Solution: **I. Table (Given)**

Marked Price ₹ (x)	1000	2000	3000	4000	5000	6000
Discounted Price ₹ (y)	500	1000	1500	2000	2500	3000

II. Variation:
When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.
Let y = kx, where k is a constant of proportionality.
From the given values, we have,

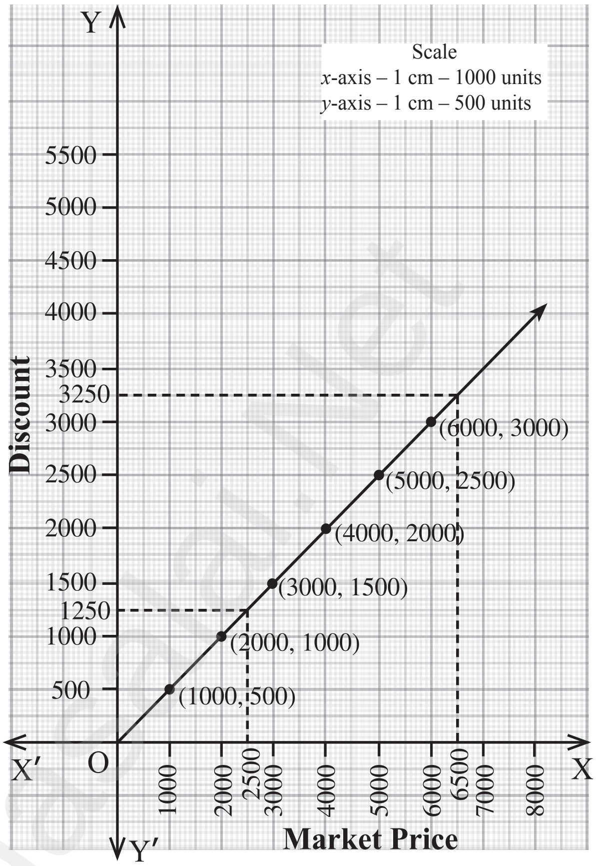
$$k = \frac{y}{x} = \frac{500}{1000} = \frac{1000}{2000} = \ldots = \frac{1}{2}$$

$\therefore$ Required Equation is $y = \frac{1}{2}x$

III. Points: (1000, 500), (2000, 1000), (3000, 1500), (4000, 2000), (5000, 2500), (6000, 3000)

IV. Solution:
(i) From the graph, when a discount price is ₹ 3250, the marked price is ₹ 6500
(ii) From the graph, when the marked price is ₹ 2500, the discounted price is ₹ 1250

Verify: When $x = 6$, y = (3.1) × 6 = 18.6



6. **Draw the graph of $xy = 24$, x, y > 0. Using the graph find,**
(i) y when $x = 3$ and (ii) x when y = 6.

Solution:

I. Table: (Given)

x	1	2	3	4	6	12	24
y	24	12	8	6	4	2	1

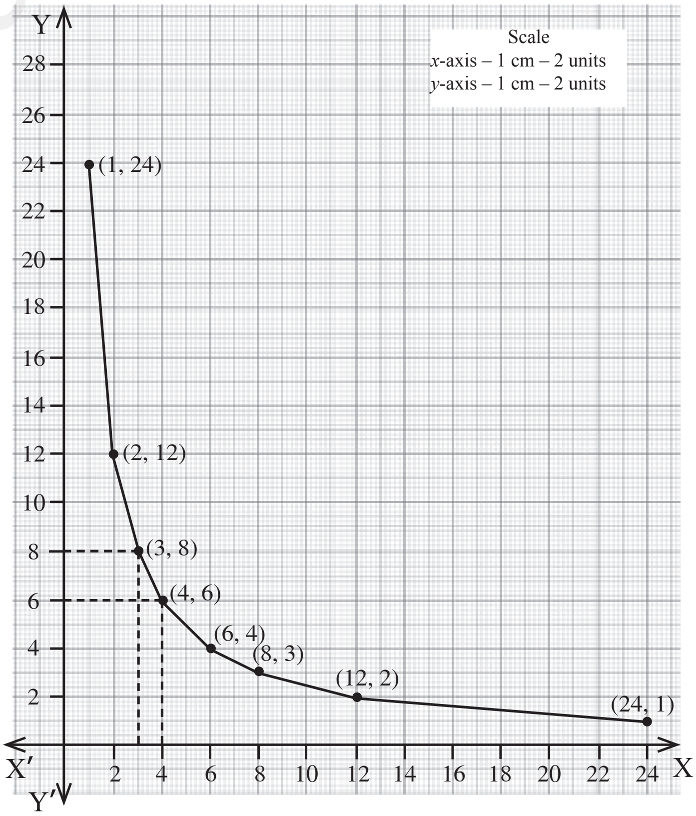
II. Variation:
When 'x' increases, 'y' also decreases.
Hence, inverse variation.
i.e. $xy = k$
$xy = 1 \times 24 = 2 \times 12 = \ldots..12\times2 = 24$
$\therefore$ Required Equation $xy = 24$

III. Points:
(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (12, 2)

IV. Solution:
(i) $x = 3 \Rightarrow 3\times y = 24 \Rightarrow y = \frac{24}{3} = 8 \Rightarrow y = 8$

(ii) $y = 6 \Rightarrow x\times6 = 24 \Rightarrow x = \frac{24}{6} = 4 \Rightarrow x = 4$

Also, Verified in the Graph.



7. Graph the following linear function y =1/2 x. Identify the constant of variation and verify it with the graph. Also (i) find y when x = 9 (ii) find x when y = 7.5.

Solution: I. Table: (Given)

x	2	4	6	8	10
y	1	2	3	4	5

II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

Let y = kx, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \ldots = \frac{1}{2}$$

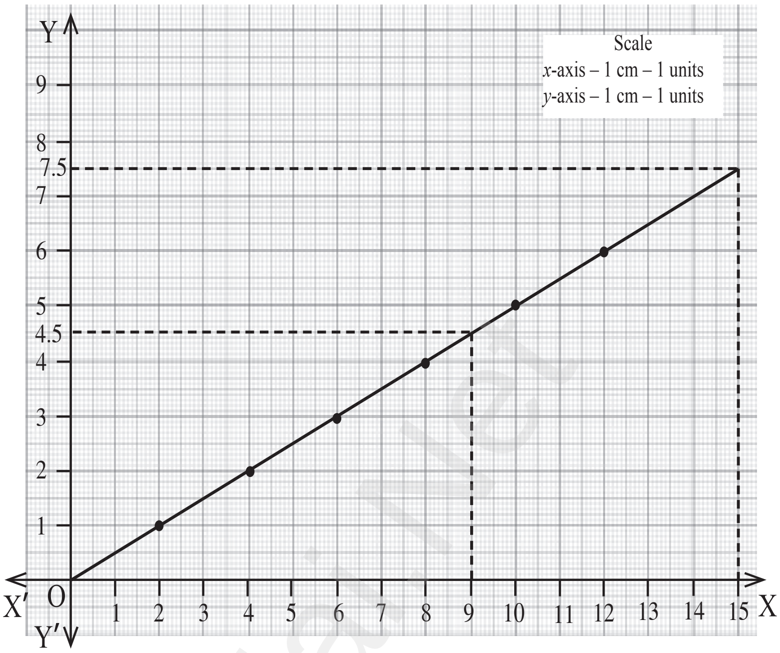
$\therefore$ Required Equation is $y = \frac{1}{2}x$

III. Points: (2, 1), (4, 2), (6, 3), (8, 4), (10, 5)

IV. Solution:

From the graph, when x = 9, y = 4.5

From the graph, when y = 7.5, x = 15



8. The following table shows the data about the number of pipes and the time taken to till the same tank.

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

(ii) find the number of pipes when the time is 9 minutes.

Solution:

I. Table (Given):

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation. i.e. $xy = k$

$xy = 2\times45 = 3\times30 = \ldots..6\times15 = 9\times10 = 90$

$\therefore$ Required Equation $xy = 90$

III. Points: (2, 45), (3, 30), (6, 15), (9, 10)

IV. Solution:

$x = 5 \Rightarrow 5 \times y = 90$

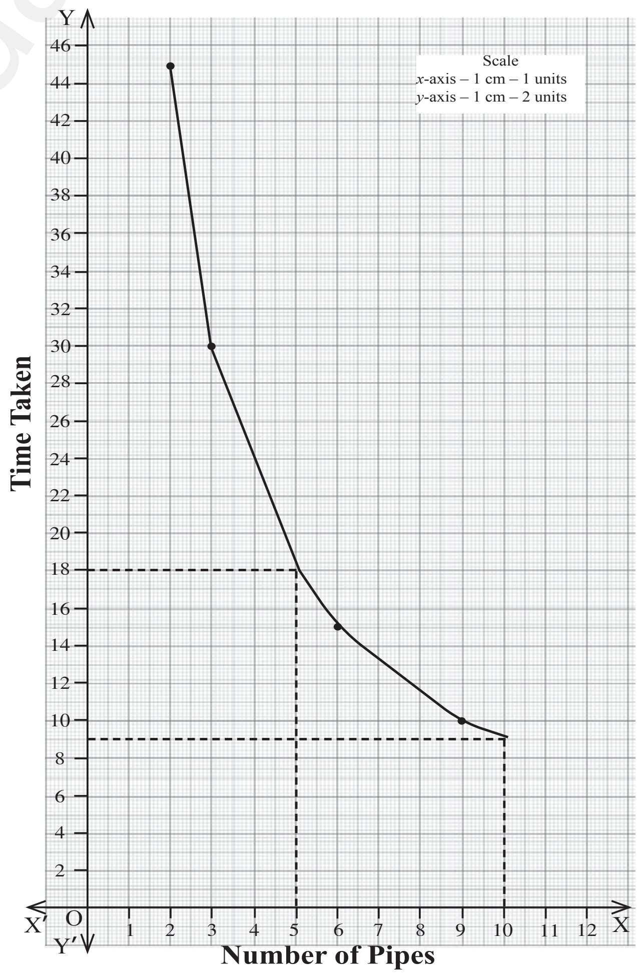
$y = \frac{90}{5} = 18$ (Verified with Graph)

Hence, the time taken to fill the tank when five pipes are used is 18.

$y = 9 \Rightarrow x \times 9 = 90$

$x = \frac{90}{9} = 10$ (Verified with Graph)

Hence, the No. of pipes when the time 9 minutes is 10

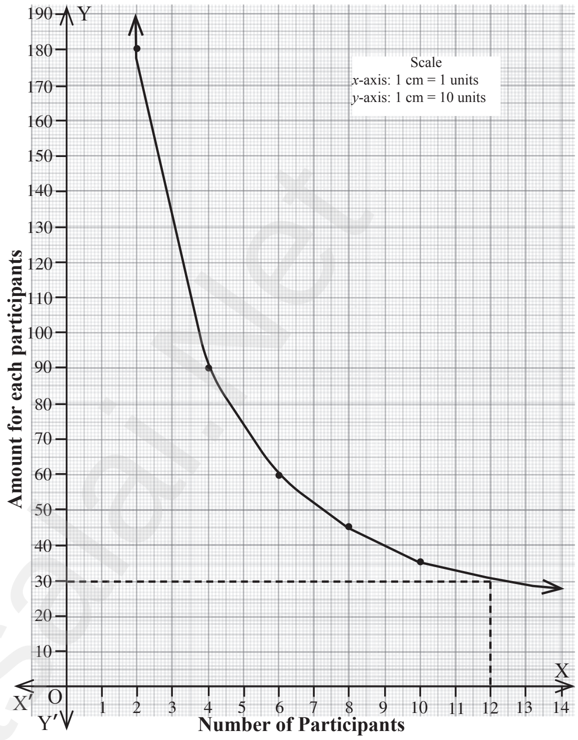


9. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.



Solution: **I. Table(Given)**

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

II. Variation:

When 'x' increases, 'y' also decreases.
Hence, inverse variation. i.e. $xy = k$
$xy = 2\times180 = 4\times90 = ...10\times36 = 360 = k$
$\therefore$ Required Equation $xy = 360$

III. Points: (2,180), (4,90), (6,60), (8,45), (10,36)

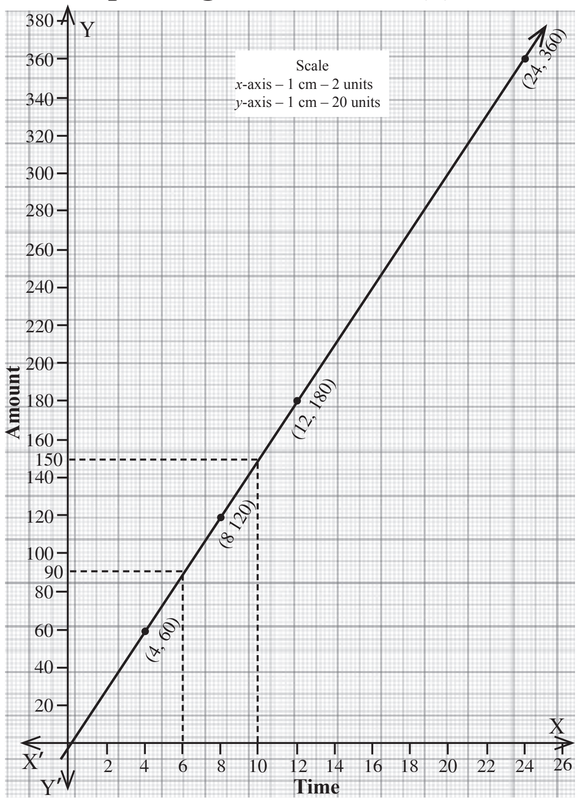
IV. Solution:

Constant of Variation: k = 360
When $x = 12 \Rightarrow xy = 360 \Rightarrow 12y = 360$
$y = \dfrac{360}{12} = 30$ (Verified with Graph)
Hence, When the number of participants are 12, then each participant will get ₹30

10. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr; (ii) find the parking duration when the amount paid is ₹150.



Solution: **I. Table (Given):**

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

II. Variation:

When 'x' increases, 'y' also increases.
Thus, the variation is a direct variation.
Let $y = kx$, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{60}{4} = \frac{120}{8} = ... \frac{180}{12} = \frac{360}{24} = 15 = k$$

$\therefore$ Required Equation is $y = 15x$

III. Points: (4, 60), (8, 120), (12, 180), (24, 360)

IV. Solution:

From the graph, when parking time is 6 hours, then the amount to be paid is ₹ 90.

From the graph, when them amount paid is 1510, then the parking duration is 10 hours.

1. Relations and Functions

2 Marks

1. If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B. SEP-20

Solution:

A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then

A = {Set of all first coordinates of elements of A × B} $\therefore$ A = {3, 5}

B = {Set of all second coordinates of elements of A × B} $\therefore$ B = {2, 4}

Thus A = {3, 5} and B = {2, 4}

2. Find A × B, A × A and B × A

i) A={2, −2, 3} and B = {1,−4}

ii) A = B = {p, q}

iii) A = {m, n} ; B = f

Solution:

i. A × B = {2, –2, 3} × {1, –4}

= {(2, 1), (2, –4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {2, –2, 3} × {2, –2, 3}

= {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {1, –4} ×{2, –2, 3}

= {(1, 2), (1, –2), (1, 3), (–4, 2), (–4, –2), (–4, 3)}

ii. Given A = B = {p, q}

A × B = {p, q} × {p, q}

= {(p, p), (p, q), (q, p),(q ,q)}

A × A = {p, q} × p, q}

= {(p, p), (p, q), (q, p),(q ,q)}

B × A = {p, q} × {p, q}

= {(p, p), (p, q), (q, p),(q ,q)}

iii. A = {m, n}, B = φ

A × B = {(m, n) × { } = { }

A × A = {(m, n) } × {m, n}

= {(m, m), (m, n), (n, m), (n,n)}

B × A = { } × {m, n} = { }

3. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A. MAY-22

Solution:

A = {1, 2, 3} B = {2, 3, 5, 7}

A × B = {1, 2, 3} × {2, 3, 5, 7}

= {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {2, 3, 5, 7} × {1, 2, 3}

= {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

4. If B × A = {(−2, 3), (−2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.

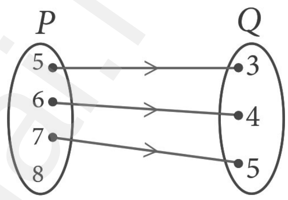
Solution:

A = {Set of all second coordinates of elements of B × A} $\therefore$ A = {3, 4}

B = {Set of all first coordinates of elements of B × A} $\therefore$ B = {–2, 0, 3}

Thus, A = {3, 4} B = {–2, 0, 3}

5. The arrow diagram shows a relationship between the sets P and Q. Write the relation in



(i) Set builder form (ii) Roster form

(iii) What is the domain and range of R.

Solution:

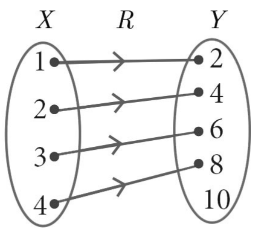
i. Set builder form of R = {(x, y) | y = x – 2, $x \in$ P, y $\in$ Q}

ii. Roster form R = {(5, 3), (6, 4), (7, 5)}

iii. Domain of R = {5, 6, 7} and range of R = {3, 4, 5}

6. Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range?

Solution:



Pictorial representation of R is given diagram, From the diagram, we see that for each $x \in$ X, there exists only one y $\in$ Y.

Thus all elements in X have only one image in Y.

Therefor R is a function.

Domain X = {1, 2, 3, 4}

Co-domain Y = { 2, 4, 6, 8, 10}

Range of f ={2, 4, 6, 8}

7. Let A = {1, 2, 3, 4, ..., 45} and R be the relation defined as "is square of a number" on A. Write R as a subset of A×A. Also, find the domain and range of R. SEP-21

Solution:

A = {1, 2, 3, …., 45}

R = {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)}

$R \subset (A \times A)$

∴ Domain of R = {1, 2, 3, 4, 5, 6}

Range of R = {1, 4, 9, 16, 25, 36}

8. A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution:

$x = \{0, 1, 2, 3, 4, 5\}$

$f(x) = y = x + 3$

f(0) = 3; f(1) = 4; f(2) = 5;

f(3) = 6; f(4) = 7; f(5) = 8

∴ R = {(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)}

Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {3, 4, 5, 6, 7, 8}

9. Given the function $f: x \to x^2 - 5x + 6$, evaluate (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x-1)$

Solution:

Given: $f: x \to x^2 - 5x + 6$

$\Rightarrow f(x) = x^2 - 5x + 6$

i. $f(-1) = (-1)^2 - 5(-1) + 6$
$= 1 + 5 + 6$
$= 12$

ii. $f(2a) = (2a)^2 - 5(2a) + 6$
$= 4a^2 - 10a + 6$

iii. $f(2) = (2)^2 - 5(2) + 6$
$= 4 - 10 + 6$
$= 0$

iv. $f(x-1) = (x-1)^2 - 5(x-1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

10. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution:

$f(x) = 3 - 2x$

$$f(x^2) = [f(x)]^2$$
$$3 - 2x^2 = [3 - 2x]^2$$
$$\Rightarrow 3 - 2x^2 = 9 + 4x^2 - 12x$$
$$3 - 2x^2 - 9 - 4x^2 + 12x = 0$$
$$\Rightarrow -6x^2 + 12x - 6 = 0 \div -6$$
$$x^2 - 2x + 1 = 0$$

$(x-1)(x-1) = 0 \quad x = 1, 1$

11. Let A = {1, 2, 3, 4} and B = N. Let $f: A \to B$ be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function.

Solution:

A = {1, 2, 3, 4}, B = N

$f: A \to B, f(x) = x^3$

$f(1) = (1)^3 = 1$; $f(2) = (2)^3 = 8$;

$f(3) = (3)^3 = 27$; $f(4) = (4)^3 = 64$

i) Range of f = {1, 8, 27, 64}

ii) It is one-one and into function.

5 Marks

1. If A = {1, 3, 5} and B = {2, 3} then
(i) find A×B and B×A.
(ii) Is A×B = B×A? If not why?
(iii) Show that n(A×B) = n(B×A) = n(A)× n(B) SEP-21

Solution:

Given that A = {1, 3, 5} and B = {2, 3}

i. A × B = {1, 3, 5} × {2 × 3}
= {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)} ....(1)

B × A = {2 × 3} × {1, 3, 5}
= {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)} ....(2)

ii. From (1) and (2) we conclude that
$A \times B \neq B \times A$ as $(1, 2) \neq (2, 1)$ and $(1, 3) \neq (3, 1)$ etc

iii. n(A) = 3; n(B) = 2
From (1) and (2) we observe that,
n(A × B) = n(B × A) = 6;
We see that, n(A) × n(B) = 3 × 2 = 6
Thus, n(A × B) = n(B × A) = n(A) × n(B).

2. Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \le x < 2\}$ and $C = \{x \in N \mid x < 3\}$. Then verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:

Given $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$,
$B = \{x \in W \mid 0 \le x < 2\} = \{0, 1\}$,
$C = \{x \in N \mid x < 3\} = \{1, 2\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0,1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C)$

$= \{2, 3\} \times \{0, 1, 2\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= (2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C)$

$= \{(2,0), (2,1), (3, 0), (3,1)\} \cup \{(2,1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3,0), (3, 1), (3, 2)\}$ ..... (2)

From (1) = (2).

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\}$

$= \{(2, 1), (3, 1)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C)$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$ ..... (2)

(1) = (2)

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence it is Verified

3. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$, Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution:

Given $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$

LHS:

$A \times A = \{5, 6\} \times \{5, 6\}$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (1)

RHS = $(B \times B) \cap (C \times C)$.

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B \times B) \cap (C \times C)$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (2)

$\therefore$ From (1) and (2). LHS = RHS

4. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution:

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$

$A \cap C = \{3\}$,

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D)$

$= \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ ..... (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1,5), (2, 2), (2, 3), (2, 5),(3, 2), (3, 3), (3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ..... (2)

(1), (2) are equal.

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

Hence it is verified.

5. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ SEP-21

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution:

Given:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \leq 4\}$

$\Rightarrow B = \{2, 3, 4\}$; $C = \{3, 5\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C)$

$= \{(0, 2), (0, 3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ..... (2)

$\therefore (1) = (2)$ Hence Verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{0,1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ..... (2)

$\therefore$ (1) = (2). Hence Proved.

iii. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$

$= \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$ ..... (1)

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\}$

$= \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$

$\therefore (A \times C) \cup (B \times C)$

$= \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ..... (2)

$\therefore$ From (1) and (2) LHS = RHS.

6. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ SEP-20

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ MAY-22

Solution:

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$ $C = \{2\}$

To verify $(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\}$

$= \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$

$\therefore (A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ .....(1)

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\}$

$= \{(2,2), (3,2), (5,2), (7,2)\}$

$(A \times C) \cap (B \times C)$

$= \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ..... (2)

$\therefore$ From (1) and (2), LHS = RHS

ii. To verify $A \times (B - C) = (A \times B) - (A \times C)$

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3, 7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (1)

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5), (6,7), (7,2), (7,3), (7,5), (7,7)\}$

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$(A \times B) - (A \times C)$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (2)

(1), (2) are equal.

$\therefore A \times (B - C) = (A \times B) - (A \times C)$.

Hence it is verified.

7. Let A = {3, 4, 7, 8} and B = {1, 7, 10}. Which of the following sets are relations from A to B?

(i) $R_1 = \{(3, 7), (4, 7), (7, 10), (8, 1)\}$

(ii) $R_2 = \{(3, 1), (4, 12)\}$

(iii) $R_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$

Solution:

$A \times B = \{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)\}$

i. We note that, $R_1 \subseteq A \times B$.

Thus R_1 is a relation from A and B.

ii. Here $(4, 12) \in R_2$, but $(4, 12) \notin A \times B$.

So R_2 is not a relation from A to B.

iii. Here $(7, 8) \in R_3$, but $(7, 8) \notin A \times B$.

So R_3 is not a relation from A to B.

8. Let A = {1, 2, 3, 7} and B = {3, 0, –1, 7}, which of the following are relation from A to B ?

(i) $R_1 = \{(2, 1), (7, 1)\}$ (ii) $R_2 = \{(-1, 1)\}$

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$

Solution:

Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

$\therefore$ A × B

= {1, 2, 3, 7} × {3, 0, –1, 7}

= {(1, 3),(1, 0), (1, –1), (1, 7), (2, 3), (2, 0), (2, –1), (2, 7), (3, 3), (3, 0), (3, –1), (3, 7), (7, 3), (7, 0), (7, –1), (7, 7)}

i. $R_1 = \{(2, 1), (7, 1)\}$, $(2, 1) \in R_1$
but $(2, 1) \notin A \times B$
$\therefore R_1$ is not a relation from A to B.

ii. $R_2 = \{(-1, 1)\}$, $(-1, 1) \in R_2$
but $(-1, 1) \notin A \times B$
$\therefore R_2$ is not a relation from A to B.

iii. $R_3 = \{(2, -1), (7, 7), (1, 3)\}$
We note that $R_3 \subseteq A \times B$
$\therefore R_3$ is a relation.

iv. $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$, (0, 3), (0, 7) $\in R_4$ but not in A × B.
$\therefore R_4$ is not a relation from A to B.

9. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

(ii) $\{(x, y) | y = x + 3$, x, y are natural numbers < 10}

Solution:

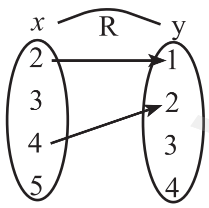
i. $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

$x = 2y$

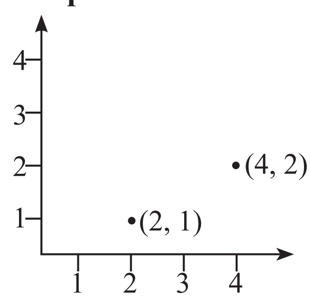
$f(x) = \frac{x}{2}$; $f(2) = \frac{2}{2} = 1$; $f(3) = \frac{3}{2}$;

$f(4) = \frac{4}{2} = 2$; f(5) = –

a) An Arrow diagram



b) Graph



c) Roster Form

{(2, 1), (4, 2)}

ii. $\{(x, y) | y = x + 3$, x, y are natural numbers < 10}

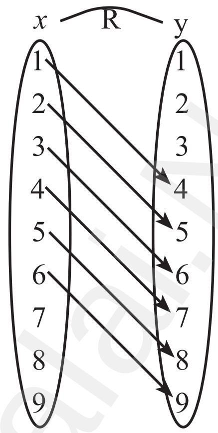
Solution:

$f(x) = x + 3$;

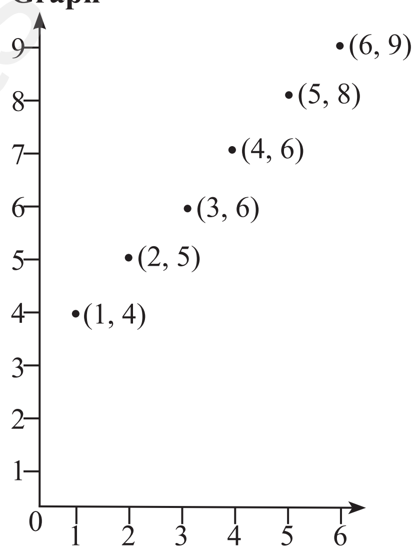
f (1) = 4; f (2) = 5; f (3) = 6;

f (4) = 7; f (5) = 8 ; f (6) =9

a) An Arrow diagram



b) Graph



c) Roster Form

(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

10. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to

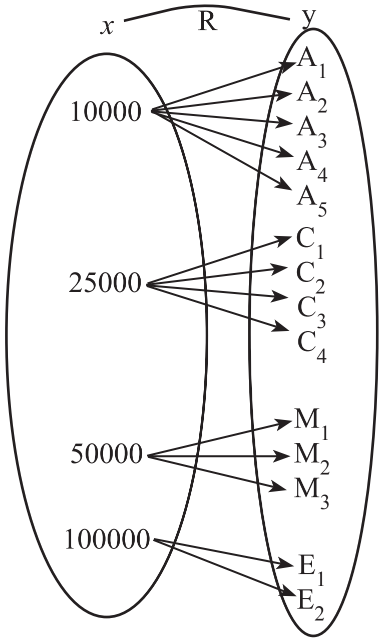
person y, express the relation R through an ordered pair and an arrow diagram.

Solution:

a) **Ordered Pair:**

{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5) (25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, C_3) (100000, E_1), (100000, E_2)}

b) **Arrow Diagram:**



11. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by f(x) = 3x − 1. Represent this function

(i) by arrow diagram

(ii) in a table form

(iii) as a set of ordered pairs

(iv) in a graphical form SEP-20

Solution:

A = {1, 2, 3, 4}, B = {2, 5, 8, 11, 14}

f(x) = 3x – 1

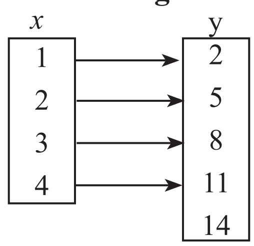
f(1) = 3(1) – 1 = 3 – 1 = 2;

f(2) = 3(2) – 1 = 6 – 1 = 5 f(3) = 3(3) – 1 = 9 – 1 = 8;

f(4) = 3(4) – 1 = 12 – 1 = 11.

R = {(1, 2), (2, 5), (3, 8), (4, 11)}

i) **Arrow Diagram**



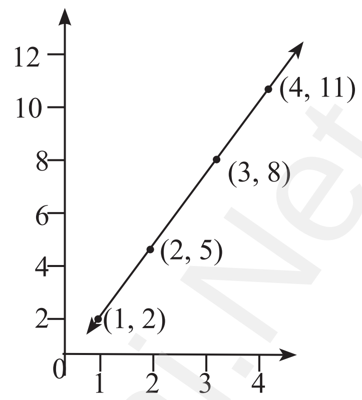
ii) **Table**

x	1	2	3	4
y	2	5	8	11

iii) **Set of Ordered pairs**

{(1, 2), (2, 5), (3, 8), (4, 11)}

iv) **Graphical Form**



12. Let f be a function f : N → N be defined by $f(x) = 3x + 2, x \in$ N

(i) Find the images of 1, 2, 3

(ii) Find the pre-images of 29, 53

(iii) Identify the type of function

Solution:

The function f : N→ N be defined by

f(x) = 3x + 2

i. If x = 1, f(1) = 3(1) + 2 = 5

If x = 2, f(2) = 3(2) + 2 = 8; If x = 3, f(3) = 3(3) + 2 = 11

The images of 1, 2, 3 are 5, 8, 11 respectively.

ii. If x is the pre-image of 29, then f(x) = 29. Hence 3x + 2 = 29 ; 3x = 27 ⇒ x = 9.

Similarly, if x is the pre-image of 53 then f(x) = 53. Hence 3x + 2 = 53

3x = 53 – 2 ⇒ 3x = 51 ⇒ x = 17.

Thus the pre-image of 29 and 53 are 9 and 17 respectively.

iii. Since different elements of N have different images in the co-domain, the function f is one-one function. The co-domain of f is N. But the range of f = {5, 8, 11, 14, 17,….}is a proper subset of N . Therefore f is not an onto function. That is, f is an into function. Thus f is one-one and into functions.

13. Let f : A → B be a function defined by f(x) = $\frac{x}{2}$ − 1 where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9} . Represent f by

i) set of ordered pairs ii) a table
iii) an arrow diagram iv) a graph

Solution:

Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0$

$x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2$

$x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

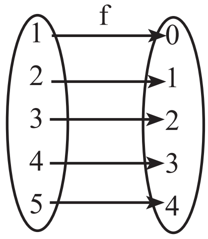
i) Set of Ordered Pairs:

f = {(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

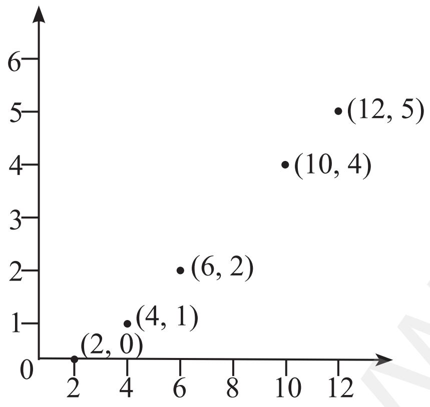
ii) Table

x	2	4	6	10	12
f(x)	0	1	2	4	5

iii) Arrow Diagram



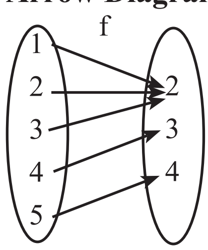
iv) Graph



14. Represent the function f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Solution:

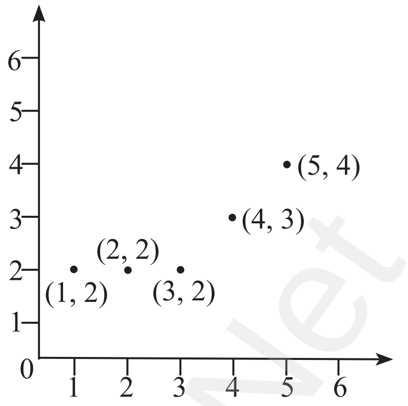
i) Arrow Diagram



ii) Table Form:

x	1	2	3	4	5
f(x)	2	2	2	3	4

iii) Graph



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2. Numbers and Sequences

2 Marks

1. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution:

$800 = a^b \times b^a$

2	800
2	400
2	200
2	100
2	50
5	25
	5

$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

$\therefore a = 2, b = 5$ (or) $a = 5, b = 2$

2. Find the HCF of 252525 and 363636.

Solution:

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
13	481
37	37
	1

5	252525
5	50505
3	10101
7	3367
13	481
37	37
	1

$252525 = 3 \times 5^2 \times 7 \times 13 \times 37$

$363636 = 2^3 \times 3^3 \times 7 \times 13 \times 37$

H.C.F of 252525 and 363636

$= 3 \times 7 \times 13 \times 37$

$= 10101.$

3. If $13824 = 2^a \times 3^b$ then find a and b. MAY-22

Solution:

$$\begin{array}{r|l} 2 & 13824 \\ \hline 2 & 6912 \\ \hline 2 & 3456 \\ \hline 2 & 1728 \\ \hline 2 & 864 \\ \hline 2 & 432 \\ \hline 2 & 216 \\ \hline 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$\Rightarrow 13824 = 2^9 \times 3^3$

$\therefore a = 9, b = 3$

4. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Solution:

$$\begin{array}{r|l} 2 & 408 \\ \hline 2 & 204 \\ \hline 2 & 102 \\ \hline 3 & 51 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

$408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

H.C.F. of 408 & 170 $= 2 \times 17 = 34$

L.C.M. of 408 & 170 $= 2^3 \times 3 \times 5 \times 17$

$= 2040$

5. The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3);\ n \in N \text{ is odd} \\ n^2+1 \ \ ;\ n \in N \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

Solution:

To find a_{11}, since 11 is odd,

we put n = 11 in

$a_n = n(n+3)$

Thus,

the eleventh term $a_{11} = 11(11+3) = 154.$

To find a_{18}, since 18 is even,

we put n = 18 in

$a_n = n^2 + 1$

Thus, the eighteenth term $a_{18} = 18^2 + 1 = 325.$

6. Find the indicated terms of the sequences whose n^{th} terms are given by

(i) $a_n = \dfrac{5n}{n+2}$; a_6 and a_{13}

(ii) $a_n = -(n^2-4)$; a_4 and a_{11}

Solution:

i. $a_n = \dfrac{5n}{n+2}$

$a_6 = \dfrac{30}{8} = \dfrac{15}{4}$; $\quad a_{13} = \dfrac{65}{15} = \dfrac{13}{3}$

ii. $a_n = -(n^2-4)$

$a_4 = -(16-4) = -12;$

$a_{11} = -(121-4) = -117$

7. Find a_8 and a_{15} whose nth term is

$$a_n = \begin{cases} \dfrac{n^2-1}{n+3} & ; n \text{ is even}, \quad n \in N \\ \dfrac{n^2}{2n+1} & ; n \text{ is odd}, \quad n \in N \end{cases}$$

Solution:

To find a_8 here n is even, so $a_n = \dfrac{n^2-1}{n+3}$

$a_8 = \dfrac{64-1}{11} = \dfrac{63}{11}$

To find a_{15}, here n is odd, so $a_n = \dfrac{n^2}{2n+1}$

$a_{15} = \dfrac{(15)^2}{30+1} = \dfrac{225}{31}$

8. Find the 19th term of an A.P. –11, –15, –19, ...

Solution:

General Form of an A.P. is $t_n = a + (n-1)d$

$a = -11;\ d = -15+11 = -4;\ n = 19$

$t_{19} = -11 + 18(-4)$

$= -11 - 72$

$t_{19} = -83$

9. Which term of an A.P. 16, 11, 6, 1,... is –54 ? MAY-22

Solution:

$n = \left(\dfrac{l-a}{d}\right) + 1$

$a = 16;\ d = 11 - 16 = -5;\ l = -54$

$n = \dfrac{-54-16}{-5} + 1 = \dfrac{-70}{-5} + 1$

$n = 14 + 1$

$n = 15$

10. Find the middle term(s) of an A.P. 9, 15, 21, 27, …, 183.

Solution:

$a = 9, \ d = 6, \ l = 183$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1 = 30$$

$\therefore$ 15 and 16 are the middle terms.

$t_n = a + (n-1)d$

$\therefore t_{15} = a + 14d = 9 + 14(6) = 9 + 84 = 93$

$t_{16} = a + 15d = 9 + 15(6) = 9 + 90 = 99$

$\therefore$ 93, 99 are the middle terms of A.P.

11. If 3 + k, 18 – k, 5k + 1 are in A.P. then find k. **SEP-21**

Solution:

3 + k, 18 – k, 5k + 1 is a A.P

$$t_2 - t_1 = t_3 - t_2$$
$$(18-k) - (3+k) = (5k+1) - (18-k)$$
$$15 - 2k = 6k - 17$$
$$-2k - 6k = -17 - 15$$
$$-8k = -32$$
$$k = 4$$

12. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution:

First Term, a = 20

Common Difference, d = 2

$\therefore$ Number of seats in the last row

$= t_n = a + (n-1)d$

$t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

13. Write an A.P. whose first term is 20 and common difference is 8.

Solution:

First Term, a = 20;

Common Difference, d = 8

Arithmetic Progression is a, a+d, a+3d, …

In this case,

we get 20, 20 + 8, 20 + 2(8), 20 + 3(8), ...

So, the required A.P. is 20, 28, 36, 44, ...

14. Find the number of terms in the A.P. 3, 6, 9, 12, …, 111. **SEP-21**

Solution:

First term a = 3,

Common difference d = 6 – 3 = 3,

Last term, $l = 111$

We know that, $n = \left(\frac{l-a}{d}\right) + 1$

$$n = \left(\frac{111-3}{3}\right) + 1 = 37$$

Thus the A.P. contains 37 terms.

15. Write the first three terms of the G.P. whose first term and the common ratio are given below.

(i) a = 6, r = 3 **(ii) $a = \sqrt{2}, r = \sqrt{2}$**

(iii) $a = 1000, r = \frac{2}{5}$

Solution:

i. General Form of an G.P. $\Rightarrow a, ar, ar^2, \ldots$..

$a = 6, r = 3$ G.P. $\Rightarrow 6, 6(3), 6(3)^2 \ldots$

$\Rightarrow 6, 18, 54, \ldots$.

ii. G.P. $\Rightarrow a, ar, ar^2, \ldots$

$a = \sqrt{2}, \ r = \sqrt{2}$

G.P. $\Rightarrow \sqrt{2}, \sqrt{2}\ \sqrt{2}, \sqrt{2}\left(\sqrt{2}\right)^2$

$\Rightarrow \sqrt{2}, 2, 2\sqrt{2}$

iii. G.P. $\Rightarrow a, ar, ar^2, \ldots$..

$a = 1000, \ r = \frac{2}{5}$

G.P. $\Rightarrow 1000, 1000 \times \frac{2}{5}, 1000 \times \left(\frac{2}{5}\right)^2 \ldots$

G.P. $\Rightarrow$ 1000, 400, 160,….

16. In a G.P. 729, 243, 81, … find t_7.

Solution:

$t_n = ar^{n-1}$

$a = 729, r = \frac{243}{729} = \frac{1}{3}, n = 7$

$$t_7 = 729 \times \left(\frac{1}{3}\right)^{7-1}$$

$$t_7 = 729 \times \left(\frac{1}{3}\right)^{6}$$

$$t_7 = 729 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

17. Find x so that $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution:

Given $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a G.P.

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$(x+12)^2 = (x+6)(x+15)$$

$$x^2+24x+144 = x^2+21x+90$$

$$24x-21x = 90-144$$

$$3x = -54$$

$$x = -\frac{54}{3} = -18$$

18. Find the number of terms in the following G.P.

(i) 4, 8, 16, …, 8192?

(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

Solution:

i. G.P. $\Rightarrow$ 4, 8, 16, …, 8192.

Here $a = 4$, $r = 2$, $t_n = 8192$

$ar^{n-1} = t_n \Rightarrow 4(2)^{n-1} = 8192$;

$$2^{n-1} = \frac{8192}{4} = 2048$$

$2^{n-1} = 2^{11}$; $n - 1 = 11$

$\Rightarrow n = 12$

ii. G.P. $\Rightarrow \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$.

Here $a = \frac{1}{3}$, $r = \frac{1}{3}$, $t_n = \frac{1}{2187}$

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187} \times 3$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729} = \left(\frac{1}{3}\right)^6;$$

$n - 1 = 6 \Rightarrow n = 7$

19. In a G.P. the 9th term is 32805 and 6th term is 1215. Find the 12th term.

Solution:

From the given

$t_9 = 32805 \Rightarrow ar^8 = 32805$ ..... (1)

$t_6 = 1215 \Rightarrow ar^5 = 1215$ ..... (2)

$(1) \div (2) \Rightarrow r^3 = 27 \Rightarrow r = 3$

$(2) \Rightarrow a(3)^5 = 1215 \Rightarrow a = 5$

To find t_{12},

$t_n = ar^{n-1}$

$t_{12} = (5)(3)^{11}$

20. Find the first term of a G.P. in which $S_6 = 4095$ and $r = 4$.

Solution:

Common ratio, $= 4 > 1$,

Sum of first 6 terms $S_6 = 4095$

Hence, $S_n = \frac{a(r^n-1)}{r-1} = 4095$

$r = 4$, $\frac{a(4^6-1)}{4-1} = 4095$

$\Rightarrow a \times \frac{4095}{3} = 4095$

First term, $a = 3$.

21. Find the value of

1 + 2 + 3 + ... + 50

Solution:

$1 + 2 + 3 + \ldots + 50$

Using $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$1 + 2 + 3 + \ldots + 50 = \frac{50 \times (50+1)}{2} = 1275$

22. Find the sum of the following series

1 + 2 + 3 + ... + 60

Solution:

$$1+2+3+\ldots+60 = \frac{n(n+1)}{2}$$

$$= \frac{60 \times 61}{2}$$

$$= 30 \times 61 = 1830$$

23. Find the sum of

(i) 1 + 3 + 5 + ... to 40 terms

(ii) 2 + 4 + 6 + ... 80

(iii) 1 + 3 + 5 + ... + 55

Solution:

i. $1 + 3 + 5 + \ldots +$ n terms $= n^2$

$1 + 3 + 5 + \ldots + 40$ terms $= (40)^2 = 1640$

ii. $2 + 4 + 6 + \ldots + 80$

$= 2[1 + 2 + 3 + \ldots + 40]$

$= 2\left[\frac{n(n+1)}{2}\right] = 40 \times 41 = 1640$

iii. $1 + 3 + 5 + \ldots + 55$

Here the number of terms is not given.

Now, we have to find the number of terms using the formula.

$n = \frac{(55-1)}{2} + 1 = 28$

Therefore,

$1 + 3 + 5 + \ldots + 55 = (28)^2 = 784$

24. Find the sum of

(i) $1^2 + 2^2 + ... + 19^2$

(ii) $5^2 + 10^2 + 15^2 + ... + 105^2$

Solution:

i. $1^2 + 2^2 + ... + 19^2$

$= \frac{n(n+1)(2n+1)}{6}$

$= \frac{19\times(19+1)(2\times19+1)}{6}$

$= \frac{19\times20\times39}{6} = 2170$

ii. $5^2 + 10^2 + 15^2 + ... + 105^2$

$= 5^2(1^2 + 2^2 + 3^2 + ... + 21^2)$

$= 25 \times \frac{21\times(21+1)\times(2\times21+1)}{6}$

$= 25 \times \frac{21\times22\times43}{6} = 82775$

25. Find the sum of $1^3 + 2^3 + 3^3 + ... + 16^3$

Solution:

$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$1^3 + 2^3 + 3^3 + ... + 16^3 = \left[\frac{16\times17}{2}\right]^2$

$= [136]^2 = 18496$

26. If $1 + 2 + 3 + ... + n = 666$ then find n.

Solution:

$1 + 2 + 3 + ... + n = 666$

$\frac{n(n+1)}{2} = 666$

$n^2 + n = 1332$

$n^2 + n - 1332 = 0$

$(n - 36)(n + 37) = 0$

$n = -37$ or $n = 36$

But $n \neq -37$ (Since n is a natural number)

Hence $n = 36$.

27. If $1 + 2 + 3 + ... + k = 325$, then find $1^3 + 2^3 + 3^3 + ... + k^3$.

Solution:

$1 + 2 + 3 + ... + k = \frac{k(k+1)}{2} = 325$

$1^3 + 2^3 + 3^3 + ... + k^3$

$= \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$

28. If $1^3 + 2^3 + 3^3 + ... + k^3 = 44100$ then find $1 + 2 + 3 + ... + k$.

Solution:

$1^3 + 2^3 + 3^3 + ... + k^3 = 44100 = \left[\frac{k(k+1)}{2}\right]^2$

$1 + 2 + 3 + ... + k = \frac{k(k+1)}{2} = 210$

29. How many terms of the series $1^3 + 2^3 + 3^3 + ...$ should be taken to get the sum 14400?

Solution:

$1^3 + 2^3 + 3^3 + ... + k^3 = \left[\frac{k(k+1)}{2}\right]^2 = 14400$

$\Rightarrow \frac{k(k+1)}{2} = \sqrt{14400} = 120$

$k(k + 1) = 240$

$k^2 + k - 240 = 0$

$(k - 15)(k + 16) = 0$

$k = +15$ or $k = -16$

k can't be negative

$\therefore k = 15$

5 Marks

1. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Solution:

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$11340 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore P_1 = 2, \ P_2 = 3, \ P_3 = 5, \ P_4 = 7$

$x_1 = 3, \ x_2 = 4, \ x_3 = 2, \ x_4 = 1$

2. If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3, n \in N$, then find the first six terms of the sequence.

Solution:

Given $a_1 = a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_2 + a_1 = 2(1) + 1 = 3;$

$a_4 = 2a_3 + a_2 = 2(3) + 1 = 7$

$a_5 = 2a_4 + a_3 = 2(7) + 3 = 17;$

$a_6 = 2a_5 + a_4 = 2(17) + 7 = 41$

3. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution:

A.P. $\Rightarrow x$, 10, y, 24, z

That is $y = \frac{10+24}{2} = \frac{34}{2} = 17$

$\therefore$ A.P $= x$, 10, 17, 24, z

Here we know that d = 17 – 10 = 7

$\therefore x = 10 - 7 = 3$

$z = 24 + 7 = 31$

$\therefore x = 3$, y = 17, z = 31.

4. Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution:

$S_n = 5 + 55 + 555 + \ldots + n$ terms

$= 5\,[1 + 11 + 111 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[9 + 99 + 999 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[10 - 1 + 100 - 1 + 1000 - 1 + \ldots n \text{ terms}]$

$= \frac{5}{9}\,[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$= \frac{50}{81}\left[(10^n - 1) - \frac{5}{9}n\right]$

5. Find the sum to n terms of the series
(i) 0.4 + 0.44 + 0.444 + ... to n terms
(ii) 3 + 33 + 333 + ... to n terms

Solution:

i. 0.4 + 0.44 + 0.444 + ... n terms

$= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots n\,terms$

$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n\,terms\right]$

$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n\,terms\right]$

$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots n\,terms\right]$

$= \frac{4}{9}\left[(1+1+1+\ldots n \text{ terms}) - \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n\,terms\right)\right]$

$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1 - \left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}\right]\right] = \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right]$

ii. 3 + 33 + 333 + ... n

$= 3(1 + 11 + 111 + \ldots + n \text{ terms})$

$= \frac{3}{9}\,(9 + 99 + 999 + \ldots + n \text{ terms})$

$= \frac{3}{9}\,((10-1) + (100-1) + (1000-1) + \ldots + n \text{ terms})$

$= \frac{3}{9}\,(10 + 100 + 1000 + \ldots + n \text{ terms}) - (1 + 11 + 111 + \ldots + n \text{ terms})$

$= \frac{3}{9}\left(10\left(\frac{10^n - 1}{9}\right) - n\right)$

$= \frac{30}{81}\,(10n - 1) - \frac{3n}{9}$

6. Find the sum of the Geometric series 3 + 6 + 12 + ... + 1536

Solution:

3 + 6 + 12 + ... + 1536

a = 3, r = 2

$t_n = 1536$

$ar^{n-1} = 1536$

$3(2)^{n-1} = 1536$

$3(2)^{n-1} = 3(2)^9$

$2^{n-1} = 2^9$

$n-1 = 9$

$\therefore \quad n = 10$

To find S_n,

$S_n = \frac{a(r^n - 1)}{r - 1} \quad \Rightarrow \quad S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$

$= 3\,(1023) = 3069$

7. Find the value of 16 + 17 + 18 + ... + 75

Solution:

16 + 17 + 18 + … + 75

$= (1 + 2 + 3 + \ldots + 75) - (1 + 2 + 3 + \ldots + 15)$

$= \frac{75(75+1)}{2} - \frac{15(15+1)}{2}$

$= 2850 - 120$

$= 2730$

8. Find the sum of $9^3 + 10^3 + ... + 21^3$

Solution:

$9^3 + 10^3 + ... + 21^3$

$= (1^3 + 2^3 + 3^3 ... + 21^3) - (1^3 + 2^3 + 3^3 ... + 8^3)$

$= \left[\frac{21 \times (21+1)}{2}\right]^2 - \left[\frac{8 \times (8+1)}{2}\right]^2$

$= (231)^2 - (36)^2$

$= 52065$

9. Find the sum of the following series
(i) $6^2 + 7^2 + 8^2 + ... + 21^2$
(ii) $10^3 + 11^3 + 12^3 + ... + 20^3$

Solution:

i. $6^2 + 7^2 + 8^2 + ... + 21^2$

$= (1^2 + 2^2 + 3^2 ... + 21^2) - (1^2 + 2^2 + 3^2 + ... + 5^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{21 \times (21+1)(42+1)}{6} - \frac{5 \times (5+1)(10+1)}{6}$

$= \frac{21 \times 22 \times 43}{6} - \frac{5 \times 6 \times 11}{6}$

$= 3311 - 55 = 3256$

ii. $10^3 + 11^3 + 12^3 + ... + 20^3$

$= 1^3 + 2^3 + 3^3 + ... + 20^3 - 1^3 + 2^3 + 3^3 + ... + 9^3$

$= \left[\frac{n(n+1)}{2}\right]^2 - \left[\frac{n(n+1)}{2}\right]^2$

$= \left[\frac{20 \times 21}{6}\right]^2 - \left[\frac{9 \times 10}{3}\right]^2$

$= [210]^2 - (45)^2$

$= 44100 - 2025 = 42075$

10. The sum of the cubes of the first *n* natural numbers is 2025, then find the value of *n*.

Solution:

$1^2 + 2^2 + 3^2 + ... + n^2 = 285$

$\frac{n(n+1)(2n+1)}{2 \times 3} = 285$

$\frac{n(n+1)(2n+1)}{6} = 285$

$n(n+1)(2n+1) = 285 \times 6$ ..... (1)

$1^3 + 2^3 + 3^3 + ... + n^3 = 2025$

$\left[\frac{n(n+1)}{2}\right]^2 = 2025$

$\frac{n(n+1)}{2} = \sqrt{2025} = 45$

$n(n+1) = 45 \times 2$ ..... (2)

$\frac{(1)}{(2)} \Rightarrow \frac{n(n+1)(2n+1)}{n(n+1)} = \frac{258 \times 6}{45 \times 2}$

$2n + 1 = 19$

$2n = 19 - 1$

$\Rightarrow 2n = 18$

$\therefore n = 9$

11. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, …, 24 cm. How much area can be decorated with these colour papers?

Solution:

The Required Area

$= 10^2 + 11^2 + 12^2 + ... + 24^2$

Area $= (1^2 + 2^2 + 3^2 + ... + 24^2)$
$- (1^2 + 2^2 + ... + 9^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$

$= 4900 - 285 = 4615 \text{ cm}^2$

Therefore Rekha has 4615 cm^2 colour paper. She can decorate 4615cm^2 area with these colour papers.

12. Find the sum of $15^2 + 16^2 + 17^2 + ... + 28^2$

$15^2 + 16^2 + 17^2 + ... + 28^2$

$= (1^2 + 2^2 + 3^2 ... + 28^2)$
$- (1^2 + 2^2 + 3^2 ... + 14^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{28 \times 29 \times 57}{2 \times 3} - \frac{14 \times 15 \times 29}{2 \times 3}$

$= 14 \times 29 \times 19 - 7 \times 5 \times 29$

$= 7714 - 1015 = 6699$

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3. Algebra

2 Marks

1. Find the LCM of the given polynomials

(i) $4x^2y$, $8x^3y^2$

(ii) $9a^3b^2$, $12a^2b^2c$

(iii) $16m$, $12m^2n^2$, $8n^2$

(iv) $p^2 - 3p + 2$, $p^2 - 4$

(v) $2x^2 - 5x - 3$, $4x^2 - 36$

(vi) $(2x^2 - 3xy)^2$, $(4x - 6y)^3$, $8x^3 - 27y^3$

Solution:

i. $4x^2y$, $8x^3y^2$

$4x^2y = 2^2x^2y$

$8x^3y^2 = 2^3x^3y^2$

$\therefore$ LCM$(4x^2y, 8x^3y^2) = 2^3x^3y^2 = 8x^3y^2$

ii. $9a^3b^2$, $12a^2b^2c$

$9a^3b^2 = (1)(3)^2\, a^3b^2$

$12a^2b^2c = 2^2 \times 3 \times a^2 \times b^2 \times c$

$\therefore$ LCM$(9a^3b^2, 12a^2b^2c)$

$= (1) \times 2^2 \times 3^2 \times a^3 \times b^2 \times c = 36a^3b^2c$

iii. $16m$, $12m^2n^2$, $8n^2$

$16m = 2^4 \times m$

$12m^2n^2 = 2^2 \times 3 \times m^2 \times n^2$

$8n^2 = 2^3 \times n^2$

$\therefore$ LCM$(16m, 12m^2n^2, 8n^2)$

$= 2^4 \times 3 \times m^2 \times n^2 = 48m^2n^2$

iv. $p^2 - 3p + 2$, $p^2 - 4$

$p^2 - 3p + 2 = (p - 1)(p - 2)$

$p^2 - 4 = (p + 2)(p - 2)$

$\therefore$ LCM$(p^2 - 3p + 2, p^2 - 4)$

$= (p - 1)(p + 2)(p - 2)$

v. $2x^2 - 5x - 3$, $4x^2 - 36$

$2x^2 - 5x - 3 = (x - 3)(2x + 1)$

$4x^2 - 36 = 4(x + 3)(x - 3)$

$\therefore$ LCM$(2x^2 - 5x - 3, 4x^2 - 36)$

$= 4(x - 3)(x + 3)(2x + 1)$

vi. $(2x^2 - 3xy)^2$, $(4x - 6y)^3$, $8x^3 - 27y^3$

$(2x^2 - 3xy)^2 = x^2(2x - 3y)^2$

$(4x - 6y)^3 = 2^3(2x - 3y)^3$

$8x^3 - 27y^3 = (2x)^3 - (3y)^3$

$= (2x - 3y)(4x^2 + 6xy + 9y^2)$

$\therefore$ LCM$((2x^2 - 3xy)^2, (4x - 6y)^3,$
$(8x^3 - 27y^3)$

$= 2^3 \times x^2 \times (2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

$= 8x^2(2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

2. Simplify:

i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4}$

ii) $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p-3)^2}$

iii) $\dfrac{5t^3}{4t - 8} \times \dfrac{6t - 12}{10t}$

Solution:

i. $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4} = \dfrac{3x^3z}{5y^3}$

ii. $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p-3)^2}$

$= \dfrac{(p-7)(p-3)}{(p-7)} = \dfrac{(p+4)(p-3)}{(p-3)^2} = (p + 4)$

iii. $\dfrac{5t^3}{4t - 8} \times \dfrac{6t - 12}{10t}$

$= \dfrac{5t^3}{4(t-2)} \times \dfrac{6(t-2)}{10t} = \dfrac{3t^2}{4}$

3. Simplify: $\dfrac{x^3}{x - y} + \dfrac{y^3}{y - x}$

Solution:

$\dfrac{x^3}{x - y} + \dfrac{y^3}{y - x} = \dfrac{x^3 - y^3}{x - y}$

$= \dfrac{(x^2 + xy + y^2)(x - y)}{(x - y)}$

$= x^2 + xy + y^2$

4. Find the excluded values of the following expressions (if any). MAY-22

i) $\dfrac{x + 10}{8x}$ ii) $\dfrac{7p + 2}{8p^2 + 13p + 5}$

Solution:

i. The expression $\dfrac{x+10}{8x}$ is undefined when $8x = 0$ or $x = 0$.

When the excluded value is 0.

ii. The expression $\dfrac{7p+2}{8p^2+13p+5}$ is undefined when $8p^2 + 13p + 5 = 0$ that is

$(8p + 5)(p + 1) = 0$ $p = \dfrac{-5}{8}$, $p = -1$.

The excluded values are $\dfrac{-5}{8}$ and -1.

5. Find the excluded values, if any of the following expressions.

i) $\dfrac{y}{y^2-25}$ ii) $\dfrac{t}{t^2-5t+6}$

iii) $\dfrac{x^2+6x+8}{x^2+x-2}$ iv) $\dfrac{x^3-27}{x^3+x^2-6x}$

Solution:

i. The expression $\dfrac{y}{y^2-25}$ is undefined

when $y^2-5^2=0$

$y^2-5^2=0$

$(y+5)(y-5)=0$

$y+5=0, y-5=0$

$y=-5, y=5$

Hence the excluded values are –5 and 5.

ii. The expression $\dfrac{t}{t^2-5t+6}$ is undefined

when $t^2-5t+6=0$

$t^2-5t+6=0$

$(t-2)(t-3)=0$

$t-2=0, t-3=0$

$t=2, t=3$

Hence the excluded values are 2 and 3.

iii. $\dfrac{x^2+6x+8}{x^2+x-2}=\dfrac{(x+4)(x+2)}{(x+2)(x-1)}=\dfrac{x+4}{x-1}$

The expression $\dfrac{x+4}{x-1}$ is undefined when $x-1=0$. Hence the excluded value is 1.

iv. $\dfrac{x^3-27}{x^3+x^2-6x}=\dfrac{(x-3)(x^2+3x+9)}{x(x^2+x-6)}$

$=\dfrac{(x-3)(x^2+3x+9)}{(x)(x+3)(x-2)}$

The expression $\dfrac{x^3-27}{x^3+x^2-6x}$ is undefined

when $x^3+x^2-6x=0$

$\Rightarrow (x)(x+3)(x-2)=0$

$\Rightarrow x=0$ or $x=-3$ or $x=2$

Hence the excluded values are 0, –3, 2

6. Find the square root of the following rational expression.

$\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

Solution:

$\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}=\sqrt{\dfrac{4y^8z^{12}}{x^4}}=2\left|\dfrac{y^4z^6}{x^2}\right|$

7. Find the square root of the following expressions

i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

ii) $\dfrac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Solution:

i. $\sqrt{\left(256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}\right)}$

$=16\,|(x-a)^4(x-b)^2(x-c)^8(x-d)10|$

ii. $\sqrt{\dfrac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}}=\dfrac{4}{3}\left|\dfrac{a^4b^6c^8}{f^6g^2h^7}\right|$

8. Find the square root of the following rational expression.

$\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Solution:

$\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}=$

$\sqrt{\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$

$=\dfrac{11}{9}\left|\dfrac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$

9. Determine the quadratic equations, whose sum and product of roots are

(i) –9, 20 (ii) $\dfrac{5}{3}$, 4 SEP-21

Solution:

i. –9, 20

$x^2-[\alpha+\beta]x+\alpha\beta=0$

$x^2-[-9]x+20=0 \Rightarrow x^2+9x+20=0$

ii. $\dfrac{5}{3}$, 4

Required Quadratic Equations

x^2 – (Sum of the roots)x + product of the roots $=0$

$x^2-\dfrac{5}{3}x+4=0$

Multiply 3 on both sides

$3x^2-5x+12=0$

10. Find the sum and product of the roots for each of the following quadratic equations
(i) $x^2 + 3x - 28 = 0$ (ii) $x^2 + 3x = 0$

Solution:

i. $x^2 + 3x - 28 = 0$

$a = 1, b = 3, c = -28$

Sum of the roots $= \alpha+\beta = -\dfrac{b}{a} = -\dfrac{3}{1} = -3$

Product of the roots $= \alpha\beta = \dfrac{c}{a}$

$= -\dfrac{28}{1} = -28$

ii. $x^2 + 3x = 0$

$a = 1, b = 3, c = 0$

Sum of the roots $= \alpha+\beta = -\dfrac{b}{a} = -\dfrac{3}{1} = -3$

Product of the roots $= \alpha\beta = \dfrac{c}{a} = \dfrac{0}{1} = 0$

11. In the matrix $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$, write

(i) The number of elements

(ii) The order of the matrix

(iii) Write the elements a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}.

Solution:

i) Number of elements $= 4 \times 4 = 16$

ii) Order of matrix $= 4 \times 4$

iii) $a_{22} = \sqrt{7}$; $a_{23} = \dfrac{\sqrt{3}}{2}$; $a_{24} = 5$;

$a_{34} = 0$; $a_{43} = -11$; $a_{44} = 1$

12. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:

Matrix having 18 elements 1×18 (or) 2×9 (or) 3×6 (or) 6×3 (or) 9×2 (or) 18×1

Matrix having 6 elements 1×6 (or) 2×3 (or) 3×2 (or) 6×1

13. Construct a 3×3 matrix whose elements are given by

(i) $a_{ij} = i - 2j$ (ii) $a_{ij} = \dfrac{(i+j)^3}{3}$

Solution:

i. $a_{ij} = | i - 2j |$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} |1-2| & |1-4| & |1-6| \\ |2-2| & |2-4| & |2-6| \\ |3-2| & |3-4| & |3-6| \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

ii. $a_{ij} = \dfrac{(i+j)^3}{3}$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$

14. Construct a 3×3 matrix whose elements are $a_{ij} = i^2j^2$

Solution:

The general 3×3 matrix is given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = 1^2 \times 1^2 = 1\times1 = 1$; $a_{12} = 1^2 \times 2^2 = 1\times4 = 4$;
$a_{13} = 1^2 \times 3^2 = 1\times9 = 9$; $a_{21} = 2^2 \times 1^2 = 4\times1 = 4$;
$a_{22} = 2^2 \times 2^2 = 4\times4 = 16$; $a_{23} = 2^2 \times 3^2 = 4\times9 = 36$
$a_{31} = 3^2 \times 1^2 = 9\times1 = 9$; $a_{32} = 3^2 \times 2^2 = 9\times4 = 36$;
$a_{33} = 3^2 \times 3^2 = 9\times9 = 81$

Hence the required matrix is $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$

15. If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

Solution:

$$A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

16. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ **then find the transpose of –A.** SEP-20

Solution:

$$A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix} \quad -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

17. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ **then verify** $(A^T)^T = A$

Solution:

$$A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$\therefore (A^T)^T = A$

18. Find the values of x, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:

i. $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$\Rightarrow$ 12 = y; 3 = z; $x = 3$

ii. $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

$\Rightarrow$ 5 + z = 5 x + y = 6;

z = 5 – 5 y = 6 – x;

z = 0

$xy = 8$

$x(6-x) = 8$

$6x - x^2 - 8 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x - 2 = 0$ (or) $x - 4 = 0$

$x = 2$ (or) $x = 4$

If $x = 2$ then $y = \frac{8}{x} = \frac{8}{2} = 4$;

If $x = 4$ then $y = \frac{8}{4} = 2$

iii. $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$x + y + z = 9$ ..... (1)

$x + z = 5$ ..... (2)

$y + z = 7$ ..... (3)

Substitute (3) in (1)

$x + 7 = 9 \Rightarrow x = 9 - 7 = 2$

Substitute x =2 in (2)

$2 + z = 5 \Rightarrow z = 5 - 2 = 3$

Substitute z = 3 in (3)

$y + 3 = 7 \Rightarrow y = 7 - 3 \Rightarrow y = 4$

19. If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ **then Find 2A+B.**

Solution:

$$2A + B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

20. If $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$,

find 4A–3B.

Solution:

$$4A - 3B = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} - 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} + \begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}$$

$$= \begin{pmatrix} 20+21 & 16-12 & -8+9 \\ 2-\frac{3}{4} & 3-\frac{21}{2} & 4\sqrt{2}-9 \\ 4-15 & 36+18 & 16-27 \end{pmatrix}$$

$$= \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

21. If $A = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ **then verify that (i) A+B = B +A**

(ii) A+(–A) = (–A)+A =O.

Solution:

i. $A + B = B + A$

L.H.S.

$$A + B = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix} \quad ..... (1)$$

R.H.S.

$$B + A = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix} \quad ..... (2)$$

(1), (2) $\Rightarrow A + B = B + A$

ii. $A + (-A) = (-A) + A = 0$

$$A + (-A) = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad ..... (1)$$

$$(-A) + A = \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad ..... (2)$$

(1), (2) $\Rightarrow A + (-A) = (-A) + A = 0$

22. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$

find the value of (i) B – 5A (ii) 3A – 9B

Solution:

$$A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix},\ B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

i. $B - 5A$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - 5\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} + \begin{pmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

ii. $3A - 9B$

$$= 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} - 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} + \begin{pmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{pmatrix}$$

$$= \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

5 Marks

1. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Solution:

	8	−1	1		
8	64	−16	17	−2	1
	(–) 64				
16 −1		−16	17		
		(+) −16	(–) 1		
16 −2 1			16	−2	1
			(–) 16	(+) −2	(–) 1
				0	

Required Square root $= |8x^2 - x + 1|$

2. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution:

	3	2	4		
3	9	12	28	a	b
	(–) 9				
6 2		12	28		
		(–) 12	(–) 4		
6 4 4			24	a	b
			(–) 24	(–) 16	(–) 16
			a = 16,	b = 16	

3. Find the square root of the following polynomials by division method

Solution:

i. $x^4 - 12x^3 + 42x^2 - 36x + 9$

	1	−6	3		
1	1	−12	42	−36	9
	(–) 1				
2 −6		−12	42		
		(+) −12	(–) 36		
2 −12 3			6	−36	9
			(–) 6	(+) −36	(–) 9
				0	

Required Square root $= |x^2 - 6x + 3|$

ii. $37x^2 - 28x^3 + 4x^4 + 42x + 9$

	2	−7	−3		
2	4	−28	37	42	9
	(–) 4				
4 −7		−28	37		
		(+) −28	(–) 49		
4 −14 −3			−12	42	9
			(+) −12	(–) 42	(–) 9
				0	

Required Square root $= |2x^2 - 7x - 3|$

iii. $16x^4 + 8x^2 + 1$

	4	0	1		
4	16	0	8	0	1
	(–) 16				
8 0		0	8		
		0	0		
8 0 1			8	0	1
			(–) 8	(–) 0	(–) 1
				0	

Required Square root $= |4x^2 + 1|$

iv. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

	11	−9	−12		
11	121	−198	−183	216	144
	(–) 121				
22 −9		−198	−183		
		(+) −198	(–) 81		
22 −18 −12			−264	216	144
			(+) −264	(–) 216	(–) 144
				0	

Required Square root $= |11x^2 - 9x - 12|$

4. Find the values of a and b if the following polynomials are perfect squares

i. $4x^4 - 12x^3 + 37x^2 + bx + a$

Solution:

	2	−3	7		
2	4	−12	37	b	a
	(–) 4				
4 −3		−12	37		
		(+) −12	(–) 9		
4 −6 7			28	b	a
			(–) 28	(+) −42	(–) 49
			a = 49,	b = −42	

ii. $\mathbf{ax^4 + bx^3 + 361x^2 + 220x + 100}$

Solution:

	10	11	12		
10	100	220	361	b	a
	(–)100				
20 11		220	361		
		(–) 220	(–)121		
20 22 12			240	b	a
			(–) 240	(–)264	(–)144
	a = 144 , b = 264				

5. Find the values of m and n if the following polynomials are perfect sqaures

i. $\mathbf{36x^4 - 60x^3 + 61x^2 - mx + n}$ **MAY-22**

Solution:

	6	–5	3		
6	36	–60	61	–m	n
	(–) 36				
12 –5		–60	61		
		(+) –60	(–)25		
12 –10 3			36	–m	n
			(–) 36	(+)– 30	(–) 9
	–m = –30, m = 30				
	n = 9				

ii. $\mathbf{x^4 - 8x^3 + mx^2 + nx + 16}$

Solution:

	1	–4	4		
1	1	–8	m	n	16
	(–) 1				
2 –4		–8	m		
		(+) –8	(–)16		
2 –8 4			m–16	n	16
			(–) 8	(+)– 32	(–) 16
				0	

$\frac{m-16}{2} = 4$

$m - 16 = 8, n = -32$

$m = 8 + 16$

$m = 24$

6. If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ then verify that $A + (B + C) = (A + B) + C$.

Solution:

$$A + (B + C) = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \left(\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad ..... (1)$$

$(A + B) + C$

$$= \left(\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}\right) + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad ..... (2)$$

From (1) & (2) LHS = RHS

7. If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ **verify that $A(B + C) = AB + AC$.**

Solution:

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-7 & 2+6 \\ -4+3 & 2+2 \end{pmatrix} = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

LHS = $A(B + C)$

$$= \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

RHS = AB + AC

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4 & 4+8 \\ -13+16 & 4+0 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$\therefore$ LHS = RHS

8. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T A^T$ SEP-20

Solution:

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$AB^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$\therefore$ LHS = RHS

9. Given that $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$ verify that A(B +C) = AB + AC.

Solution:

$$A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

To verify that A (B + C) = AB + AC

LHS

$$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad .....(1)$$

$$AB = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad .....(2)$$

(1), (2) $\Rightarrow$ A (B + C) = AB + AC.

10. If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$

Solution:

$$A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix} = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

(1), (2) $\Rightarrow (AB)^T = B^T A^T$

11. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$

Solution:

$$A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$A^2 - 5A + 7I_2$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, $A^2 - 5A + 7I_2 = 0$

***

4. Geometry

2 Marks

1. If ΔABC is similar to ΔDEF such that BC = 3 cm, EF= 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution:

Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

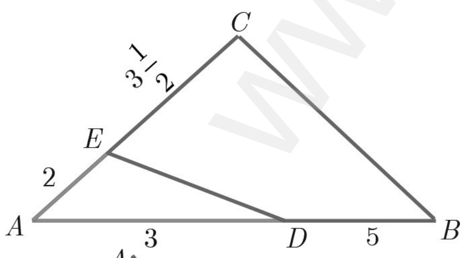
$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

gives $$\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

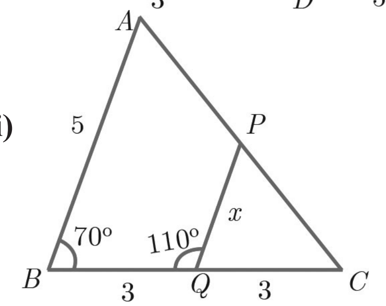
$$\text{Area }(\Delta DEF) = \frac{16\times 54}{9} = 96 \text{ cm}^2$$

2. Check whether the which triangles are similar and find the value of *x*.

i)



ii)



Solution:

i. From the figure, in ΔABC and ADE

$$\frac{AC}{AE} = \frac{3\frac{1}{2}+2}{2} = \frac{\frac{7}{2}+2}{2} = \frac{\frac{7+4}{2}}{2}$$

$$= \frac{11}{2}\times\frac{1}{2} = \frac{11}{4} \quad \text{..... (1)}$$

$$\frac{AB}{AD} = \frac{3+5}{3} = \frac{8}{3} \quad \text{..... (2)}$$

From (1), (2) $\Rightarrow \dfrac{AC}{AE} \neq \dfrac{AB}{AD}$

∴ ΔABC and ΔADE are not similar

ii. From the figure, in ΔABC and ΔPQC

∠ABC= ∠PQC = 70° ..... (1)

(Corresponding angles are equal)

∠C = ∠C (Common Angles) ..... (2)

∴ ∠A = ∠QPC (∵ AAA criterian)

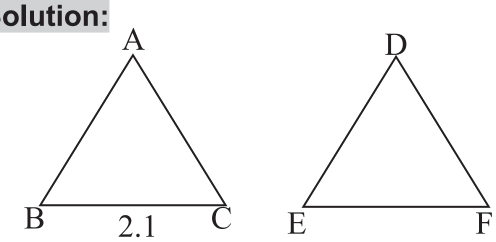
Hence, ΔABC and ΔPQC are similar triangles

Then, $\dfrac{AB}{PQ} = \dfrac{BC}{QC} \Rightarrow \dfrac{5}{x} = \dfrac{6}{3} = 2$

$\therefore x = \dfrac{5}{2} = 2.5$

3. If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm² and BC = 2.1 cm. Find the length of EF.

Solution:



Given ΔABC ~ ΔDEF

$$\frac{Area\ of(\Delta ABC)}{Area\ of(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

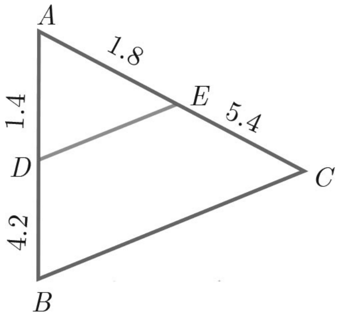
$$\Rightarrow \quad \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow \quad EF^2 = (2.1)^2 \times \frac{16}{9}$$

$$\Rightarrow \quad EF = 2.1 \times \frac{4}{3} = 2.8 \text{ cm}$$

4. D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC .

Solution:



AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

BD = AB – AD = 5.6 – 1.4 = 4.2 cm and

EC = AC – AE = 7.2 – 1.8 = 5.4 cm

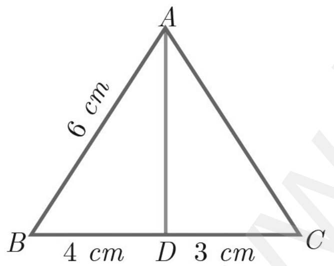
$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC.

Hence Proved.

5. In the Figure, AD is the bisector of ∠A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC. **MAY-22**



Solution:

In ΔABC, AD is the bisector of ∠A.

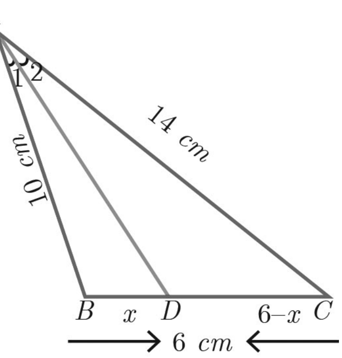
Therefore by Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{4}{3} = \frac{6}{AC} \text{ gives } 4AC = 18$$

$$\text{Hence } AC = \frac{9}{2} = 4.5 \text{ cm}$$

6. In the Figure, AD is the bisector of ∠BAC, if AB = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.



Solution:

AD is the bisector of ∠BAC

AB =10 cm, AC = 14 cm, BC = 6 cm

By Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{x}{6-x} = \frac{10}{14}$$

$$\frac{x}{6-x} = \frac{5}{7}$$

$$7x = 30 - 5x$$

$$12x = 30$$

$$x = \frac{30}{12} = 2.5 \text{ cm}$$

∴ BD = 2.5 cm DC = 3.5 cm

7. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC **SEP-21**

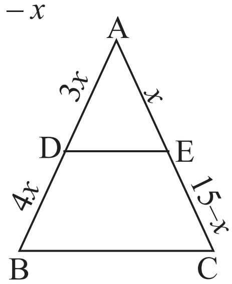
(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.

(ii) If AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1, find the value of x.

Solution:

i. If $\frac{AD}{DB} = \frac{3}{4}$, AC = 15 cm , AE = x,

EC = 15 – x



DE || BC then by basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{3}{4} = \frac{x}{15-x}$$

$$3(15-x) = 4x$$
$$45 - 3x = 4x$$
$$45 = 7x$$
$$x = \frac{45}{7} = 6.43 \text{ cm}$$

ii. Given $AD = 8x-7$, $DB = 5x-3$, $AE = 4x-3$ and $EC = 3x-1$

By basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$
$$\Rightarrow (8x-7)(3x-1) = (5x-3)(4x-3)$$
$$\Rightarrow 24x^2 - 29x + 7 = 20x^2 - 27x + 9$$
$$24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$
$$\Rightarrow 4x^2 - 2x - 2 = 0$$
$$\Rightarrow 2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0$$

$x = 1$, $x = -\frac{1}{2}$ (Not Admissible).

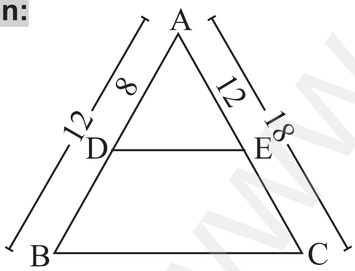
$\therefore x = 1$

8. In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC.

(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

Solution:

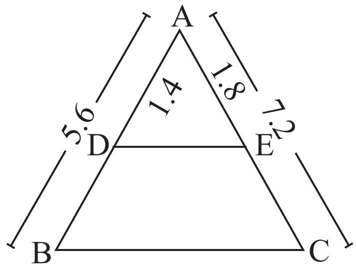


i. AB = 12 cm, AD = 8 cm AE = 12 cm and AC = 18cm

$$\frac{AD}{AB} = \frac{8}{12} = \frac{2}{3} \quad \text{..... (1)}$$
$$\frac{AE}{AC} = \frac{12}{18} = \frac{2}{3} \quad \text{..... (2)}$$

From (1) & (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\therefore$ DE || BC



ii. AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

$$\frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4} \quad \text{..... (1)}$$
$$\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4} \quad \text{..... (2)}$$

(1), (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

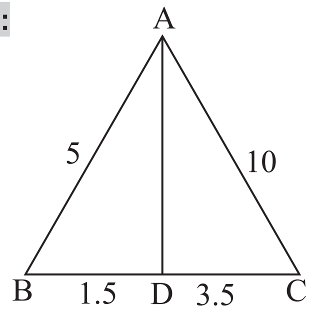
$\therefore$ DE || BC

9. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following

(i) AB = 5 cm, AC = 10 cm, BD= 1.5 cm and CD= 3.5 cm. SEP-20

(ii) AB= 4 cm, AC = 6 cm, BD = 1.6 cm and CD= 2.4 cm.

Solution:

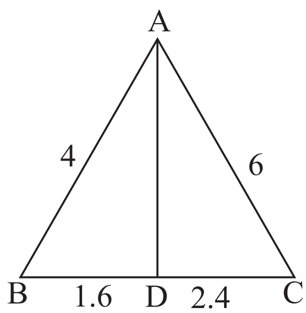


i. AB = 5cm , AC =10 cm, BD =1.5 cm and CD = 3.5 cm

$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2} \quad \text{..... (1)}$$
$$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7} \quad \text{..... (2)}$$

(1), (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{CD}$ ($\because$ By ABT)

AD is not a bisector of $\angle A$ in ΔABC



ii. AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$ ..... (1)

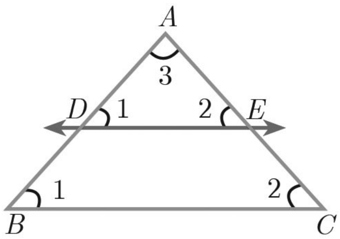
$\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$ ..... (2)

(1), (2) $\Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$ ($\because$ By ABT)

AD is a bisector of ∠A in ΔABC

5 Marks

1. State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem. MAY-22



Statement:

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given:

In ΔABC, D is a point on AB and E is a point on AC

To Prove:

$\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line DE || BC

No.	Statement	Reason
1.	∠ABC = ∠ADE $\longrightarrow$ 1	Corresponding angles are equal because DE || BC
2.	∠ACB = ∠AED $\longrightarrow$ 2	Corresponding angles are equal because DE || BC
3.	∠DAE = ∠BAC $\longrightarrow$ 3	Both triangles have a common angle.
	ΔABC ~ ΔADE	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD\ \ DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E
	$1+\frac{DB}{AD} = 1+\frac{EC}{AE}$	On Simplication
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
	Hence Proved	

2. State and Prove Angle Bisector Theorem.

Statement: SEP-20

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

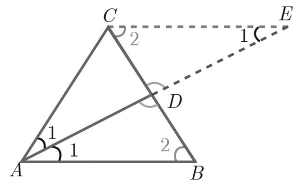
Proof

Given:

In ΔABC, AD is the internal bisector

To Prove:

$\frac{AB}{AC} = \frac{BD}{CD}$.



Construction:

Draw a line through C parallel to AB.

Extend AD to meet line through C at E.

No.	Statement	Reason
1.	∠AEC = ∠BAE = ∠1	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles AC = CE .... (1)	In ΔACE ∠CAE = ∠CEA.
3.	ΔABD ~ ΔECD $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE. Hence Proved.

3. State and Prove Pythagoras Theorem.

Statement:

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

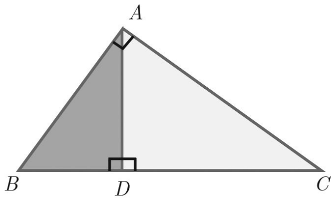
Proof

Given:

In ΔABC, $\angle A = 90°$

To Prove:

$AB^2 + AC^2 = BC^2$



Construction: Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ..... (1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity

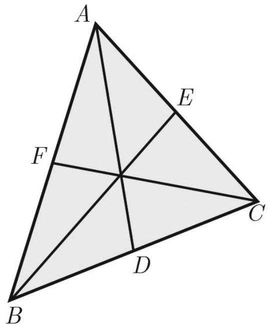
Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC\,(BD + DC)$$
$$AB^2 + AC^2 = BC \times BC = BC^2$$

Hence the theorem is proved.

4. Show that in a triangle, the medians are concurrent. SEP-21

Solution:



Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is midpoint of BC,

$BD = DC$. So $\frac{BD}{DC} = 1$ ..... (1)

Since E is midpoint of CA,

$CE = EA$. So $\frac{CE}{EA} = 1$ ..... (2)

Since F is midpoint of AB,

$AF = FB$. So $\frac{AF}{FB} = 1$ ...... (3)

Thus, multiplying (1), (2), (3) we get

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB}$$

$= 1 \times 1 \times 1 = 1$

And so, Ceva's theorem is satisfied.

Hence the Medians are concurrent.

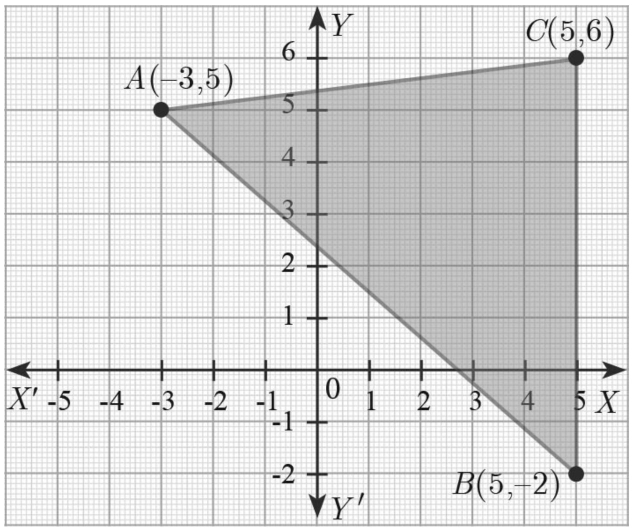
***

5. Coordinate Geometry

2 Marks

1. Find the area of the triangle whose vertices are (–3, 5), (5, 6) and (5, –2)

Solution:



A(–3, 5), B(5, –2), C(5,6)

$x_1 y_1$ $\quad x_2 y_2$ $\quad x_3 y_3$

$$\text{Area of } \Delta = -\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -3 & 5 \\ 5 & -2 \\ 5 & 6 \\ -3 & 5 \end{vmatrix}$$

$$= \frac{1}{2}[(6+30+25)-(25-10-18)]$$

$$= \frac{1}{2}[61+3]$$

$$= \left|\frac{64}{2}\right| = 32 \text{ sq. units.}$$

2. Show that the points P (–1.5, 3), Q (6, –2), R (–3, 4) are collinear. **MAY-22**

Solution:

Area of ΔPQR = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} -1.5 & 3 \\ 6 & -2 \\ -3 & 4 \\ -1.5 & 3 \end{vmatrix} = 0$$

$$\frac{1}{2}[(3+24-9)-(18+6-6)] = 0$$

$$\frac{1}{2}[18-18] = 0$$

∴ Therefore, the given points are collinear.

3. If the area of the triangle formed by the vertices A (–1, 2), B (*k*, –2) and C (7, 4) (taken in order) is 22 sq. units, find the value of *k*.

Solution:

The vertices are A (–1, 2), B (k, –2) and C (7, 4)

Area of ΔABC is 22 sq.units

$$\frac{1}{2}\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 22$$

$$\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 44$$

$$\{(2+4k+14)-(2k-14-4)\} = 44$$

$$4k+16-2k+18 = 44$$

$$2k+34 = 44$$

$$2k = 10$$

Therefore $k = 5$

4. Find the area of the triangle formed by the points (i) (1, –1), (–4, 6) and (–3, –5) (ii) (ii) (–10, –4), (–8, –1) and (–3, –5)

Solution:

i. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2}[(6+20+3)-(4-18-5)]$$

$$= \frac{1}{2}[6+20+3-4+18+5]$$

$$= \frac{1}{2}[(6+20+3+18+5)-4]$$

$$= \frac{1}{2}[52-4]$$

$$= \frac{1}{2}[48] = 24 \text{ sq.units.}$$

ii. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -10 & -4 \\ -8 & -1 \\ -3 & -5 \\ -10 & -4 \end{vmatrix}$$

$$= \frac{1}{2}[(10+40+12)-(32+3+50)]$$

$$= \frac{1}{2}[62-85]$$

$$= \frac{1}{2}[-23] = -11.5 \text{ sq.units.}$$

∴ Area of the Triangle = 11.5 sq.units

5. Determine whether the sets of points are collinear?

(i) $\left(-\frac{1}{2}, 3\right)$ **(–5, 6) and (–8, 8)**

Solution:

$\left(-\frac{1}{2}, 3\right)$ (–5, 6) and (–8, 8)

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -\frac{1}{2} & 3 \\ -5 & 6 \\ -8 & 8 \\ -\frac{1}{2} & 3 \end{vmatrix}$$

$$= \frac{1}{2}[(-3-40-24)-(-15-48-4)]$$

$$= \frac{1}{2}[(-67)-(-67)] = 0$$

∴ The given points are collinear.

(ii) (a, b+c), (b, c+a) and (c, a+b)

Solution:

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$$

$$= \frac{1}{2}[(ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ca + a^2 + ab]$$

$$= \frac{1}{2}[ac + a^2 + ab + b^2 + bc + c^2 - b^2 - bc - c^2 - ca - a^2 - ab]$$

$$= \frac{1}{2}[0] = 0 \text{ sq.units.}$$

Aliter:

(a, b+c), (b, c+a), (c, a+b)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ b+c-c-a & b+c-a-b \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ -(a-b) & -(a-c) \end{vmatrix}$$

$$= \frac{1}{2}[(a-b)(a-c) + (a-b)(a-c)]$$

$$= \frac{1}{2}[0] = 0$$

$\therefore$ The given points are collinear.

6. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p'.

S.No.	Vertices	Area (sq.units)
(i)	(0, 0), (p, 8), (6, 2)	20
(ii)	(p, p), (5, 6), (5, –2)	32

Solution:

i. A (0, 0), B (p, 8), C (6, 2)

Area of ΔABC = 20 sq.units.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \text{Area of }\Delta ABC$$

$$\frac{1}{2}\begin{vmatrix} 0 & 0 \\ p & 8 \\ 6 & 2 \\ 0 & 0 \end{vmatrix} = 20$$

$$(0+2p+0) - (0+48+0) = 40$$

$$2p - 48 = 40$$

$$2p = 88$$

$$p = 44$$

ii. A (p, p), B (5, 6), C (5, –2)

Area of Δ = 32 sq.units

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32$$

$$\frac{1}{2}\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 32$$

$$\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 64$$

$$(6p-10+5p) - (5p+30-2p) = 64$$

$$6p - 10 + 5p - 5p - 30 + 2p = 64$$

$$8p - 40 = 64$$

$$\Rightarrow 8p = 64 + 40$$

$$8p = 104$$

$$\Rightarrow p = \frac{104}{8}$$

$$\Rightarrow p = 13$$

7. In each of the following, find the value of '*a*' for which the given points are collinear.

(i) (2, 3), (4, a) and (6, –3)

(ii) (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

Solution:

i. (2, 3), (4, a) and (6, –3)

$\Delta = 0$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} 2 & 3 \\ 4 & a \\ 6 & -3 \\ 2 & 3 \end{vmatrix} = 0$$

$$[(2a-12+18) - (12+6a-6)] = 0$$

$$2a - 12 + 18 - 12 - 6a + 6 = 0$$

$$-4a = 0$$

$$\therefore a = 0$$

ii. (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

$\Delta = 0$ sq.units.

$(2a^2–6a+2a^2+6–2a–8+8a–2a+2a^2) –$
$(–2a+2a^2+2–2a–8a–2a^2+6a–2a^2) = 0$

$\Rightarrow (6a^2–2a–2) – (–2a^2–6a+2) = 0$

$\Rightarrow 8a^2 + 4a – 4 = 0 \div 4$

$2a^2 + a – 1 = 0$

$(a+1)(2a–1) = 0$

$\Rightarrow \therefore a = +\frac{1}{2}$ and $a = –1$

Aliter:

(a, a–2a), (–a+1, 2a), (–4–a, 6–2a)

$x_1,\ y_1 \quad x_2,\ y_2 \quad x_3,\ y_3$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a+a-1 & a+4+a \\ 2-2a-2a & 2-2a-6+2a \end{vmatrix} = 0$$

$$\begin{vmatrix} 2a-1 & 2a+4 \\ 2-4a & -4 \end{vmatrix} = 0$$

$–4(2a–1) – (2–4a)(2a+4) = 0$

$–8a+4 – [4a+8–8a^2–16a] = 0$

$–8a+4–4a–8+8a^2+16a = 0$

$8a^2+4a–4 = 0$

$2a^2+a–1 = 0$

$(a+1)(2a–1) = 0$

$a = –1$ (or) $a = \frac{1}{2}$

5 Marks

1. The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (–3, 2), (–1, –1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Solution:

Vertices of one triangular tile are at (–3, 2), (–1, –1) (1, 2)

$$\text{Area of this tile} = \frac{1}{2}\begin{vmatrix} -3 & 2 \\ -1 & -1 \\ 1 & 2 \\ -3 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\{(3–2+2)–(–2–1–6)\}$$

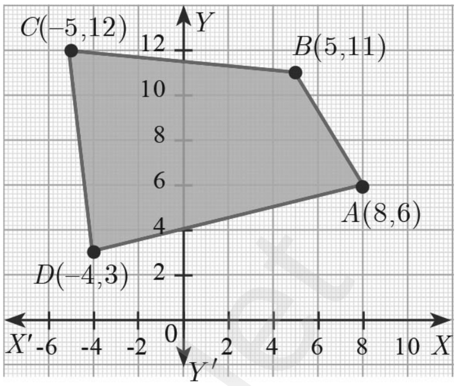
$$= \frac{1}{2}(12) = 6 \text{ sq.units}$$

Since the floor is covered by 110 triangle shaped identical tiles,

Area of the floor = 110 × 6 = 660 sq. units

2. Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (–5, 12) and (–4, 3).

Solution:



Before determining the area of the quadrilateral, plot the vertices in a graph A (8, 6), B (5, 11), C (–5, 12) and D (–4, 3).

Therefore, area of the quadrilateral ABCD

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & 6 \\ 5 & 11 \\ -5 & 12 \\ -4 & 3 \\ 8 & 6 \end{vmatrix}$$

$$= \frac{1}{2}[(88 + 60 – 15 – 24) – (30 – 55 – 48 + 24)]$$

$$= \frac{1}{2}[88 + 60 – 15 – 24 – 30 + 55 + 48 – 24]$$

$$= \frac{1}{2}[88 + 60 + 55 + 48 – 15 – 24 – 30 – 24]$$

$$= \frac{1}{2}[251 – 93]$$

$$= \frac{1}{2}[158] = 79 \text{ sq.units.}$$

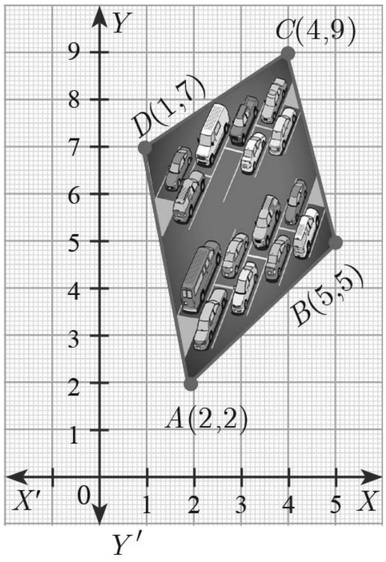
3. The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Solution:

The parking lot is a quadrilateral whose vertices A (2, 2), B (5, 5), C (4, 9) and D (1, 7).

Therefore, Area of parking lot is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 2 \\ 5 & 5 \\ 4 & 9 \\ 1 & 7 \\ 2 & 2 \end{vmatrix}$$



$$= \frac{1}{2}[(10 + 45 + 28 + 2) - (10 + 20 + 9 + 14)]$$

$$= \frac{1}{2}[85 - 53]$$

$$= \frac{1}{2}[32] = 16 \text{ sq.units}$$

So, Area of parking lot = 16 sq.feets.

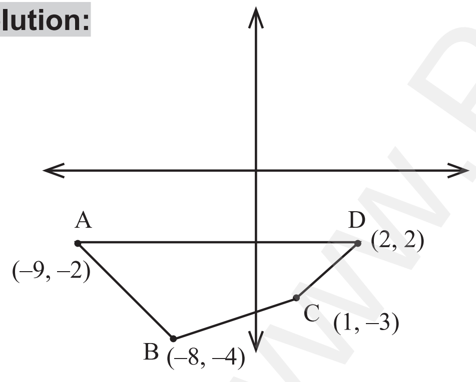
Construction rate per square fee = ₹ 1300

Therefore, total cost for constructing the parking lot = 16 × 1300 = ₹ 20,800

4. Find the area of the quadrilateral whose vertices are at

(i) (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution:



Let A (–9, –2), B (–8, –4), C (1, –3), D (2, 2)

$$\text{Area of the quadrilateral} = \frac{1}{2}\begin{vmatrix} -9 & -2 \\ -8 & -4 \\ 1 & -3 \\ 2 & 2 \\ -9 & -2 \end{vmatrix}$$

$$= \frac{1}{2}[(36+24+2-4) - (16-4-6-18)]$$

$$= \frac{1}{2}[(36+24+2-4-16+4+6+18)]$$

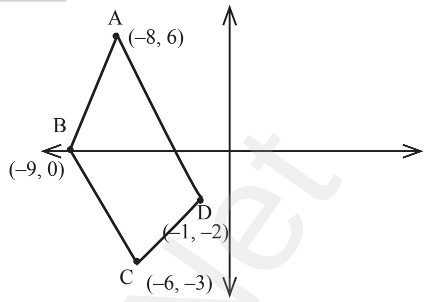
$$= \frac{1}{2}[(36+24+2+4+6+18) - (4+16)]$$

$$= \frac{1}{2}[90-(20)]$$

$$= \frac{1}{2}[70] = 35 \text{ sq. units}$$

(ii) (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution:



A (–8, 6), B (–9, 0), C (–6, –3), D (–1, –2)

$$\text{Area of the quadrilateral} = \frac{1}{2}\begin{vmatrix} -8 & 6 \\ -9 & 0 \\ -6 & -3 \\ -1 & -2 \\ -8 & 6 \end{vmatrix}$$

$$= \frac{1}{2}[(0+27+12-6) - (-54+0+3+16)]$$

$$= \frac{1}{2}[27+12-6+54-3-16]$$

$$= \frac{1}{2}[(27+12+54) - (6+3+16)]$$

$$= \frac{1}{2}[93-25] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

Aliter:

A(–8, 6), B(–9, 0), C(–6, –3), D(–1, –2)

x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8-(-6) & -9-(-1) \\ 6-(-3) & 0-(-2) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8+6 & -9+1 \\ 6+3 & 0+2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -8 \\ 9 & 2 \end{vmatrix}$$

$$= \frac{1}{2}[-4+72] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

5. Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3) **SEP-20**

Solution:

$$\frac{1}{2}\begin{vmatrix} -4 & -2 \\ -3 & k \\ 3 & -2 \\ 2 & 3 \\ -4 & -2 \end{vmatrix} = 28$$

$\Rightarrow (-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12) = 56$

$\Rightarrow (11 - 4k) - (3k - 10) = 56$

$\Rightarrow 11 - 4k - 3k + 10 = 56$

$\Rightarrow 21 - 7k = 56$

$\Rightarrow 7k = -35$

$\Rightarrow k = -5$

6. If the points A (–3, 9), B (a, b) and C (4, –5) are collinear and if a + b = 1, then find a and b.

Solution:

Given A (–3, 9), B(a, b), C(4, –5) are collinear and a + b = 1 ..... (1)

Area of the triangle formed by 3 points = 0

$$\frac{1}{2}\begin{vmatrix} -3 & 9 \\ a & b \\ 4 & -5 \\ -3 & 9 \end{vmatrix} = 0$$

$\Rightarrow (-3b - 5a + 36) - (9a + 4b + 15) = 0$

$\Rightarrow -5a - 3b + 36 - 9a - 4b - 15 = 0$

$\Rightarrow -14a - 7b + 21 = 0$

$\Rightarrow -14a - 7b = -21$

$\Rightarrow 14a + 7b = 21 \ (\div 7)$

$\Rightarrow 2a + b = 3$ ..... (2)

Given $a + b = 1$ ..... (1)

(1) – (2) $\Rightarrow a = 2 \quad b = -1$

7. A triangular shaped glass with vertices at A (–5, –4) , B (1, 6) and C (7, –4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution:

The required number of buckets = $\dfrac{\text{Area of the } \Delta ABC}{\text{Area of the paint covered by one bucket}}$

Area of the $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -5 & -4 \\ 1 & 6 \\ 7 & -4 \\ -5 & -4 \end{vmatrix}$

$= \frac{1}{2}[(-30 - 4 - 28) - (-4 + 42 + 20)]$

$= \frac{1}{2}[-62 - 58]$

$= \frac{1}{2}[-120]$

$= 60$ sq. units.

$\therefore$ The required number of buckets $= \frac{60}{6} = 10$

***

6. Trigonometry

2 Marks

1. Prove that $\dfrac{\sin A}{1+\cos A} = \dfrac{1-\cos A}{\sin A}$

Solution:

$$\frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A} = \frac{1-\cos A}{\sin A}$$

2. Prove that $1+\dfrac{\cot^2\theta}{1+cosec\theta} = cosec\theta$

Solution:

$$1 + \frac{\cot^2\theta}{1+cosec\theta} = 1+\frac{cosec^2\theta-1}{cosec\theta+1}$$

$[\because cosec^2\theta - 1 = \cot^2\theta]$

$$= 1+\frac{(cosec\theta+1)(cosec\theta-1)}{cosec\theta+1}$$

$= 1+(cosec\theta - 1) = cosec\theta$

3. Prove that $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = cosec\theta + \cot\theta$

Solution:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$= \sqrt{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

LHS = cosecθ + cotθ

∴ LHS = RHS

4. Prove the following identities.

(i) cotθ + tanθ = secθ cosecθ

Solution:

LHS = cotθ + tanθ

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

= secθ cosecθ

∴ LHS = RHS.

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution:

LHS = $\tan^4\theta + \tan^2\theta = \tan^2\theta\,(\tan^2\theta + 1)$

= $\tan^2\theta\,(\sec^2\theta)$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

= $(\sec^2\theta - 1)(\sec^2\theta)$ ($\because \tan^2\theta = \sec^2\theta - 1$)

= $\sec^4\theta - \sec^2\theta$

∴ LHS = RHS

5. Prove the following identities.

(i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ = secθ tanθ SEP-20

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

= secθ + tanθ = RHS

Hence Proved.

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = 2secθ

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

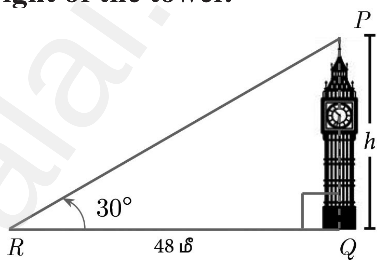
$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

= 2secθ

Hence Proved.

6. A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.



Solution:

In ΔPQR $\tan\theta = \frac{PQ}{QR}$

$$\tan 30° = \frac{h}{48}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

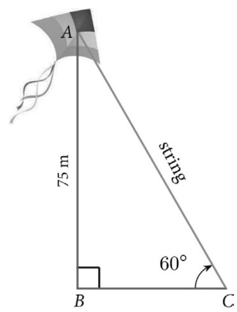
$$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3} = 16\sqrt{3}$$

Therefore the height of the tower is,

$h = 16\sqrt{\ }$ m

7. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution:



In ΔABC $\sin\theta = \frac{AB}{AC}$

$$\sin 60° = \frac{75}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$

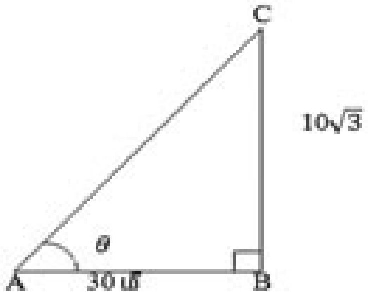
$$AC = \frac{75\times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$$

$$AC = 50\sqrt{3} \text{ m}$$

$\therefore$ Hence, the length of the string is $50\sqrt{3}$ m.

8. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m. **SEP-21**

Solution:



In ΔABC

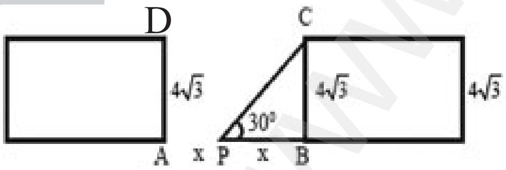
$\tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}} \Rightarrow \tan\theta = \frac{10\sqrt{3}}{30}$

$\tan\theta = \frac{\sqrt{3}}{3} \qquad \Rightarrow \tan\theta = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}} \qquad \theta = 30°$

9. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution:



In the figure, BC – House, AB – Width of Road, P – Median of Road

$AP = PB = x$

In ΔPBC, $\tan 30° = \frac{BC}{PB}$

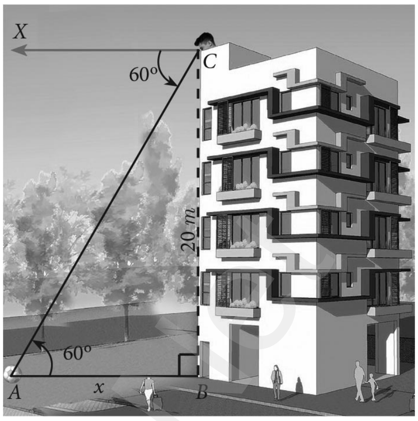
$\Rightarrow \tan 30° = \frac{4\sqrt{3}}{PB} \qquad \Rightarrow \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{PB}$

$PB = 4\sqrt{3} \times \sqrt{3} = 4 \times 3 = 12$

Hence, Width of Road

$= AP + PB = 12 + 12 = 24$ m

10. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3} = 1.732$)



Solution:

Let BC be the height of the tower and A be the position of the ball lying on the ground.

Then, BC = 20 m and

$\angle XCA = 60° = \angle CAB$

Let AB = x metres.

In the right angled triangle ABC,

$$\tan 60° = \frac{20}{AB}$$

$$\sqrt{3} = \frac{20}{AB}$$

$$AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

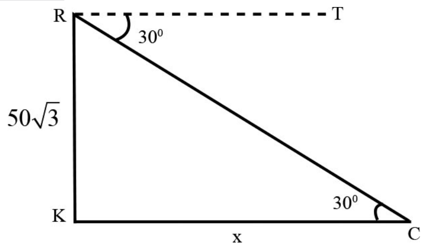
$$AB = \frac{20\sqrt{3}}{3} = \frac{20\times 1.732}{3}$$

$$= \frac{34.640}{3} = 11.54 \text{ m}$$

Hence, the distance between the foot of the tower and the ball is 11.55 m.

11. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. **MAY-22**

Solution:



In ΔABC, $\tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}}$

$$\tan 30° = \frac{50\sqrt{3}}{BC}$$

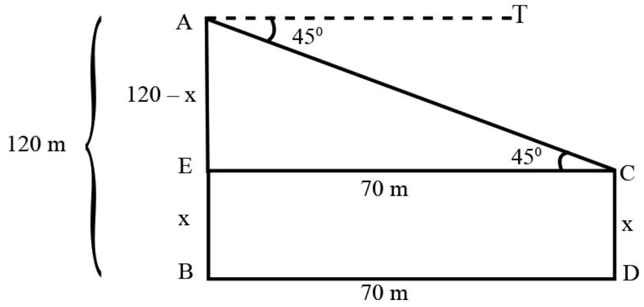
$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$$

$$BC = 50\sqrt{3} \times \sqrt{3}$$

$$= 50\,(3) = 150 \text{ m}$$

12. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building. ($\sqrt{3}$ = 1.732)

Solution:



CD – First Building,

AB – Second Building

From the figure AB = 120 m,

EB = CD = x, AE=120 – x,

EC = BD = 70 m

In ΔACE, $\tan 45° = \frac{AE}{EC}$

$$\Rightarrow \quad 1 = \frac{120 - x}{70}$$

$$\Rightarrow \quad 120 - x = 70 \text{ m}$$

$$\therefore x = 50 \text{ m}$$

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7. Mensuration

2 Marks

1. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution:

l = 5 cm, R = 4 cm, r = 1 cm

C.S.A of the frustum = π (R + r)l sq.units

$$= \frac{22}{7}(4+1) \times 5$$

$$= \frac{22 \times 5 \times 5}{7} = \frac{550}{7}$$

$$= 78.57 \text{ cm}^2$$

2. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Solution:

r : h = 5 : 7 $\Rightarrow$ r = 5x cm , h = 7x cm

CSA = 5500 sq.cm

$$2\pi rh = 5500 \Rightarrow 2 \times \frac{22}{7} \times 5x \times 7x = 5500$$

$$x^2 = \frac{5500}{2 \times 22 \times 5} = 25 \Rightarrow x = 5$$

Hence, Radius = 5 × 5 = 25 cm,

Height = 7 × 5 = 35 cm

3. The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights. **|May 22|**

Solution:

Ratio of the volumes of two cones

$$= \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2$$

$$= h_1 : h_2$$

$$= 3600 : 5040$$

$$= 360 : 504$$

$$= 40 : 56$$

$$= 5 : 7$$

4. If the ratio of radii of two spheres is 4:7, find the ratio of their volumes.

Solution:

The ratio of radii of two spheres = 4 : 7

Let radius of first sphere is 4x,

that is $r_1 = 4x$

Let radius of second sphere is 7x,

that is $r_2 = 7x$

The ratio of their volumes

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{(4x)^3}{(7x)^3} = \frac{4^3 \times x^3}{7^3 \times x^3}$$

$$= \frac{4^3}{7^3} = \frac{64}{343}$$

Hence the ratio of the volumes is 64 : 343

5. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$:4.

Solution:

Given

Total Surface Area of a solid Sphere

= Total surface Area of a solid hemisphere

$\Rightarrow 4\pi R^2 = 3\pi r^2$

$\Rightarrow \therefore \frac{R^2}{r^2} = \frac{3}{4} \quad \Rightarrow \quad \therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$

$\therefore$ Ratio of their volumes

$$= \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left[\frac{\sqrt{3}}{2}\right]^3$$

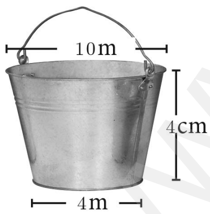
$$\Rightarrow 2 \times \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$$

$\therefore$ Ratio of their volumes = $3\sqrt{3}$: 4

5 Marks

1. An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

Solution:



Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

Given that, diameter of the top =10 m;

radius of the top R = 5 m.

diameter of the bottom = 4 m;

radius of the bottom r = 2 m, height h = 4 m

Now, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{4^2 + (5-2)^2}$

$l = \sqrt{16+9} = \sqrt{25}$ =5 m

C.S.A. = $\pi(R + r)l$ sq. units

$= \frac{22}{7}(5 + 2) \times 5$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 5$

$= 110$ m^2

T.S.A. = $\pi(R + r)l + \pi R^2 + \pi r^2$ sq. units

$= \pi[(R+r)l + R^2+r^2]$

$= \frac{22}{7}[(5+2)5+5^2+2^2]$

$= \frac{22}{7}(35+25+4) = \frac{1408}{7} = 201.14$m^2

Therefore, C.S.A. = 110 m^2 and

T.S.A. = 201.14 m^2

2. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Solution:

From the given figure, r = 6m, R = 12m and h = 8m.

But, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{10} = 10$

l = 10 m

The required total arc of table lamp

= CSA of frustrum + Area of the top

$= \pi(R + r)l + \pi r^2$

$= \frac{22}{7} \times 18 \times 10 + \frac{22}{7} \times 6 \times 6$

$= \frac{22}{7} \times 6[30+6] = \frac{22}{7} \times 6 \times 36$

= 678.86 m^2

Cost of painting for 1 sq.m. is ₹ 2.

$\therefore$ The total cost of painting

= 678.86 × 2 = ₹1357.72.

3. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre. MAY-22

Solution:

h = 16 cm, r = 8 cm, R = 20 cm,

Volume of the frustum

$= \frac{1}{3} \pi h[R^2 + Rr + r^2]$ cu. units

$= \frac{1}{3} \times \frac{22}{7} \times 16\,[20^2 + 20(8) + 8^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 16\,[400 + 160 + 64]$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 624$

$= 10459$ cm^3

= 10.459 litre

The cost of milk is ₹ 40 per litre

The cost of 10.459 litres milk = 10.459 × 40

= ₹ 418.36

4. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum. SEP-21

Solution:

height of the frustum, h = 45 cm,

bottom radii, R = 28 cm,

top radii, r = 7 cm

Volume of the frustum

$= \frac{1}{3} \pi h\,[R^2 + Rr + r^2]$ cu.units

$= \frac{1}{3} \times \frac{22}{7} \times 45\,[28^2 + 28 \times 7 + 7^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 45\,[784 + 196 + 49]$

$= \frac{1}{3} \times \frac{22}{7} \times 45 \times 1029$

$= 22 \times 15 \times 147 = 48510$ cm^3

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8. Statistics and Probability

2 Marks

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:

Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

Coefficient of range $= \frac{L-S}{L+S}$

Coefficient of range $= \frac{67-18}{67+18} = \frac{49}{85} = 0.576$

2. Find the range of the following distribution.

Age (in years)	**16-18**	**18-20**	**20-22**	**22-24**	**24-26**	**26-28**
Number of students	**0**	**4**	**6**	**8**	**2**	**2**

Solution:

Here

Largest value, L = 28

Smallest Value, S = 18

Range R = L – S

R = 28 – 18 = 10 Years.

3. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution:

Range R = 13.67

Largest value L = 70.08

Range R = L − S

13.67 = 70.08 −S

S = 70.08 −13.67 = 56.41

Therefore, the smallest value is 56.41

4. Find the range and coefficient of range of following data SEP-20

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

i. 63, 89, 98, 125, 79, 108, 117, 68

L = 125, S = 63

Range, R = L – S = 125 – 63 = 62

Coefficient of Range $= \frac{L-S}{L+S}$

$= \frac{125-63}{125+63} = \frac{62}{188} = 0.33$

ii. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4, S = 13.6

Range, R = L – S = 61.4 – 13.6 = 47.8

Coefficient of Range $= \dfrac{L-S}{L+S}$

$= \dfrac{47.8}{61.4+13.6} = \dfrac{47.8}{75.0} = 0.64$

5. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution:

Range, R = 36.8

Smallest Value, S = 13.4

Largest Value, L = R + S

= 36.8 + 13.4 = 50.2

6. Calculate the range of the following data.

Income	400-450	450-500	500-550
Number of workers	8	12	30
Income	550-600	600-650	
Number of workers	21	6	

Solution:

Given: Largest Value, L = 650

Smallest Value,, S = 400

∴ Range = L – S = 650 – 400 = 250

7. Find the standard deviation of first 21 natural numbers.

Solution:

Standard Deviation of first 21 natural numbers,

$\sigma = \sqrt{\dfrac{n^2-1}{12}}$

$= \sqrt{\dfrac{(21)^2-1}{12}} = \sqrt{\dfrac{441-1}{12}} = \sqrt{\dfrac{440}{12}}$

$= \sqrt{36.66} = 6.05$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution:

standard deviation of a data, σ = 4.5

each value of the data decreased by 5,

the new standard deviation does not change and it is also 4.5.

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

The new standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.

The new standard deviation $= \dfrac{3.6}{3} = 1.2$

The new variance = (Standard Deviation)^2

$= \sigma^2 = (1.2)^2 = 1.44$

10. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution:

Mean $\bar{x} = 25.6$

Coefficient of variation, C.V. = 18.75

$C.V = \dfrac{\sigma}{\bar{x}} \times 100$

$18.75 = \dfrac{\sigma}{25.6} \times 100$

$\sigma = \dfrac{18.75 \times 25.6}{100} = 4.8$

11. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution:

Co-efficient of variation C.V. $= \dfrac{\sigma}{\bar{x}} \times 100.$

$\sigma = 6.5,\ \bar{x} = 12.5$

$CV = \dfrac{\sigma}{\bar{x}} \times 100 = \dfrac{6.5}{12.5} \times 100$

$= \dfrac{6500}{125} = 52\ \%$

12. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution:

$\bar{x} = 15$, C.V. = 48,

$CV = \dfrac{\sigma}{\bar{x}} \times 100$

$\sigma = \dfrac{C.V \times \bar{x}}{100} = \dfrac{48 \times 15}{100} = \dfrac{720}{100} = 7.2$

13. If n = 5 , x = 6 , x^2 = 765, then calculate the coefficient of variation.

Solution:

$n = 5,\ \bar{x} = 6,\ \Sigma x^2 = 765$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{153 - 36} = \sqrt{117}$$

$$= 10.8$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{10.8}{6} \times 100 = \frac{1080}{6} = 180\%$$

14. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution:

Total number of possible outcomes

n(S) = 5 + 4 = 9

i) Let A be the event of getting a blue ball.

Number of favourable outcomes for the event A. Therefore, n(A) = 5

Probability that the ball drawn is blue.

Therefore, $P(A) = \frac{n(A)}{n(S)} = -$

ii) A will be the event of not getting a blue ball.

So $P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

15. Two coins are tossed together. What is the probability of getting different faces on the coins? **MAY-22**

Solution:

When two coins are tossed together, the sample space is

S = {HH,HT,TH,TT}; n (S) = 4

Let A be the event of getting different faces on the coins.

A = {HT,TH}; n (A) = 2

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

16. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

n(S) = 8

Event A :

Two Consecutive tails = {HTT, TTH, TTT}

n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

17. What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution:

A leap year has 366 days.

So it has 52 full weeks and 2 days.

52 Saturdays must be in 52 full weeks.

S = {(Sun - Mon, Mon - Tue, Tue - Wed, Wed - Thu, Thu - Fri, Fri - Sat, Sat - Sun)}

n(S) = 7

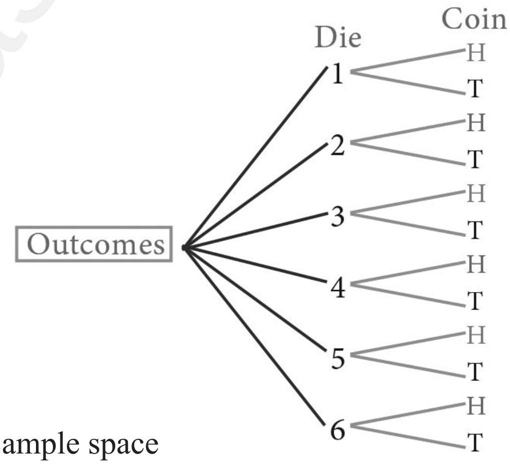
Let A be the event of getting 53rd Saturday.

Then A = {Fri - Sat, Sat - Sun} n(A) = 2

Probability of getting 53 Saturdays in a leap year is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

18. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head. **SEP-21**

Solution:



Sample space

S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T};

n(S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

19. If P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09 then find P(A∪B) .

Solution:

$P(A) = 0.37 , P(B) = 0.42 , P(A \cap B) = 0.09$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.37 + 0.42 - 0.09 = 0.7$

20. What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution:

Total number of cards = 52

A = Number of King cards

n(A) = 4 P(A) = $\frac{4}{52}$

B = Number of Queen cards = 4

n(B) = 4 P(B) = $\frac{4}{52}$

Both the events of drawing a king and a queen are mutually exclusive $P(A \cup B) = P(A) + P(B)$

$\therefore$ Probability of drawing either a king or a queen = $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

21. If P(A) = 23, P(B) =25, P(A∪B) = 13 then find P(A∩B).

Solution:

$P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$
$$= \frac{10+6-5}{15}$$
$$P(A \cap B) = \frac{11}{15}$$

22. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\overline{A})+P(\overline{B})$.

Solution:

Given $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$
$$= 0.6 + 0.2$$
$$= 0.8$$
$$\therefore P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$
$$= 2 - [P(A) + P(B)]$$
$$= 2 - 0.8$$
$$= 1.2$$

5 Marks

1. Find the mean and variance of the first *n* natural numbers.

Solution:

Mean $\overline{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+...+n}{n} = \frac{\cancel{n}(n+1)}{2 \times \cancel{n}}$$
$$\overline{x} = \frac{n+1}{2}$$

Variance σ^2

$$= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \left|\begin{array}{l} \Sigma x_i^2 = 1^2 + 2^2 + 3^2 + ... + n^2 \\ (\Sigma x_i)^2 = (1+2+3+...+n)^2 \end{array}\right|$$
$$= \frac{n(n+1)(2n+1)}{6 \times n} - \left|\frac{n(n+1)}{2 \times n}\right|^2$$
$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$
$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

Variance $\sigma^2 = \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$

2. Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13 SEP-21

Solution:

When we roll two dice, the sample space is given by

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) };

n (S) = 36

i) Let A be the event of getting the sum of outcome values equal to 4.

Then A = {(1, 3), (2, 2), (3, 1)}; n(A)= 3.

Probability of getting the sum of outcomes equal to 4 is

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5, 6), (6, 5), (6, 6)}; n(B) = 3

Probability of getting the sum of outcomes greater than 10 is

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

iii) Let C be the event of getting the sum of outcomes less than 13. Here all the outcomes have the sum value less than 13. Hence C = S .

Therefore, n(C) = n(S) = 36

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

3. From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution:

Suits of playing cards	Spade ♠	Heart ♥	Clavor ♣	Diamond ♦
Cards of each suit	A	A	A	A
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9
	10	10	10	10
	J	J	J	J
	Q	Q	Q	Q
	K	K	K	K
Set of playing cards in each suit	13	13	13	13

$n(S) = 52$

i) Let A be the event of getting a red card.

$n(A) = 26$

Probability of getting a red card is

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

ii) Let B be the event of getting a heart card.

$n(B) = 13$

Probability of getting a heart card is

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king. $n(C) = 2$

Probability of getting a red king card is

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K).

$n(D) = 4\times3 = 12$

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} \quad \frac{3}{13}$$

v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

$n(E) = 4\times9 = 36$

Probability of getting a number card is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

4. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

$n(S) = 36$

When two dice are rolled together, there will be $6\times6 = 36$ outcomes.

Let S be the sample space. Then $n(S) = 36$

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then $A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$B = \{(1,3), (2,2), (3,1)\}$

$\therefore A \cap B = \{(2,2)\}$

Then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A\cup B)$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = -$$

Hence, the required probability is $\frac{2}{9}$.

5. If A is an event of a random experiment such that $P(A) : P(\overline{A})=17:15$ and $n(S) = 640$ then find (i) $P(\overline{A})$ (ii) $n(A)$.

Solution:

Given $n(S) = 640$

$$\frac{P(A)}{P(\overline{A})} = \frac{17}{15}$$

$$\frac{1-P(\overline{A})}{P(\overline{A})} = \frac{17}{15}$$

$$15[1-P(\overline{A})] = 17P(\overline{A})$$
$$15-15P(\overline{A}) = 17P(\overline{A})$$
$$15 = 15P(\overline{A}) + 17P(\overline{A})$$
$$32P(\overline{A}) = 15$$
$$P(\overline{A}) = \frac{15}{32}$$
$$P(A) = 1-P(\overline{A})$$
$$= 1-\frac{15}{32}$$
$$= \frac{32-15}{32} = \frac{17}{32}$$
$$P(A) = \frac{n(A)}{n(S)}$$
$$\frac{17}{32} = \frac{n(A)}{640}$$
$$n(A) = \frac{17 \times \cancel{640}^{20}}{\cancel{32}}$$
$$n(A) = 340$$

6. Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1 SEP-20

Solution:

$n(S) = 36$

i) A = Probability of getting Doublets (Equal numbers on both dice)

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

$n(A) = 6; P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

B = Probability of getting the product of the prime number

ii) B = {(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)}

$n(B) = 6; P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

C = Probability of getting sum of the prime number.

iii) C ={(1,1), (2,1), (1,2), (1,4), (4,1), (1,6), (6,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5)}

$n(C) = 14; P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

iv) D = Probability of getting the sum as 1

$n(D) = 0; P(D) = \frac{n(D)}{n(S)} = 0$

7. Three fair coins are tossed together. Find the probability of getting
(i) all heads
(ii) atleast one tail
(iii) atmost one head
(iv) atmost two tails

Solution:

Possible Outcomes = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No.of possible outcomes,

$n(S) = 2\times2\times2 = 8$

i) A = Probability of getting all heads

A = {HHH} n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

ii) B = Probability of getting atleast one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(B) = 7 \quad P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

iii) C = Probability of getting atmost one head.

C = {TTT, TTH, THT, HTT}

$n(C) = 4 \quad P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

iv) D = Probability of getting atmost two tails.

D = {TTH, THT, HTT, HHT, HTH, THH, HHH}

$n(D) = 7 \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

8. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) white
(ii) black or red
(iii) not white
(iv) neither white nor black

Solution:

S = {5 Red, 6 White, 7 Green, 8 Black}

$n(S) = 26$

i) A – probability of getting white balls

$n(A) = 6; \quad P(A) = \frac{6}{26} = \frac{3}{13}$

ii) B – Probability of getting black (or) red balls

$n(B) = 8 + 5 = 13; P(B) = \frac{13}{26} = \frac{1}{2}$

iii) C – Probability of not getting white balls

$n(C) = 20;\ P(C) = \frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting of neither white nor black

$n(D) = 12;\ P(D) = \frac{12}{26} = \frac{6}{13}$

9. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Solution:

In a box there are 20 non – defective and x defective bulbs

$n(S) = x + 20$

Let A – probability of getting Defective Bulbs

$n(A) = x$

$P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+20}$

From Given data

$$\frac{x}{x+20} = \frac{3}{8}$$
$$8x = 3x + 60$$
$$5x = 60$$
$$x = 12$$

$\therefore$ Number of defective bulbls = 12

10. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Solution:

Removed cards:

The King and Queen of diamonds,

Queen and Jack of hearts, and King of spades

(i.e) remaining number of cards

$= 52 - 6 = 46$

$n(S) = 46$

i) A is probability of getting Clavor Cards

$n(A) = 13 \quad P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$

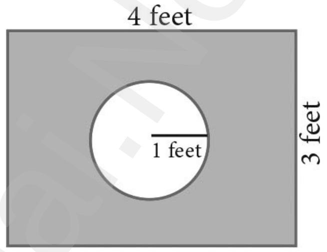
ii) B is probability of getting a queen of red card.

$n(B) = 0 \quad P(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$

iii) C is probability of getting King of black card.

$n(C) = 1 \quad P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$

11. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? ($\pi = 3.14$)



Solution:

Total Region = $4 \times 3 = 12$ sq.ft

$\therefore n(S) = 12$

Winning Region = Area of circle

$= \pi r^2 = \pi(1)^2$

$= \pi = 3.14$ sq. unit

$n(A) = 3.14$

$P(\text{Winning the Game}) = \frac{n(A)}{n(S)}$

$= \frac{3.14}{12} = \frac{314}{1200}$

$= \frac{157}{600}$

12. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution:

$\sigma = 1.2,\ CV = 25.6,\ c = ?$

$CV = \frac{\sigma}{\bar{x}} \times 100$

$\bar{x} = \frac{\sigma}{C.V} \times 100 = \frac{1.2}{25.6} \times 100 = \frac{1200}{256}$

$\bar{x} = 4.7$

13. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

(i) the same day
(ii) different days
(iii) consecutive days?

Solution:

n(S) = 36

i) A be the Probability of Priya and Amuthan to visit shop on same day

A = {(Mon, Mon), (Tue, Tue), (Wed, Wed), (Thurs, Thurs), (Fri, Fri), (Sat, Sat)}

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

ii) P (Priya and Amuthan Visit on Different Days)

$$= P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

C be the Probability of Priya and Amuthan to visit on Consequent days

iii) C = {(Mon, Tue), (Tue, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat) (Tue, Mon), (Wed, Tue), (Thurs, Wed), (Fri, Thurs), (Sat, Fri,)}

n(C) = 10

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

14. In a game, the entry fee is ₹ 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

i) For Receiving double entry Fees have to get Three Heads

A = Probability of Getting three Heads

A = {HHH}

n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) For getting Entry Fess getting atleast one Head

B = Probability of Getting One or Two Heads

B = {TTH, THT, HTT, HHT, HTH, THH}

n(B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

iii) To loss the entry fees, she have to get no Heads

C = Probability of Getting No Heads

C = {TTT}

n(C) = 1

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

15. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

n(S) = 36

When two dice are rolled together, there will be 6 × 6 = 36 outcomes.

Let S be the sample space.

Then n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

B = {(1,3),(2,2),(3,1)}

$\therefore A \cap B$ = {(2,2)}

Then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$.

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

16. If A and B are two events such that $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{2}$ and P(A and B) = –, find

(i) P (A or B)

(ii) P(not A and not B).

Solution:

i. $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$

ii. $P(\text{not } A \text{ and not } B) = P(\bar{A} \cap \bar{B})$

$= P\left(\overline{A \cup B}\right)$

$= 1 - P(A \cup B)$

$P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{3}{8}$

17. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution:

Total number of cards = 52 ; n(S) = 52.

Let A be the event of getting a king card.

$n(A) = 4;\quad P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

Let B be the event of getting a heart card

$n(B) = 13;\quad P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$

Let C be the event of getting a red card

$n(C) = 26;\quad P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$

$P(A \cap B) = P(\text{getting heart king}) = \frac{1}{52}$

$P(B \cap C) = P(\text{getting red and heart}) = \frac{13}{52}$

$P(A \cap C) = P(\text{getting red king}) = \frac{2}{52}$

$P(A \cap B \cap C) = P(\text{getting heart, king which is red}) = \frac{1}{52}$

Therefore, required probability is

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

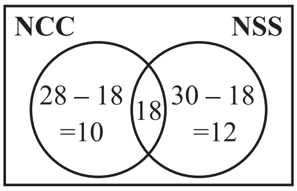
$= \frac{28}{52} = \frac{7}{13}$

18. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them. **MAY-22**



Solution:

Total number of students n(S) = 50

i. A : A : opted only NCC but not NSS

$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50} = -$

ii. B : opted only NSS but not NCC

$P(B) = \frac{n(B)}{n(S)} = \frac{12}{50} = \frac{6}{25}$

iii. C : opted only one

$P(C) = \frac{n(C)}{n(S)} = \frac{(10+12)}{50} = \frac{22}{50} = \frac{11}{25}$

19. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) =36

A = Probability of getting an even number in the first die.

A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6))}

$n(A) = 18;\quad P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

B = Probability of getting a total face sum is 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

$n(B) = 5;\quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B$ = {(2,6), (4, 4), (6, 2)}

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36} = \frac{5}{9}$

20. A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution:

S = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Let A = Multiple of 7

A= {7, 21, 35}, n(A) = 3

$P(A) = \frac{3}{18}$

Let B = a Prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

$n(B) = 11; P(B) = \frac{11}{18}$

$A \cap B = \{7\} \Rightarrow n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{18}$

$P(\text{Either A or B}) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

21. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Probability of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

$n(A) = 7 P(A) = \frac{7}{8}$

B = Probability of getting atmost 2 heads

B = {HHT, HTH, THH, HHH}

$n(B) = 4 P(B) = \frac{4}{8}$

$P(A \cap B) = \frac{4}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \frac{4}{8} - - = \frac{7}{8}$

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GOVT QUESTION PAPER - SEPTEMBER 2020

CLASS: X

MATHEMATICS

Time allowed: 3.00 Hours Maximum Marks: 100

Instructions : 1. Check the question paper for fairness of printing. If there is any lack of fairness inform the hall supervisor immediately.

2. Use Blue or Black ink to write and underline and pencil to draw diagrams.

Note: This question paper contains **four** parts.

PART - I

Note : (i) Answer all the questions. **14×1=14**

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. **If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}, then state which of the following statement is true?**
   a) $(A\times C) \subset (B\times D)$ b) $(B\times D) \subset (A\times C)$ c) $(A\times B) \subset (A\times D)$ d) $(D\times A) \subset (B\times A)$

2. **Let $f(x) = x^2 - x$, then $f(x-1) - f(x+1)$ is**
   a) $4x$ b) $2-2x$ c) $2-4x$ d) $4x-2$

3. **Using Euclid's division lemma, if the cube of any positive integer is divided by 9, then the possible remainders are**
   a) 0, 1, 8 b) 1, 4, 8 c) 0, 1, 3 d) 1, 3, 5

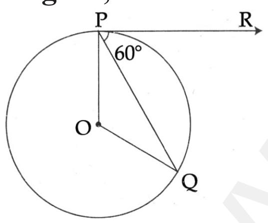
4. **If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \ldots\ldots. + 2^0$, which of the following is true?**
   a) B is 2^{64} more than A b) A and B are equal c) B is larger than A by 1 d) A is larger than B by 1

5. $\dfrac{a^2}{a^2-b^2} + \dfrac{b^2}{b^2-a^2} =$
   a) a – b b) a + b c) $a^2 - b^2$ d) 1

6. **Transpose of a column matrix is**
   a) unit matrix b) diagonal matrix c) column matrix d) row matrix

7. **In ΔLMN, $\angle L=60°$, $\angle M = 50°$. If $\Delta LMN \sim \Delta PQR$, then the value of $\angle R$ is**
   a) 40° b) 70° c) 30° d) 110°

8. **In the figure, if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is**



   a) 120°
   b) 100°
   c) 110°
   d) 90°

9. **The straight line given by the equation $x = 11$ is**
   a) Parallel to x-axis b) Parallel to y-axis
   c) Passing through the origin d) Passing through the point (0, 11)

10. **If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is**
    a) 0 b) 1 c) 2 d) 4

11. **A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere, then the radius of sphere is**
    a) 24 cm b) 12 cm c) 6 cm d) 48 cm

12. **A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is**
    a) 2 : 1 b) 1 : 2 c) 4 : 1 d) 1 : 4

13. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

a) 40000 b) 160900 c) 160000 d) 30000

14. If a letter is chosen at random from the English alphabets (a, b, c, ..., z), then the probability that the letters chosen precedes *x*, is

a) $\frac{12}{13}$ b) $\frac{1}{13}$ c) $\frac{23}{26}$ d) $\frac{3}{26}$

PART - II

Answer any 10 questions. Question No. 28 is compulsory. **10×2=20**

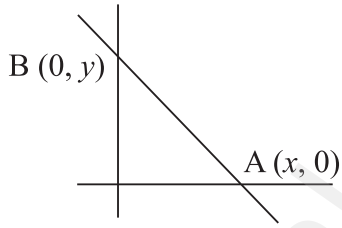
15. If A × B = {(3, 2) (3, 4) (5, 2) (5, 4)}, then find A and B.
16. Show that the function $f : N \to N$ defined by $f(m) = m^2 + m + 3$ is one-one function.
17. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ end in 5?
18. Find the 3rd and 4th terms of a sequence, if $a_n = \begin{cases} n^2 & if \quad n\, is\, odd \\ \frac{n^2}{2} & if \quad n\, is\, even \end{cases}$
19. Find the value of $1^2 + 2^2 + 3^2 + \ldots\ldots + 10^2$ and hence deduce $2^2 + 4^2 + 6^2 + \ldots.. + 20^2$.
20. Find the value of k for which the equation $9x^2 + 3kx + 4 = 0$ has real and equal roots.
21. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A.
22. Check whether AD is bisector of $\angle A$ of ΔABC in the following: AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm.
23. Find the slope of a line joining the points (14, 10) and (14, –6).
24. Prove $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$
25. Find the diameter of a sphere whose surface area is 154 m^2.
26. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area.
27. Find the range and coefficient of range of the data. 63, 89, 98, 125, 79, 108, 117, 68.
28. Find the volume of the iron used to make a hollow cylinder of height 9cm and whose internal and external radii are 3 cm and 5 cm respectively.

PART - III

Answer any 10 questions. Question No. 42 is compulsory. **10×5=50**

29. Let A = The set of all natural numbers less than 8
    B = The set of all prime numbers less than 8
    C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
30. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f : A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function i) by Arrow diagram ii) in a table form iii) as a set of ordered pairs iv) in a graphical form
31. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11.
32. Solve: $6x + 2y - 5z = 13$; $3x + 3y - 2z = 13$; $7x + 5y - 3z = 26$
33. Find the GCD of the polynomials, $x^4 + 3x^3 - x - 3$ and $x^3 + x^2 - 5x + 3$.
34. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$
35. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^TA^T$.

36. State and prove Angle Bisector theorem.
37. Find the value of k, if the area of a quadrilateral is 29 sq. units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3).
38. From the top of a tower 60m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. (tan 38° = 0.7813, $\sqrt{3}$ = 1.732)
39. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small non-hollow cylindrical metal of radius 5 cm and height 4 cm is immersed in it completely. Calculate the rise of water in the glass.
40. The scores of a cricketer in 7 matches are 70, 80, 60, 50, 40, 90, 95. Find the standard deviation.
41. Two unbiased dice are rolled once. Find the probability of getting:
    i) a doublet (equal numbers on both dice)
    ii) the product as a prime number
    iii) the sum as a prime number
    iv) the sum as 1
42. A straight line AB cuts the co-ordinate axes at A and B. If the mid-point of AB is (2, 3), find the equation of AB.



PART - IV

Answer all the questions. **2×8=16**

43. a) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC. $\left(\text{scale factor } \frac{6}{5}\right)$

OR

b) Draw two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also measure the lengths of the tangents.

44. a) Graph the quadratic equation $x^2 - 8x + 16 = 0$ and state the nature of their solution.

OR

b) A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount.

Hence find

(i) the marked price when a customer gets a discount of ₹ 3250 (from graph)

(ii) the discount when the marked price is ₹ 2500

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