DEPARTMENT OF EDUCATION

VIRUDHUNAGAR DISTRICT.

LATE BLOOMERS - GUIDE

10

MATHS

2022 - 2023

Chief

Tmt. A Gnanagowri, M.Sc., B.Ed., M.Phil., PGDCA.,
Chief Educational Officer, Virudhunagar.

Administrators

Tmt. T Mahalakshmi, M.Sc., B.Ed., M.Phil.,
District Educational Officer
(Secondary Education)

Mr. C Raja, M.Sc., M.Ed., M.Phil.,
District Educational Officer [Incharge]
(Secondary Education)

Tmt. A Pandichelvi, M.Sc., M.Ed.,
District Educational Officer
(Private Schools)

Integrator

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Hedmaster
Govt. High School, Udaiyanampatti.

Faculty Panel

Mr. A Subbaraj, M.Sc., B.Ed., M.Phil.,
Graduate Teacher(Maths)
Govt. High School,
Irunchiri.

Mr. P Senthilkumar, B.Sc., B.Ed.,
Graduate Teacher(Maths)
Govt. Hr. Sec. School,
Panaiyur.

Mr. C Amalraj, M.Sc., B.Ed., M.Phil.,
Graduate Teacher(Maths)
Govt. High School,
T Managaseri.

Mr. S Balaguru, M.Sc., B.Ed., PGDCA.,
Graduate Teacher(Maths)
Govt. Hr. Sec. School,
Viswanatham.

Mr. S Manohar, M.Sc., B.Ed., M.Phil., PGDCA.,
Graduate Teacher(Maths)
Govt. Model Hr. Sec. School,
Thiyagarajapuram.

CONTENTS

The only way to learn mathematics is to do mathematics

NUMBER OF QUESTIONS WHICH APPERED IN THE GOVERNMENT EXAMINATION

CHAPTER	TITLE	2 MARKS				5 MARKS			
		Sep-20	Sep-21	May-22	Aug-22	Sep-20	Sep-21	May-22	Aug-22
1	Relations and Functions	2	2	2	2	2	1	1	1
2	Numbers and Sequences	3	2	2	2	1	2	2	2
3	Algebra	2	2	2	2	4	3	3	3
4	Geometry	1	1	1	1	1	2	2	2
5	Coordinate Geometry	1	3	3	3	2	2	2	2
6	Trigonometry	1	1	1	1	1	1	1	1
7	Mensuration	3	2	2	2	1	2	2	2
8	Statistics and Probability	1	1	1	1	2	1	1	1

TWO MARK QUESTIONS

1) $A = \{1,3,5\}$ $B = \{2,3\}$then find (i) $A \times B$and $B \times A$.(ii) Is $\times B = B \times A$? If not why? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

1) [S-21]

Solution:-

(i) $A \times B = \{(1,2),(1,3),(3,2),(3,3),(5,2),(5,3)\}$

$B \times A = \{(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)\}$

(ii) $A \times B \neq B \times A \therefore (1,2) \neq (2,1)$

(iii) $n(A \times B) = 6, n(B \times A) = 6$

$n(A) = 3, n(B) = 2$

$n(A) \times n(B) = 3 \times 2 = 6$

$\therefore n(A \times B) = n(B \times A) = n(A) \times n(B)$

2) Let $A = \{1,2,3\}$ and $B = \{x|x$is a prime number less than 10$\}$. Find $A \times B$and $B \times A$.[M-22]

Solution:-

$A = \{1,2,3\}, B = \{2,3,5,7\}$

$A \times B = \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),$
$(3,2),(3,3),(3,5),(3,7)\}$

$B \times A = \{(2,1),(3,1),(5,1),(7,1),(2,2),(3,2),(5,2),(7,2),$
$(2,3),(3,3),(5,3),(7,3)\}$

3) $A = \{m,n\}$ and $B = \emptyset$ then find (i)A X B and (ii) A X A. காண்க. [PTA-1]

Solution:-

(i) $A \text{ X } B = \{m,n\} \text{X } \emptyset = \emptyset$

(ii) $A \text{ X } A = \{m,n\}\text{X}\{m,n\}$

$= \{(m,m),(m,n),(n,m),(n,n)\}$

4) $If\ A\ X\ B = \{(3,2),(3,4),(5,2),(5,4)\}$ then find A and B - S-20 , A-22]

Soluation:-

$A = \{3,5\}$

$B = \{2,4\}$

5) $If\ B\ X\ A = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$ then find A and B

Solution:-

$A = \{3,4\}$

$B = \{-2,0,3\}$

6) $If\ A = \{5,6\},\ B = \{4,5,6\},\ C = \{5,6,7\}$ then verify that $A\ X\ A = \{(B\ X\ B) \cap (C\ X\ C)$ [A-22]

Soluation

$A = \{5,6\}, B = \{4,5,6\}$, C= $\{5,6,7\}$

LHS:-

$\boldsymbol{A \times A = \{5,6\} \times \{5,6\}}$

$= \{(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{6}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6})\} \rightarrow (1)$

RHS:-

$B \times B = \{4,5,6\} \times \{4,5,6\}$

$= \{(4,4),(4,5),(4,6),(5,4),(5,5),(5,6),(6,4),(6,5),(6,6)\}$

$C \times C = \{5,6,7\} \times \{5,6,7\}$

$= \{(5,5),(5,6),(5,7),(6,5),(6,6,),(6,7),(7,5),(7,6),(7,7)\}$

$(\boldsymbol{B \times B}) \cap (\boldsymbol{C \times C}) = \{(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{6}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6})\} \rightarrow (2)$

$\therefore$ From (1) and (2) $\boldsymbol{A \times A = (B \times B) \cap (C \times C)}$

7) A relation R is given by the set $\{(x,y)|y = x + 3, x \in \{0,1,2,3,4.5\}\}$. Determine its domain and range.[PTA-5]

Soluation:-

$$y = 0 + 3 = 3 \quad y = 3 + 3 = 6$$
$$y = 1 + 3 = 4 \quad y = 4 + 3 = 7$$
$$y = 2 + 3 = 5 \quad y = 5 + 3 = 8$$

$\therefore R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain= $\{0,1,2,3,4,5\}$

Range $= \{3,4,5,6,7,8\}$

8) A relation R is given by the set $\{(x,y)/\, y = x^2 + 3, x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range.

Soluation-

$$y = f(x) = x^2 + 3$$

$$f(0) = (0)^2 + 3 = 0 + 3 = 3$$
$$f(1) = (1)^2 + 3 = 1 + 3 = 4$$
$$f(2) = (2)^2 + 3 = 4 + 3 = 7$$
$$f(3) = (3)^2 + 3 = 9 + 3 = 12$$
$$f(4) = (4)^2 + 3 = 16 + 3 = 19$$
$$f(5) = (5)^2 + 3 = 25 + 3 = 28$$

Domain $= \{0,1,2,3,4,5\}$

Range $= \{3,4,7,12,19,28\}$

9) Let $A=\{1,2,3,4,...,45\}$ and R be the relation defined as "is square of a number" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R. **[S-21]**

Spolution:-

$$A = \{1,2,3,4 \dots ,45\}$$
$$A \times A = \{1,2,3 \dots ,45\} \times \{1,2,3 \dots ,45\}$$
$$= \{(1,1), (1,2) \dots (2,1) \dots (3,1) \dots \dots (45,45)\}$$

R be the relation defined as "is square of a number" on A.

$\therefore R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$

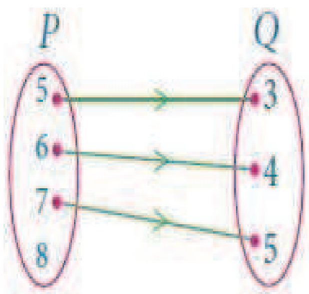
$R \subseteq A \times A$

Domain $= \{1,2,3,4,5,6\}$

Range $= \{1,4,9,16,25,36\}$

10) [M-22]

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form



Solution:-

(i) Set builder form $R = \{(x,y)/\, y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5,3), (6,4), (7,5)\}$

11) Let $X = \{3, 4, 6, 8\}$. Determine whether the relation. $R = \{(x, f(x))/\, x \in X, f(x) = x^2 + 1\}$ is a function from X to N?

Solution:-

$f(3) = 3^2 + 1 = 9 + 1 = 10$

$f(4) = 4^2 + 1 = 16 + 1 = 17$

$f(6) = 6^2 + 1 = 36 + 1 = 37$

$f(8) = 8^2 + 1 = 64 + 1 = 65$

$\therefore R = \{(3,10), (4,17), (6,37), (8,65)\}$

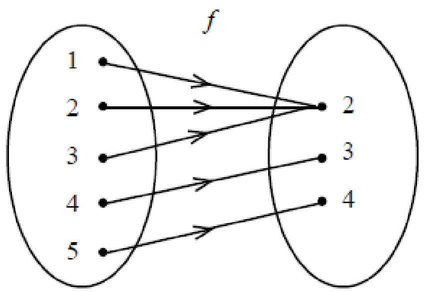
$R: X \to N$ is a function $from\ X\ to\ N$

12) Represent the function $f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$ through (i) an arrow diagram (ii) a table form (iii) a graph.

Solution:-

$$f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$$

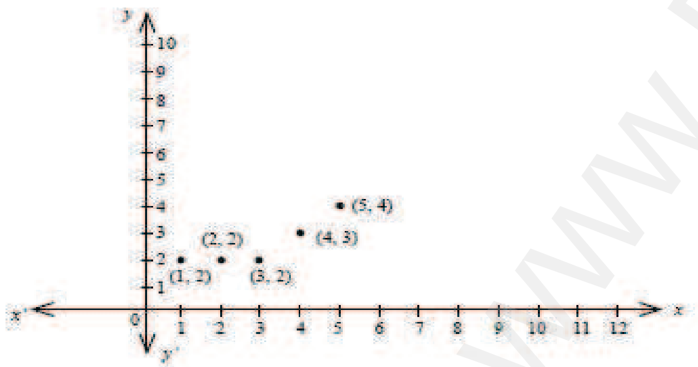
(i) An arrow diagaram



(ii) A table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) A graph



13) *If $R = \{(x,-2),(-5,y)\}$ is a identity function then find the value of x and y.* **[PTA-6]**

Solution:-

$R = \{(x,-2),(-5,y)\}$ *is a identity function*

$x = -2$ மற்றும் $y = -5$

14) Show that the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one-one function. **[S-20]**

solution:-

$$f(x) = m^2 + m + 3$$

domain , $\mathbb{N} = \{1, 2, 3, ,4, \ldots\}$

Codomain , $\mathbb{N} = \{1, 2, 3, ,4, \ldots\}$

$f(1) = (1)^2 + 1 + 3 = 1 + 1 + 3 = 5$

$f(2) = (2)^2 + 2 + 3 = 4 + 2 + 3 = 9$

$f(3) = (3)^2 + 3 + 3 = 9 + 3 + 3 = 15$

...

...

...

distinct elements of *DOMAIN* have distinct images in *CODOMAIN*.

$\therefore$ f is one-one function

15) Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$- as a composition of two functions.

Solution:-

$h(x) = 2x^2 - 5x + 3$ and $g(x) = \sqrt{x}$

Now $f(x) = \sqrt{2x^2 - 5x + 3}$

$= \sqrt{h(x)}$

$= g[h(x)]$

$= g\ o\ h(x)$

16) If $f(x) = 2x + 1, g(x) = x^2 - 2$ then find fog and gof.

Solution:-

$$f\ o\ g = (2x+1)\ o\ (x^2-2)$$
$$= 2(x^2-2)+1$$
$$= 2x^2-4+1$$
$$= 2x^2-3$$
$$g\ o\ f = (x^2-2)\ o\ (2x+1)$$
$$= (2x+1)^2-2$$
$$= 4x^2+4x+1-2$$
$$= 4x^2+4x-1$$

17) $If\ f(x) = 3 + x\ , g(x) = x - 4$ then verify that $fog = gof$
PTA-1]

Solution:-

$$fog = (3+x)\ o\ (x-4)$$
$$= 3+(x-4)$$
$$= 3+x-4$$
$$= x-1 \longrightarrow (1)$$

$$gof = (x-4)\ o\ (3+x)$$
$$= (3+x)-4$$
$$= 3+x-4$$
$$= x-1 \longrightarrow (2)$$

$\therefore from\ (1) and\ (2)\ fog = gof$

18) $If\ a^b x b^a = 800$ then find a and b.

Solution:-

$$a^b\text{ x }b^a = 800$$
$$= 2\text{ x }2\text{ x }2\text{ x }2\text{ x }2\text{ x }5\text{ x }5$$
$$a^b\text{ x }b^a = 2^5\text{ x }5^2$$

$a = 2, b = 5$ (or) $a = 5, b = 2$

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

19) $If\ 13824 = 2^a\text{ x }3^b$ then find a and b.
[M-22]

Solution:-

$$2^a\text{ x }3^b = 13824$$
$$\Longrightarrow 2^a\text{ x }3^b = 2^9\text{ x }3^3$$
$$\therefore\ a = 9 \qquad b = 3$$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

20) Find the least number that is divisible by the first ten natural numbers. [A-22]

Solution:-

lcm

2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	1, 1, 3, 2, 5, 3, 7, 4, 9, 5
2	1, 1, 3, 1, 5, 3, 7, 2, 9, 5
3	1, 1, 3, 1, 5, 3, 7, 1, 9, 5
3	1, 1, 1, 1, 5, 1, 7, 1, 3, 5
5	1, 1, 1, 1, 5, 1, 7, 1, 1, 5
7	1, 1, 1, 1, 1, 1, 7, 1, 1, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1, 1

$= 2^3 x 3^2 x 5 x 7$

$= 8 x 9 x 35$

$= 72 x 35$

$= 2520$

∴ least number that is divisible by the first ten natural numbers = 2520

21) What is the time 100 hours after 7 a.m.?

Solution:-

	4
24	107
	96
	11

$$7 + 100 = 107$$

$$107 \; mod \; 24 \equiv 11$$

∴ 100 hours after 7 a.m is 11 a.m.

22) What is the time 15 hours before 11p.m.??

Solution:-

11 p.m = 23 hours

$$23 - 15 = 8$$

8 mod $24 \equiv 8$

∴15 hours before 11 p.m. is 8 a.m

23) Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Solution:-

If Tuesday is 2nd of the week

	6
7	47
	42
	5

$\therefore 2 + 45 = 47$

$\rightarrow 47$ mod $7 \equiv 5$

∴ 5th day of the week is Friday.

24) Find the number of terms in the A.P 3,6,9,12,. . . . ,111.

Solution:-

$a = 3$ and $d = 6 - 3 = 3$

$$n = \frac{l - a}{d} + 1$$

$$= \frac{111 - 3}{3} + 1$$

$$= \frac{108}{3} + 1$$

$$= 36 + 1$$

$$n = 37$$

25) Find the 19th term of an A.P $-11, -15, -19, \ldots$.

Solution:-

$a = -11$ and$d = -15 - (-11) = -4$

$$t_n = a + (n - 1)d$$

$$t_{19} = -11 + (19 - 1)(-4)$$

$$= -11 + 18 \times (-4)$$

$$= -11 + (-72)$$
$$t_{19} = -83$$

26) Which term of an A.P $16, 11, 6, 1, \ldots$ is -54?

Solution:-

$a = 16$ and $d = 11 - 16 = -5$ then $t_n = -54$

$$t_n = a + (n-1)d$$
$$-54 = 16 + (n-1) \times -5$$
$$-54 - 16 = (n-1) \times -5$$
$$-70 = (n-1) \times -5$$
$$\frac{-70}{-5} = n - 1$$
$$14 = n - 1$$
$$14 + 1 = n$$
$$\therefore n = 15$$

27) Find x, y and z given that the numbers $x, 10, y, 24, z$ are in A.P.

Solution:-

In given sequence $10, y, 24$ are in A.P

$$\therefore 2y = 10 + 24$$
$$2y = 34$$
$$y = \frac{34}{2}$$
$$y = 17$$
$$\therefore d = 17 - 10 = 7$$
$$\therefore x = 10 - 7 = 3 \text{ and } z = 24 + 7 = 31$$
$$\therefore x = 3, y = 17, z = 31$$

28) In a G.P. 729, 243, 81,… find t_7.

Solution:-

$$a = 729$$
$$r = \frac{t_2}{t_1} = \frac{243}{729} = \frac{81}{243} = \frac{27}{81} = \frac{9}{27} = \frac{1}{3}$$
$$n = 7$$
$$t_n = ar^{n-1}$$
$$\therefore t_7 = ar^6$$
$$= (729)\left(\frac{1}{3}\right)^6$$
$$= 729 \text{ x} \frac{1^6}{3^6}$$
$$= \frac{729}{3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3}$$
$$t_7 = 1$$

29) Find the sum to an infifinity of $3 + 1 + \frac{1}{3} + \cdots \infty$

Solution:-

$$a = 3, \quad r = \frac{1}{3}$$
$$S_\infty = \frac{a}{1-r}$$

$\therefore\ S_\infty = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{3-1}{3}} = \frac{3}{\frac{2}{3}} = 3 \text{ x} \frac{3}{2} = \frac{9}{2}$

30) Find the sum to an infinifity of

(i) $9+3+1+\cdots$

Solution:-

(i) $9+3+1+\cdots$

$$a = 9,\ \ r = \frac{3}{9} = \frac{1}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$\therefore\ S_\infty = \frac{9}{1-\frac{1}{3}} = \frac{9}{\frac{3-1}{3}} = \frac{9}{\frac{2}{3}} = 9 \text{ x} \frac{3}{2} = \frac{27}{2}$

31) Find the sum of $1+3+5+\ \ldots + 55.$**[PTA-6]**

தீர்வு:-

WKT, $l = 55$ (odd number)

$$1+3+5+\cdots+l = \left(\frac{l+1}{2}\right)^2$$

$$\therefore\ \ 1+3+5+\cdots+55 = \left(\frac{55+1}{2}\right)^2$$

$$= \left(\frac{56}{2}\right)^2$$

$$= (28)^2$$

$= 784$

32) Find the sum of

(i) $1^3+2^3+3^3+\cdots+16^3$

(ii) $9^3+10^3+11^3+\cdots+21^3$

solution:-

(i) $1^3+2^3+3^3+\cdots+16^3$

WKT, $1^3+2^3+3^3+\cdots+n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore\ 1^3+2^3+3^3+\cdots+16^3 = \left[\frac{16 \text{ x } 17}{2}\right]^2$

$= [8 \text{ x } 17]^2$

$= (136)^2$

$= 18496$

(ii) $9^3+10^3+11^3+\cdots+21^3$

WKT, $1^3+2^3+3^3+\cdots+n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore\ 9^3+10^3+11^3+\cdots+21^3$

$= (1^3+2^3+3^3+\cdots+21^3) - (1^3+2^3+3^3+\cdots+8^3)$

$= \left(\frac{21 \text{ x } 22}{2}\right)^2 - \left(\frac{8 \text{ x } 9}{2}\right)^2$

$= (231)^2 - (36)^2$

$= 53361 - 1296$

$= 52065$

33) $If\ 1+2+3+\cdots+k = 325$ then find $1^3+2^3+3^3+\cdots+k^3$

Solution:-

$$1+2+3+\cdots+k = 325$$

$$\Rightarrow\ \ \frac{k(k+1)}{2} = 325$$

$\therefore\ 1^3+2^3+3^3+\cdots+k^3 = \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$

34) $If\ 1^3+2^3+3^3+\cdots+k^3 = 44100$ then find $1+2+3+\cdots+k$

Solution:-

$1^3+2^3+3^3+\ \ldots + k^3 = 44100$

$$\left[\frac{k(k+1)}{2}\right]^2 = 44100$$

$$\left[\frac{k(k+1)}{2}\right]^2 = (210)^2$$

Take square root on both sides

$$\frac{k(k+1)}{2} = 210$$

$$1 + 2 + 3 + \cdots + k = 210$$

35) *Find the excluded values of* $\frac{7p+2}{8p^2+13p+5}$ **[M-22]**

solution:-

$$\frac{7p+2}{8p^2+13p+5}$$

$$= \frac{7p+2}{(p+1)(8p+5)}$$

+40	+13
$\frac{+8}{8p}$	$\frac{+5}{8p}$

$let\ (p+1)(8p+5) = 0$

$p + 1 = 0$ (or) $8p + 5 = 0$

$p = -10$ (or) $8p = -5$

$p = -10$ (or) $p = \frac{-5}{8}$

the excluded values -1 and $\frac{-5}{8}$

36) Find the square root of $\frac{144\,a^8b^{12}c^{16}}{81\,f^{12}g^4h^{14}}$ **[PTA-5]**

solution:-

$$\sqrt{\frac{144\,a^8b^{12}c^{16}}{81\,f^{12}g^4h^{14}}} = \left|\frac{12\,a^4b^6c^8}{9\,f^6g^2h^7}\right| = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

37) Find the square root of $\frac{400\,x^4y^{12}z^{16}}{100\,x^8y^4z^4}$ **[A-22]**

Solution:-

$$\sqrt{\frac{400\,x^4y^{12}z^{16}}{100\,x^8y^4z^4}} = \sqrt{\frac{4\,y^8z^{12}}{x^4}} = 2\left|\frac{y^4z^6}{x^2}\right|$$

38) Determine the quadratic equations, whose sum and product of roots are -9 and 20 **[S-21]**

Solution:-

sum of the roots $= -9$

product of the roots $= 20$

$x^2 -$ (sum of the roots)x + product of the roots $= 0$

$x^2 - (-9)x + 20 = 0$

$x^2 + 9x + 20 = 0$

39) Determine the quadratic equations, whose sum and product of roots are $\frac{-3}{2}$ and -1 **[PTA-4]**

Solution:-

sum of the roots $= \frac{-3}{2}$

product of the roots $= -1$

$x^2 -$ (sum of the roots)x + product of the roots $= 0$

$= 0.$

$$x^2 - \left(\frac{-3}{2}\right)x + (-1) = 0$$

$$x^2 + \frac{3}{2}x - 1 = 0$$

$$\frac{2x^2 + 3x - 2}{2} = 0$$

$$2x^2 + 3x - 2 = 0$$

40) Determine the nature of the roots for the following quadratic equation $15x^2 + 11x + 2 = 0$ **[S-21]**

Solution:-

$$15x^2 + 11x + 2 = 0$$

$$a = 15 , \quad b = 11 , \quad c = 2$$

$$\Delta = b^2 - 4ac$$
$$= (11)^2 - 4(15)(2)$$
$$= 121 - 120$$
$$\Delta = 1 > 0$$

Real and Unequal roots

41) If a matrix has 16 elements, what are the possible orders it can have?

Solution:-

the possible orders of a matrix with 16 elements

1×16 $\quad$ 8×2

2×8 $\quad$ 16×1

4×4

42) If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:-

the possible orders of a matrix with **18 elements**	the possible orders of a matrix with **6 elements**
1×18	1×6
2×9	2×3
3×6	3×2
6×3	6×1
9×2	
18×1	

43) Construct a 3 x 3 matrix whose elements are given by

$$A = a_{ij} = i^2j^2$$

Solution:-

$$a_{11} = 1^21^2 = 1 \times 1 = 1$$
$$a_{12} = 1^22^2 = 1 \times 4 = 4$$
$$a_{13} = 1^23^2 = 1 \times 9 = 9$$
$$a_{21} = 2^21^2 = 4 \times 1 = 4$$
$$a_{22} = 2^22^2 = 4 \times 4 = 16$$
$$a_{23} = 2^23^2 = 4 \times 9 = 36$$
$$a_{31} = 3^21^2 = 9 \times 1 = 9$$
$$a_{32} = 3^22^2 = 9 \times 4 = 36$$
$$a_{33} = 3^23^2 = 9 \times 9 = 81$$

$$\therefore A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$$

44) Construct a 3 x 3 matrix whose elements are given by

$$a_{ij} = |i - 2j|$$

Solution:-

The general 3 x 3 matrix is given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{ij} = |i - 2j|$$

$$a_{11} = |1 - 2(1)| = |1 - 2| = |-1| = 1$$
$$a_{12} = |1 - 2(2)| = |1 - 4| = |-3| = 3$$
$$a_{13} = |1 - 2(3)| = |1 - 6| = |-5| = 5$$
$$a_{21} = |2 - 2(1)| = |2 - 2| = |0| = 0$$
$$a_{22} = |2 - 2(2)| = |2 - 4| = |-2| = 2$$
$$a_{23} = |2 - 2(3)| = |2 - 6| = |-4| = 4$$

$a_{31} = |3-2(1)| = |3-2| = |1| = 1$
$a_{32} = |3-2(2)| = |3-4| = |-1| = 1$
$a_{33} = |3-2(3)| = |3-6| = |-3| = 3$
Hence the required matrix is

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$$

45) Find the values of x, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:-

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$x = 3$, $y = 12$, $z = 3$

(ii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$x + y + z = 9 \longrightarrow (1)$
$x + z = 5 \longrightarrow (2)$
$y + z = 7 \longrightarrow (3)$

Substitute (2) in (1)

$5 + y = 9$
$y = 9 - 5$
$y = 4$

$put\ y = 4$ in(3)

$4 + z = 7$
$z = 7 - 4$
$z = 3$

$put\ z = 3$ in(2)

$x + 3 = 5$
$x = 5 - 3$
$x = 2$

$\therefore x = 2,\ y = 4,\ z = 3$

46) $If\ A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

Solution:-

$$A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

47) $If A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of -A.

[PTA-2 , S-20]

Solution:-

$$-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

48) $If\ A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

Solution:-

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$\therefore (A^T)^T = A$ is verified.

49) If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then find $2A + B$ **[PTA-3]**

Solution:-

$$2A + B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

50) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then verify $AA^T = I$

Solution:-

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$AA^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$\cos\theta \quad \sin\theta \qquad -\sin\theta \quad \cos\theta$

$$= \begin{matrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{matrix} \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + -\sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$AA^T = I$ is verified

51) If $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$ then verify $A^2 = I$

Solution:-

$$A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$A^2 = A \times A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$5 \quad 6 \qquad -4 \quad -5$

$$= \begin{matrix} 5 & -4 \\ 6 & -5 \end{matrix} \begin{pmatrix} 25-24 & -20+20 \\ 30-30 & -24+25 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 = I$$

52) (Show that the points $(-3,-4), (7,2)$ and $(12,5)$ are collinear. **[S-21]**

Solution:-

$$(x_1, y_1) = (-3,-4)$$

$$(x_2, y_2) = (7,2)$$

$$(x_3, y_3) = (12,5)$$

$\therefore$ Area ofa a triangle

$$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -3 & 7 & 12 & -3 \\ -4 & 2 & 5 & -4 \end{Bmatrix}$$

$$= \frac{1}{2}(-6 + 35 - 48 + 28 - 24 + 15)$$

$$= \frac{1}{2}(-78 + 78)$$

$= 0$ Therefore, the given points are collinear.

53) *Show that the points* $P(-1.5, 3), Q(6, -2), R(-3, 4)$ are collinear .**[M-22]**

Solution:-

$$(x_1, y_1) = P(-1.5, 3)$$

$$(x_2, y_2) = Q(6, -2)$$

$$(x_3, y_3) = R(-3, 4)$$

Area ofa a ΔPQR

$$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{Bmatrix}$$

$$= \frac{1}{2}(3 + 24 - 9 - 18 - 6 + 6)$$

$$= \frac{1}{2}(27 - 27)$$

$= 0$ Therefore, the given points are collinear.

54) Find the slope of a line joining the points $(5, \sqrt{5})$ with the origin **[A-22]**

Solution:-

$(x_1, y_1) = (5, \sqrt{5})$ and $(x_2, y_2) = (0, 0)$

WKT, slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ slope $m = \frac{0-\sqrt{5}}{0-5} = \frac{\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{1}{\sqrt{5}}$

55) *The line p passes through the points* $(3, -2), (12, 4)$ *and the points* $(6, -2)$ *and* $(12, 2)$ *. Is p parralel to q?* **[M-22 , A-22]**

Solution:-

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

The slope of line p is

$x_1 \to 3$ $y_1 \to -2$	$x_2 \to 12$ $y_2 \to 4$

$$m_1 = \frac{4 + 2}{12 - 3} = \frac{6}{9} = \frac{2}{3}$$

The slope of line $q$is

$x_1 \to 6$ $y_1 \to -2$	$x_2 \to 12$ $y_2 \to 2$

$$m_2 = \frac{2 + 2}{12 - 6} = \frac{4}{6} = \frac{2}{3}$$

$$m_1 = m_2$$

Therefore, line p is parallel to the line q.

56) Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$ **[S-21]**

Solution:-

$$8x - 7y + 6 = 0$$

$$-7y = -8x - 6$$

$$7y = 8x + 6$$

$$y = \frac{8}{7}x + \frac{6}{7}$$

Comparing with $y = mx + c$

Slope $m = \frac{8}{7}$ and y intercept $c = \frac{6}{7}$

57) *Find the equationofa aline passing through the point* $(-1, 2)$ and having slope $\frac{-5}{4}$ **[M-22]**

Solution:-

$(x_1, y_1) = (-1,\ 2);\ = \frac{-5}{4}$.

The equation of the point-slope form of the straight line is $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{-5}{4}[x - (-1)]$$

$$4(y-2) = -5[x+1]$$

$$4y - 8 = -5x - 5$$

$$4y - 8 + 5x + 5 = 0$$

$$5x + 4y - 3 = 0$$

58) Show that the straight lines $2x + 3y = 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution:-

Slope $= \frac{-\text{co efficient of x}}{\text{co efficient of y}}$

Slope of the line $2x + 3y + 8 = 0$

$$= \frac{-2}{3}$$

Slope of the line $4x + 6y + 18 = 0$

$$= \frac{-4}{6} = \frac{-2}{3}$$

Here, $m_1 = m_2$ $\therefore$ Two lines are parallel.

59) Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution:-

Slope $= \frac{-\text{co efficient of x}}{\text{co efficient of y}}$

Slope of the line$x - 2y + 3 = 0$

$$= \frac{-1}{-2} = \frac{1}{2}$$

Slope of the line $6x + 3y + 8 = 0$

$$= \frac{-6}{3} = -2$$

$$\therefore m_1 \times m_2 = \frac{1}{2} \times -2 = -1$$

$\therefore$ Two lines are perpendicular

60) Find the equation of the straight line which is parallel to the line $3x - 7y = 12$ and passing through the point $(6,4)$.

Solution:-

Equation of the straight line parallel to $3x - 7y = 12$ is $3x - 7y + k = 0$

The line passes through $(6,4)$ $\therefore 3(6) - 7(4) + k = 0$

$$18 - 28 + k = 0 \quad \rightarrow -10 + k = 0$$

$$\therefore k = 10$$

Equation of the parallel line $3x - 7y + 10 = 0$

61) Prove that $\sqrt{\frac{1+cos\theta}{1-cos\theta}} = cosec\theta + cot\theta$ **[GMQ]**

solution:-

$$LHS = \sqrt{\frac{1+cos\theta}{1-cos\theta}}$$

$$= \sqrt{\frac{1+cos\theta}{1-cos\theta} \text{x} \frac{1+cos\theta}{1+cos\theta}}$$

$$= \sqrt{\frac{(1+cos\theta)^2}{1^2 - cos^2\theta}} [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \sqrt{\frac{(1+cos\theta)^2}{sin^2\theta}} [\because sin^2\theta + cos^2\theta = 1]$$

$$= \frac{1+cos\theta}{sin\theta}$$

$$= \frac{1}{sin\theta} + \frac{cos\theta}{sin\theta}$$

$$= cosec\theta + cot\theta \left[\because cosec\theta = \frac{1}{sin\theta} \,\&\, cot\theta = \frac{cos\theta}{sin\theta}\right]$$

$$= RHS$$

62) *Prove that* $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution:-

$$LHS = \sqrt{\frac{1+sin\theta}{1-sin\theta}} + \sqrt{\frac{1-sin\theta}{1+sin\theta}}$$

$$= \frac{\sqrt{1+sin\theta}}{\sqrt{1-sin\theta}} + \frac{\sqrt{1-sin\theta}}{\sqrt{1+sin\theta}}$$

$$= \frac{\left(\sqrt{1+sin\theta}\right)^2 + \left(\sqrt{1-sin\theta}\right)^2}{\sqrt{1-sin\theta}\sqrt{1+sin\theta}}$$

$$= \frac{1+sin\theta+1-sin\theta}{\sqrt{1^2 - sin^2\theta}} [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{2}{\sqrt{cos^2\theta}} [\because sin^2\theta + cos^2\theta = 1]$$

$$= \frac{2}{cos\theta}$$

$$= 2\,sec\theta$$

$$= RHS$$

63) A tower stands vertically on the ground. From a point on the ground which is 48m away from the foot of the tower, the angle of elevation of the top of the tower is 30^0. Find the height of the tower.

Solution:-

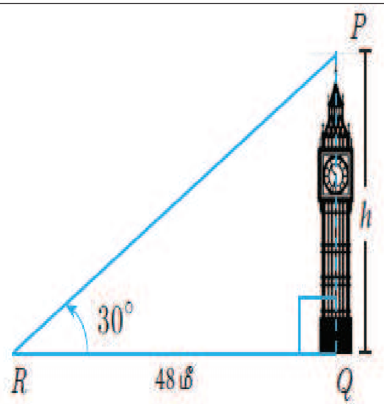
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{PQ}{RQ}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$\therefore h = \frac{48}{\sqrt{3}} = \frac{3 \times 16}{\sqrt{3}}$$

$$\therefore h = 16\sqrt{3}\text{ m}$$

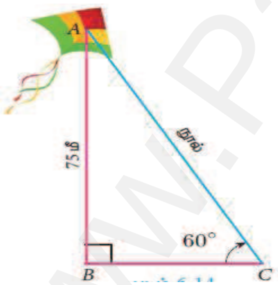


$$h = \frac{\sqrt{3}\text{x}\sqrt{3}\text{x } 16}{\sqrt{3}}$$

$$h = 16\sqrt{3}\text{மீ}$$

64) A kite is flying at a straight of 75m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^0. Find the length of the string, assuming that there is no slack in the string.

Solution:-



$$sin\theta = \frac{opp\ side}{hyp\ side}$$

$$sin60^0 = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$

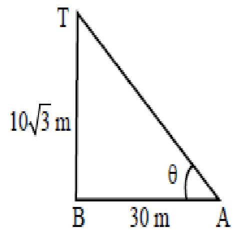
$$\therefore AC = 75 \times \frac{2}{\sqrt{3}}$$

$$= \frac{3 \times 25 \times 2}{\sqrt{3}}$$

$$\therefore AC = 50\sqrt{3}\text{ m}$$

65) Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$m.

Solution:-



$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan\theta = \frac{BT}{AB}$$

$$tan\theta = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3}$$

$$\therefore tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^0$$

66) A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30^0. Find the width of the road.

Solution:-

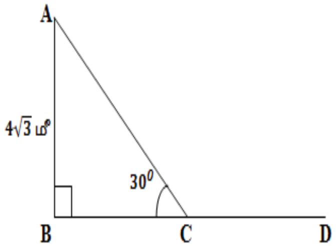
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{BC}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x}$$

$$\therefore x = 4\sqrt{3} \times \sqrt{3}$$

$$\therefore x = 4 \times 3 = 12 \text{ m}$$



67) A player siting on the top of a tower of height 20m observes the angle of depression of a ball on the ground as 60^0. Find the distance between the foot of the tower and the ball.($\sqrt{3} = 1.732$).
[PTA-3]

Solution:-

$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan60^0 = \frac{BC}{AB}$$

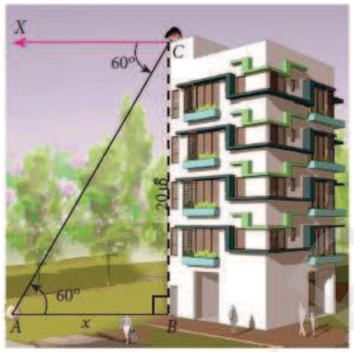
$$\sqrt{3} = \frac{20}{x}$$

$$\therefore x = \frac{20}{\sqrt{3}} = \frac{20 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\therefore x = \frac{20 \times 1.732}{3}$$

$$\therefore x = 20 \times 0.5773$$

$$x = 11.55 \text{ m}$$



68) From the top of a rock $50\sqrt{3}$m high, the angle of depression of a car on the ground is observed to be 30^0. Find the distance of the car from the rock.

Solution:-

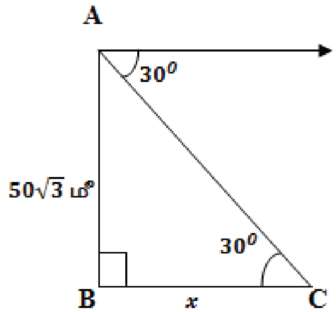
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{KR}{CK}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$

$$\therefore x = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$

$$\therefore x = 150 \text{ m}$$



69) The curved surface area of a right circular cylinder of height 14cm is 88cm^2. Find the diameter of the cylinder.

Solution:-

$h = 20$ cm

CSA of a cylinder $= 88$ sq.cm

$$2\pi rh = 88$$

$$2 \times \frac{22}{7} \times r \times 14 = 88$$

$$r = 88 \times \frac{1}{2} \times \frac{7}{22} \times \frac{1}{14}$$

$r = 1$ cm

diameter$d = 2$cm

70) The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.
[A-22]

Solution:-

Let radius $r = 5x$ height $h = 7x$

Surface Area of a Cylinder= 5500

$$\Rightarrow 2\pi rh = 5500$$

$$2 \times \frac{22}{7} \times 5x \times 7x = 5500$$

$$x^2 = \frac{5500}{2 \times 22 \times 5}$$

$$x^2 = 25$$

$$x = 5$$

∴ radius of a cylinder $= 5x = 5 \times 5 = 25$cm.

height of a cylinder $= 7x = 7 \times 5 = 35$cm.

71) If the total surface area of a cone of radius 7cm is 704 cm², then find its slant height. **[A-22]**

solution:-

radius, $r = 7$cm.

Total surface area of a cone = 704

$$\Rightarrow \pi r(l + r) = 704$$

$$\frac{22}{7} \times 7 \times (l + 7) = 704$$

$$l + 7 = \frac{704}{22}$$

$$l + 7 = 32$$

$$l = 32 - 7$$

$$l = 25\text{cm}$$

72) Find the diameter of a sphere whose surface area is 154 m² **[S-20]**

Solution:-

Let radius $= r$

surface area of sphere= 154

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{4 \times 22}$$

$$r^2 = \frac{7 \times 7}{2 \times 2}$$

$$r = \frac{7}{2}$$

∴ the diameter of a sphere = $2r = 2 \times \frac{7}{2} = 7$cm

73) If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area? **[S-20]**

Solution:-

Base area of a hemisphere=Area of a circle

$$\therefore \pi r^2 = 1386$$

TSA of a hemisphere= $3\pi r^2$

$$= 3 \times 1386$$

$$= 4158 \text{ sq.m}$$

74) Find the volume of a cylinder whose height is 2 m and whose base area is 250 m². **[S-21]**

Solution:-

height $h = 2$மீ

base area $= 250$

$\pi r^2 = 250 \text{ m}^2$

volume of a cylinder $= \pi r^2 h$

$= 250 \times 2$

$= 500 \text{ m}^3$

75) The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.

Solution:-

$h = 24$ cm

volume of a cone $= 11088$ cu.cm

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$\therefore r^2 = 11088 \times 3 \times \frac{7}{22} \times \frac{1}{24}$$

$$r^2 = 63 \times 7 = 3 \times 3 \times 7 \times 7$$

$$\therefore r = 3 \times 7 = 21$$

76) A metallic sphere of radius 16cm is melted and recast into small spheres each of radius 2cm. Howmany small spheres can be obtained?

Solution:-

Big sphere

$r = 16$cm

volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (16)^3$

$= \frac{4}{3}\pi \times 16 \times 16 \times 16$

Small sphere

$r = 2$cm

volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (2)^3$

$= \frac{4}{3}\pi \times 2 \times 2 \times 2$

No. of required sphere $= \dfrac{\text{volume of big sphere}}{\text{volume of small sphere}}$

$$= \frac{\frac{4}{3}\pi \times 16 \times 16 \times 16}{\frac{4}{3}\pi \times 2 \times 2 \times 2}$$

$= 8 \times 8 \times 8$

$= 512$

77) The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights. **[PTA-4 , M-22]**

Solution:-

Cone-1:-

Let radius $= r$

height $= h_1$

Cone-2:-

radius $= r$

height $= h_2$

volumes of two cones = 3600 : 5040

$\frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2 = 3600 : 5040$

$\frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{3600}{5040}$

$\frac{h_1}{h_2} = \frac{360}{504}$

$\frac{h_1}{h_2} = \frac{30}{42}$

$\frac{h_1}{h_2} = \frac{5}{7}$

$h_1 : h_2 = 5 : 7$

78) The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. **[M-22]**

Solution:-

$r_1 = 12\text{cm}$, $r_2 = 16\text{cm}$

ratio of C.S.A. of balloons = $\frac{4\pi r_1{}^2}{4\pi r_1{}^2}$

$= \frac{4\pi \text{ x } 12 \text{ x } 12}{4\pi \text{ x } 16 \text{ x } 16}$

$= \frac{9}{16}$

$= 9 : 16$

79) Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:-

Desending order 67,53,48,44,39,25,18

$\therefore L = 67, S = 18$

Range $= L - S = 67 - 18$

Range= 49

Co efficient of Range= $\frac{L-S}{L+s}$

$= \frac{67-18}{67+18}$

$= \frac{49}{85} = 0.576$

80) Find the range and coefficient of range of the following data.(i) 63, 89, 98, 125, 79, 108, 117, 68. (ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8.

Solution:-

(i) Decending order 125,117,108,98,89,79,68,63

$\therefore L = 125, S = 63$

Range= $L - S = 125 - 63$

$= 62$

Co efficient of Range= $\frac{L-S}{L+s}$

$= \frac{125-63}{125+63}$

$= \frac{62}{188} = 0.33$

(ii) Decending order 61.4, 43.5, 38.4, 29.8,18.9, 13.6

$$\therefore L = 61.4, S = 13.6$$

Range $= L - S = 61.4 - 13.6$

$$= 47.8$$

Co efficient of Range$= \frac{L-S}{L+s}$

$$= \frac{61.4 - 13.6}{61.4 + 13.6}$$

$$= \frac{47.8}{75} = 0.64$$

81) If the range and the smallest value of a set of data are 36.8 an 13.4 respectively, then find the largest value.

Solution:-

$\therefore$Range$= 36.8, S = 13.4$

Range$= L - S$

$36.8 = L - 13.4$

$$L = 36.8 + 13.4$$

$\therefore L = 50.2$

82) The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value. **[PTA-4]**

Solution:-

Range$= 13.67, \quad L = 70.08$

$$\text{Range} = L - S$$

$$13.67 = 70.08 - S$$

$$S = 70.08 - 13.67$$

$$S = 56.41$$

83) Calculate the range of the following data

வயது (வருடங்களில்)	16-18	18-20	20-22	22-24	24-26	26-28
மாணவர்களின் எண்ணிக்கை	0	4	6	8	2	2

Solution:-

$$L = 28, S = 18$$

Range$= L - S = 28 - 18$

$\therefore$Range$= 10\ years$

84) Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution:-

$$L = 450, S = 650$$

Range$= L - S = 650 - 450$

$\therefore$Range$= 250$

85) If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

solution:-

standard deviation, $\sigma = 4.5$

the standard deviation will not change when we add 5 to all the values.

$\therefore$ the new standard deviation.,$\sigma = 4.5$

86) If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

solution:-

standard deviation, $\sigma = 3.6$

we divide each data by 3 the standard deviation also get divided by *3*. ∴ new standard deviation

$$\sigma = \frac{3.6}{3} = 1.2$$

new variance $\sigma^2 = (1.2)^2 = 1.44$

87) If the standard deviation of 20 datas is $\sqrt{6}$. And each value of data is multiple by 3 then find the new variance and new standard deviation. **[PTA-1]**

Solution:- standard deviation $\sigma = \sqrt{6}$

each value of data is multiple by 3

∴ new standard deviation $= 3\sqrt{6}$

new variance $= (3\sqrt{6})^2 = 9 \text{ x } 6 = 54$

88) Find the standard deviation of first 21 natural numbers. **[PTA-6]**

Solution:-

standard deviation of first 'n' natural numbers $\sigma = \sqrt{\frac{n^2-1}{12}}$

$$\therefore \sigma = \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$$

$$= \sqrt{36.67} = 6.06$$

89) The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation. **[PTA-3]**

Solution:-

$\bar{x} = 25.6$, $C.V = 18.75$

$$C.V = \frac{\sigma}{\bar{x}} \text{x } 100\%$$

$$18.75 = \frac{\sigma}{25.6} \text{x } 100$$

$$\frac{18.75 \text{ x } 25.6}{100} = \sigma$$

$$\sigma = \frac{480}{100}$$

$$\sigma = 4.8$$

90) The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation. **[GMQ]**

Solution:-

$$\sigma = 6.5\,, \quad \bar{x} = 12.5$$

$$C.V = \frac{\sigma}{\bar{x}} \text{x } 100\%$$

$$= \frac{6.5}{12.5} \text{ x } 100$$

$$= \frac{650}{12.5}$$

$$= \frac{6500}{125}$$

$$= 52 \ \%$$

91) The standard deviation and coefficient of variation of a data

are 1.2 and 25.6 respectively. Find the value of mean.

Solution:-

$$\sigma = 1.2\,, \quad C.V = 25.6$$

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$25.6 = \frac{1.2}{\bar{x}} \text{ x } 100$$

$$\bar{x} = \frac{1.2 \text{ x } 100}{25.6}$$

$$\bar{x} = \frac{120}{25.6}$$

$$\bar{x} = 4.6875$$

92) If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

solution:-

$$\bar{x} = 15 \;\; , \; C.V = 48$$

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$48 = \frac{\sigma}{15} \text{ x } 100$$

$$\frac{48 \text{ x } 15}{100} = \sigma$$

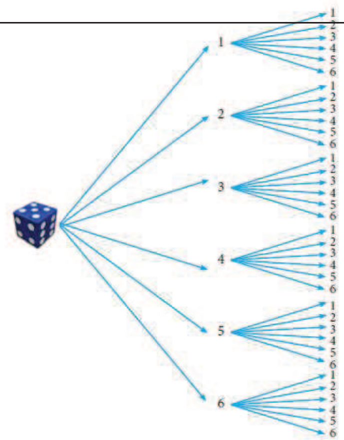
$$\sigma = \frac{720}{100}$$

$$\sigma = 7.2$$

93) Express the sample space for rolling two dice using tree diagram.

Solution:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$



$\therefore\; n(S) = 36$

94) Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:-

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

$A = \{\text{Getting Different Faces}\}$

$$A = \{HT, TH\}$$

$$n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

95) A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head [**S-21**]

Solution:-

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$n(S) = 12$

$A =$ {getting the die shows odd and coin shows head}

$A = \{1H, 3H, 5H\}$

$n(A) = 3$

$\therefore P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{12} = \dfrac{1}{4}$

96) What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution:-

Leap year 366 = 52 weeks+ 2 days

$S =$ {(sun,mon), (mon, tue), (tue.wed), (wed,thu), (thu,fri), (fri,sat), (sat,sun)}

$n(S) = 7$

$A =$ {getting 53 Saturdays in a leap year}

= {(fri,sat),(sat,sun) }

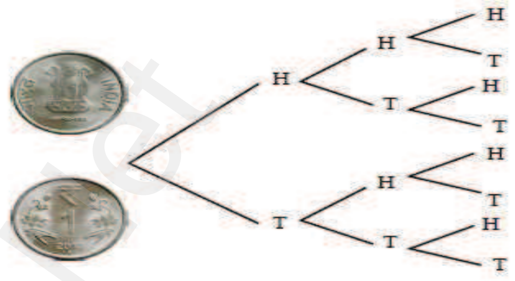
$n(A) = 2$

$P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{2}{7}$

97) Write the sample space for tossing three coins using tree diagram.

Solution:-



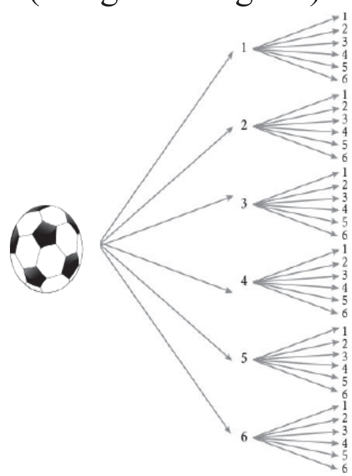
$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\therefore n(S) = 8$

98) Write a sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram). **[PTA-4]**

Solution:-

$S = \{(1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5)\}$



$\therefore n(S) = 30$

99) A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black **[A-22]**

Solution:-

No. of red balls = 5

No. of white balls = 6

No. of green balls = 7

No. of black balls = 8

Total balls = 26

$\therefore n(S) = 26$

(i) Let A be the event of getting a white ball

. $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$$

(ii) Let B be the event of getting a black or red ball

$n(B) = 8 + 5 = 13$

$$P(B) = \frac{\text{n(B)}}{\text{n(S)}} = \frac{13}{26} = \frac{1}{2}$$

(iii) Let C be the event of getting a not white ball

$n(C) = 5 + 7 + 8 = 20$

$$P(C) = \frac{n(C)}{n(S)} = \frac{20}{26} = \frac{10}{13}$$

(iv) Let D be the event of getting a neither white nor black ball

$n(D) = 5 + 7 = 12$

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{26} = \frac{6}{13}$$

100) two coins are tossed together. What is the probability of getting different faces on the coins? **[M-22]**

Solution:-

$$S = \{HH, HT, TH, TT\}$$

$n(S) = 4$

Let A be the event of getting different faces

$A = \{HT, TH\}$

$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

101) A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:-

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

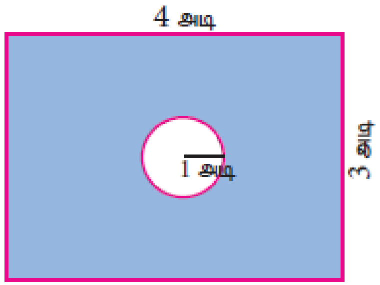
Let A be the event of getting two consecutive tails

$A = \{HTT, TTH, TTT\}$

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

102) Some boys are playing a game, in which the stone thrown by them landing ina circular region (given in figure) is considered as win and landind other than the circular region is considered as loss. What is the probability to win the game?



Solution:-

Area of Rectangle$= l \times b = 4 \times 3 = 12$ Sq.ft

Area of circle$= \pi r^2 = \pi \times 1^2 = \pi$

Probability of win the game$= \frac{\pi}{12} = \frac{3.14}{12}$

$$= \frac{314}{1200} = \frac{157}{600}$$

103) $If\ P(A) = 0.37,\ P(B) = 0.42,\ P(A \cap B) = 0.09$ then find $P(A \cup B))$

Solution:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.37 + 0.42 - 0.09$$

$$= 0.79 - 0.09$$

$$= 0.7$$

104) From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting a king or queen card.

Solution:-

$$n(S) = 52$$

Let A be the event of getting a king card

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a queen card

$$n(B) = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$$

A, B are mutually exclusive events, then

$\therefore$ $$P(A \cup B) = P(A) + P(B)$$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{4+4}{52}$$

$$= \frac{8}{52}$$

$$= \frac{2}{13}$$

105) $If\ P(A) = \frac{2}{3},\ P(B) = \frac{2}{5},\ P(A \cap B) = \frac{1}{3}$ then find $P(A \cap B))$ **[PTA-1]**

Solution:-

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10 + 6 - 5}{15}$$

$$= \frac{16 - 5}{15}$$

$$= \frac{11}{15}$$

106) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Solution:-

$$P(A) = 0.5, \quad P(B) = 0.3, \quad P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.3 - 0$$

$$= 0.8$$

∴ the probability that neither A nor B happen

$$= P(\overline{A \cup B})$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.8$$

$$= 0.2$$

107) Check whether AD is bisector of $\angle A$of ΔABC in each of the following

(i) $AB = 5$ cm, $AC = 10$ cm, $BD\_ = 1.5$ cm and $CD = 3.5$ cm. **[S-20]**

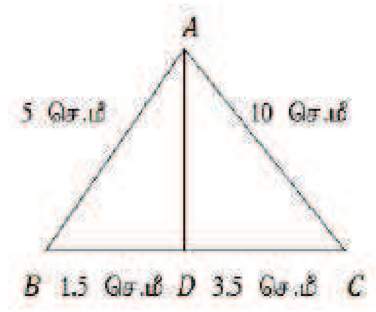
Solution:-

$AB = 5$ cm

$AC = 10$ cm

$BD = 1.5$ cm

$CD = 3.5$ cm



$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2} \dashrightarrow (1)$$

$$\frac{BD}{DC} = \frac{1.5}{3.5} = \frac{15}{35} = \frac{3}{7} \dashrightarrow (2)$$

From (1)and (2)

$$\frac{AB}{AC} \neq \frac{BD}{DC}$$

∴ AD is not a bisector of $\angle A$

108) In the Fig. AD is the bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC. **[PTA-5 , M-22]**

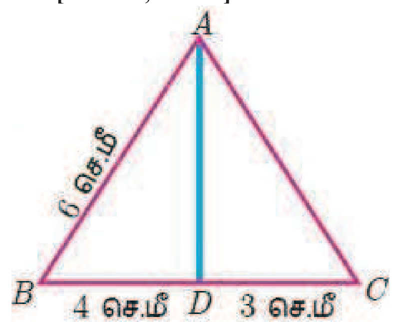
Solution:- $BD = 4$ cm

$DC = 3$ செ.மீ

$AB = 6$செ.மீ

$AC = ?$

By Angle Bisector Theorem



$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{6}{AC} = \frac{4}{3}$$

$$\frac{6 \text{ x } 3}{4} = AC$$

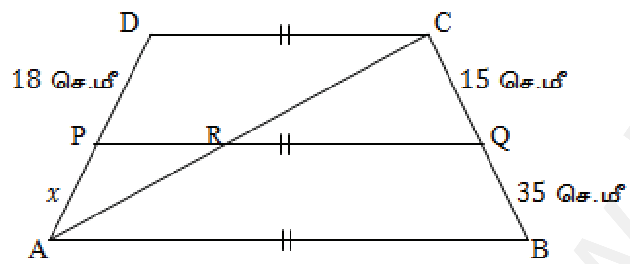
$$AC = \frac{3 \text{ x } 3}{2}$$

$$AC = \frac{9}{2}$$

$AC = 4.5$செ.மீ

109) ABCD is a trapezium in which $AB \parallel DC$ and P, Q are points on AD and BC respectively, such that $PQ \parallel DC$ if $PD = 18$ cm, $BQ = 35$ cm and $QC = 15$ cm, find AD. **[A-22]**

Solution:-



In ΔACD, $PR \parallel CD$

By **Thales theorem**

$$\frac{AP}{PD} = \frac{AR}{RC}$$

$$\frac{x}{18} = \frac{AR}{RC}$$

$$\frac{AR}{RC} = \frac{x}{18} \longrightarrow (1)$$

In ΔABC, $RQ \parallel AB$

By **Thales theorem**

$$\frac{AR}{RC} = \frac{BQ}{QC}$$

$$\frac{AR}{RC} = \frac{35}{15}$$

$$\frac{AR}{RC} = \frac{7}{3} \longrightarrow (2)$$

From (1) and (2)

$$\frac{x}{18} = \frac{7}{3}$$

$$x = \frac{7 \text{ x } 18}{3}$$

$$x = 7 \text{ x } 6$$

$$x = 42$$

$\therefore AP = x = 42$

$AD = AP + PD = 42 + 18 = 60$cm.

110) A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point? **[A-22]**

Solution:-

In right angled ΔABC –

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 24^2 + 18^2$$

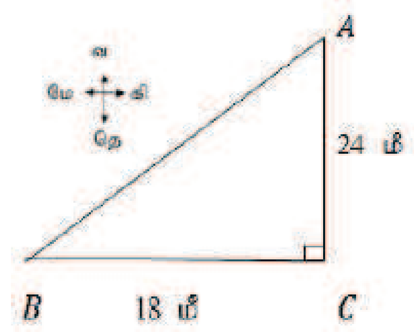
$$AB^2 = 576 + 324$$

$$AB^2 = 900$$

$$AB = \sqrt{900}$$

$$AB = \sqrt{30 \text{ x } 30}$$

$AB = 30$மீ

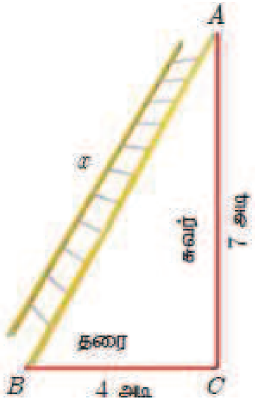


$\therefore$ தொட distance of his current position from the starting point $= 30$ m

111) What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution:-

In right angled ΔABC $AB^2 = BC^2 + AC^2$



$AB^2 = 4^2 + 7^2$

$AB^2 = 16 + 49$

$AB^2 = 65$

$AB = \sqrt{65}$

$AB = 8.062$

$AB = 8.1$அடி

Therefore, the length of the ladder is approximately 8.1 ft.

Five Mark Questions

1) $A = \{x \in \mathbb{W} / x < 2\}$, $B = \{x \in \mathbb{N} / 1 < x \leq 4\}$ and $C = \{3, 5\}$ then, verify that $AX(B \cup C) = (AXB) \cup (AXC)$.

Soln:-

Given, $A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{3, 5\}$

LHS: $B \cup C = \{2, 3, 4, 5\}$

$AX\ (B \cup C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\} \rightarrow (1)$

RHS: $AXB = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$AXC = \{(0,3), (0,5), (1,3), (1,5)\}$

$(AXB) \cup (AXC) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\} \rightarrow (2)$

∴from (1) and (2) we see that, $AX\ (B \cup C) = (AXB) \cup (AXC)$.

2) $A = \{x \in \mathbb{W} / x < 2\}$, $B = \{x \in \mathbb{N} / 1 < x \leq 4\}$ and $C = \{3, 5\}$ then, verify that $AX(B \cap C) = (AXB) \cap (AXC)$.

Soln:-

Given, $A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{3, 5\}$

LHS: $B \cap C = \{3\}$

$AX\ (B \cap C) = \{(0, 3), (1,3)\} \rightarrow (1)$

RHS: $AXB = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$AXC = \{(0,3), (0,5), (1,3), (1,5)\}$

$(AXB) \cap (AX\ C) = \{(0, 3), (1,3)\} \rightarrow (2)$

∴from (1) and (2) we see that, $AX\ (B \cap C) = (AXB) \cap (AXC)$.

3) $A = \{x \in \mathbb{W} / x < 2\}$, $B = \{x \in \mathbb{N}, 1 < x \leq 4\}$ and $C = \{3, 5\}$ then, verify that $(A \cup B) X\ C = (A\ X\ C) \cup (B\ X\ C)$.

Soln:-

Given, $A = \{0, 1\}$, $B = \{2, 3, 4\}$, $C = \{3, 5\}$

LHS:

$A \cup B = \{0, 1, 2, 3, 4\}$

$(A \cup B) X\ C) = \{(0,3), (0, 5), (1,3), (1,5), (2,3, (2,5), (3,3)(3,5), (4,3), (4,5)\} \rightarrow$ **(1)**

RHS:

$A\ X\ C = \{(0,3), (0,5), (1, 3), (1,5)\}$

$B\ X\ C = \{(2,3), (2,5), (3,3), (3,5), (4,3), (4, 5)\}$

$(A\ X\ C) \cup (B\ X\ C) = \{(0,3), (0, 5), (1,3), (1,5), (2,3, (2,5), (3,3)(3,5), (4,3), (4,5)\} \rightarrow$ **(2)**

∴from (1) and (2) we see that, $(A \cup B) X\ C = (A\ X\ C) \cup (B\ X\ C)$.

4) $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}, C = \{3, 4\}$ and $D = \{1, 3, 5\}$ then verify that, $(A \cap C) X (B \cap D) = (AXB) \cap (CXD)$.

Soln:-

LHS: $A \cap C = \{3\}$

$B \cap D = \{3, 5\}$

$(A \cap C) X (B \cap D) = \{(3, 3), (3, 5)\} \rightarrow (1)$

RHS: $A\ X\ B = \{(1,2), (1,3), (1,5), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5)\}$

$C\ X\ D = \{(3,1), (3,3), (3,5), (4,1), (4,3), (4,5)\}$

$(A\ X\ B) \cap (C\ X\ D) == \{(3, 3), (3, 5)\} \rightarrow (2)$

∴from (1) and (2), $(A \cap C) X (B \cap D) = (A\ X\ B) \cap (C\ X\ D)$.

5) $A = \{x \in \mathbb{N} / 1 < x < 4\}$, $B = \{x \in \mathbb{W} / 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} / x < 3\}$ then verify that, $AX(B \cup C) = (AXB) \cup (AXC)$.

Soln:-

Given, $A = \{2, 3\}$

$B = \{0, 1\}$

$C = \{1, 2\}$

LHS: $B \cup C = \{0, 1, 2\}$

$AX\ (B \cup C) = \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\} \rightarrow (1)$

RHS: $AXB = \{(2,0),(2,1),(3,0),(3,1)\}$

$AXC = \{(2,1),(2,2),(3,1),(3,2)$

$(AXB) \cup (AXC) = \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\} \rightarrow$ (2)

$\therefore$from (1) and (2) we see that, $AX\,(B \cup C) = (AXB) \cup (AXC)$.

6) $A = \{x \in \mathbb{N}/\ 1 < x < 4\}$, $B = \{x \in \mathbb{W}/\ 0 \leq x < 2\}andC = \{x \in \mathbb{N}/\ x < 3\}$ then that, $AX(B \cap C) = (AXB) \cap (AX\,C$.

Soln:-

Given, $A = \{2,3\}$

$B = \{0,1\}$

$C = \{1,2\}$

LHS: $B \cap C = \{1\}$

$AX\,(B \cap C) = \{(2,1),(3,1)\} \rightarrow$ (1)

RHS: $AXB = \{(2,0),(2,1),(3,0),(3,1)\}$

$AXC = \{(2,1),(2,2),(3,1),(3,2)$

$(AXB) \cap (AXC) =) = \{(2,1),(3,1)\} \rightarrow$ (2)

$\therefore$from (1) and (2) we see that, $AX\,(B \cap C) = (AXB) \cap (AXC)$.

7) Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8 and C = The set of even prime number. Verify that $(A \cap B)XC = (AXC) \cap (BXC)$.

Soln:-

Given, $A = \{1,2,3,4,5,6,7\}$

$B = \{2,3,5,7\}$

$C = \{2\}$

LHS: $A \cap B = \{2,3,5,7\}$

$(A \cap B)XC = \{(2,2),(3,2),(5,2),(7,2)\} \rightarrow$ (1)

RHS: $AXC = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$BXC = \{(2,2),(3,2),(5,2),(7,2)$

$(AXC) \cap (BXC) = \{(2,2),(3,2),(5,2),(7,2)\} \rightarrow$ (2)

$\therefore$from (1) and (2) we see that, $(A \cap B)XC = (AXC) \cap (BXC)$.

8) Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8 and C = The set of even prime number. Verify that $A\,X(B - C) = (A\,X\,B) - (A\,X\,C)$.

Soln:-

Given, $A = \{1,2,3,4,5,6,7\}$

$B = \{2,3,5,7\}$

$C = \{2\}$

LHS: $B - C = \{3,5,7\}$

$A\,X\,(B - C)$
$= \{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),(3,7),$
$(4,3),(4,5),(4,7),(5,3),(5,5),(5,7),(6,3),(6,5),(6,7),(7,3),$
$(7,5),(7,7)\} \rightarrow$ (1)

RHS: $A\,X\,B =$
$\{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),$
$(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),$
$(6,5),(6,7),(7,2),(7,3),(7,5),(7,7)\} \rightarrow$ **(1)**

$A\,X\,C = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A\,X\,B) - (A\,X\,C) =$
$\{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),(3,7),(4,3),$
$(4,5),(4,7),(5,3),(5,5),(5,7),(6,3),(6,5),(6,7),$
$(7,3),(7,5),(7,7)\} \rightarrow$ **(2)**

$\therefore$ (from (1) and (2) we see that, $A\,X(B - C) = (A\,X\,B) - (A\,X\,C)$.

9) A relation is given by the set $\{(x,y)/\ y = x + 3,\ x,\ y$ are natural numbers $< 10\}$ Represent the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible. **[A-22]**

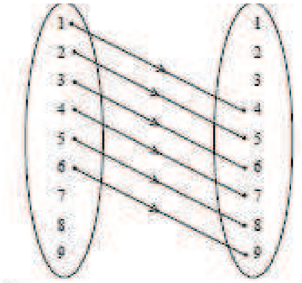
Soln:-

Given, $\{(x,y)/\ y = x + 3,\ x,\ y$ are natural numbers $< 10\}$

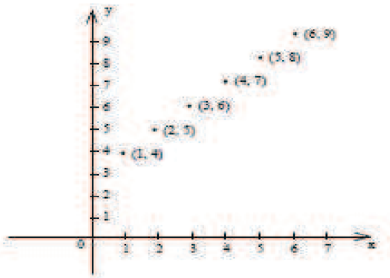
$x = \{1,2,3,4,5,6,7,8,9\}andy = \{1,2,3,4,5,6,7,8,9\}$

If $x = 1$, then $y = 1 + 3 = 4$
If $x = 2$, then $y = 2 + 3 = 5$
If $x = 3$, then $y = 3 + 3 = 6$
If $x = 4$, then $y = 4 + 3 = 7$
If $x = 5$, then $y = 5 + 3 = 8$
If $x = 6$, then $y = 6 + 3 = 9$
If $x = 7$, then $y = 7 + 3 = 10$
If $x = 8$, then $y = 8 + 3 = 11$
If $x = 9$, then $y = 9 + 3 = 12$

(i)<u>an arrow diagram</u>:-



(ii)<u>a graph</u>:-



(iii)<u>a set in roster form</u>:-

$$R = \{(1,4)),(2,5),(3,6),(4,7),(5,8),(6,9)\}$$

10) Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f : A \longrightarrow B$ be a function given by $f(x) = 3x - 1$. Represent this function (i) as a set of ordered pairs (ii) in a table form (iii) by arrow diagram (iv) in a graphical form.

<u>Soln</u>:-<u>Given</u>, $f(x) = 3x - 1$

$$f(1) = 2, f(2) = 5, f(3) = 8, f(4) = 11$$

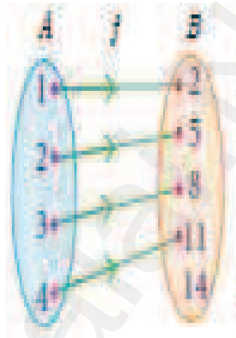
<u>(i) Set of ordered pairs:-</u>

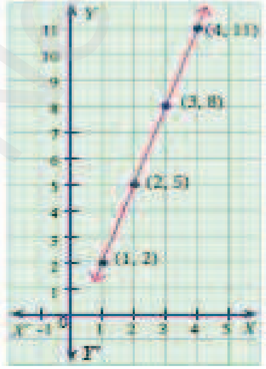
$$f(x) = \{(1,2),(2,5),(3,8),(4,11)\}$$

<u>(ii) Table form:-</u>

x	1	2	3	4
$f(x)$	2	5	8	11

<u>(iii) Arrow diagram:-(iv) Graphical Form</u>:-





11) Let $A = \{2, 4, 6, 10, 12\}$ and $B = \{0, 1, 2, 4, 5, 9\}$ be two sets. Let $f : A \longrightarrow B$ be a function given by $f(x) = \frac{x}{2} - 1$. Represent this function (i) as a set of ordered pairs (ii) in a table form (iii) by arrow diagram (iv) in a graphical form.

<u>Soln</u>:-<u>Given</u>, $f(x) = \frac{x}{2} - 1$

$$f(2) = 0, f(4) = 1, f(6) = 2, f(10) = 4, f(12) = 5$$

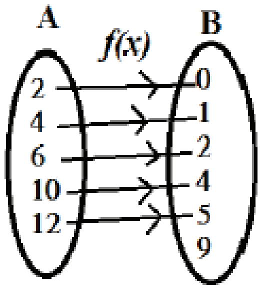
<u>(i) Set of ordered pairs:-</u>

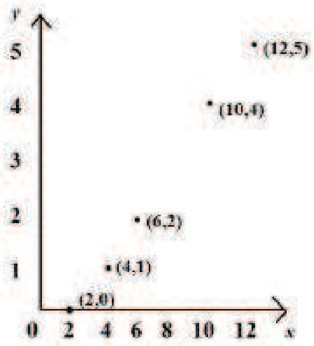
$$f(x) = \{(2,0),(4,1),(6,2),(10,4),(12,5)\}$$

<u>(ii) Table form:-</u>

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) Arrow diagram:-(iv) Graphical form:-





12) Let f be a function $f:\mathbb{N}\longrightarrow\mathbb{N}$ defined by $f(x)=3x+2, x\epsilon\mathbb{N}$ (i) Find the images of 1, 2, 3 (ii) Find the pre-images of 29, 53.

Soln:-**Given**, $f(x)=3x+2$

(i) $f(1)=3(1)+2=3+2=5$

$f(2)=3(2)+2=6+2=8$

$f(3)=3(3)+2=9+2=11$

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii)

Given, $f(x)=29$

$3x+2=29$

$3x=29-2$

$3x=27$

$x=\frac{27}{3}$

$x=9$

Given, $f(x)=53$

$3x+2=53$

$3x=53-2$

$3x=51$

$x=\frac{51}{3}$

$x=17$

(iii) Since different elements of $\mathbb{N}$ have different images in the co-domain, the function f is one-one function.

The co-domain of f is $\mathbb{N}$.

But the range of $f=\{5,8,11,14,17,...\}$ is a proper subset of $\mathbb{N}$.

Therefore f is not an onto function. That is f is an into function.

Thus f is one-one and into function.

13) If $f(x)=3x-2,\ g(x)=2x+k$ and $fog=gof$, then find the value of k.

Soln:-

$fog=(3x-2)o\,(2x+k)$

$=3(2x+k)-2$

$=6x+3k-2$

$gof=(2x\mp k)o\,(3x-2)$

$=2(3x-2)+k$

$=6x-4+k$

Given, $fog=gof$

$$6x+3k-2=6x-4+k$$
$$3k-k=2-4$$
$$2k=-2$$
$$k=\frac{-2}{2}$$
$$k=-1$$

14) If $f(x)=3x+2,\ g(x)=6x-k$ and $fog=gof$, then find the value of k.

Soln:-

$fog=(3x+2)o\,(6x-k)$

$=3(6x-k)+2$

$=18x-3k+2$

$gof=(6x-k)o\,(3x+2)$

$=6(3x+2)-k$

$=18x+12-k$

Given, $fog=gof$

$$18x-3k+2=18x+12-k$$
$$-3k+k=12-2$$
$$-2k=10$$
$$k=\frac{10}{-2}$$
$$k=-5$$

15) If $f(x)=2x-k,\ g(x)=4x+5$ and $fog=gof$, then find the value of k.

Soln:-

$fog=(2x-k)o\,(4x+5)$

$=2(4x+5)-k$

$=8x+10-k$

$gof=(4x+5)o\,(2x-k)$

$=4(2x-k)+5$

$=8x-4k+5$

Given, $fog = gof$

$$8x + 10 - k = 8x - 4k + 5$$
$$4k - k = 5 - 10$$
$$3k = -5$$
$$k = \frac{-5}{3}$$

16) If $f(x) = 2x + 3,\ g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $fo(goh) = (fog)oh$.

Soln:-

$$fo(goh) = (2x+3)o\,[(1-2x)o(3x)]$$
$$= (2x+3)o\,[1-2(3x)]$$
$$= (2x+3)o\,(1-6x)$$
$$= 2(1-6x)+3$$
$$= 2-12x+3$$
$$= 5-12x \rightarrow (1)$$
$$(fog)oh = [(2x+3)o(1-2x)]o(3x)]$$
$$= [2(1-2x)+3]o\,(3x)$$
$$= (2-4x+3)o\,(3x)$$
$$= (5-4x)+(3x)$$
$$= 5-4(3x)$$
$$= 5-12x \rightarrow (2)$$

∴from (1) and (2) we see that, $fo(goh) = (fog)oh$.

17) If $f(x) = x - 1,\ g(x) = 3x + 1$ and $h(x) = x^2$ then verify that $fo(goh) = (fog)oh$.

Soln:-

$$fo(goh) = (x-1)o\,[(3x+1)o(x^2)]$$
$$= (x-1)o\,(3x^2+1)$$
$$= 3x^2+1-1$$
$$= 3x^2 \rightarrow (1)$$
$$(fog)oh = [(x-1)o(3x+1)]o(x^2)]$$
$$= (3x+1-1)o\,(x^2)$$
$$= (3x)o\,(x^2)$$
$$= 3x^2 \rightarrow (2)$$

∴from (1) and (2) we see that, $fo(goh) = (fog)oh$.

18) $f(x) = x^2,\ g(x) = 2x$ and $h(x) = x + 4$ then verify that $fo(goh) = (fog)oh$.

Soln:-

$$fo(goh) = (x^2)\,o\,[(2x)\,o\,(x+4)]$$
$$= (x^2)\,o\,2(x+4)$$
$$= (x^2)\,o\,(2x+8)$$
$$= (2x+8)^2 \rightarrow \textbf{(1)}$$
$$(fog)oh = [(x^2)\,o\,(2x)]\,o\,(x+4)]$$
$$= (2x)^2\,o\,(x+4)$$
$$= [2(x+4)]^2$$
$$= (2x+8)]^2 \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that, $fo(goh) = (fog)oh$.

19) If $f(x) = x - 4,\ g(x) = x^2$ and $h(x) = 3x - 5$ then verify that $fo(goh) = (fog)oh$.

Soln:-

$$fo(goh) = (x^2)\,o\,[(2x)\,o\,(x+4)]$$
$$= (x^2)\,o\,2(x+4)$$
$$= (x^2)\,o\,(2x+8)$$
$$= (2x+8)^2 \rightarrow (1)$$
$$(fog)oh = [(x^2)\,o\,(2x)]\,o\,(x+4)]$$
$$= (2x)^2\,o\,(x+4)$$
$$= [2(x+4)]^2$$
$$= (2x+8)]^2 \rightarrow (2)$$

∴from (1) and (2) we see that, $fo(goh) = (fog)oh$.

20) $f(x) = x^2,\ g(x) = 3x$ and $h(x) = x - 2$hen verify that $fo(goh) = (fog)oh$.

Soln:-

$$fo(goh) = (x^2)\ o\ [\,(3x)\ o\ (x-2)]$$
$$= (x^2)\ o\ [3(x-2)]$$
$$= [3(x-2)]^2$$
$$= 9(x-2)^2 \rightarrow \textbf{(1)}$$

$$(fog)oh = [(x^2)\ o\ (3x)]\ o\ (x-2)]$$
$$= (3x)^2\ o\ (x-2)$$
$$= [3(x-2)]^2$$
$$= 9(x-2)^2 \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that, $fo(goh) = (fog)oh$.

21) Let $f(x) = 3x + 1$, $g(x) = x + 3$ be any two functions. Also if $gff(x) = fgg(x)$, find the value of $'x'$.

Soln:-$gff(x) = g\ o\ f\ o\ f$

$$= (x+3)\ o\ (3x+1)\ o\ (3x+1)$$
$$= (x+3)\ o\ [3(3x+1)+1]$$
$$= (x+3)\ o\ (9x+3+1]$$
$$= (x+3)\ o\ (9x+4]$$
$$= 9x+4+3$$
$$= 9x+7$$

$fgg(x) = f\ o\ g\ o\ g$

$$= (3x+1)\ o\ (x+3)\ o\ (x+3)$$
$$= (3x+1)\ o\ (x+3+3)$$
$$= (3x+1)\ o\ (x+6]$$
$$= 3(x+6)+1$$
$$= 3x+18+1$$
$$= 3x+19$$

Given, $gff(x) = fgg(x)$

$$9x+7 = 3x+19$$
$$9x-3x = 19-7$$
$$6x = 12$$
$$x = \frac{12}{6}$$
$$x = 2$$

22) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ then prove that $A^2 - 5A + 7I_2 = 0$.

Soln:-

$A^2 = A \times A$

$$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \times \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (3\ \ 1)\begin{pmatrix} 3 \\ -1 \end{pmatrix} & (3\ \ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ (-1\ \ 2)\begin{pmatrix} 3 \\ -1 \end{pmatrix} & (-1\ \ 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$$-5A = -5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix}$$

$$7I_2 = 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} + \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\therefore A^2 - 5A + 7I_2 = 0$ Hence Proved

23) $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then prove that $A^2 - 4A + 5I_2 = 0$ **[PTA-5]**

Soln:-

$$A^2 = A \times A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ -1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1\ \ -1)\begin{pmatrix} -1 \\ 3 \end{pmatrix} \\ (2\ \ 3)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (2\ \ 3)\begin{pmatrix} -1 \\ 3 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$$

$$4A = 4\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix}$$

$$5I_2 = 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$A^2 - 4A + 5\,I = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} - \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} + \begin{pmatrix} -4 & +4 \\ -8 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= 0$$

$\therefore$ $A^2 - 4A + 5I_2 = 0$ Hence Proved.

24) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ then verify that $(AB)^T =$

$B^T A^T$.

Soln:-

LHS:-

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \text{ x } \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 2\ \ 1)\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} & (1\ \ 2\ \ 1)\begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} \\ (2\ \ -1\ \ 1)\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} & (2\ \ -1\ \ 1)\begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$(AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$ → **(1)**

RHS:-

$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}$ and $A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \text{ x } \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (2\ \ -1\ \ 0)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (2\ \ -1\ \ 0)\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \\ (-1\ \ 4\ \ 2)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (-1\ \ 4\ \ 2)\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$B^T A^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$ → **(2)**

∴from (1) and (2) we see that $(AB)^T = B^T A^T$.

25) If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ then verify that $(AB)^T = B^T A^T$.

Soln:-

LHS:-

$$AB = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix} \text{ x } \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (5\ \ 2\ \ 9)\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} & (5\ \ 2\ \ 9)\begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} \\ (1\ \ 2\ \ 8)\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} & (1\ \ 2\ \ 8)\begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$ → **(1)**

RHS:-

$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}$ and $A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \text{ x } \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 1\ \ 5)\begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & (1\ \ 1\ \ 5)\begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix} \\ (7\ \ 2\ \ -1)\begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & (7\ \ 2\ \ -1)\begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix} \end{pmatrix}$$

$B^T A^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \rightarrow$ **(2)**

$\therefore$from (1) and (2) we see that $(AB)^T = B^T A^T$.

26) If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ then prove that prove that $A(B + C) = AB + AC$.

Soln:-

LHS:-

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - 7 & 2 + 6 \\ -4 + 3 & 2 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$A(B + C) = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \text{x} \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{array}{c} 1 \quad 1 \\ -6 \\ -1 \end{array} & \begin{array}{c} 1 \quad 1 \\ 8 \\ 4 \end{array} \\ \begin{array}{c} -1 \quad 3 \\ -6 \\ -1 \end{array} & \begin{array}{c} -1 \quad 3 \\ 8 \\ 4 \end{array} \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix} \rightarrow \textbf{(1)}$$

RHS:-

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \text{x} \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{array}{c} 1 \quad 1 \\ 1 \\ -4 \end{array} & \begin{array}{c} 1 \quad 1 \\ 2 \\ 2 \end{array} \\ \begin{array}{c} -1 \quad 3 \\ 1 \\ -4 \end{array} & \begin{array}{c} -1 \quad 3 \\ 2 \\ 2 \end{array} \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \text{x} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{array}{c} 1 \quad 1 \\ -7 \\ 3 \end{array} & \begin{array}{c} 1 \quad 1 \\ 6 \\ 2 \end{array} \\ \begin{array}{c} -1 \quad 3 \\ -7 \\ 3 \end{array} & \begin{array}{c} -1 \quad 3 \\ 6 \\ 2 \end{array} \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3 - 4 & 4 + 8 \\ -13 + 16 & 4 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix} \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that $A(B+C) = AB + AC$.

27) If $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$ than, prove that $A(B+C) = AB + AC$.

Soln:-

LHS:-

$$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+1 & -1+3 & 2+2 \\ 3-4 & 5+1 & 2+3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \text{ x } \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 3)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 2 \\ 6 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 4 \\ 5 \end{pmatrix} \\ (5\ \ -1)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} 2 \\ 6 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} 4 \\ 5 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \rightarrow \textbf{(1)}$$

RHS:-

$$AB = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \text{ x } \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 3)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} -1 \\ 5 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ (5\ \ -1)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} -1 \\ 5 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \text{ x } \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 3)\begin{pmatrix} 1 \\ -4 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ (5\ \ -1)\begin{pmatrix} 1 \\ -4 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (5\ \ -1)\begin{pmatrix} 2 \\ 3 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix} + \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 10-11 & 14+6 & 8+11 \\ 2+9 & -10+14 & 8+7 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that $A(B+C) = AB + AC$.

28) If $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ then prove that $(A-B)C = AC - BC$.

Soln:-

LHS:-

$$A - B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} -4 & 0 \\ -1 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 2+0 \\ 1-1 & 3-5 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}$$

$$(A-B)C = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix} \text{ x } \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix}-3 & 2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}-3 & 2\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \\ \begin{pmatrix}0 & -2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}0 & -2\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix}-4 & 4\\ -2 & -4\end{pmatrix} \rightarrow \textbf{(1)}$$

RHS:-

$$AC = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix} \text{ x } \begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \\ \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix}$$

$$BC = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix} \text{ x } \begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix}4 & 0\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}4 & 0\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \\ \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$$

$$AC - BC = \begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix} - \begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$$

$$= \begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix} + \begin{pmatrix}-8 & 0\\ -7 & -10\end{pmatrix}$$

$$= \begin{pmatrix}4-8 & 4+0\\ 5-7 & 6-10\end{pmatrix}$$

$$= \begin{pmatrix}-4 & 4\\ -2 & -4\end{pmatrix} \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that$(A-B)C = AC - BC$

29) If $A = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ and $B = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ then verify that $(A-B)^T = B^T - A^T$.

Soln:-

LHS:-

$$A - B = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix} - \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$$

$$= \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix} + \begin{pmatrix}-4 & 0\\ -1 & -5\end{pmatrix}$$

$$= \begin{pmatrix}1-4 & 2+0\\ 1-1 & 3-5\end{pmatrix}$$

$$= \begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}$$

$$(A-B)^T = \begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}^T$$

$$= \begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix} \rightarrow \textbf{(1)}$$

RHS:-

$$A^T = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}^T = \begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}$$

$$B^T = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}^T = \begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}$$

$$A^T - B^T = \begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix} - \begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}$$

$$= \begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix} + \begin{pmatrix}-4 & -1\\ 0 & -5\end{pmatrix}$$

$$= \begin{pmatrix}1-4 & 1-1\\ 2+0 & 3-5\end{pmatrix}$$

$$A^T - B^T = \begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix} \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that $(A-B)^T = A^T - B^T$.

30) If $A = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ and $C = \begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ then prove that $A(BC) = (AB)C$.

Soln:-

LHS:-

$$BC = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix} \text{ x } \begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix}4 & 0\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}4 & 0\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \\ \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \end{pmatrix}$$

RHS:-

$$AB = \begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix} \text{ x } \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix} & \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}0\\5\end{pmatrix} \\ \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix} & \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}0\\5\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 2)\begin{pmatrix} 8 \\ 7 \end{pmatrix} & (1\ \ 2)\begin{pmatrix} 0 \\ 10 \end{pmatrix} \\ (1\ \ 3)\begin{pmatrix} 8 \\ 7 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 0 \\ 10 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+14 & 0+20 \\ 8+21 & 0+30 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix} \rightarrow \textbf{(1)}$$

$$= \begin{pmatrix} 4+2 & 0+10 \\ 4+3 & 0+15 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 10 \\ 7 & 15 \end{pmatrix}$$

$$(AB)C) = \begin{pmatrix} 6 & 10 \\ 7 & 15 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (6\ \ 10)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (7\ \ 15)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (7\ \ 15)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (7\ \ 15)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 12+10 & 0+20 \\ 14+15 & 0+30 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix} \rightarrow \textbf{(2)}$$

∴from (1) and (2) we see that$A(BC) = (AB)C$

31) If $A = (1 \quad -1 \quad 2), B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ then prove that $(AB)C = A(BC)$.

Soln:-

LHS:-

$$AB = (1 \quad -1 \quad 2) \times \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ -1\ \ 2)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (1\ \ -1\ \ 2)\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \end{pmatrix}$$

$$= (1-2+2 \quad -1-1+6)$$

$$= (1 \quad 4)$$

RHS:-

$$BC = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ -1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1\ \ -1)\begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ (2\ \ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (2\ \ 1)\begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ (1\ \ 3)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 2 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$(AB)C) = (1 \quad 4) \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ 4)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1\ \ 4)\begin{pmatrix} 2 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$= (1+8 \quad 2-4)$$

$$= (9 \quad -2) \dashrightarrow (1)$$

$$= \begin{pmatrix} 1-2 & 2+1 \\ 2+2 & 4-1 \\ 1+6 & 2-3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$A(BC)$

$$= (1 \quad -1 \quad 2) \times \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ \ -1\ \ 2)\begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix} & (1\ \ -1\ \ 2)\begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} \end{pmatrix}$$

$$= (-1-4+14 \quad 3-3-2)$$

$$= (-5+14 \quad -2)$$

$$= (9 \quad -2) \dashrightarrow (2)$$

∴from (1) and (2) we see that$(AB)C = A(BC)$

32) If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ then prove that$A + (B + C) = (A + B) + C$.

Soln:-

LHS:-

$$B + C = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+8 & 3+3 & 4+4 \\ 1+1 & 9-2 & 2+3 \\ -7+2 & 1+4 & -1-1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$A+(B+C) = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 4+10 & 3+6 & 1+8 \\ 2+2 & 3+7 & -8+5 \\ 1-5 & 0+5 & -4-2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \dashrightarrow (1)$$

RHS:-

$$A+B = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+2 & 3+3 & 1+4 \\ 2+1 & 3+9 & -8+2 \\ 1-7 & 0+1 & -4-1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 6 & 5 \\ 3 & -6 & -6 \\ -6 & 1 & -5 \end{pmatrix}$$

$$(A+B)+C = \begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6+8 & 6+3 & 5+4 \\ 3+1 & 12-2 & -6+3 \\ -6+2 & 1+4 & -5-1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \dashrightarrow (2)$$

∴from (1) and (2) we see that $A+(B+C) = (A+B)+C$.

33) Find the square root of : $64x^4 - 16x^3 + 17x^2 - 2x + 1$ **[S-21]**

Soln:- **Given** $64x^4 - 16x^3 + 17x^2 - 2x + 1$

			8	−1	1		
		8	64	−16	17	−2	1
			64				
			(−)				
	16	−1		−16	17		
				−16	1		
				(+)	(−)		
16	−2	1			16	−2	1
					16	−2	1
					(−)	(+)	(−)
							0

$$\therefore \sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$$

34) Find the square root of : $x^4 - 12x^3 + 42x^2 - 36x + 9$

Soln:- **Given** $x^4 - 12x^3 + 42x^2 - 36x + 9$

		1	−6	3		
	1	1	−12	42	−36	9
		1				
		(−)				
2	−6		−12	42		

				-12	36		
				$(+)$	$(-)$		
2	-12	3			6	-36	9
					6	-36	9
					$(-)$	$(+)$	$(-)$
							0

$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$

35) Find the square root of : $4x^4 - 28x^3 + 37x^2 + 42x + 9$.

Soln:- Given, $4x^4 - 28x^3 + 37x^2 + 42x + 9$

			2	-7	-3		
		2	4	-28	37	42	9
			4				
			$(-)$				
	4	-7		-28	37		
				-28	49		
				$(+)$	$(-)$		
4	-14	-3			-12	42	9
					-12	42	9
					$(-)$	$(+)$	$(-)$
							0

$\therefore \sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = |2x^2 - 7x - 3|$

36) Find the square root of : $16x^4 + 0x^3 + 8x^2 + 0x + 1$.

Soln:- Given, $16x^4 + 0x^3 + 8x^2 + 0x + 1$

			4	0	1		
		4	16	0	8	0	1
			16				
			$(-)$				
	8	0		0	8		
				0	0		
				$(+)$	$(-)$		
8	0	1			8	0	1
					8	0	1
					$(-)$	$(+)$	$(-)$
							0

$\therefore \sqrt{16x^4 + 0x^3 + 8x^2 + 0x + 1} = |4x^2 + 1|$

37) Find the square root of : $121x^4 - 198x^3 - 183x^2 + 216x + 144$.

Soln:- Given, $121x^4 - 198x^3 - 183x^2 + 216x + 144$

			11	-9	12		
		11	121	-198	-183	216	144
			121				
			$(-)$				
	22	-9		-198	-183		
				-198	81		
				$(+)$	$(-)$		
22	-18	12			-264	216	144
					-264	216	144
					$(-)$	$(+)$	$(-)$
							0

$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = |11x^2 - 9x - 12|$

38) If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, then find the values of a, b. **[PTA-5]**

Soln:- Given, $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square.

			3	2	4		
		3	9	12	28	a	b
			9				
			(−)				
	6	2		12	28		
				12	4		
				(−)	(−)		
6	4	4			24	a	b
					24	16	16
					(−)		
							0

$\therefore a = 16$ and $b = 16$

39) If $4x^4 - 12x^3 + 37x^2 + bx + a$ is a perfect square, then find the values of a, b.

Soln:- **Given**, $4x^4 - 12x^3 + 37x^2 + bx + a$ is a perfect square.

			2	−3	7		
		2	4	−12	37	b	a
			4				
			(−)				
	4	−3		−12	37		
				−12	9		
				(+)	(−)		
4	−6	7			28	b	a
					28	−42	49
					(−)		
							0

$\therefore a = 49$ and $b = -42$

40) If $ax^4 + bx^3 + 361x^2 + 220x + 100$ is a perfect square, then find the values of a, b.

Soln:- **Given**, $100 + 220x + 361x^2 + bx^3 + ax^4$ is a perfect square.

			10	11	12		
		10	100	220	361	b	a
			100				
			(−)				
	20	11		220	361		
				220	121		
				(+)	(−)		
20	22	12			240	b	a
					240	264	144
					(−)		
							0

$\therefore a = 144$ and $b = 264$

41) $36x^4 - 60x^3 + 61x^2 - mx + n$ is a perfect square, then find the values of m, n. **[M-22]**

Soln:- **Given**, $36x^4 - 60x^3 + 61x^2 - mx + n$ is a perfect square.

			6	−5	3		
		6	36	−60	61	$-m$	n
			36				
			(−)				
	12	−5		−60	61		
				−60	25		
				(+)	(−)		
12	−10	3			36	$-m$	n
					36	−30	9
					(−)		

	0

$\therefore m = 30$ and $n = 9$

42) $x^4 - 8x^3 + mx^2 + nx + 16$ is a perfect square, then find the values of m, n.

Soln:- **Given**, $x^4 - 8x^3 + mx^2 + nx + 16$ is a perfect square.

			1	-4	4		
		1	1	-8	m	n	16
			1				
			$(-)$				
	2	-4		-8	m		
				-8	16		
				$(+)$	$(-)$		
2	-8	4			$m-16$	n	16
					8	-32	16
					$(-)$		
							0

$\therefore m - 16 = 8$ and $n = -32$

$m = 16 + 8$ and $n = -32$

$m = 24$ and $n = -32$

43) Find the square root of :

$$(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$$

Soln:-

$6x^2 + x - 1 = (2x + 1)(3x - 1)$

-6	$+1$
$\frac{+3}{6x}$	$\frac{-2}{6x}$

$3x^2 + 2x - 1 = (x + 1)(3x - 1)$

-3	$+2$
$\frac{+3}{3x}$	$\frac{-1}{3x}$

$2x^2 + 3x + 1 = (x + 1)(2x + 1)$

$+2$	$+3$
$\frac{+2}{2x}$	$\frac{+1}{2x}$

$\therefore \sqrt{(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)}$

$= \sqrt{(2x + 1)(3x - 1)(x + 1)(3x - 1)(x + 1)(2x + 1)}$

$= \sqrt{(2x + 1)^2(3x - 1)^2(x + 1)^2}$

$= |\mathbf{(2x + 1)(3x - 1)(x + 1)}|$

44) Find the square root of :

$$(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$$

Soln:-

$4x^2 - 9x + 2 = (x - 2)(4x - 1)$

$+8$	-9
$\frac{-8}{4x}$	$\frac{-1}{4x}$

$7x^2 - 13x - 2 = (x - 2)(7x + 1)$

-14	-13
$\frac{-14}{7x}$	$\frac{+1}{7x}$

$28x^2 - 3x - 1$

$= (4x - 1)(7x + 1)$

-28	-3
$\frac{-7}{28x}$	$\frac{+4}{28x}$

$\therefore \sqrt{(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)}$

$= \sqrt{(x - 2)(4x - 1)(x - 2)(7x + 1)(4x - 1)(7x + 1)}$

$= \sqrt{(x - 2)^2(4x - 1)^2(7x + 1)^2}$

$= |\mathbf{(x - 2)(4x - 1)(7x + 1)}|$

45) Find the GCD of the given polynomials $x^4 + 3x^3 - x - 3$ and $x^3 + x^2 - 5x + 3$. **[S-20]**

Soln:- $f(x) = x^4 + 3x^3 - x - 3$

$g(x) = x^3 + x^2 - 5x + 3$

First we divide $f(x)$ by $g(x)$.

$$
\begin{array}{r|l}
 & x + 2 \\
\hline
x^3 + x^2 - 5x + 3 & x^4 + 3x^3 + 0x^2 - x - 3 \\
 & x^4 + x^3 - 5x^2 + 3x \\
 & (-)(-)(+)\quad(-) \\
\hline
 & \quad 2x^3 + 5x^2 - 4x - 3 \\
 & \quad 2x^3 + 2x^2 - 10x + 6 \\
 & (-)(-)(+)\,(-) \\
\hline
 & \qquad\quad 3x^2 + 6x - 9
\end{array}
$$

Remainder $= 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) \neq 0$

Then divide $x^3 + x^2 - 5x + 3$ by $x^2 + 2x - 3$.

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x^2 + 2x - 3 & x^3 + x^2 - 5x + 3 \\
 & x^3 + 2x^2 - 3x \\
 & (-)(-)(+) \\
\hline
 & \quad -x^2 - 2x + 3 \\
 & \quad -x^2 - 2x + 3 \\
 & (+)(+)(-) \\
\hline
 & \qquad\quad 0
\end{array}
$$

Here, Remainder $= 0$

$\therefore$ G.C.D $= \mathbf{x^2 + 2x - 3}$

46) Find the area of the quadrilateral whose vertices are at $(8,6), (5,11), (-5,12)$ and $(-4,3)$.

Soln:-

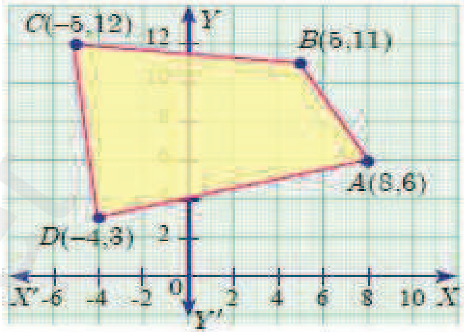
Given,

$(x_1, y_1) = (8, 6)$

$(x_2, y_2) = (5, 11)$

$(x_3, y_3) = (-5, 12)$

$(x_4, y_4) = (-4, 3)$



Area of the quadrilateral $= \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$

$= \frac{1}{2}\left\{\begin{matrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 11 & 12 & 3 & 6 \end{matrix}\right\}$

$= \frac{1}{2}(88 + 60 - 15 - 24 - 30 + 55 + 48 - 24)$

$= \frac{1}{2}(251 - 93)$

$= \frac{158}{2}$

$= 79$ Sq. Units.

47) Find the area of the quadrilateral whose vertices are at $(-9,-2), (-8,-4), (2,2)$ and $(1,-3)$.

Soln:-

Given, $(x_1, y_1) = (-9, -2)$, $(x_2, y_2) = (-8, -4)$;

$(x_3, y_3) = (1, -3)$; $(x_4, y_4) = (2, \quad 2)$

Area of the quadrilateral $= \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$

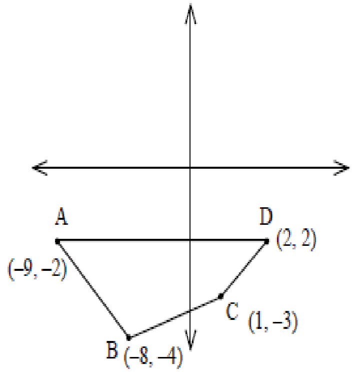
$= \frac{1}{2}\begin{Bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{Bmatrix}$

$= \frac{1}{2}(36 + 24 + 2 - 4 - 16 + 4 + 6 + 18)$

$= \frac{1}{2}(90 - 20)$

$= \frac{70}{2}$

$= 35$Sq. Units.



48) Find the area of the quadrilateral whose vertices are at$(-9,0), (-8,6), (-1,-2)$ and; $(-6,-3)$.

Soln:-

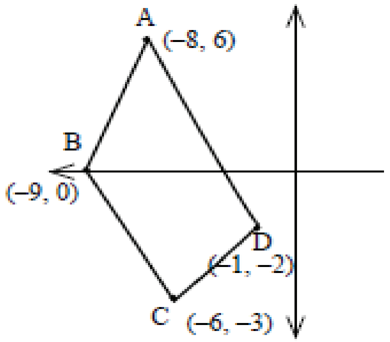
Given,

$(\boldsymbol{x_1}, \boldsymbol{y_1}) = (\boldsymbol{-9}, \boldsymbol{0})$

$(\boldsymbol{x_2}, \boldsymbol{y_2}) = (\boldsymbol{-6}, \boldsymbol{-3})$

$(\boldsymbol{x_3}, \boldsymbol{y_3}) = (\boldsymbol{-1}, \boldsymbol{-2})$

$(\boldsymbol{x_4}, \boldsymbol{y_4}) = (\boldsymbol{-8}, \boldsymbol{6})$



Area of the quadrilateral$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} -9 & -6 & -1 & -8 & -9 \\ 0 & -3 & -2 & 6 & 0 \end{Bmatrix}$

$= \frac{1}{2}(27 + 12 - 6 + 0 + 0 - 3 - 16 + 54)$

$= \frac{1}{2}(93 - 25)$

$= \frac{68}{2} = 34$Sq. Units.

49) Find the value of , if the area of a auadrilateral is 28 Sq. Units, whose vertices are $(-4,-2), (-3,k), (3,-2)$and; $(2,3)$.

Soln :-

Given , $(\boldsymbol{x_1}, \boldsymbol{y_1}) = (\boldsymbol{-9}, \boldsymbol{-2})$, $(\boldsymbol{x_2}, \boldsymbol{y_2}) = (\boldsymbol{-8}, \boldsymbol{-4})$;

$(\boldsymbol{x_3}, \boldsymbol{y_3}) = (\boldsymbol{1}, \boldsymbol{-3})$; $(\boldsymbol{x_4}, \boldsymbol{y_4}) = (\boldsymbol{2}, \quad \boldsymbol{2})$

Area of the quadrilateral= **28**Sq. Units

Area of the quadrilateral$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$

$28 = \frac{1}{2}\begin{Bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{Bmatrix}$

$2 \text{ x } 28 = -4k + 6 + 9 - 4 - 6 - 3k + 4 + 12$

$56 = -7k + 31 - 10$

$56 = -7k + 21$

$7k = 21 - 56$

$7k = -35$

$k = \frac{-35}{7}$

$k = -5$

50) If vertices of a quadrilateral are at $A(-5,7)$, $B(-4,k)$, $C(-1,-6)$ and ;$D(4,5)$ and its area is 72sq.units. Find the value of k.

Soln:-

Given, $(x_1,y_1) = A(-5,7)$, $(x_2,y_2) = B(-4,k)$;

$(x_3,y_3) = C(-1,-6)$; $(x_4,y_4) = D(4,5)$

Area of the quadrilateral $= 72$sq.units.

$$\text{Area of the quadrilateral} = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$72 = \frac{1}{2}\begin{Bmatrix} -5 & -4 & -1 & 4 & -5 \\ 7 & k & -6 & 5 & 7 \end{Bmatrix}$$

$$2 \text{ x } 72 = -5k + 24 - 5 + 28 + 28 + k + 24 + 25$$

$$144 = -4k + 129 - 5$$

$$144 = -4k + 124$$

$$4k = 124 - 144$$

$$4k = -20$$

$$k = \frac{-20}{4}$$

$$k = -5$$

51) The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost Rs.1300 per square feet. What will be the total cost for making the parking lot?

Soln :-

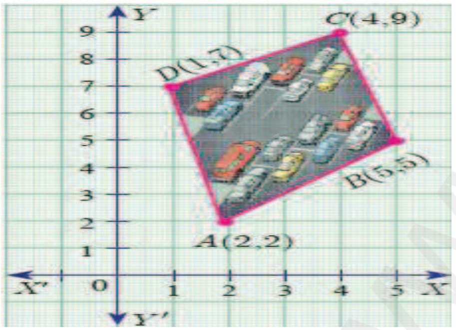
Given ,

$(x_1,y_1) = A(2,2)$

$(x_2,y_2) = B(5,5)$

$(x_3,y_3) = C(4,9)$

$(x_4,y_4) = D(1,7)$



$$\text{Area of parking lot} = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{Bmatrix}$$

$$= \frac{1}{2}(10 + 45 + 28 + 2 - 10 - 20 - 9 - 14)$$

$$= \frac{1}{2}(85 - 53)$$

$$= \frac{32}{2}$$

$= 16$ Sq. Units.

Given , Construction rate per square feet = Rs.1300.

$\therefore$ Total cost for constructing the parking lot = 16 x 1300 =Rs.20800

52) In the fig, the quadrilateral swimming pool shown is surroundedby concrete patio. Find the area of the patio. **[PTA-2]**

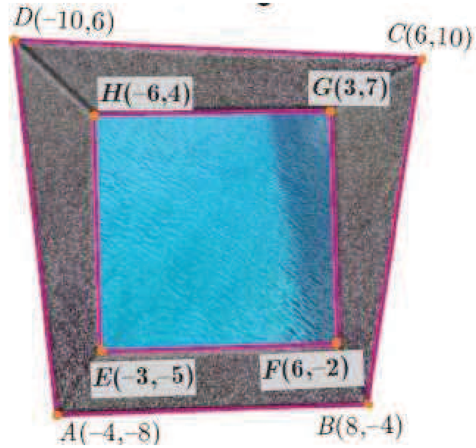
Given,

$(x_1,y_1) = A(-4,-8)$

$(x_2,y_2) = B(8,-4)$

$(x_3,y_3) = C(6,10)$

$(x_4,y_4) = D(-10,6)$



$$\text{Area of the Quadrilateral ABCD} = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -4 & 8 & 6 & -10 & 4 \\ -8 & 4 & 10 & 6 & 8 \end{Bmatrix}$$

$$= \frac{1}{2}(16 + 80 + 36 + 80 + 64 + 24 + 100 + 24)$$

$= \dfrac{424}{2}$

$= 212\text{Sq.units}$

Given,

$(x_1, y_1) = E(-3, -5)$

$(x_2, y_2) = F(6, -2)$

$(x_3, y_3) = G(3, 7)$

$(x_4, y_4) = H(-6, 4)$

Area of the Quadrilateral **EFGH** $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{Bmatrix}$

$= \dfrac{1}{2}(6 + 42 + 12 + 30 + 30 + 6 + 42 + 12)$

$= \dfrac{180}{2}$

$= 90\text{Sq.units}$

∴The area of the patio

=Area of the Quadrilateral **ABCD**– Area of the Quadrilateral **EFGH**$= 212 - 90$

$= 122\text{Sq.units.}$

53) Two dice are rololed. Find the probabilithy that the sum of outcomes **(i) equal to** 4**(ii) greater than** 10**(iii) less than** 13.**[S-21]**

Soln:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

∴ $n(S) = 36$

Let Abe the event of getting the sum of outcome values equal to 4.

$A = \{(1,3), (2,2), (3,1)\}$

$n(A) = 3$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$

Let Bbe the event of getting the sum of outcome values greater than 10.

$B = \{(5,6), (6,6), (6,6)\}$

$n(B) = 3$

$P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$

Let Cbe the event of getting the sum of outcome values less than 13.

Here,$C = S$

$n(C) = n(S) = 36$

$P(C) = \dfrac{n(C)}{n(S)} = \dfrac{36}{36} = 1$

54) Two unbiased dice are rolled once. Find the probability of getting (i) the doublet (equal numbers on both dice) (ii) the product as a prime number (iii) the sum as a prime number (iv) the sum as 1. **[S-20 , A-22]**

Soln:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

∴ $n(S) = 36$

(i) Let Abe the event of getting the doublet.

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Let B be the event of getting the product as a prime number.

$$B = \{(1,1), (1,2), (1,3), (1,5), (2,1), (3,1), (5,,1)\}$$

$$n(B) = 7$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{36}$$

(iii) Let C be the event of getting the sum as a prime number.

$$C = \left\{\begin{matrix}(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), \\ (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)\end{matrix}\right\}$$

$$n(C) = 15$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(iv) Let D be the event of getting the sum as 1.

$$D = \{\ \}$$

$$n(D) = 0$$

$$P(D) = \frac{n(D)}{n(S)} = 0$$

55) Two dice of blue color and gray color are rolled simultaneously. Write all the outcomes of this. What is the probability of getting the following addition of numbers rolled on the dice? (i) 8 (ii) 13 (iii) less than or equal to 12.

Soln:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$\therefore n(S) = 36$

(i) Let A be the event of getting the sum of outcome values equal to 8.

$$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(ii) Let B be the event of getting the sum of outcome values equal to 13.

$$B = \{\ \}$$

$$n(B) = 0$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$$

(iii) Let C be the event of getting the sum of outcome values less than or equal to 12.

$$C = S$$

$$n(C) = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

56) Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) at least one tail(iii) at most one head (iv) at most two tails.

Soln:-

Sample Space, $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$n(S) = 8$$

(i) Let A be the event of getting all heads.

$$A = \{HHH\}$$

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B be the event of getting at least one tail.

$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(B) = 7$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

(iii) Let C be the event of getting at most one head.

$C = \{HTT, THT, TTH, TTT\}$

$n(C) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let D be the event of getting at most two tails.

$D = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

$n(D) = 7$

$$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

57) From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability ofgetting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card.

Soln:-

Total Number of Cards $= 52$

$n(S) = 52$

(i) Let A be the event of getting a red card.

Number of red cards $= 26$

$n(A) = 26$

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card.

Number of heart cards $= 26$

$n(B) = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C be the event of getting a red king card.

Number of red king cards $= 2$

$n(C) = 2$

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card.

The face cards are :Jack (J), Queen (Q) and King (K).

Number of face cards $= 4 \text{ x } 3 = 12$

$n(D) = 12$

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card.

The number cards are : $2, 3, 4, 5, 6, 7, 8, 9$ and 10

Number of number cards $= 4 \text{ x } 9 = 36$

$n(E) = 36$

$$P(E) = \frac{n(E}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

58) The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) aking of black card.

Soln:-

Total Number of Cards $= 52 - 2 - 2 - 2 = 46$

$\therefore\ n(S) = 46$

(i) Let A be the event of getting a red card.

Number of clavor cards $= 13$

$n(A) = 13$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$$

(ii) Let Bbe the event of getting a red card.

Number of red queen cards= 0

$$n(B) = 0$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$$

(iii) Let Cbe the event of getting a red card.

Number of black king cards= 1

$$n(C) = 1$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$$

59) Two dice are rolled once. Find the probability of getting an even number on the first die or total offace sum 8.

<u>Soln</u>:-

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$\therefore$ $n(S) = 36$

Let Abe the event of getting an even number on the first die.

$$A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$$n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Let Bbe the event of getting the total offace sum 8.

$$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

Also, $A \cap B = \{(2,6), (4,4), (6,2)\}$

$$n(A \cap B) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

$\therefore$ By the addition theorem on probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$
$$= \frac{18 + 5 - 3}{36}$$
$$= \frac{23 - 3}{36}$$
$$= \frac{20}{36}$$
$$= \frac{5}{9}$$

60) Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

<u>Soln</u>:-

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$\therefore$ $n(S) = 36$

Let Abe the event of getting a doublet.

$$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

Let B be the event of getting the sum of faces as 4.

$$B = \{(1,3),(2,2),(3,1)\}$$
$$n(B) = 3$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

Also, $A \cap B = \{(2,2)\}$

$$n(A \cap B) = 1$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

∴ By the addition theorem on probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36}$$
$$= \frac{6+3-1}{36}$$
$$= \frac{9-1}{36}$$
$$= \frac{8}{36}$$
$$= \frac{2}{9}$$

61) If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face value 5.

Soln:-

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

∴ $n(S) = 36$

Let A be the event of getting the product of face value 6.

$$A = \{(1,6),(2,3),(3,2),(6,1)\}$$
$$n(A) = 4$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

Let B be the event of getting the difference of face value 5.

$$B = \{(6,1)\}$$
$$n(B) = 1$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{36}$$

Also, $A \cap B = \{(6,1)\}$

$$n(A \cap B) = 1$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

∴ By the addition theorem on probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{4}{36} + \frac{1}{36} - \frac{1}{36}$$
$$= \frac{4+1-1}{36}$$
$$= \frac{4}{36}$$
$$= \frac{1}{9}$$

62) From a well shuffled pack of 52 cards, a card is drawn at random. Find the probability of it beingeither a red king or a black queen.

Soln:-

Total Number of Cards $= 52$

$$n(S) = 52$$

Let A be the event of getting a red king.

Number of red red king cards= 2

$$n(A) = 2$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{2}{52}$$

Let Bbe the event of getting a black queen.

Number of red black queen cards= 2

$$n(B)=2$$

$$P(B)=\frac{\mathrm{n(B)}}{\mathrm{n(S)}}=\frac{2}{52}$$

Here the events A and B are mutually exclusive,

$$A\cap B=\{\,\}\Longrightarrow n(A\cap B)=0\Longrightarrow\ P(A\cap B)=0$$

∴ By the addition theorem on probability,

$$P(A\cup B)=P(A)+P(B)$$

$$=\frac{2}{52}+\frac{2}{52}$$

$$=\frac{2+2}{52}$$

$$=\frac{4}{52}$$

$$=\frac{1}{13}$$

63) A box contains cards numbered 3, 5, 7, 9, …, 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Soln:-

$$S=\{3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37\}$$

∴ $n(S)=18$

Let Abe the event of getting the drawn card have multiples of 7.

$$A=\{7,21,35\}$$

$$n(A)=3$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{3}{18}$$

Let Bbe the event of getting the drawn card have a prime number.

$$B=\{3,5,7,11,13,17,19,23,29,31,37\}$$

$$n(B)=11$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{11}{18}$$

Also, $A\cap B=\{7\}$

$$n(A\cap B)=1$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{1}{18}$$

∴ By the addition theorem on probability,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{3}{18}+\frac{11}{18}-\frac{1}{18}$$

$$=\frac{3+11-1}{18}$$

$$=\frac{14-1}{18}$$

$$=\frac{13}{18}$$

64) Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or at least 2 heads.

Soln:-

$$S=\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$$

$\therefore n(S)=8$

Let Abe the event of getting atmost 2 tails.

$$A=\{HHH,HHT,HTH,HTT,THH,THT,TTH\}$$

$$n(A)=7$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{7}{8}$$

Let B be the event of getting at least 2 heads.

$$B=\{HHH,HHT,HTH,THH\}$$

$$n(B)=4$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{4}{8}$$

Also, $A\cap B=\{HHH,HHT,HTH,THH\}$

$$n(A\cap B)=4$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{4}{8}$$

∴ By the addition theorem on probability,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{7}{8}+\frac{4}{8}-\frac{4}{8}$$

$$=\frac{7}{8}$$

65) A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail orconsecutive two heads.**[PTA-2, PTA-6]**

Soln:-

$$S=\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$$

∴ $n(S)=8$

Let Abe the event of getting exactly two heads.

$$A=\{HHT,HTH,THH\}$$

$$n(A)=3$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{3}{8}$$

Let Bbe the event of getting atleast one tail.

$$B=\{HHT,HTH,HTT,THH,THT,TTH,TTT\}$$

$$n(B)=7$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{7}{8}$$

Let Cbe the event of getting consecutive two heads.

$$C=\{HHH,HHT,THH\}$$

$$n(C)=3$$

$$P(C)=\frac{n(C)}{n(S)}=\frac{3}{8}$$

$A\cap B=\{HHT,HTH,THH\}$

$n(A\cap B)=3$

$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{8}$

$B\cap C=\{HHT,THH\}$

$n(B\cap C)=2$

$P(B\cap C)=\frac{n(B\cap C)}{n(S)}=\frac{2}{8}$

$A\cap C=\{HHT,THH\}$

$n(A\cap C)=2$

$P(B\cap C)=\frac{n(B\cap C)}{n(S)}=\frac{2}{8}$

Also, $A\cap B\cap C=\{HHT,THH\}$

$$n(A\cap B\cap C)=2$$

$$P(A\cap B\cap C)=\frac{n(A\cap B\cap C)}{n(S)}=\frac{2}{8}$$

∴ By the addition theorem on probability,

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

$$=\frac{3}{8}+\frac{7}{8}+\frac{3}{8}-\frac{3}{8}-\frac{2}{8}-\frac{2}{8}+\frac{2}{8}$$

$$=\frac{7+3-2}{8}$$

$$=\frac{10-2}{8}$$

$= \frac{8}{8}$

$= 1$

66) In class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted for NCC and NSS. One of the students is selected at random. Find the probability that,

(i) The selected student opted for NCC but not NSS

(ii) The selected student opted for NSS but not NCC

(iii) The selected student opted for exactly one of them.

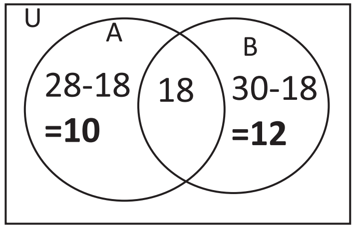
[M-22, PTA-1, PTA-4]

Soln:-

A = opted for NCC

B = opted for NSS

Given, $n(S) = 50$, $n(A) = 28$, $n(B) = 30$, $n(A \cap B) = 18$



(i) The probability that the selected student opted for NCC but not NSS

$= \frac{10}{50}$

$= \frac{1}{5}$

(ii) The probability that theselected student opted for NSS but not NCC

$= \frac{12}{50}$

$= \frac{6}{25}$

(iii) The probability that theselected student opted for exactly one of them

$= \frac{10}{50} + \frac{12}{50}$

$= \frac{22}{25}$

$= \frac{11}{25}$

67) Find the HCF of :396, 504, 636. **[S-21]**

Soln:-

First we have to find HCF of 396and504

Here, $a = 504$and$b = 396$

$504 = 396 \text{ x } 1 + 108$; Remainder$= 108 \neq 0$

$396 = 108 \text{ x } 3 + 72$; Remainder$= 72 \neq 0$

$108 = 72 \text{ x } 1 + 36$; Remainder$= 36 \neq 0$

$72 = 36 \text{ x } 2 + 0$

Here, Remainder$= 0$

Therefore, the HCF of 396 and504 $= 36$

To find the HCF of 636 and36

Here, $a = 636$and$b = 36$

$636 = 36 \text{ x } 17 + 24$; Remainder$= 24 \neq 0$

$36 = 24 \text{ x } 1 + 12$; Remainder$= 12 \neq 0$

$24 = 12 \text{ x } 2 + 0$

Here, Remainder$= 0$

Therefore, the HCF of636and36 $= 12$

∴ The HCF of396, 504and636 $= 12$

68) The sum of 3 consecutive terms that are in A.P.is 27 and their product is 288. Find the 3 terms.**[S-21]**

Soln:-

Let us take the consecutive 3 terms of an A.P in the form $a-d, a, a+d$.

Given, $a-d+a+a+d=27$

$$3a=27$$
$$a=\frac{27}{3}$$
$$a=9$$

Given, $(a-d) \text{x } a \text{ x } (a+d)=288$

$(a^2-d^2) \text{x } a=288$

$(9^2-d^2) \text{x } 9=288$

$$81-d^2=\frac{288}{9}$$
$$81-d^2=32$$
$$81-32=d^2$$
$$49=d^2$$
$$d=\pm 7$$
$$\Rightarrow a=9 \text{ and } d=\pm 7$$

(i) If $a=9$ and $d=7$ then the 3 terms are,

$9-7,\quad 9,\quad 9+7$

$2,\quad 9,\quad 16$

(ii) If $a=9$ and $d=-7$ then the 3 terms are,

$9-\!-7,\quad 9,\quad 9+(-7)$

$9+7,\quad 9,\quad 9-7$

$16,\quad 9,\quad 2$

69) Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Soln:-

Here, $a=301,\ l=595,\ d=11$

WKT, $n=\left(\frac{l-a}{d}\right)+1$

$$n=\left(\frac{595-301}{7}\right)+1$$
$$=\left(\frac{294}{7}\right)+1$$
$$=42+1$$
$$n=43$$

WKT, $S_n=\frac{n}{2}(a+l)$

$$S_{43}=\frac{43}{2}(301+595)$$
$$=\frac{43 \text{ x } 896}{2}$$
$$=43 \text{ x } 448$$
$$=19264$$

∴ The sum of all natural numbers between 300 and 600 which are divisible by 7 = 19624

70) Find the sum of all natural numbers between 100 and 1000 which are divisible by 11.**[S-20]**

Soln:-

$$\begin{array}{r|r} & 9 \\ \hline 11 & 100 \\ & 99 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} & 90 \\ \hline 11 & 1000 \\ & 99 \\ \hline & 10 \end{array}$$

Here, $a=100+11-1=111-1=110$

$l=1000-10=990$ and $d=11$

WKT, $n=\left(\frac{l-a}{d}\right)+1$

$$n=\left(\frac{990-110}{11}\right)+1$$
$$=\left(\frac{880}{11}\right)+1$$
$$=80+1$$
$$n=81$$

WKT, $S_n=\frac{n}{2}(a+l)$

$$S_{81}=\frac{81}{2}(110+990)$$
$$=\frac{81 \text{ x } 1100}{2}$$
$$=81 \text{ x } 550$$
$$=44550$$

∴ The sum of all natural numbers between 300 and 600 which are divisible by 7 = 44550

71) Find the sum to n terms of the series: $5+55+555+\cdots$.

Soln:-

$S_n=5+55+555+\cdots n$ terms

$=5(1+11+111+\cdots n \text{ terms})$

$=5 \text{ x } \frac{9}{9}\ (1+11+111+\cdots n \text{ terms})$

$=\frac{5}{9}(9+99+999+\cdots n \text{ terms})$

$= \frac{5}{9}[(10-1)+(100-1)+(1000-1)+\cdots n \text{ terms}]$

$= \frac{5}{9}[(10+100+1000+\cdots n \text{ terms}) - (1+1+1+\cdots n \text{ terms})]$

WKT,

$S_n = \frac{a(r^n-1)}{r-1}$ Here, $a = 10, r = 10$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

$$= \frac{50(10^n-1)}{9} - \frac{5n}{9}$$

72) Find the sum to n terms of the series: $3+33+333+\cdots$.

Soln:-

$S_n = 3+33+333+\cdots n\text{terms}$

$= 3(1+11+111+\cdots n\text{terms})$

$= 3 \times \frac{9}{9}(1+11+111+\cdots n\text{terms})$

$= \frac{1}{3}(9+99+999+\cdots n\text{terms})$

$= \frac{1}{3}[(10-1)+(100-1)+(1000-1)+\cdots n\text{terms})$

$= \frac{1}{3}[(10+100+1000+\cdots n\text{terms})$

$-(1+1+1+\cdots n\text{terms})$

WKT,

$S_n = \frac{a(r^n-1)}{r-1}$Here,$a = 10, r = 10$

$$= \frac{1}{3}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{1}{3}\left[\frac{10(10^n-1)}{9} - n\right]$$

$$= \frac{10(10^n-1)}{9} - \frac{n}{3}$$

73) Find the sum of : $15^2+16^2+17^2+\cdots+28^2$

Soln:-

WKT, $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore 15^2+16^2+17^2+\cdots+28^2$

$= (1^2+2^2+3^2+\cdots+28^2) - (1^2+2^2+3^2+\cdots+14^2)$

$$= \frac{28 \times 29 \times 57}{6} - \frac{14 \times 15 \times 29}{6}$$

$= 14 \times 29 \times 19 - 7 \times 5 \times 29$

$= 7714 - 1015$

$= 6699$

74) Find the sum of: $6^2+7^2+8^2+\cdots+21^2$

Soln:-

WKT, $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore 6^2+7^2+8^2+\cdots+21^2$

$= (1^2+2^2+3^2+\cdots+21^2) - (1^2+2^2+3^2+\cdots+5^2)$

$$= \frac{21 \times 22 \times 43}{6} - \frac{5 \times 6 \times 11}{6}$$

$= 7 \times 11 \times 43 - 5 \times 11$

$= 77 \times 43 - 55$

$= 3311 - 55$

$= 3256$

75) Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm,…, 24cm. How much area can bedecorated with these colour papers? **[PTA-1]**

தீர்வு:-

WKT, **1)**Area of the square = a^2

2)$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore$ Area can bedecorated with 15 square colour papers

$= 10^2 + 11^2 + 12^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2) - (1^2 + 2^2 + 3^2 + \cdots + 9^2)$

$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$

$= 100 \times 49 - 15 \times 19$

$= 4900 - 285$

$= 4615$Sq.cm

76) Find the sum of: $9^3 + 10^3 + 11^3 + \cdots + 21^3$

தீர்வு:-

WKT, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore 9^3 + 10^3 + 11^3 + \cdots + 21^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 21^3) - (1^3 + 2^3 + 3^3 + \cdots + 8^3)$

$= \left(\frac{21 \times 22}{2}\right)^2 - \left(\frac{8 \times 9}{2}\right)^2$

$= (231)^2 - (36)^2$

$= 53361 - 1296$

$= 52065$

77) Find the sum of: $10^3 + 11^3 + 12^3 + \cdots + 20^3$ **[PTA-5]**

Soln:-

WKT, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore 10^3 + 11^3 + 12^3 + \cdots + 20^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 20^3) - (1^3 + 2^3 + 3^3 + \cdots + 9^3)$

$= \left(\frac{20 \times 21}{2}\right)^2 - \left(\frac{9 \times 10}{2}\right)^2$

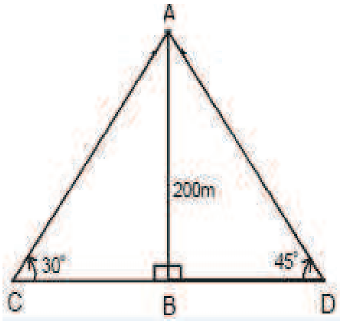
$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= 42075$

78) Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30^o and 45^o respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)

Soln:-



AB = Height of the lighthouse = 200 m

CD = Distance between the two ships

$= x + y$

In right triangle ΔABC ,

$tan30^o = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{200}{x}$

$x = 200\sqrt{3}$

$= 200 \times 1.732$

$= 346.4$ m

In right triangle ΔABD ,

$tan45^o = \frac{AB}{BC}$

$1 = \frac{200}{x}$

$y = 200$ m

$\therefore$ Distance between two ships $= x + y$

$= 346.4 + 200$

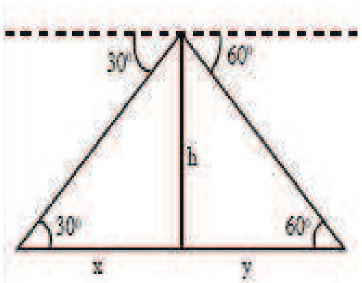
$= 546.4$ m

79) From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30^o and 60^o. If the height of the lighthouse is h metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Soln:-

AB = Height of the lighthouse = h m

CD = Distance between the two ships $= x + y$



In right triangle ΔABC,

$$tan30^o = \frac{AB}{BC}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$
$$x = h\sqrt{3} \text{ m}$$

In right triangle ΔABD,

$$tan60^o = \frac{AB}{BC}$$
$$\sqrt{3} = \frac{h}{y}$$

$y = \frac{h}{\sqrt{3}}$m

∴Distance between two ships= $x + y$

$= h\sqrt{3} + \frac{h}{\sqrt{3}} = \frac{h(\sqrt{3})^2+h}{\sqrt{3}}$m

$$= \frac{3h + h}{\sqrt{3}}$$

$= \frac{4h}{\sqrt{3}}$m

80) From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3} = 1.732$) **[M-22]**

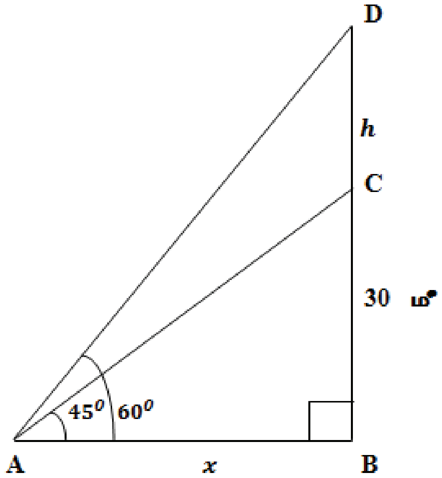
Soln:-

BC = Height of the building = 30 m

CD = Height of the tower= h

Let $AB = x$

In a right angled ΔABC,



$$tan45^o = \frac{BC}{AB}$$
$$1 = \frac{30}{x}$$
$$x = 30$$

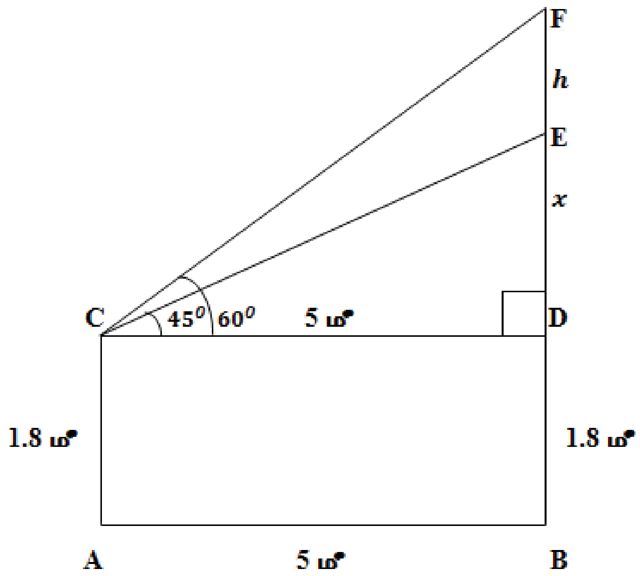
In a right angled ΔABD,

$$tan60^o = \frac{BD}{AB}$$
$$\sqrt{3} = \frac{30 + h}{x}$$
$$x\sqrt{3} = 30 + h$$
$$30\sqrt{3} = 30 + h$$
$$30\sqrt{3} - 30 = h$$
$$30(\sqrt{3} - 1) = h$$
$$h = 30(1.732 - 1)$$
$$h = 30(0.732)$$
$$h = 21.96\text{m}$$

∴ Height of the tower, $h = 21.96$m

81) To a man standing outside his house, the angles of elevation of the top and bottom of a window are60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window?($\sqrt{3} = .732$)**[A-22]**

Soln:-



$AC = BD$ =Height of the viewer= 180cm= 1.8m

$AB = CD$ =The distance between the viewer and the wall.= 5m

EF =Height of the window= h

$DE = x$

In a right angled ΔCDE,

$$tan45^o = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{ED}{CD}$$

$$1 = \frac{x}{5}$$

$$x = 5$$

In a right angled ΔCDF,

$$tan60^o = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{FD}{CD}$$

$$\sqrt{3} = \frac{x+h}{5}$$

$$5\sqrt{3} = 5 + h$$

$$5\sqrt{3} - 5 = h$$

$$5(\sqrt{3} - 1) = h$$

$$5(1.732 - 1) = h$$

$$5(0.732) = h$$

$$h = 3.66\text{m}$$

∴ Height of the window, $h = 3.66$m

82) A cylindrical drum has a height of 20cm andbase radius of 14cm. Find its curved surface area and thetotal surface area.**[A-22]**

Soln:-

Given, Height,$h = 20$ cm

Radius,$r = 14$cm

(i) Curved surface area of the cylinder= $2\pi rh$

$$= 2 \text{ x} \frac{22}{7} \text{x } 14 \text{ x } 20$$

$$= 88 \text{ x } 20$$

$$= 1760\text{Sq.cm}$$

(ii) Total surface area of the cylinder= $2\pi r(h + r)$

$$= 2 \text{ x} \frac{22}{7} \text{x } 14 \text{ x } (20 + 14)$$

$$= 88 \text{ x } 34$$

$$= 2992\text{Sq.cm}$$

83) If the circumference of a conical wooden piece is 484cm then find its volume when its height is105cm.**[A-22]**

Soln:-

Let the radius be r.

Given, Height,$h = 105$cm

Circumference of a conical wooden piece= 484cm

$$2\pi r = 484$$

$$2 \text{ x} \frac{22}{7} \text{x } r = 484$$

$$r = \frac{484 \text{ x } 7}{2 \text{ x } 22}$$
$$r = 11 \text{ x } 7$$
$$r = 77\text{cm}$$

∴ Volume of the cone= $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \text{x} \frac{22}{7} \text{x } 77 \text{ x } 77 \text{ x } 105$$
$$= 22 \text{ x } 11 \text{ x } 77 \text{ x } 35$$
$= 652190$க.செ.மீ.

84) If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

Soln:-

Given, $R = 28$ cm
$r = 7$ cm
$h = 45$ cm

Volume of the frustum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

$$= \frac{22 \text{ x } 45}{7 \text{ x } 3}(28^2 + 7^2 + 28 \text{ x } 7)$$
$$= \frac{22 \text{ x } 15}{7}(784 + 49 + 196)$$
$$= \frac{22 \text{ x } 15 \text{ x } 1029}{7}$$
$$= 22 \text{ x } 15 \text{ x } 147$$

$= 48510$ cu.cm

∴ Volume of the frustum = 48510 cu.cm

85) A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of Rs.40 per litre.

Soln:- **Given**, $R = 20$ cm, $r = 8$ cm, $h = 16$ cm

Volume of the frustum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

$$= \frac{22 \text{ x } 16}{7 \text{ x } 3}(20^2 + 8^2 + 20 \text{ x } 8)$$
$$= \frac{352}{21}(400 + 64 + 160)$$
$$= \frac{352 \text{ x } 624}{21}$$
$$= \frac{352 \text{ x } 208}{7}$$
$$= \frac{73216}{7}$$

$= 10459.4$cu.cm

$= \frac{10459.4}{1000}$Litres

$= 10.4594$Litres

Given, Cost of 1 litre milk = Rs.40

The cost of milk which can completely fill a container

= Rs.40 x 10.4594

=Rs.418.38

86) A right circular cylindrical container of base radius 6cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9cm and base radius 3cm,having a hemispherical cap. Find the number of cones needed to empty the container. **[PTA-5, PTA-6]**

Soln:-

Cylinder:-

Radius,$r = 6$cm

Height,$h = 15$cm

Volume= $\pi r^2 h$

$$= \pi \text{ x}(6)^2 \text{x } 15$$
$$= \pi \text{ x } 6 \text{ x } 6 \text{ x } 15$$

Ice cream cone:-

Radius,$r = 3$cm

Height,$h = 9$cm

Volume= $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^2(2r + h)$$
$$= \frac{1}{3}\pi (3)^2[2(3) + 9]$$
$$= \pi \text{ x } 3 \text{ x } 15$$

∴ The number of cones needed

$= \dfrac{\text{Volume of the ice cream in the container}}{\text{Volume of the ice cream in a cone}}$

$= \dfrac{\pi \text{ x } 6 \text{ x } 6 \text{ x } 15}{\pi \text{ x } 3 \text{ x } 15}$

$= 12$

87) An aluminium sphere of radius 12cm is melted to make a cylinder of radius 8cm. find the height ofthe cylinder.

Soln:-

Aluminium sphere:-

Radius, $r = 12$cm

Volume$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \text{x } \pi \text{ x} (12)^3$

$= \frac{4}{3} \text{x } \pi \text{ x } 12 \text{ x } 12 \text{ x } 12$

$= 4 \text{ x } \pi \text{ x } 4 \text{ x } 12 \text{ x } 12$

Cylinder:-

Radius, $r = 8$cm

Let the height be $'h'$cm

Volume$= \pi r^2 h$

$= \pi \text{ x} (8)^2 \text{x } h$

$= \pi \text{ x } 8 \text{ x } 8 \text{ x } h$

Here, Volume of the cylinder=Volume of the sphere

$$\pi \text{ x } 8 \text{ x } 8 \text{ x } h = 4 \text{ x } \pi \text{ x } 4 \text{ x} 12 \text{ x} 12$$

$$h = \frac{4 \text{ x } \pi \text{ x } 4 \text{ x} 12 \text{ x} 12}{\pi \text{ x } 8 \text{ x } 8}$$

$$h = 36\text{cm}$$

$\therefore$ Height of the cylinder, $h = 36$cm

88) Find the standard deviation of 70, 80, 60, 50, 40, 90, 95 **[S-20]**

Soln:-

Assumed mean, $A = 70$

x	$d = x - A$	d^2
70	0	0
80	10	100
60	-10	100
50	-20	400
40	-30	900
90	20	400
95	25	625
	-5	2525

$\sum d^2 = 2525$

$\sum d = -5$

$n = 7$

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{2525}{7} - \left(\frac{-5}{7}\right)^2}$$

$$= \sqrt{360.7 - 0.51}$$

$$= \sqrt{360.19}$$

$$= \sqrt{360.2}$$

$\sigma = 18.98$

89) Find the coefficient of variation of :24, 26, 33, 37, 29, 31.

Soln:-

$\bar{x} = \frac{\sum x}{n} = \frac{180}{6} = 30$

x	$d = x - \bar{x}$	d^2
24	-6	36
26	-4	16
33	3	9

Here,

$\sum d^2 = 112$

$n = 6$

$\therefore \sigma = \sqrt{\frac{\sum d^2}{n}}$

Therefore

$C.V = \frac{\sigma}{\bar{x}} \text{x } 100 \%$

$= \frac{4.321}{30} \text{x } 100$

$= \frac{43.21}{3}$

37	7	49
29	−1	1
31	1	1
180	$\sum d^2$	112

$= \sqrt{\frac{112}{6}}$

$= \sqrt{18.666}$

$= \sqrt{18.67}$

$= 4.321$

$= 14.4\ \%$

90) Find the coefficient of variation of :38, 40, 34, 31, 28, 26, 34 .

Soln:-

x	$d = x - \bar{x}$	d^2
38	5	25
40	7	49
34	1	1
31	−2	4
28	−5	25
26	−7	49
34	1	1
231		154

$\bar{x} = \frac{\sum x}{n}$

$= \frac{231}{7}$

$= 33$

Here,

$\sum d^2 = 154$

$n = 7$

$\therefore\ \sigma = \sqrt{\frac{\sum d^2}{n}}$

$= \sqrt{22} = 4.69$

$\therefore$ C.V= $\frac{\sigma}{\bar{x}}$ x 100%

$= \frac{4.69}{33}$ x 100

$= \frac{469}{33}$

$= 14.21\ \%$

91) The time taken (in minute) to complete a homework by 8 students in a day are given by 38, 40,47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Soln:-

x	$d = x - \bar{x}$	d^2
38	−7	49
40	−5	25
47	2	4
44	−1	1
46	1	1
43	−2	4
49	4	16
3	8	64
360		164

$\bar{x} = \frac{\sum x}{n} = \frac{360}{8} = 45$

Here,

$\sum d^2 = 164$

$n = 8$

WKT, $\sigma = \sqrt{\frac{\sum d^2}{n}}$

$\therefore\ \sigma = \sqrt{\frac{164}{8}}$

$= \sqrt{20.5}$

$= 4.53$

$\therefore$ C.V= $\frac{\sigma}{\bar{x}}$ x 100%

$= \frac{4.53}{45}$ x 100

$= \frac{453}{45}$

$= 10.066$

$= 10.07$ %

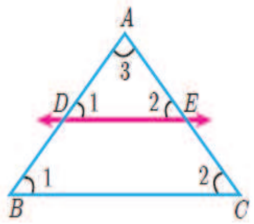
92) **Thales Theorem or Basic proportanality Theorem (BPT)**:-

Statement:-

A straight line drawn parallel to a side of triangle intersection the other two sides, divides the sides in the same ratio.

Given:-

In ΔABC, $D$is point on $AB E$is a point on AC.



To Prove:-

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction:-

Draw a kube$DE \parallel BC$.

Proof:-

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because DE || BC
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because DE || BC
3.	$\angle DAE = \angle BAC = \angle 3$	Both angles have a common angle
4.	$\Delta ABC \sim \Delta ADE$	By AAA Similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	Split AB and AC using the points D and E
	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals

Hence proved.

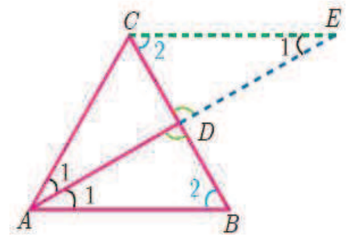
93) **Angle Bisector Theorem (ABT)**:-

Statement:-

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Given:-

In ΔABC, AD is the internal bisector.



To Prove:-

$$\frac{AB}{AC} = \frac{BD}{DC}$$

Construction:-

Draw a line through C parallel to AB.

Extend AD to meet the line through Cat E.

Proof:-

No	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal..
2.	ΔACE is an isosceles $AC = CE$→**(1)**	In ΔACE, $\angle CAE = \angle CEA$
3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE

Hence Proved.

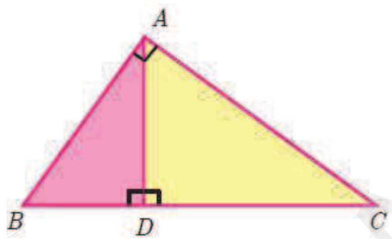
94) **PYTHAGORAS THEOREM**:-

Statement:-

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

Given:-

In ΔABC , $\angle A = 90^o$



To Prove:-

$AB^2 + AC^2 = BC^2$

Construction:-

Draw $AD \perp BC$

Proof:-

No	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common. $\angle BAC = \angle BDA = 90^o$ Therefore, $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC$xBD→**(1)**	Given $\angle BAC = 90^o$ and by construction $\angle BDA = 90^o$ By AA Similarity
2.	Compare ΔABC and ΔDAC $\angle C$ common. $\angle BAC = \angle ADC = 90^o$ Therefore, $\Delta ABC \sim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC$xDC→**(2)**	Given $\angle BAC = 90^o$ and by construction $\angle BDA = 90^o$ By AA Similarity

(1) + (2) : $AB^2 + AC^2 = BC\text{x}BD + BC\text{x}DC$

$= BC(BD + DC)$

$= BC \text{ x } BC = BC^2$

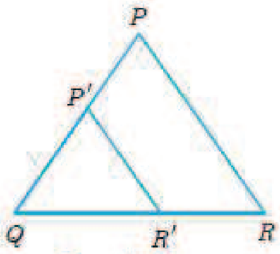
Hence Proved.

EIGHT MARK QUESTIONS

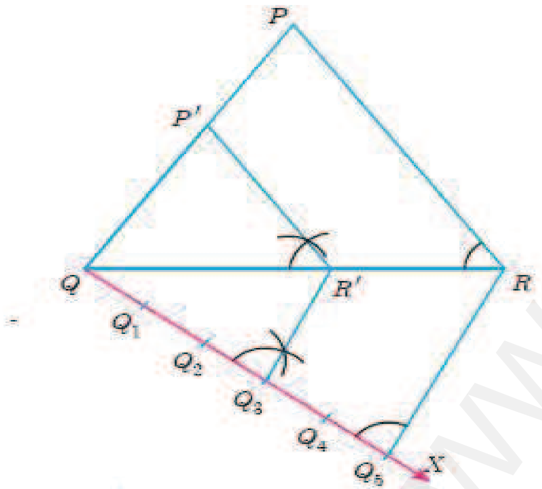
1) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR. (Scale factor $\frac{3}{5} < 1$).

Soln:-

Given, Scale factor $\frac{3}{5} < 1$



Rough diagram

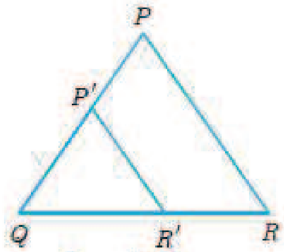


$\Delta P'QR'$ is the required similar triangle.

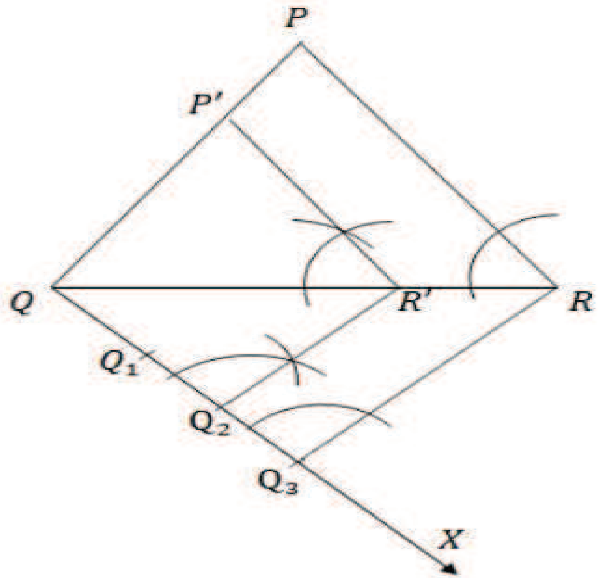
2) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR. (Scale factor $\frac{2}{3} < 1$).

Soln:-

Given, Scale factor $\frac{3}{5} < 1$



Rough diagram



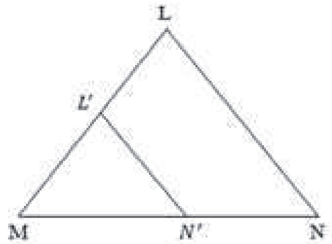
$\Delta P'QR'$ is the required similar triangle.

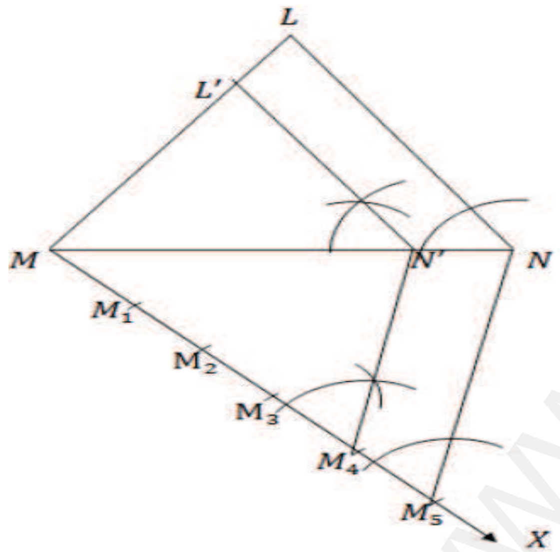
3) Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN. (Scale factor $\frac{4}{5} < 1$).

Soln:-

Rough Diagram

Given, Scale factor $\frac{4}{5} < 1$



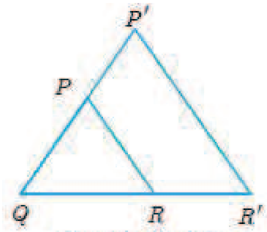


$\Delta L'MN'$ is the required similar triangle.

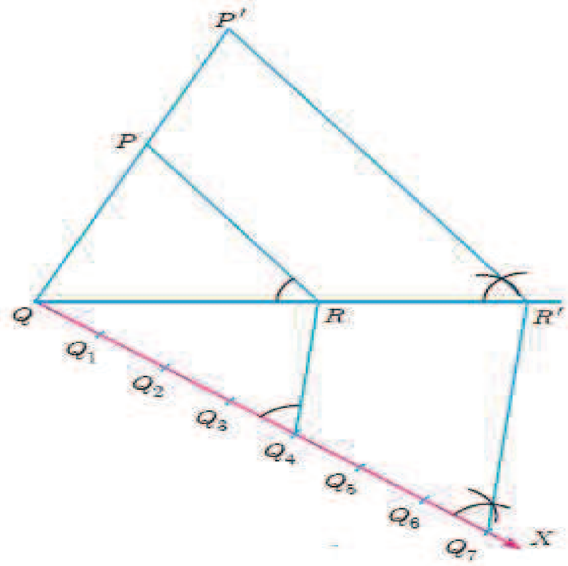
4) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR. (Scale factor $\frac{7}{4} > 1$).

Soln:-

Given, Scale factor $\frac{7}{4} > 1$



Rough diagram



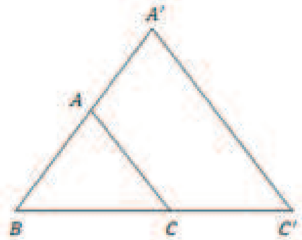
$\Delta P'QR'$ is the required similar triangle.

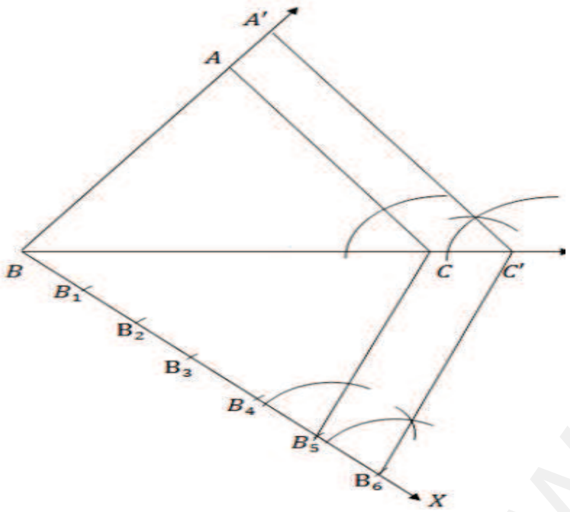
5) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$of the corresponding sides of the triangle ABC. (Scale factor $\frac{6}{5}$> 1). **[PTA-1, S-20]**

Soln:-

Rough Diagram

Given, Scale factor$\frac{6}{5} > 1$



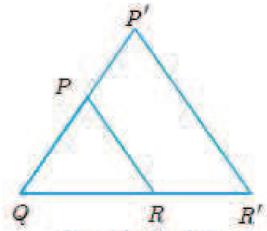


$\Delta A'BC'$is the required similar triangle.

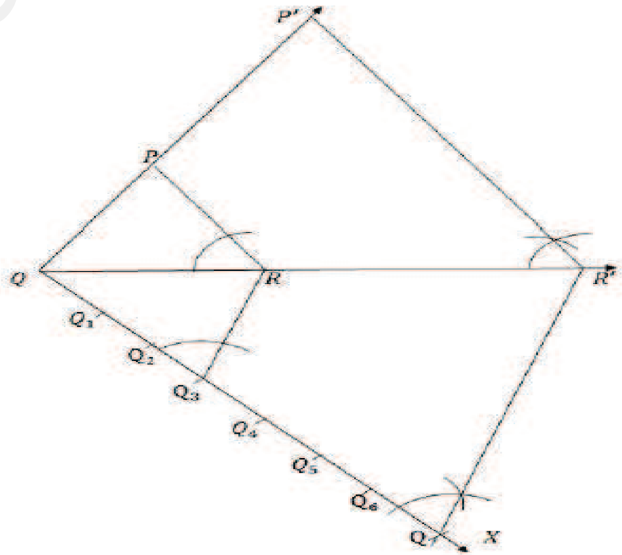
6) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$of the corresponding sides of the triangle PQR. (Scale factor $\frac{7}{3}$> 1).**[A-22]**

Soln:-

Given, Scale factor$\frac{7}{3} > 1$



Rough diagram



$\Delta P'QR'$is the required similar triangle.

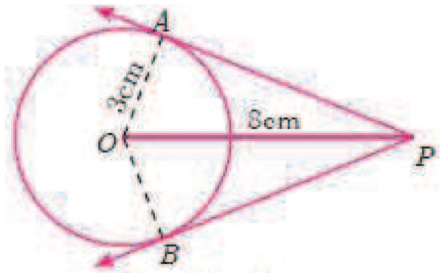
7) Draw a circle of diameter 6cm from a point P, which is 8cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.**[PTA-6, S-21, A-22]**

Soln:-

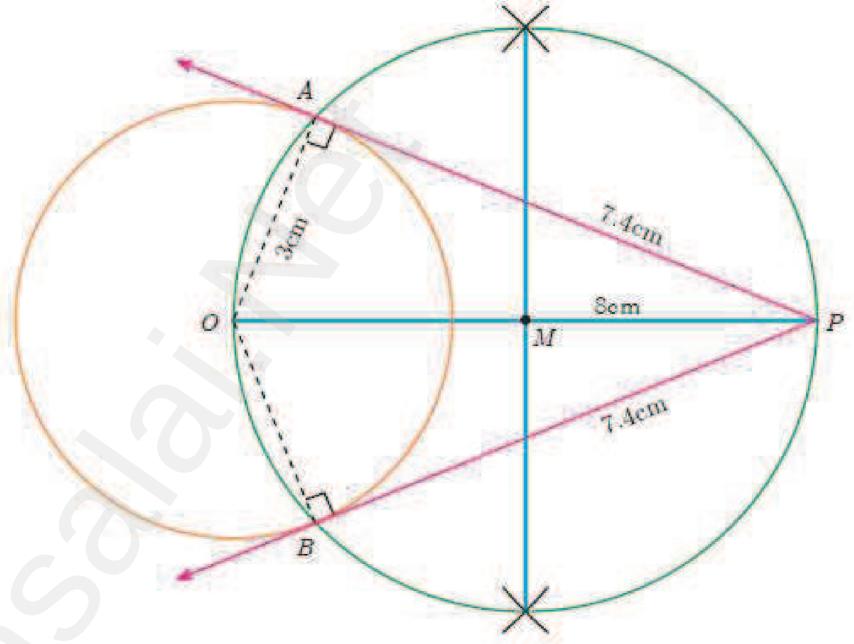
Gven, Diameter= 6cm

Radius= $\frac{6}{2}$ = 3cm

Distance= 8cm



Rough diagram



$\therefore$ PA and PB are the two required tangents.
Lengths of the tangents, $PA = PB = 7.4$ cm

Verification:-
In the right angle triangle OAP,

$$PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{8^2 - 3^2}$$
$$= \sqrt{64 - 9}$$
$$= \sqrt{55}$$
$$= \sqrt{7.4 \text{ x } 7.4}$$
$$PA = 7.4 \text{ cm}$$

8) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Alsomeasure the lengths of the tangents. **[S-20]**

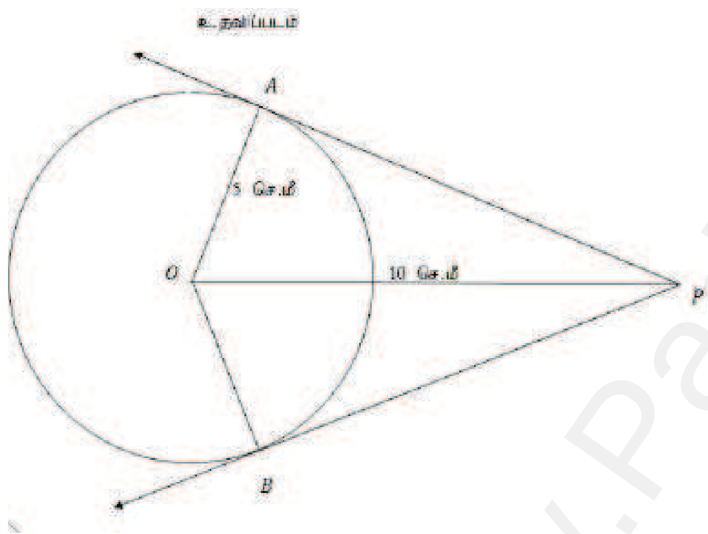
Soln:-

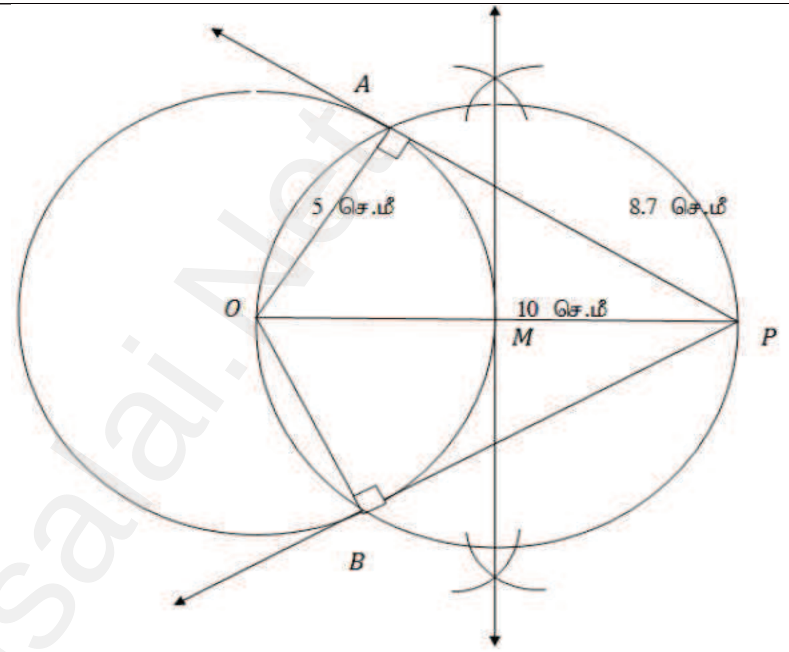
Gven, Diameter = 6 cm

Radius $= \frac{6}{2} = 3$ cm

Distance = 8 cm

Rough Diagram





∴ PAandPB are the two required tangents.
Lengths of the tangents, $PA = PB = 7.4$ cm

Verification:-

In the right angle triangle OAP,

$$PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{10^2 - 5^2}$$
$$= \sqrt{100 - 25}$$
$$= \sqrt{75}$$
$$= \sqrt{8.7 \text{ x } 8.7}$$
$$PA = 8.7\text{cm}$$

9) Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point. **[PTA-2]**

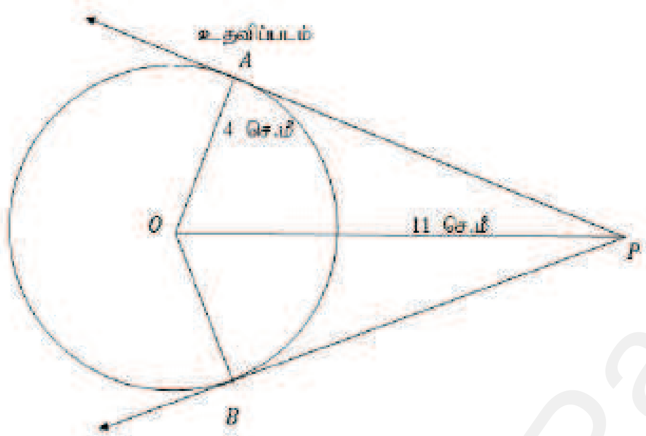
Soln:-

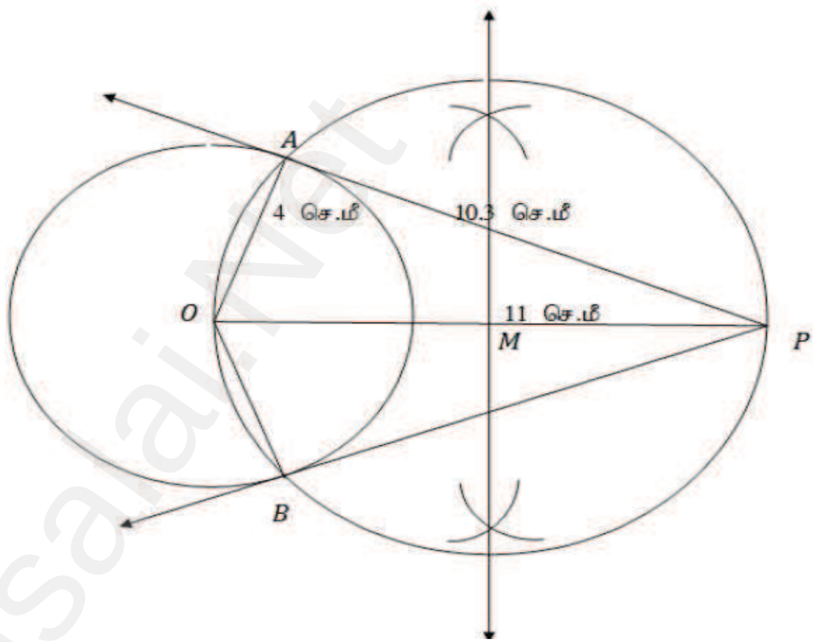
Gven,

Radius = 4 cm

Distance = 11cm

Rough Diagram





∴ PAandPB are the two required tangents.
Lengths of the tangents, $PA = PB = 7.4$ cm

Verification:-

In the right angle triangle OAP,

$$PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{11^2 - 4^2}$$
$$= \sqrt{121 - 16}$$
$$= \sqrt{105}$$
$$= \sqrt{10.3 \text{ x } 10.3}$$
$$PA = 10.3\text{cm}$$

10) Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents. **[S-22 , M-22]**

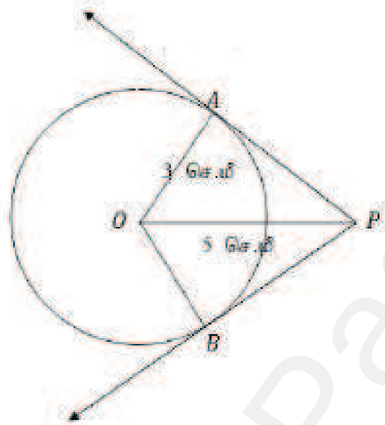
Soln:-

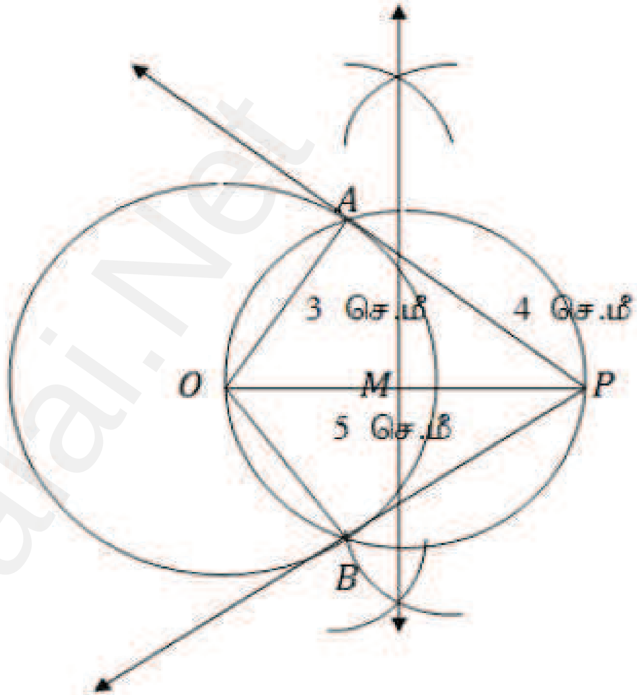
Gven, Diameter = 6 cm

Radius $= \frac{6}{2} = 3$ cm

Distance = 5 cm

Rough Diagram





∴ PA and PB are the two required tangents.

Lengths of the tangents, $PA = PB = 7.4$ cm

Verification:-

In the right angle triangle OAP,

$$PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9}$$
$$= \sqrt{16}$$
$$= \sqrt{4 \text{ x } 4}$$
$$PA = 4 \text{ cm}$$

11) Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

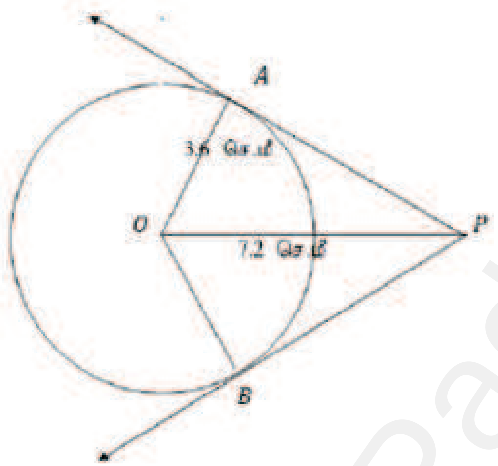
Soln:-

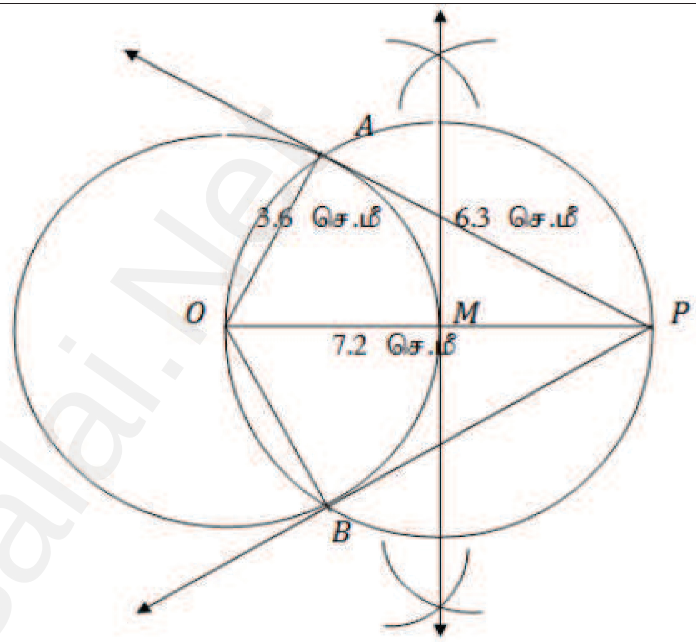
Gven,

Radius = 3.6 cm

Distance = 7.2 cm

Rough Diagram





∴ PA and PB are the two required tangents.
Lengths of the tangents, $PA = PB = 7.4$ cm

Verification:-

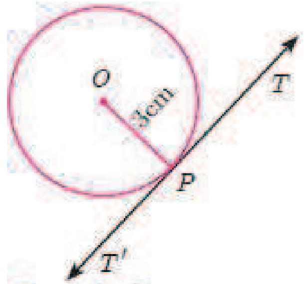
In the right angle triangle OAP,

$$PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{7.2^2 - 3.6^2}$$
$$= \sqrt{51.84 - 12.96}$$
$$= \sqrt{38.88}$$
$$= \sqrt{6.3 \text{ x } 6.3}$$
$$PA = 6.3\text{cm}$$

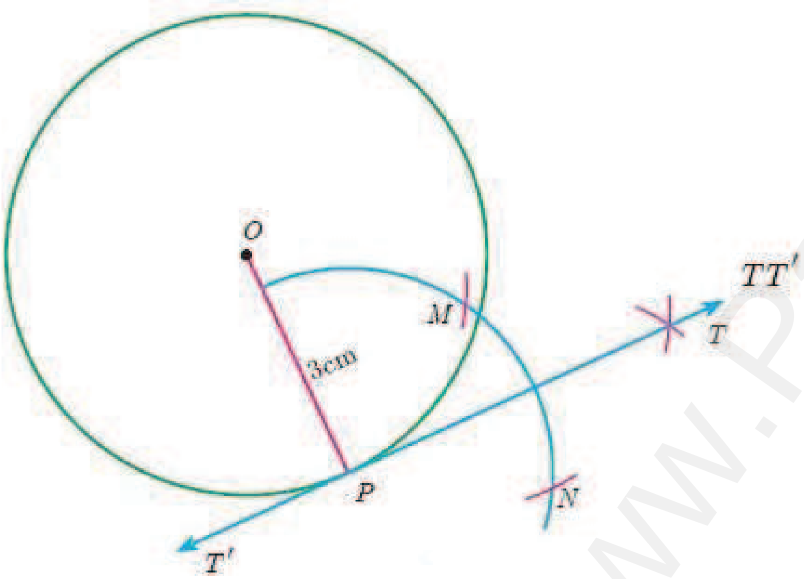
12) Draw a circle of radius 3cm. Take a point P on this circle and draw a tangent at P. (Using the centre)

Soln:-

Gven, Radius = 3 cm



Rough diagram



∴ TPT' is the required tangent.

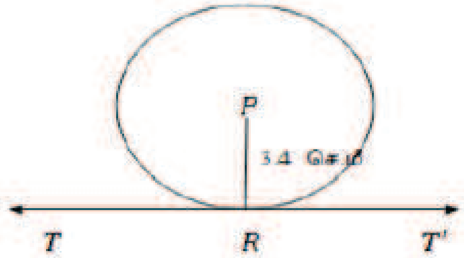
13) Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P? (Using the centre)

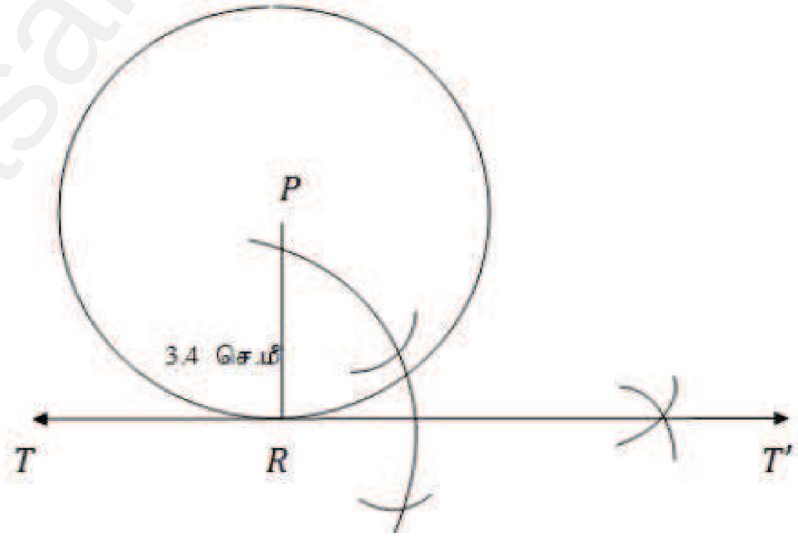
Soln:-

Gven,

Radius = 3.4 cm

Rough Diagram



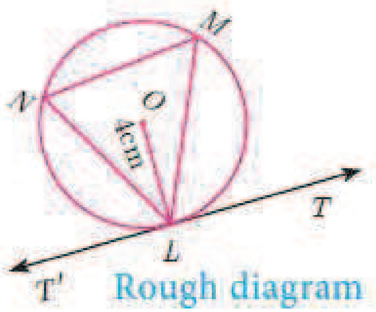


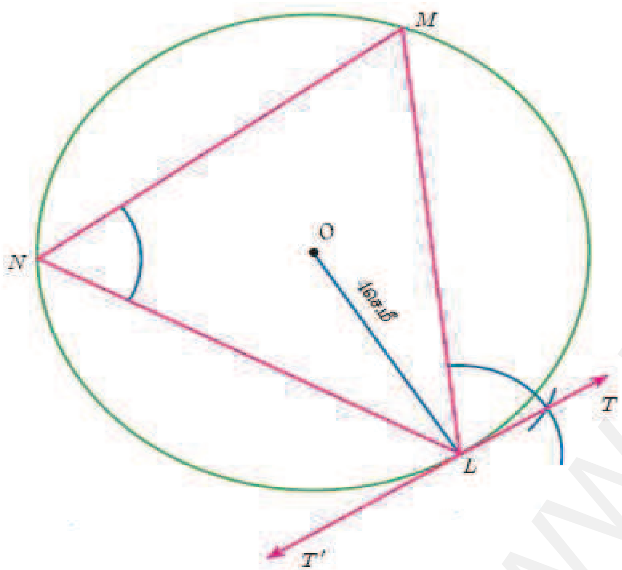
∴ TPT' is the required tangent.

14) Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternative segment.

Soln:-

Gven, Radius = 4 cm





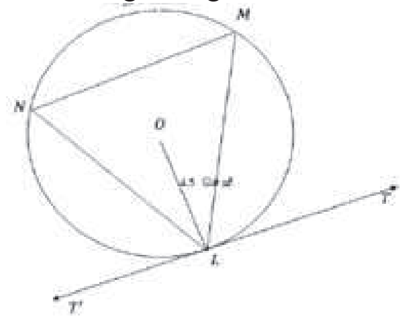
∴ TPT' is the required tangent.

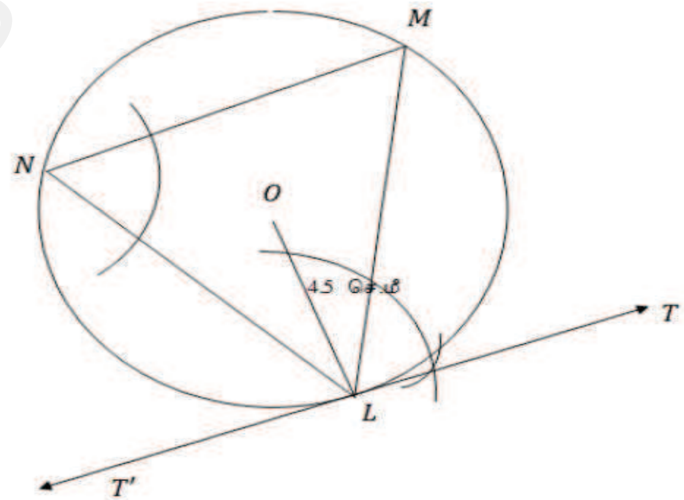
15) Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternative segment theorem

Soln:-

Gven, Radius = 4.5 cm

Rough Diagram





∴ TPT' is the required tangent.

16) Discuss the nature of solutions of the following quadratic equation $x^2 + x - 12 = 0$. **[S-21]**

Soln:-

தரவு. $y = x^2 + x - 12$

TABLE:-

x	-5	-4	-3	-2	-1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
x	-5	-4	-3	-2	-1	0	1	2	3	4
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
y	8	0	-6	-10	-12	-12	-10	-6	0	8

POINTS:-

$(-5, 8), (-4, 0), (-3, -6), (-2, -10), (-1, -12), (0, -12),$
$(1, -10), (2, -6), (3, 0), (4, 8)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm= 2units

Intersecting Points with the x - axis:-

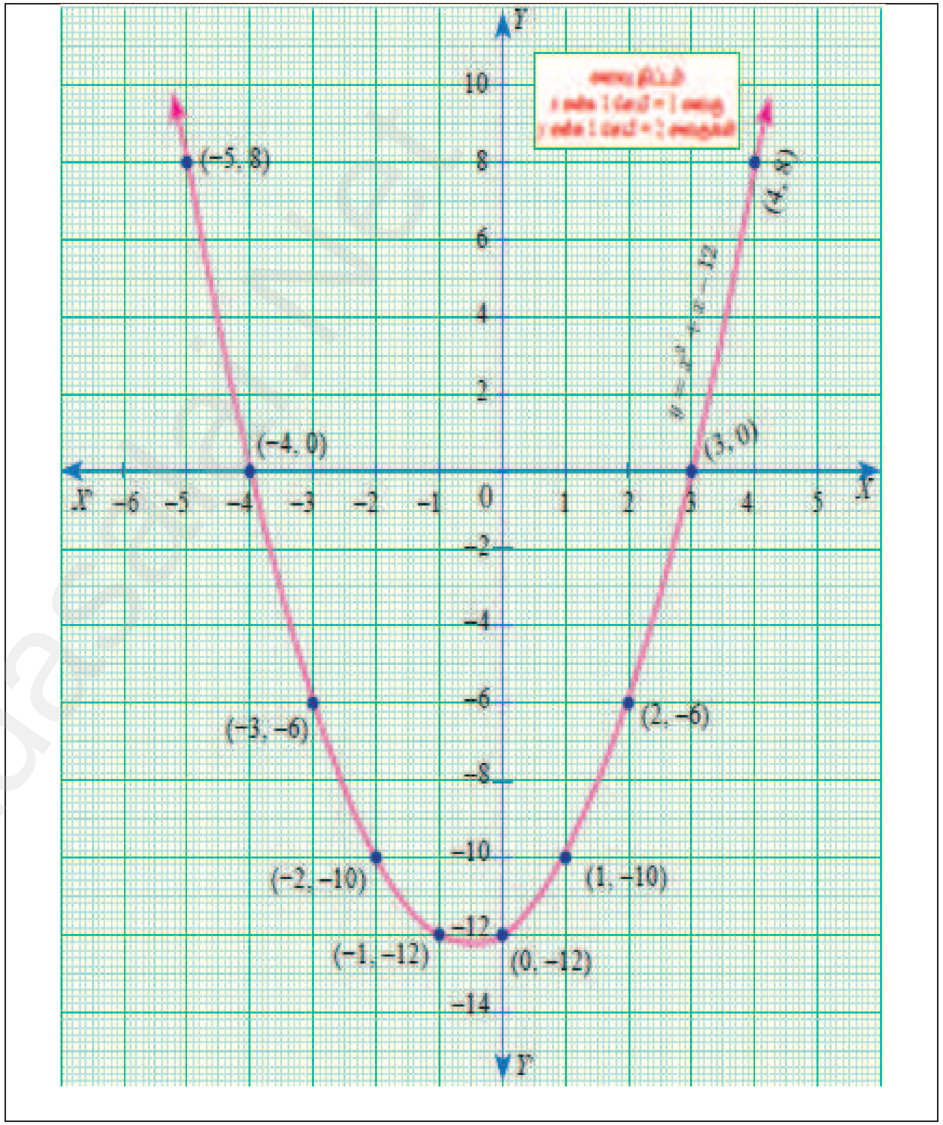
$(-4, 0)$and$(3, 0)$

Nature of Roots:-

The roots are real and unequal.

Solution:-

$x = \{-4, 3\}$



17) Discuss the nature of solutions of the following quadratic equation $x^2 - 9x + 20 = 0$ **[M-22]**

Soln:-

தரவு. $y = x^2 - 9x + 20$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$-9x$	36	27	18	9	0	-9	-18	-27	-36	-45
20	20	20	20	20	20	20	20	20	20	20
y	72	56	42	30	20	12	6	2	0	0

POINTS:-

$(-4,72), (-3,56), (-2,42), (-1,30), (0,20),$
$(1,12), (2,6), (3,2), (4,0), (5,0), (6,2), (7,6), (8,12), (9,20)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 4 units

Intersecting Points with the x - axis:-

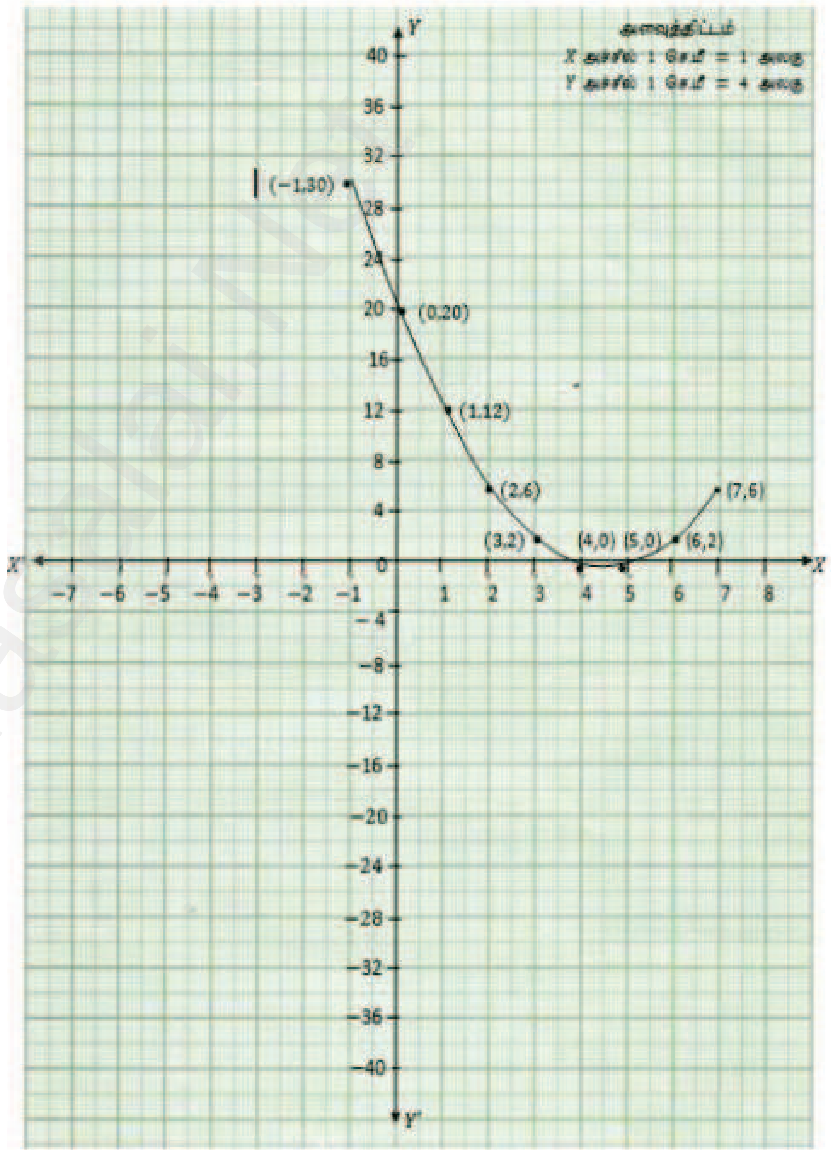
$(4, 0)$ and $(5, 0)$

Nature of Roots:-

The roots are real and unequal.

Solution:-

$x = \{4, 5\}$



18) Discuss the nature of solutions of the following quadratic equation $x^2 - 9 = 0$.

Soln:-

தரவு. $y = x^2 - 9$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
y	7	0	-5	-8	-9	-8	-5	0	7

POINTS:-

$(-4,7), (-3,0), (-2,-5), (-1,-8), (0,-9), (1,-8),$
$(2,-5), (3,0), (4,7)$

SCALE:-

x -axis:1 cm = 1unit
y -axis:1 cm = 1unit

Intersecting Points with the x - axis:-

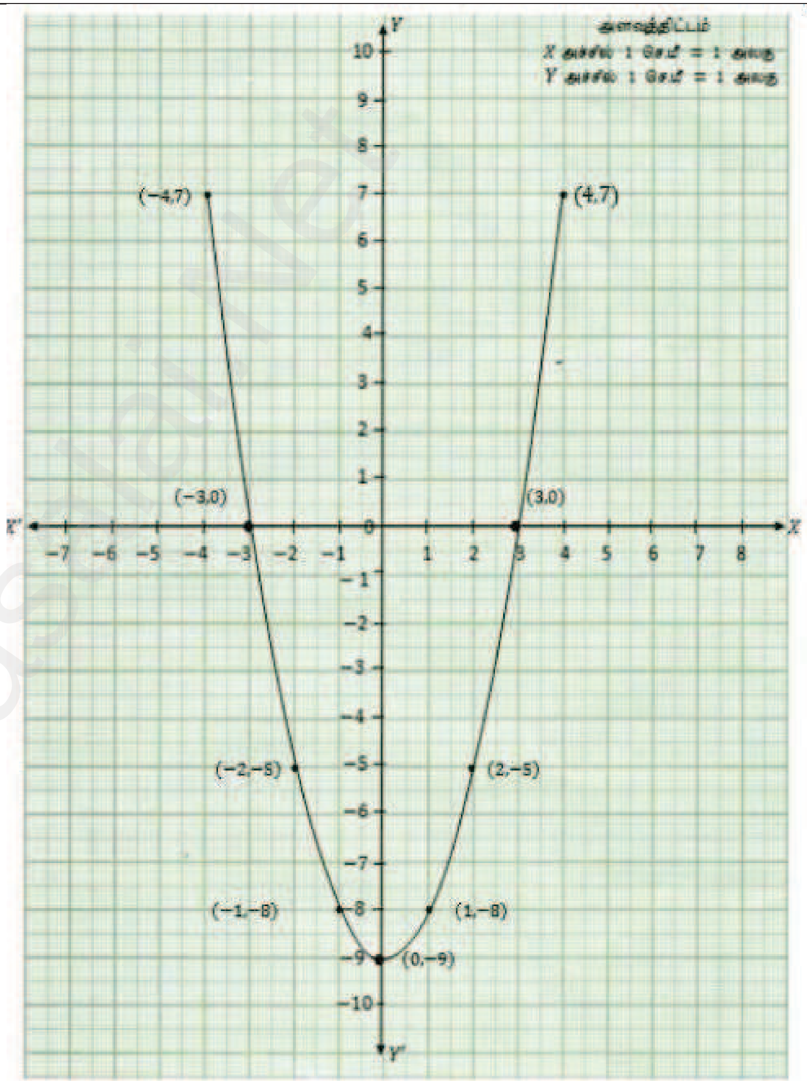
$(-3,0)$and$(3,0)$

Nature of Roots:-

The roots are real and unequal.

Solution:-

$x = \{-3, 3\}$



19) Discuss the nature of solutions of the following quadratic equation $x^2 - 8x + 16 = 0$ **[S-20]**

Soln:-

தரவு. $y = x^2 - 8x + 16$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$-8x$	32	24	16	8	0	-8	-16	-24	-32	-40
16	16	16	16	16	16	16	16	16	16	16
y	64	49	36	25	16	9	4	1	0	1

POINTS:-

$(-4, 64), (-3,49), (-2, 36), (-1,25), (0, 16), (1, 9),$
$(2, 4), (3, 1), (4, 0), (5,1)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 2 units

Intersecting Points with the x - axis:-

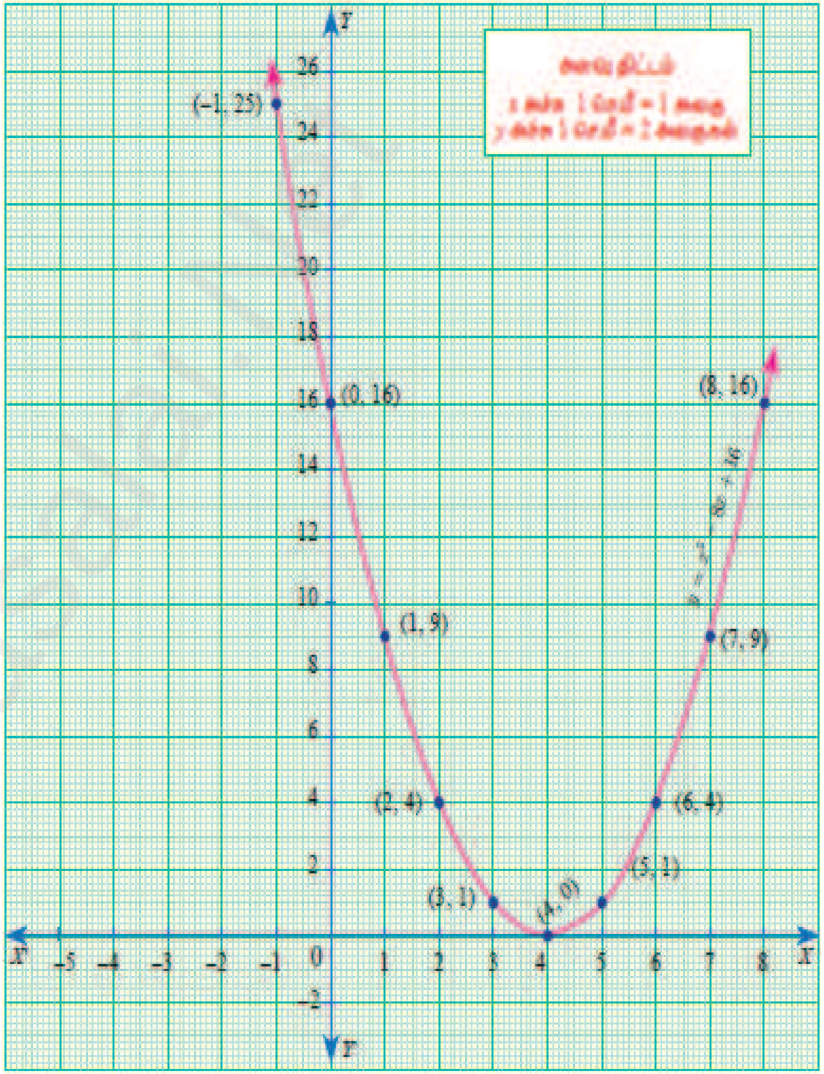
$(4, 0)$

Nature of Roots:-

The roots are real and equal.

Solution:-

$x = \{4, 4\}$



20) Discuss the nature of solutions of the following quadratic equation $x^2 - 4x + 4 = 0$ **[S-22 , M-22]**

Soln:-

தரவு. $y = x^2 - 4x + 4$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	-4	-8	-12	-16
4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	$\mathbf{0}$	1	4

POINTS:-

$(-4, 36), (-3, 25), (-2, 16), (-1, 9), (0, 4), (1, 1), (2, 0), (3, 1), ($

SCALE:-

x -axis:1 cm $=$ 1unit

y -axis:1 cm $=$ 2 units

Intersecting Points with the x - axis:-

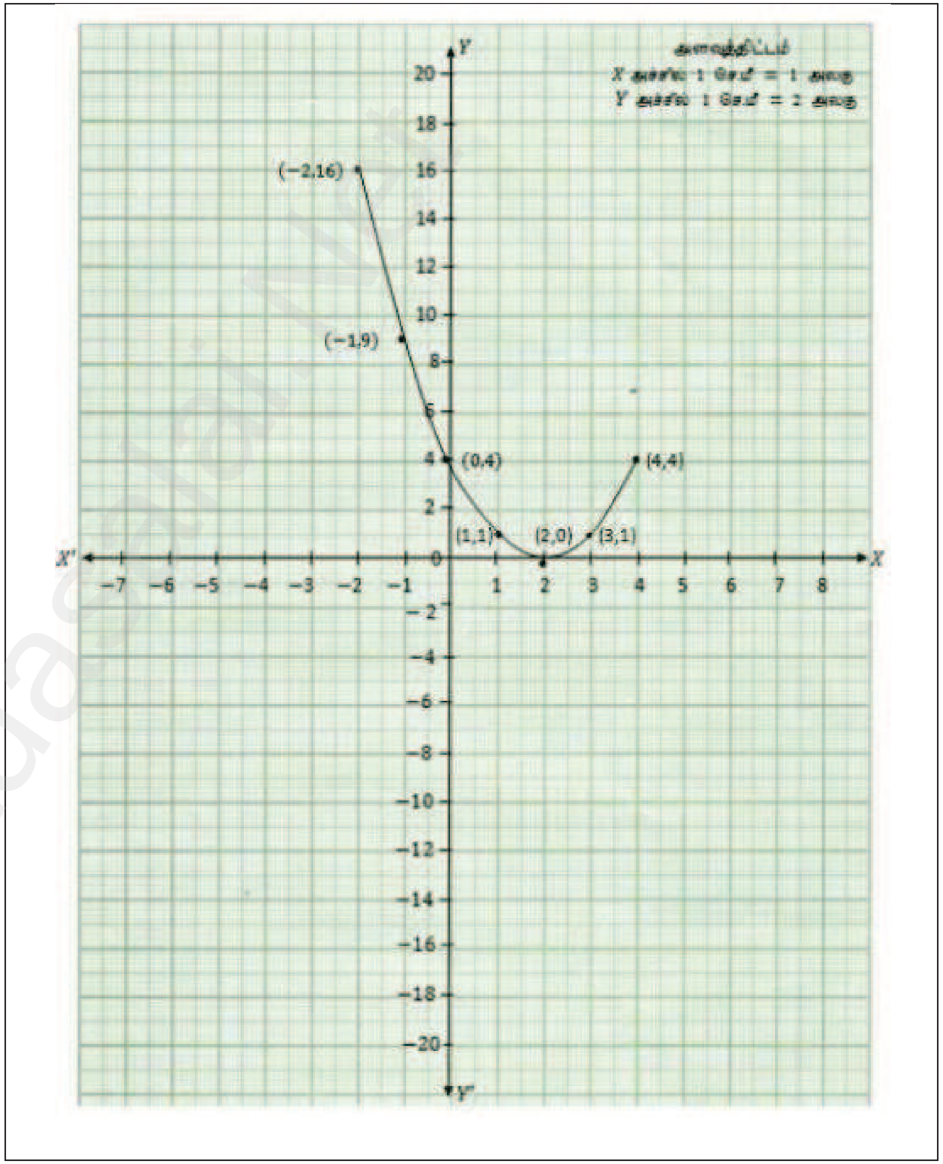
$(2, 0)$

Nature of Roots:-

The roots are real and equal.

Solution:-

$x = \{2, 2\}$



21) Discuss the nature of solutions of the following quadratic equation $x^2 - 6x + 9 = 0$.

Soln:-

தரவு. $y = x^2 - 6x + 9$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$-6x$	24	18	12	6	0	-6	-12	-18	-24	-30
9	9	9	9	9	9	9	9	9	9	9
y	49	36	25	16	9	4	1	**0**	1	4

POINTS:-

$(-4, 39), (-3, 36), (-2, 25), (-1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1), ($

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 2 units

Intersecting Points with the x - axis:-

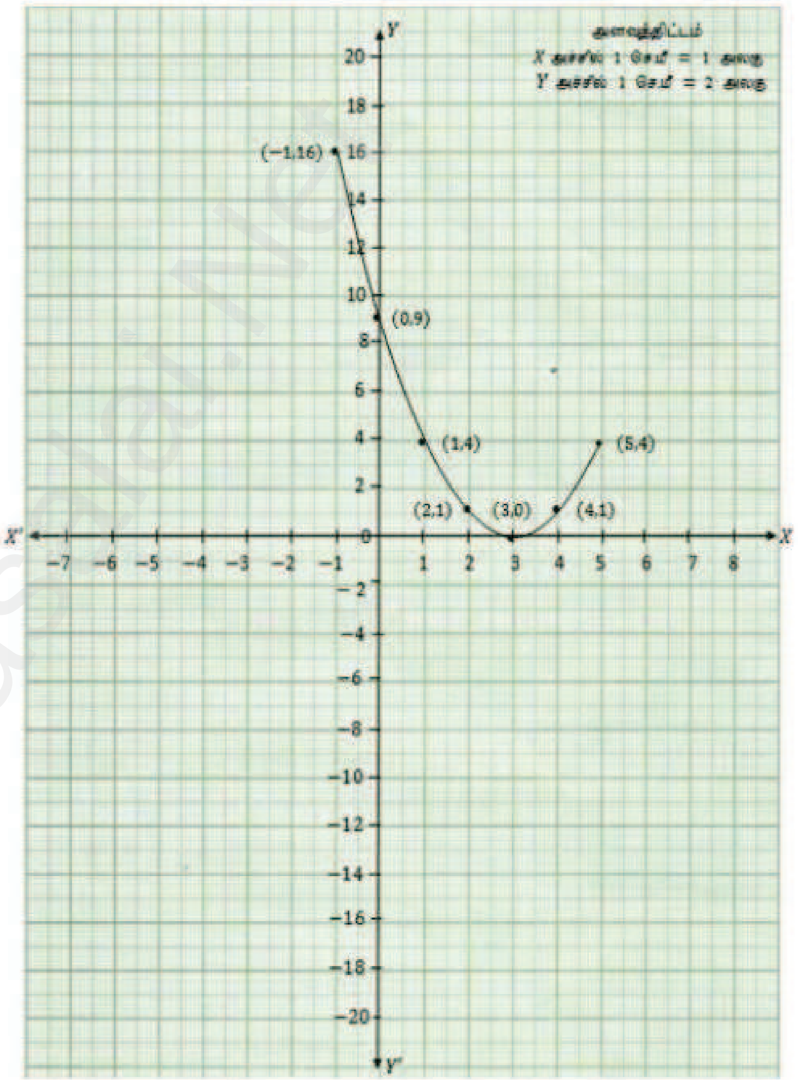
$(3, 0)$

Nature of Roots:-

The roots are real and equal.

Solution:-

$x = \{3, 3\}$



22) Discuss the nature of solutions of the following quadratic equation $x^2 + 2x + 5 = 0$.

Soln:-

தரவு. $y = x^2 + 2x + 5$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x$	-8	-6	-4	-2	0	2	4	6	8
5	5	5	5	5	5	5	5	5	5
y	13	8	5	4	5	8	13	20	25

POINTS:-

$(-4, 13), (-3, 8), (-2, 5), (-1, 4), (0, 5), (1, 8), (2, 13), (3, 20), (4, 25)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 2 units

Intersecting Points with the x - axis:-

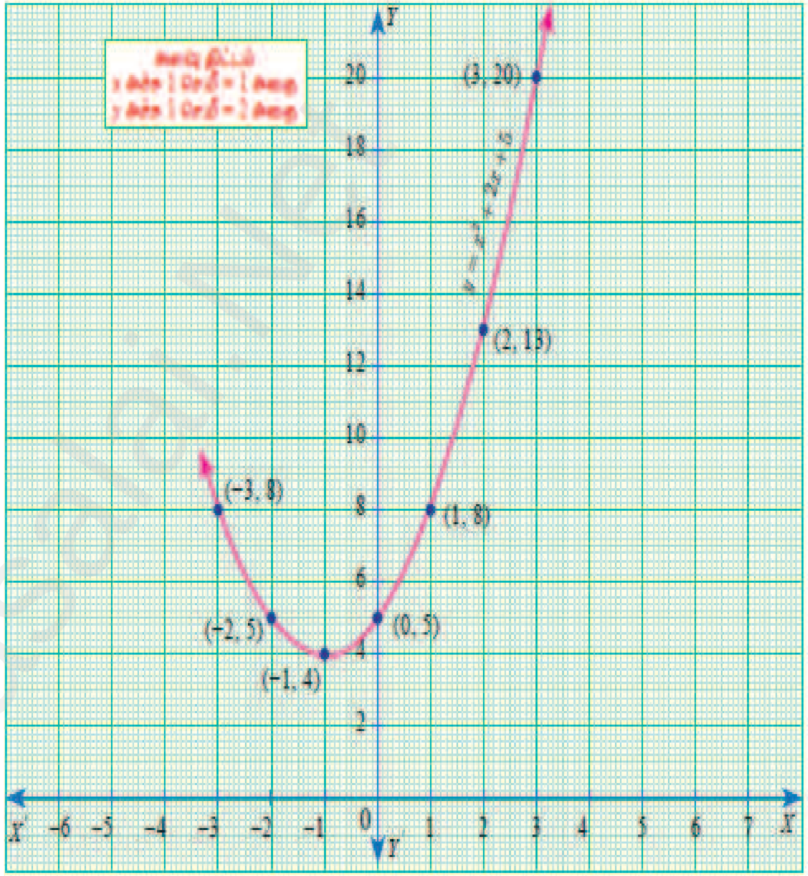
No intersectin points with the x–axis.

Nature of Roots:-

The roots are unreal.

Solution:-

No real roots.



23) Discuss the nature of solutions of the following quadratic equation $x^2 + x + 7 = 0$.

Soln:-

Given, $y = x^2 + x + 7$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	-4	-3	-2	-1	0	1	2	3	4
7	7	7	7	7	7	7	7	7	7
y	19	13	9	7	7	9	13	19	27

POINTS:-
$(-4,19), (-3,13), (-2,9), (-1,7), (0,7), (1,9), (2,13), (3,19), (4,27)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 2 units

Intersecting Points with the x - axis:-

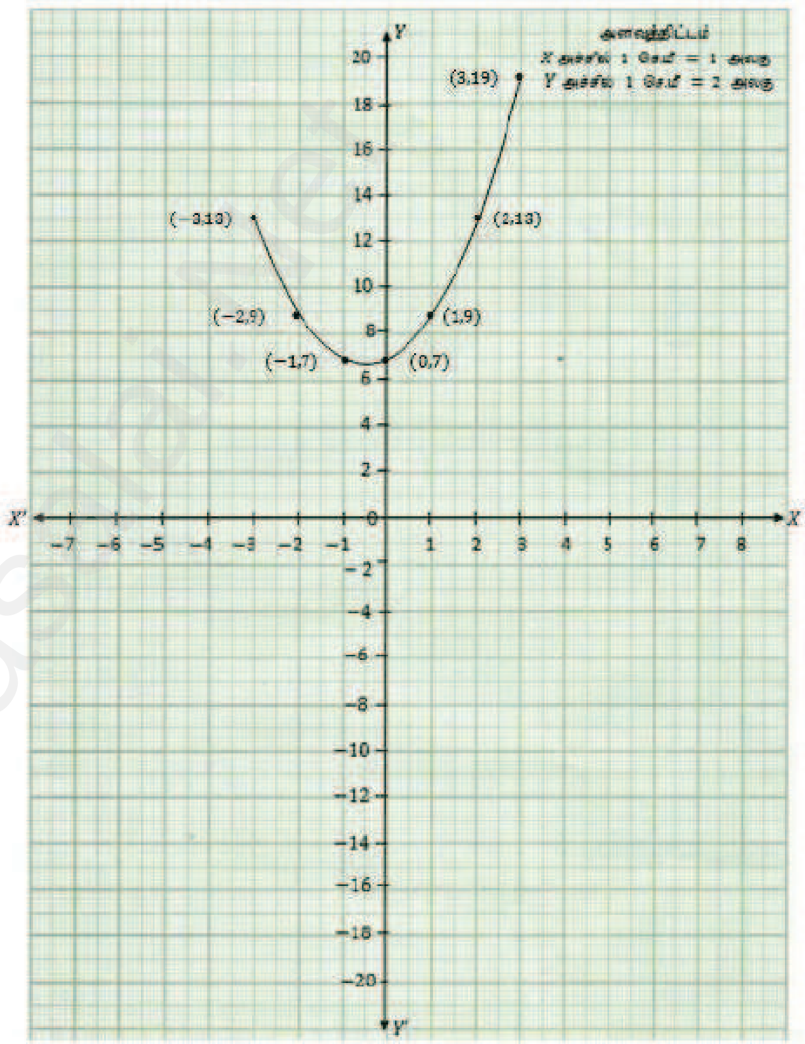
No intersectin points with the x–axis.

Nature of Roots:-

The roots are unreal.

Solution:-

No real roots.



24) Discuss the nature of solutions of the following quadratic equation $(2x-3)(x+2)=0$.

Soln:-

தரவு. $y=(2x-3)(x+2)$

$$=2x(x+2)-3(x+2)$$

$$=2x^2+4x-3x-6$$

$$y=2x^2+x-6$$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
x	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	22	9	0	-5	-6	-3	4	15	30

POINTS:-

$(-4,22),(-3,9),(-2,0),(-1,-5),(0,-6),(1,-3),(2,4),(3,15),(4,3$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 2 units

Intersecting Points with the x - axis:-

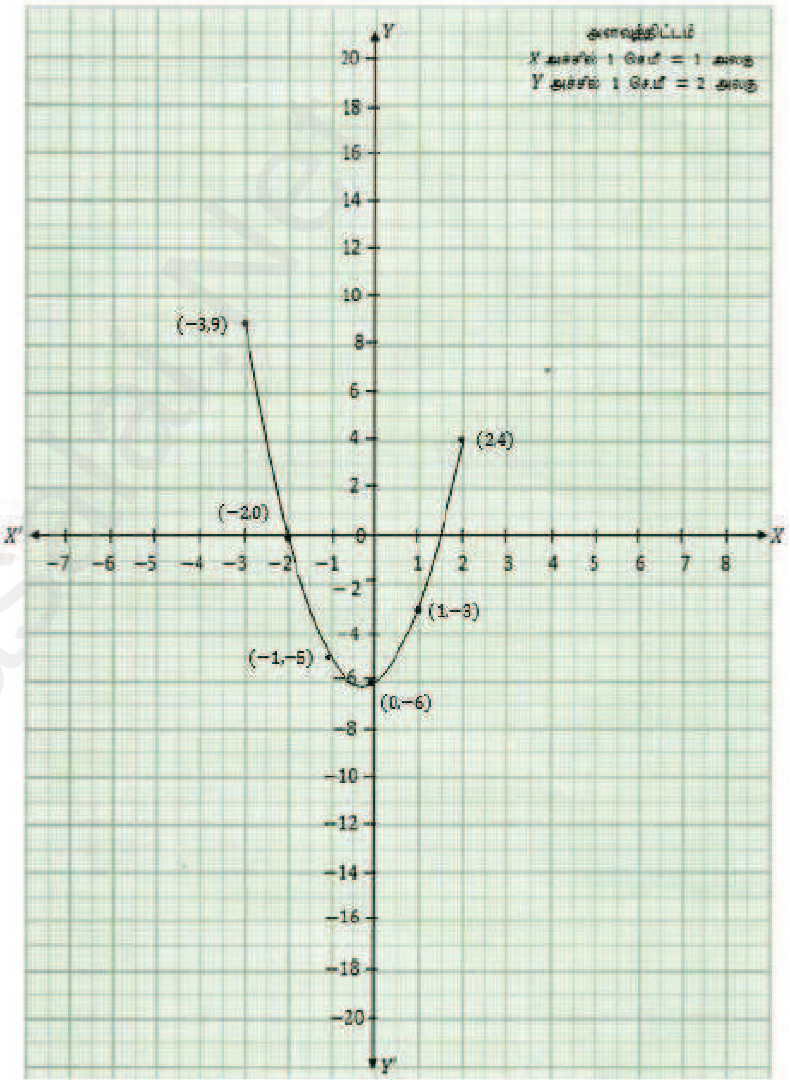
$(-2,0)$and $(1.5,0)$

Nature of Roots:-

The roots are real and unequal.

Solution:-

$x=\{-2,1.5\}$



25) Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$. **[PTA-1]**

Soln:-

தரவு. $y = x^2 + x - 2$

TABLE:-

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	-4	-3	-2	-1	0	1	2	3	4
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	10	4	0	-2	-2	0	4	10	18

POINTS:-

$(-4, 10), (-3, 4), (-2, 0), (-1, -2), (0, -2), (1, 0), (2, 4), (3, 10), (4, 18)$

SCALE:-

x -axis: 1 cm = 1 unit

y -axis: 1 cm = 1 unit

SUBTRACTION:-

$$\begin{array}{c} y = x^2 + x - 2 \\ 0 = x^2 + x - 2 \\ (-)(-)(-)(+) \\ \hline y = 0 \\ \hline \end{array}$$

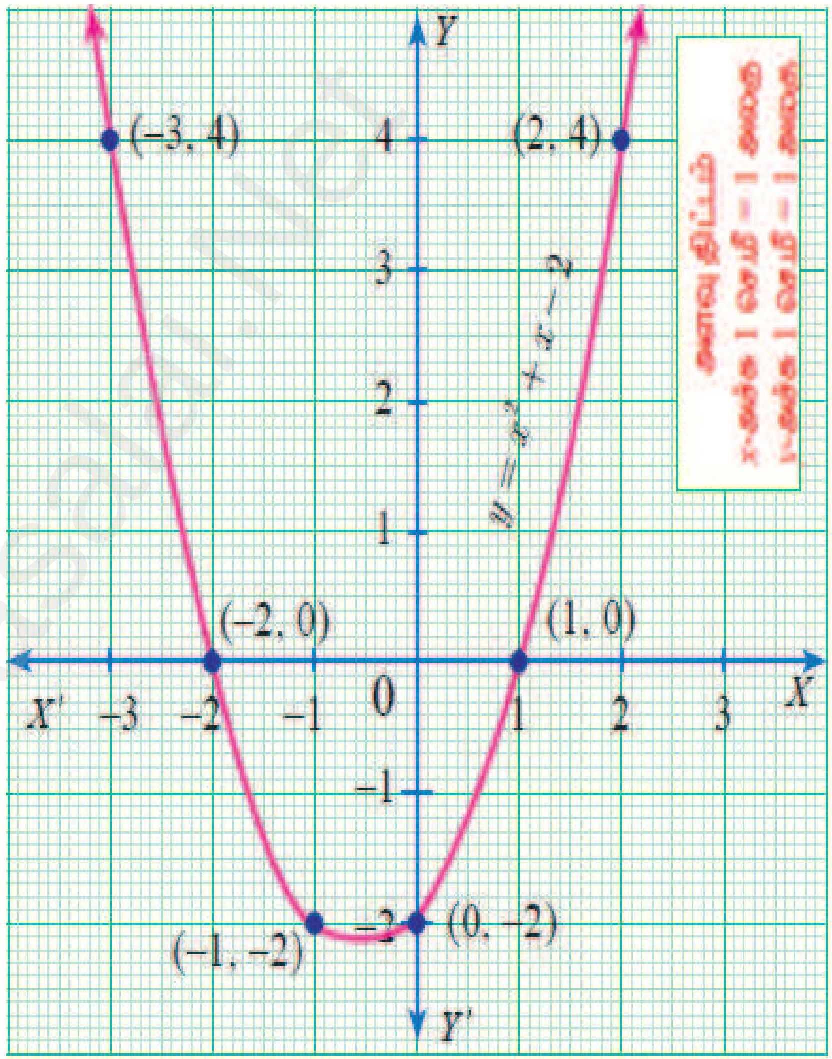
$y = 0$ is the equation of x - axis.

Intersecting Points with the x - axis:-

$(-2, 0)$ and $(1, 0)$

Solution:-

$x = \{-2, 1\}$



26) Draw the graph of $y = x^2 + 3x - 4$ and hence solve $x^2 + 3x - 4 = 0$. **[GMQ, S-21]**

Soln:-

Given, $y = x^2 + 3x - 4$

TABLE:-

x	-5	-4	-3	-2	-1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
$3x$	-15	-12	-9	-6	-3	0	3	6	9	12
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	6	0	-4	-6	-6	-4	0	6	14	24

POINTS:-

$(-5,6), (-4,0), (-3,-4), (-2,-6), (-1,-6), (0,-4), (1,0), (2,6), (3,14), (4,24)$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 1unit

SUBTRACTION:-

$$y = x^2 + 3x - 4$$
$$0 = x^2 + 3x - 4$$
$$(-)(-)(-)(+)$$
$$y = 0$$

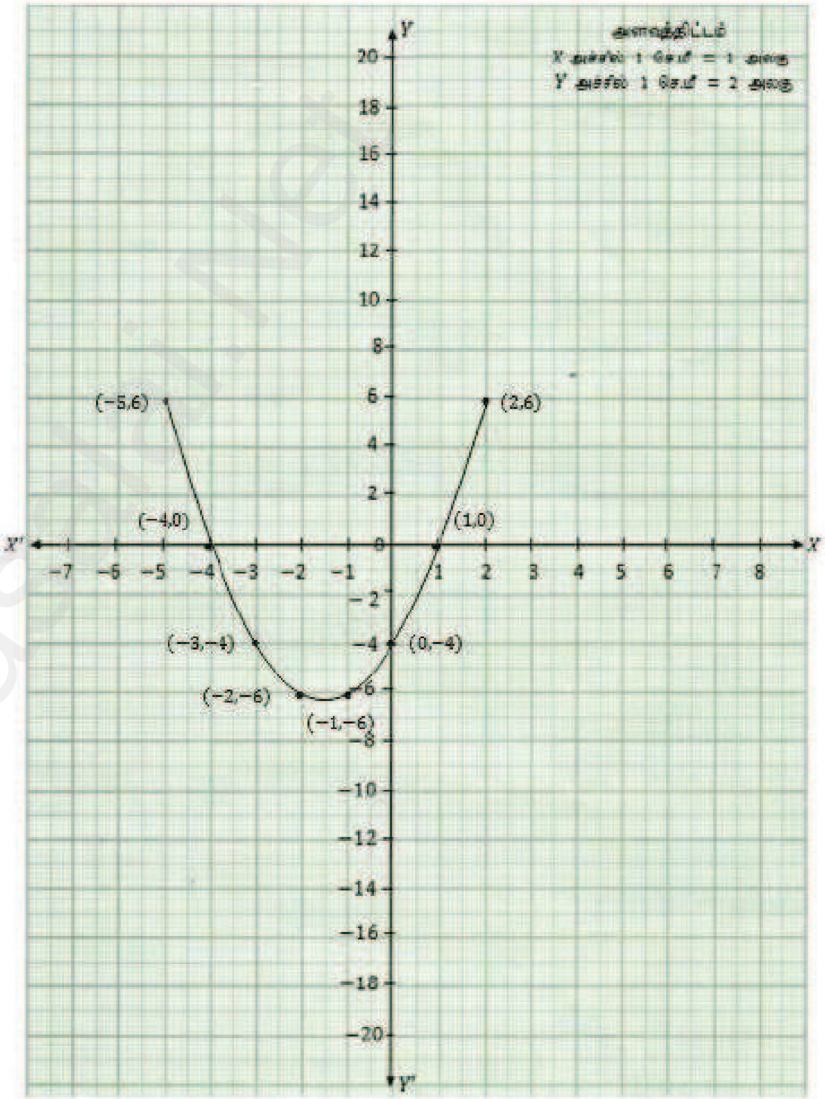
$y = 0$ is the equation of x - axis.

Intersecting Points with the x - axis:-

$(-4,0)$ and $(1,0)$

தீர்வு:-

$x = \{-4, 1\}$



27) Graph the following linear function $= \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$(ii) find x when $y = 7.5$.

Soln:-

Given, $y = \frac{1}{2}x$

VARIATION:- DIRECT VARIAION.

TABLE:-

x	2	4	6	8	10
y	1	2	3	4	5

POINTS:-

$(2,1),(4,2),(6,3),(8,4),(10,5)$

CONSTANT OF VARIATION:-

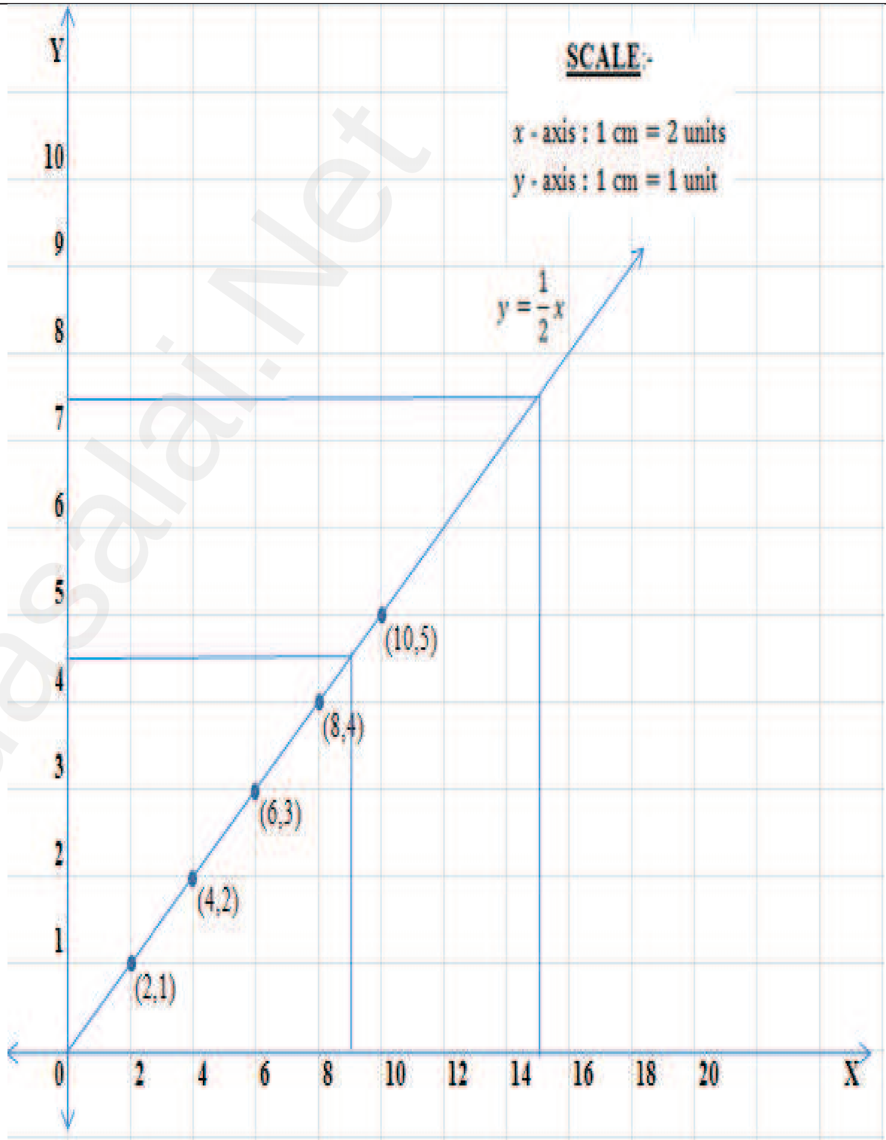
$$k = \frac{y}{x} = \frac{1}{2}$$

SCALE:-

x - axis: 1 cm = 2 units
y - axis: 1 cm = 1 unit

FROM THE GRAPH,

(i) If $x = 9$then $y = 4.5$
(ii) If $y = 7.5$ then $x = 15$



28) Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter x (cm)	1	2	3	4	5
circumference y (cm)	3.1	6.2	9.3	12.4	15.5

Soln:-

GIVEN:-

Diameter x (cm)	1	2	3	4	5
circumference y (cm)	3.1	6.2	9.3	12.4	15.5

VARIATION:-Direct Variation.

EQUATION:-$y = kx$

$$k = \frac{y}{x} = \frac{3.1}{1} = 3.1$$

$$y = (3.1)\,x$$

POINTS:-

$$(1, 3.1), (2, 6.2), (3, 9.3), (4, 12.4), (5, 15.5)$$

SCALE:-

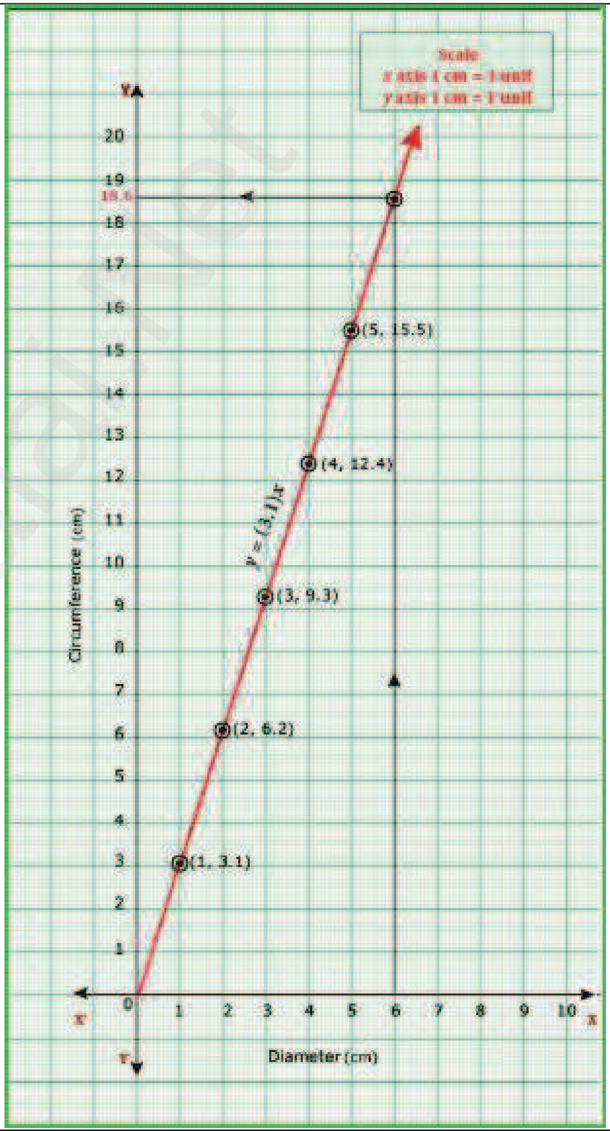
x -axis:1 cm = 1unit

y -axis:1 cm = 1unit

FROM THE GRAPH,

If $x = 6$ then $y = 18.6$

The circumference of a circle when its diameter is 6 cm« 18.6 cm.



29) A two wheeler parking zone near bus stand charges as below.

Time (in hours)(x)	4	8	12	24
Amount (Rs.) (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 *hr;* (ii) find the parking duration when the amount paid is ₹150.

Soln:-

GIVEN:-

Time (in hours)(x)	4	8	12	24
Amount (Rs.) (y)	60	120	180	360

VARIATION:-Direct Variation.

EQUATION:-$y = kx$

$$k = \frac{y}{x} = \frac{60}{4} = 15$$

$$y = 15\,x$$

POINTS:-

$$(4, 60), (8, 120), (12, 180), (24, 360)$$

SCALE:-

x -axis:1 cm = 2unit2

y -axis:1 cm = 30 units

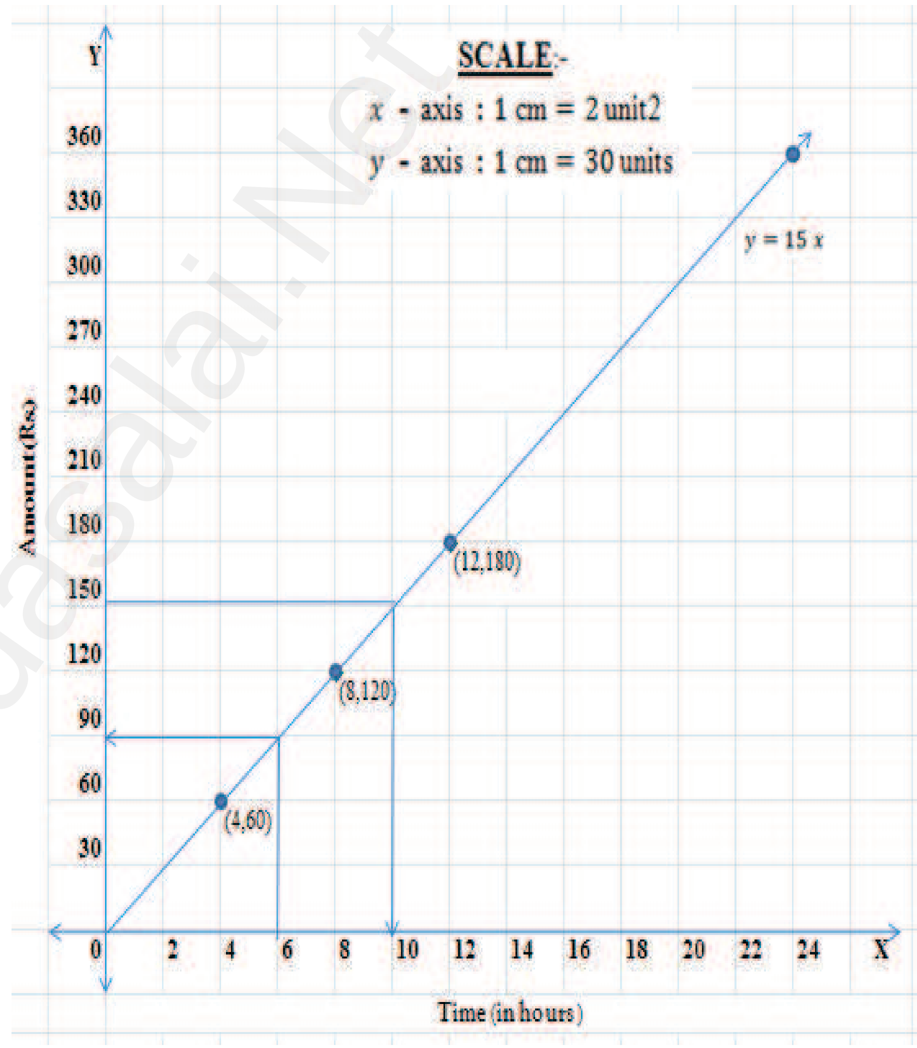
FROM THE GRAPH,

(i) If $x = 6$ then $y = 90$

The amount to be paid when parking time is 6 hrs is Rs.90

(ii) If $y = 150$ then $x = 10$

The parking duration when the amount paid is ₹150 is 10 hours.



30) A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

(i) the constant of variation.

(i) how far will it travel in 90 minutes?

(ii) the time required to cover a distance of 300 km from the graph.

Soln:-

x =Marked Price and y =Discount

TABLE:-

Time (in minutes)(x)	60	120	180	240	300
Distance (kms)(y)	50	100	150	200	250

VARIATION:-Direct Variation.

EQUATION:-$y = kx$

$$k = \frac{y}{x} = \frac{50}{60} = \frac{5}{6}$$

$$y = \frac{5}{6}x$$

POINTS:-

$(60, 50), (120, 100), (180, 150), (240, 200), (300, 250)$

SCALE:-

x -axis:1 cm = 60unit

y -axis:1 cm = 50 units

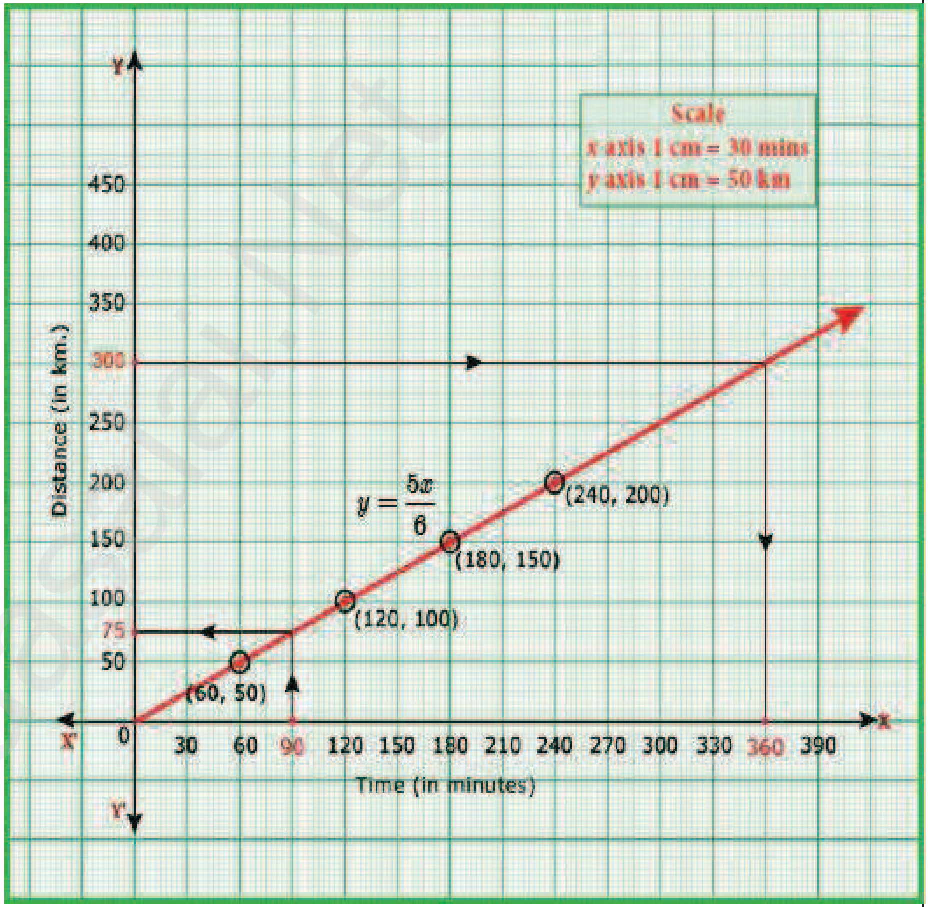
FROM THE GRAPH,

(i) If $x = 1\frac{1}{2}$then $y = 90$

The bus will travel in $1\frac{1}{2}$ hours be 75kms.

(ii) If $y = 360$ then $x = 6$

The time required to cover a distance of 300 km is 6 hours.



31) A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find

(i) the marked price when a customer gets a discount of ₹3250 (from graph)

(ii) the discount when the marked price is ₹2500

Soln:-

x =Marked Price and y =Discount

TABLE:-

Marked Price (Rs.)(x)	1000	2000	3000	4000	5000	6000	7000
Discount (Rs.)(y)	500	1000	1500	2000	2500	3000	3500

VARIATION:-Direct Variation.

EQUATION:-$y = kx$

$$k = \frac{y}{x} = \frac{500}{1000} = \frac{1}{2}$$

$$y = \frac{1}{2}x$$

POINTS:-

$(1000, 500), (2000, 1000), (3000, 1500), (4000, 2000), (5000, 2500)$

SCALE:-

x -axis:1000 cm = 1units

y -axis:1 cm = 500 units

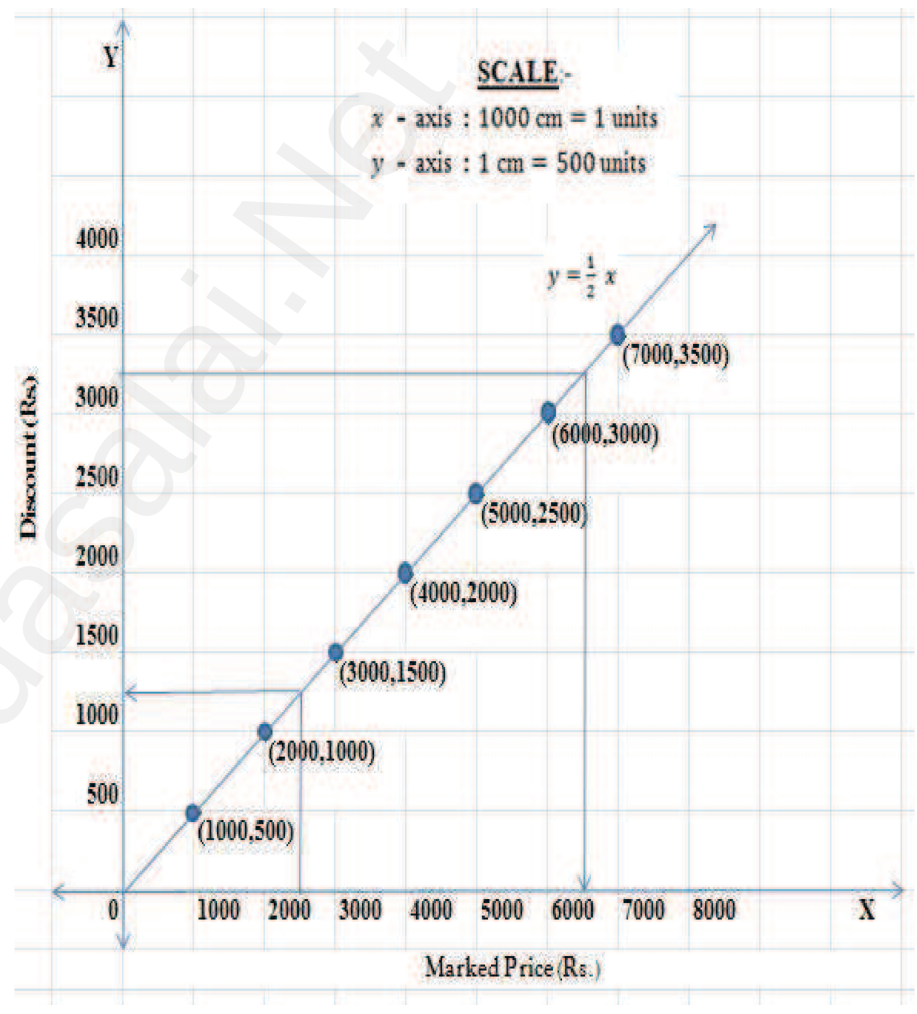
FROM THE GRAPH,

(i) If $y = 3250$ then $x = 6500$

When a customer gets a discount of ₹3250 the marked price is Rs. 6500

(ii) If $x = 2500$ then $y = 1250$

When the marked price is ₹2500 the discount is Rs. 1250



32) Draw the graph of $xy = 24,\ x, y > 0$,. Using the graph find,

(i) y when x =3 and

(ii) $x$when y = 6.

Soln:-

Given, $xy = 24$

VARIATION:- INDIRECT VARIAION.

TABLE:-

x	1	2	3	4	6	8	12	24
y	24	12	8	6	4	3	2	1

POINTS:-

$(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1)$

SCALE:-

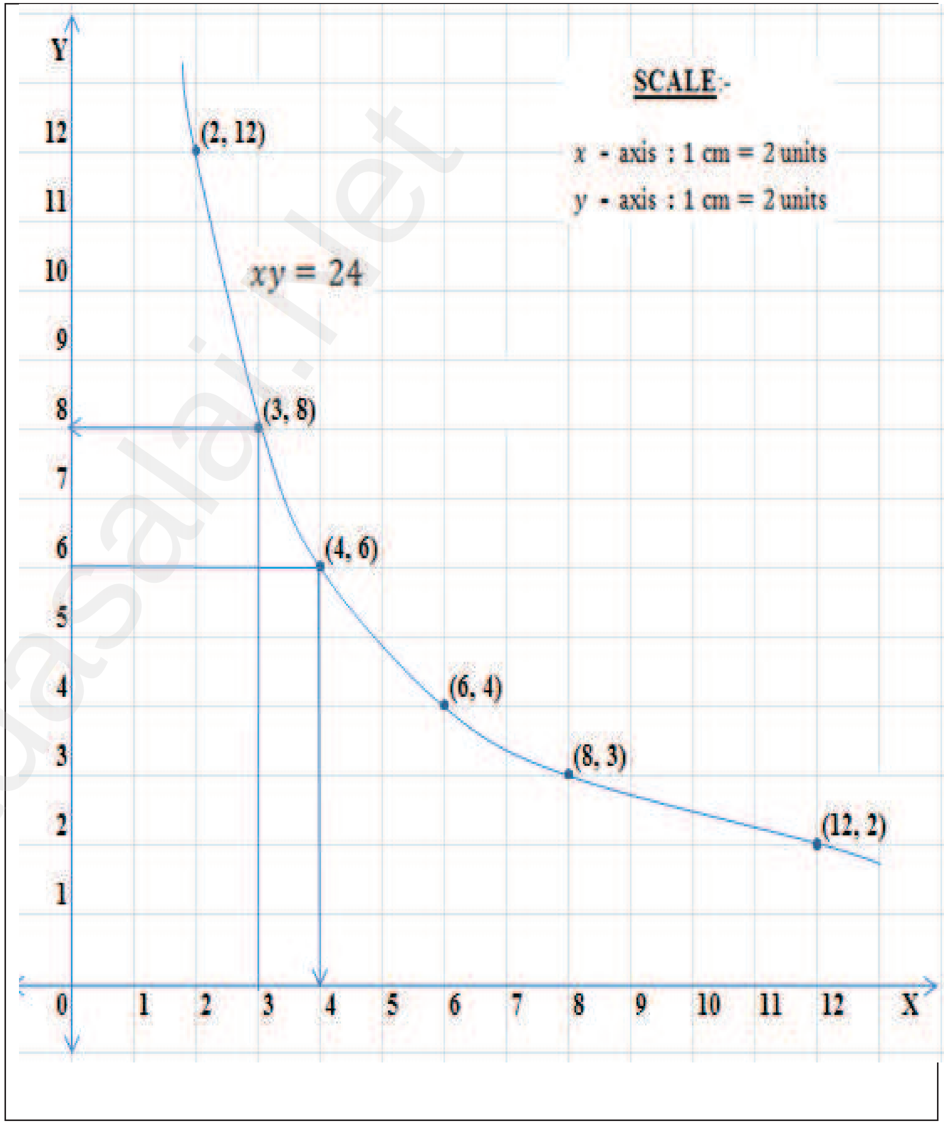
x -axis:1 cm = 2units

y -axis:1 cm = 2 units

FROM THE GRAPH,

(i) If$x = 3$then $y = 8$

(ii) If$y = 6$ then $x = 4$



33) A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation.

(ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers?

(iii) If the work has to be completed by 200 days, how many workers are required?

Soln:-

VARIATION:-Indirect Variation.

CONSTANT OF VARIATION:-

$$xy = k$$

$$k = x40 \text{ x } 150 = 6000$$

EQUATION:-$xy = 6000$

POINTS:-

$$(40, 150), (50, 120), (60, 100), (75, 80)$$

SCALE:-

x -axis:1 cm = 10units

y -axis:1 cm = 10 units

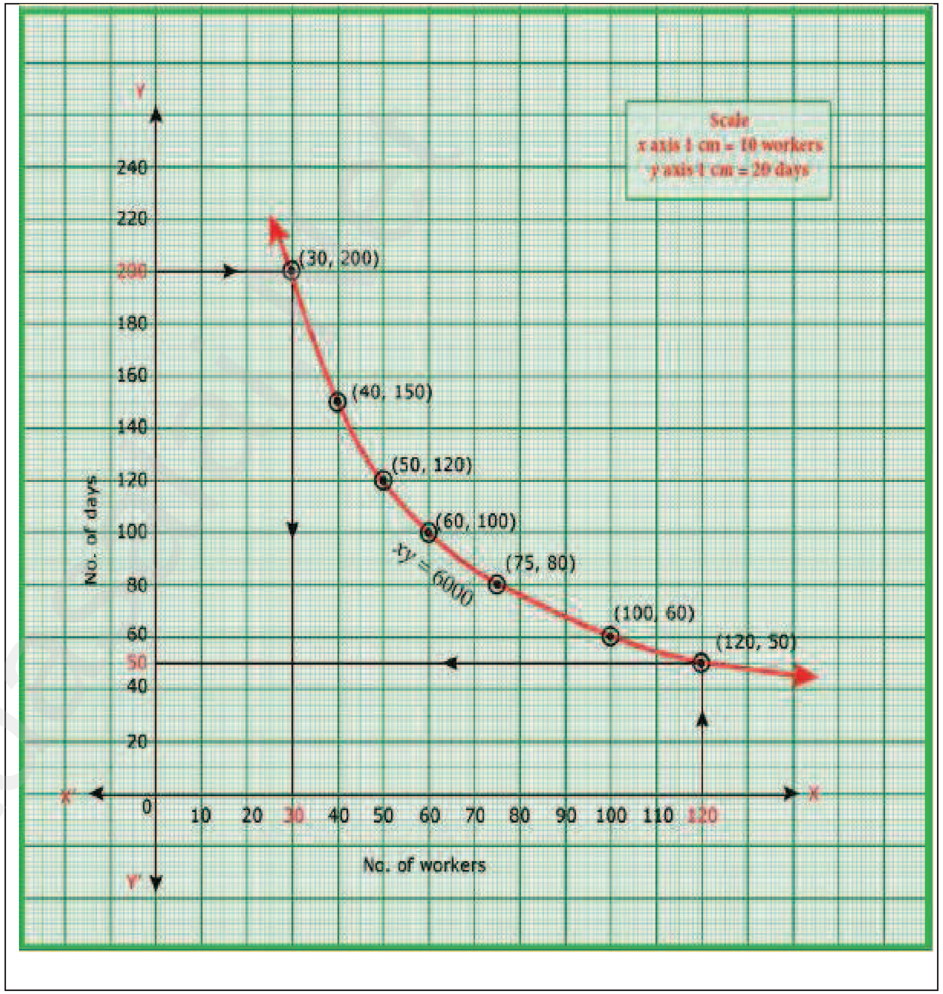
From the graph,

(i) If $x = 120$ then $y = 50$

120days are required to complete the work if the company decides to opt for 120 workers

(ii) If $y = 30$ then $x = 200$

200workers are requiredto complete the work by 30 days.



34) The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of Pipes (x)	2	3	6	9
Time taken(y) (in minutes)	45	30	15	10

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

(ii) Find the number of pipes when the time is 9 minutes.

Soln:-

VARIATION:-Indirect Variation.

CONSTANT OF VARIATION:-

$$xy = k$$

$$k = xy = 2 \text{ x} 45 = 90$$

EQUATION:-$xy = 90$

POINTS:-

$$(2, 45), (3, 30), (6, 15), (9, 10)$$

SCALE:-

x -axis:1 cm = 1unit

y -axis:1 cm = 5 units

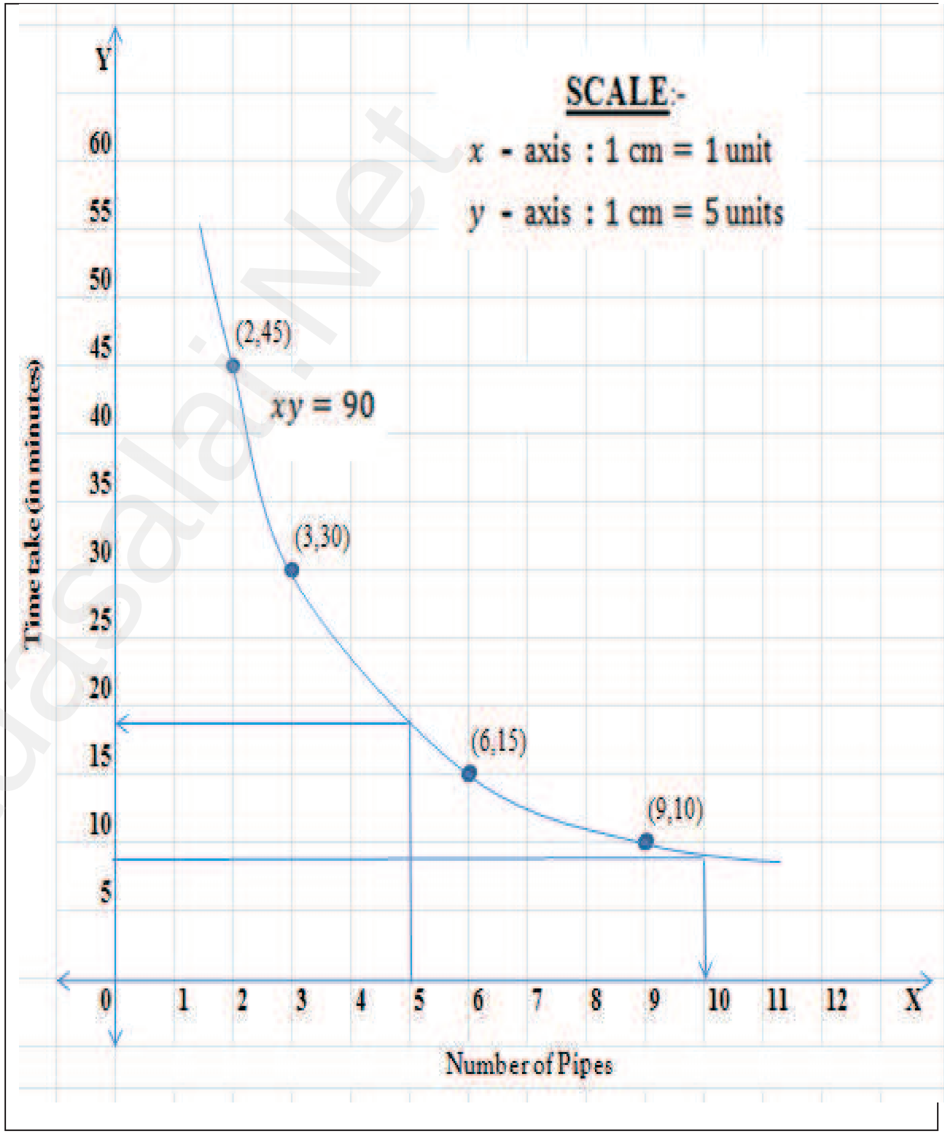
From the graph,

(i) If $x = 5$ then$y = 18$

The time taken to fill the tank when five pipes are used = 18 minutes.

(ii) If $y = 9$ then$x = 10$

10 pipes are required to fill the tank when the time is 9 minutes.



35) A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of Participants (x)	2	4	6	8	10
Amount for each participant (Rs.) (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Soln:-

GIVEN:-

No. of Participants (x)	2	4	6	8	10
Amount for each participant (Rs.) (y)	180	90	60	45	36

POINTS:-

$(2, 180), (4, 90), (6, 60), (8, 45), (10, 36)$

VARIATION:-Indirect Variation.

CONSTANT OF VARIATION:-

$$xy = k$$

$$k = xy = 2 \text{ x } 180 = 360$$

EQUATION:-$xy = 180$

SCALE:-

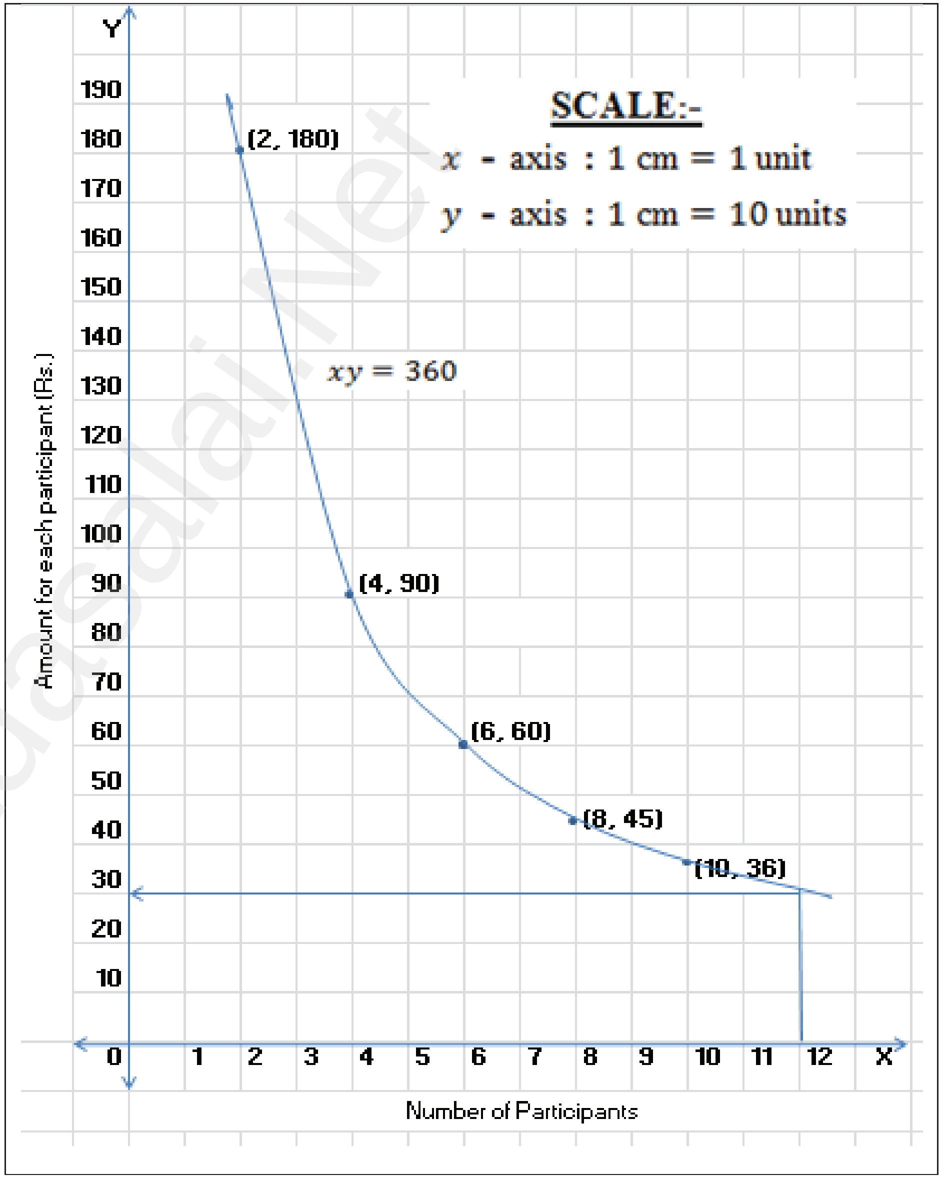
x -axis:1 cm = 1unit

y -axis:1 cm = 10 units

From the graph,

If $x = 12$, then $y = 30$

∴ If The number of participants are12, each participant will get Rs.30.



36) Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively.
Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

Soln:-

x = Speed (Km), y =Time (Hour)

TABLE:-

Speed(x)(Km/Hr)	12	6	4	3	2
Time(y)(Hour)	1	2	3	4	6

VARIATION:-Direct Variation.

EQUATION:-$xy = k$

$$k = xy = 12 \text{ x } 1 = 12$$

$$xy = 12$$

POINTS:-

$(12, 1), (6, 2), (4, 3), (3, 4), (2, 6)$

SCALE:-

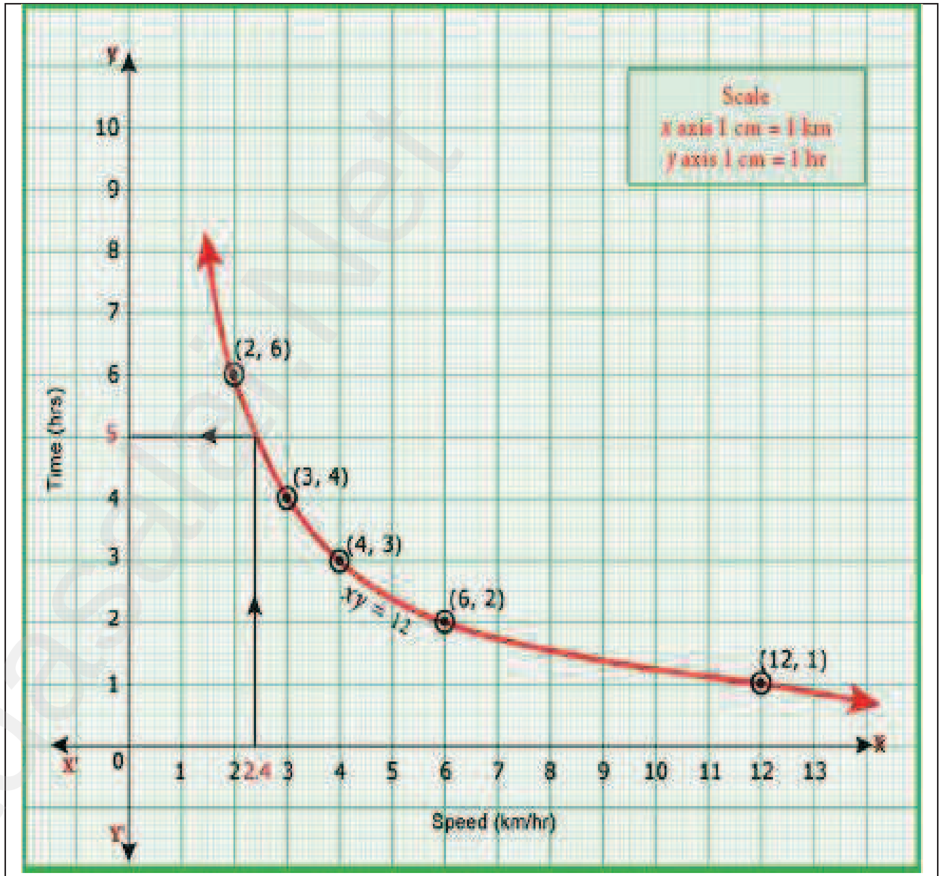
x -axis:1 cm = 1unit

y -axis:1 cm = 1unit

From the graph,

If $x = 2.4$, then $y = 5$

∴Kaushik takes 5 hrs with a speed of 2.4 km/hr.



SSLC – MATHS
BOOK BACK ONE MARK QUESTIONS

UNIT – 1 : RELATIONS AND FUNCTIONS

1) If $n(A \text{ X } B) = 6$ and $A = \{1, 3\}$ then, $n(B)$ is
(1) 1 (2) 2 (3) 3 (4) 6

2) $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then, $n[(A \cup C) \text{ X } B]$ is
(1) 8 (2) 20 (3) 12 (4)16

3) If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true?
(1) $(A \text{ X } C) \subset (B \text{ X } D)$ (2) $(B \text{ X } D) \subset (A \text{ X } C)$
(3) $(A \text{ X } B) \subset (A \text{ X } D)$ (4) $(D \text{ X } A) \subset (B \text{ X } A)$

4) If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements of B is
(1) 3 (2) 2 (3) 4 (4) 8

5) The range of the relation $R = \{(x, x^2) / \ x \ isa \ prime \ number \ less \ than \ 13\}$ is
(1) $\{2, 3, 5, 7\}$ (2) $\{2, 3, 5, 7, 11\}$ (3) $\{4, 9, 25, 49, 121\}$ (4) $\{1, 4, 9, 25, 49, 121\}$

6) If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
(1) $(2, -2)$ (2) $(5, 1)$ (3) $(2, 3)$ (4) $(3, -2)$

7) Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is
(1) m^n (2) n^m (3) $2^{mn} - 1$ (4) 2^{mn}

8) If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively
(1) (8, 6) (2) (8, 8) (3) 96, 8) (4) (6, 6)

9) Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10$. A function $f: A \longrightarrow B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a
(1) Many-one-function (2) Identity function
(3) One-to-one function (4) Into function

10) If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$ then fog is
(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$ (3) $\frac{2}{9x^2}$ (4) $\frac{1}{6x^2}$

11) If $f: A \longrightarrow B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
(1) 7 (2) 49 (3) 1 (4) 14

12) Let f and g be two functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$,
$g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7,0)\}$ then the range of fog is
(1) $\{0, 2, 3, 4, 5\}$ (2) $\{-4, 1, 0, 2, 7\}$ (3) $\{1, 2, 3, 4, 5\}$ (4) $\{0, 1, 2\}$

13) Let $f(x) = \sqrt{1 + x^2}$ then
(1) $f(xy) = f(x). f(y)$ (2) $f(xy) \geq f(x). f(y)$
(3) $f(xy) \leq f(x). f(y)$ (4) None of these

14) If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
(1) $(-1, 2)$ (2) $(2, -1)$ (3) $(-1, -2)$ (4) $(1, 2)$

15) $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is
(1) linear (2) cubic (3) reciprocal (4) quadratic

UNIT – 2 : NUMBERS AND SEQUENCES

1) Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
(1) $1 < r < b$ (2) $0 < r < b$ (3) $0 \leq r < b$ (4) $0 < r \leq b$

2) Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible reminders are
(1) 0, 1, 8 (2) 1, 4, 8 (3) 0, 1, 3 (4) 1, 3, 5

3) If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
(1) 4 (2) 2 (3) 1 (4) 3

4) The sum of the exponents of the prime factors in tbhe prime factorization of 1729 is
(1) 1 (2) 2 (3) 3 (4) 4

5) The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
(1) 2025 (2) 5220 (3) 5025 (4) 2520

6) $7^{4k} \equiv$ _____ $(mod\ \ 100)$
(1) 1 (2) 2 (3) 3 (4) 4

7) Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
(1) 3 (2) 5 (3) 8 (4) 11

8) The first term of an arithmetic progression is unity and the common difference is 4 which of the following will be a term of this A.P?
(1) 4551 (2) 10091 (3) 7881 (4) 13531

9) If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th}, then the 13^{th} term of the A.P. is
(1) 0 (2) 6 (3) 7 (4) 13

10) An A.P. consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is
(1) 16m (2) 62m (3) 31m (4) $\frac{31}{2}m$

11) In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?
(1) 6 (2) 7 (3) 8 (4) 9

12) If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \ ... + \ 2^0$ which of the following is t rue?
(1) B is 2^{64} more than (2) A and B are equal
(3) B is larger than A by 1 (4) A is larger than B by 1

13) The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, ...$ is
(1) $\frac{1}{24}$ (2) $\frac{1}{27}$ (3) $\frac{2}{3}$ (4) $\frac{1}{81}$

14) If the sequence $t_1, t_2, t_3, ...$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, ...$ is
(1) a Geometric Progression
(2) an Arithmetic Progression
(3) neither an Arithmetic Progression nor a Geometric Progression
(4) a constant sequence

15) The value of $(1^3 + 2^3 + 3^3 + \ ... + 15^3) - (1 + 2 + 3 + \cdots + 15)$ is
(1) 14400 (2) 14200 (3) 14280 (4) 14520

UNIT – 3 : ALGEBRA

1) A system of three linear equations in three variables is inconsistent if their planes
(1) intersect only at a point (2) intersect in a line
(3) coincides with each other (4) do not intersect

2) The solution of the system $x + y - 3x = -6, -7y + 7z = 7, 3z = 9$ is
(1) $x = 1, y = 2, z = 3$ (2) $x = -1, y = 2, z = 3$
(3) $x = -1, y = -2, z = 3$ (4) $x = 1, y = 2, z = -3$

3) If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
(1) 3 (2) 5 (3) 6 (4) 8

4) $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is
(1) $\frac{9y}{7}$ (2) $\frac{9y^3}{(21y-21)}$ (3) $\frac{21y^2-42y+21}{3y^3}$ (4) $\frac{7(y^2-2y+1)}{y^2}$

5) $y^2 + \frac{1}{y^2}$ - is not equal to
(1) $\frac{y^4+1}{y^2}$ (2) $\left(y + \frac{1}{y}\right)^2$ (3) $\left(y - \frac{1}{y}\right)^2 + 2$ (4) $\left(y + \frac{1}{y}\right)^2 - 2$

6) $\frac{x}{x^2-25} - \frac{8}{x^2+6x+5}$ gives
(1) $\frac{x^2-7x+40}{(x-5)(x+5)}$ (2) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$ (3) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$ (4) $\frac{x^2+10}{(x^2-25)(x+1)}$

7) The square root of $\frac{256\, x^8y^4z^{10}}{25\, x^6y^6z^6}$ is equal to
(1) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (2) $16\left|\frac{y^2}{x^2z^4}\right|$ (3) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ (4) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

8) Which of the following should be added to make $x^4 + 64$ a perfect square?
(1) $4x^2$ (2) $16x^2$ (3) $8x^2$ (4) $-8x^2$

9) The solution of $(2x - 1)^2 = 9$ is equal to
(1) -1 (2) 2 (3) $-1, 2$ (4) None of these

10) The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square
(1) $100, 120$ (2) $10, 12$ (3) $-120, 100$ (4) $12, 10$

11) If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in
(1) A.P (2) G.P (3) Both A.P and G.P (4) None of these

12) Graph of a linear polynomial is a
(1) Straight line (2) circle (3) parabola (4) hyperbola

13) The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X-axis is
(1) 0 (2) 1 (3) 0 or1 (4) 2

14) For the given matrix $A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ the order of the matrix A^T is
(1) 2 x 3 (2) 3 x 2 (3) 3 x 4 (4) 4 x 3

15) If A is a 2 x 3 matrix and B is a 3 x 4 matrix, how many columns does AB have
(1) 3 (2) 4 (3) 2 (4) 5

16) If number of columns and rows are not equal in a matrix then it is said to be a
(1) diagonal matrix (2) rectangular matrix (3) square matrix (4) unit matrix

17) Transpose of a column matrix is
(1) Unit matrix (2) diagonal matrix (3) column matrix (4) row matrix

18) Find the matrix X, if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$
(1) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (4) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19) Which of the following can be calculated from the given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$,
$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ (i) A^2 (ii) B^2 (iii) AB (iv) BA

(1) (i), (ii) only (2) (ii), (iii) only (3) (ii), (iv) only (4) all of these

20) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?
(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$ (iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

(1) (i) and (ii) only (2) (ii) and (iii) only
(3) (iii) and (iv) only (4) All of these

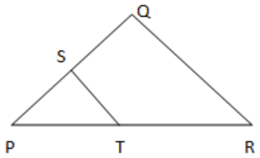
UNIT – 4 : GEOMETRY

1) If in triangles ABC and EDF, $\frac{AB}{DB} = \frac{BC}{FD}$ then they will be similar, when
(1) $\angle B = \angle E$ (2) $\angle A = \angle D$ (3) $\angle B = \angle D$ (4) $\angle A = \angle F$

2) In $\Delta LMN, \angle L = 60^o, \angle M = 50^o$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is
(1) 40^o (2) 70^o (3) 30^o (4) 110^o

3) If ΔABC is an isosceles triangle with $\angle C = 90^o$ and AC = 5 cm, then AB is
(1) 2.5 cm (2) 5 cm (3) 10 cm (4) $5\sqrt{2}$ cm

4) In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is



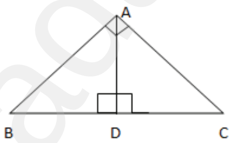
(1) 25 : 4 (2) 25 : 7 (3) 25 : 11 (4) 25 : 13

5) The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ=10 cm, then the length of AB is
(1) $6\frac{2}{3}$ cm (2) $\frac{10\sqrt{6}}{3}$ cm (3) $66\frac{2}{3}$ cm (4) 15 cm

6) If in ΔABC, DE || BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is
(1) 1.4 cm (2) 1.8 cm (3) 1.2 cm (4) 1.05 cm

7) In a ΔABC , AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is
(1) 6 cm (2) 4 cm (3) 3 cm (4) 8 cm

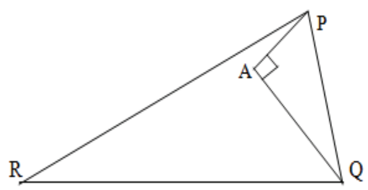
8) In the adjacent figure, $\angle BAC = 90^o$ and $AD \perp BC$ then



(1) $BD \cdot CD = BC^2$ (2) $AB \cdot AC = BC^2$ (3) $BD \cdot CD = AD^2$ (4) $AB \cdot AC = AC^2$

9) Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?
(1) 13 m (2) 14 m (3) 15 m (4) 12.8 m

10) In the given figure, PR= 26 cm, QR = 24 cm, $\angle PAQ = 90^o$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$.



(1) 80^o (2) 85^o (3) 75^o (4) 90^o

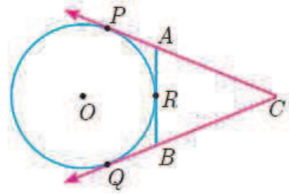
11) A tangent is perpendicular to the radius at the
(1) centre (2) point of contact (3) infinity (4) chord

12) How many tangents can be drawn to the circle from an exterior point?
(1) one (2) two (3) infinite (4) zero

13) The two tangents from an exterior points P to a circle with centre at O are PA and PB. $\angle APB = 70^o$ then the value of $\angle AOB$ is

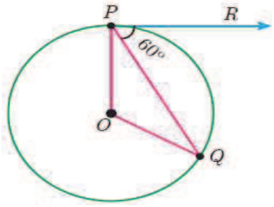
(1) 100^o (2) 110^o (3) 120^o (4) 130^o

14) In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is



(1) 6 cm (2) 5 cm (3) 8 cm (4) 4 cm

15) In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is



(1) 120^o (2) 100^o (3) 110^o (4) 90^o

UNIT – 5 : COORDINATE GEOMETRY

1) The area of triangle formed by the points $(-5,0),(0,-5)$ and $(5,0)$ is
(1) 0 sq.units (2) 25 sq.units (3) 5 sq.units (4) none of these

2) A man walks near a wall, such that the distance between him and the wall is 20 units. Consider the wall to be the Y axis. The path travelled by the man is
(1) $x = 10$ (2) $y = 10$ (3) $x = 0$ (4) $y = 0$

3) The straight line given by the equation $x = 11$ is
(1) parallel to X - axis (2) parallel to Y - axis
(3) passing through the origin (4) passing through the point $(0, 11)$

4) If $(5,7),(3,p)$ and $(6,6)$ are collinear, then the value of p is
(1) 3 (2) 6 (3) 9 (4) 12

5) The point of intersection of $3x - y = 4$ and $x + y = 8$ is
(1) $(5,3)$ (2) $(2,4)$ (3) $(3,5)$ (4) $(4,4)$

6) The slope of the line joining $(12,3),(4,a)$ is $\frac{1}{8}$. Then value of $'a'$ is
(1) 1 (2) 4 (3) – 5 (4) 2

7) The slope of the line which is perpendicular to line joining the points $(0, 0)$ and $(-8,8)$ is
(1) -1 (2) 1 (3) $\frac{1}{3}$ (4) -8

8) If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of the perpendicular bisector of PQ is
(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 0

9) If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissa is 5 then the equation of the line AB is
(1) $8x + 5y = 40$ (2) $8x - 5y = 40$ (3) $x = 8$ (4) $y = 5$

10) The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is
(1) $7x - 3y + 4 = 0$ (2) $3x - 7y + 4 = 0$ (3) $3x + 7y = 0$ (4) $7x - 3y = 0$

11) Consider four straight lines
(i) $l_1 : 3y = 4x + 5$ (ii) $l_2 : 4y = 3x - 1$ (iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$
Which of the following statement is true?
(1) l_1 and l_2 are perpendicular (2) l_1 and l_4 are parallel
(3) l_2 and l_4 are perpendicular (4) l_2 and l_3 are parallel

12) A straight line has equation $8y = 4x + 21$. Which of the following is true?
(1) The slope is **0.5** and y–intercept **2.6** (2) The slope **5** and y–intercept **1.6**
(3) The slope **0.5** and y–intercept **1.6** (4) The slope **5** and y–intercept **2.6**

13) When proving that a quadrilateral is a trapezium, it is necessary to show
(1) Two sides are parallel
(2) Two parallel and two non-parallel sides
(3) Opposite sides are parallel
(4) All sides are of equal length

14) When proving that a quadrilateral is a parallelogram by using slopes you must find
(1) The slopes are parallel (2) The slopes of two pair of opposite sides
(3) The lengths of all sides (4) Both the lengths and slopes of two sides

15) (2, 1) is the point of intersection of two lines
(1) $x - y - e = 0 ; 3x - y - 7 = 0$ (2) $x + y = 3 ; 3x + y = 7$
(3) $3x + y = 3 ; x + y = 7$ (4) $x + 3y - 3 = 0 ; x - y - 7 = 0$

UNIT – 6 : TRIGONOMETRY

1) The value of $sin^2\theta + \frac{1}{1+tan^2\theta}$ is equal to
(1) $tan^2\theta$ (2) 1 (3) $cot^2\theta$ (4) 0

2) The value of $tan\theta\ cosec^2\theta -\ tan\theta$ is equal to
(1) $sec\theta$ (2) $cot^2\theta$ (3) $sin\theta$ (4) $cot\theta$

3) If $(sin\alpha + cosec\alpha)^2 + (cos\alpha + sec\alpha)^2 = k + tan^2\alpha + cot^2\alpha$, then k is equal to
(1) 9 (2) 7 (3) 5 (4) 3

4) If $sin\theta + cos\theta = a$ and $sec\theta + cosec\theta = b$, then $b\ (a^2 - 1)$ is equal to
(1) $2a$ (2) $3a$ (3) 0 (4) $2ab$
(2)

5) If $5x = sec\theta$ and $\frac{5}{x} = tan\theta$ then $x^2 - \frac{1}{x^2}$ is equal to
(1) 25 (2) $\frac{1}{25}$ (3) 5 (4) 1

6) If $sin\theta = cos\theta$, then $2tan^2\theta + sin^2\theta - 1$ is equal to
(1) $\frac{-3}{2}$ (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{-2}{3}$

7) If $x = atan\theta$ and $y = bsec\theta$ then
(1) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (2)) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (3)) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

8) $(1 + tan\theta + sec\theta)(1 + cot\theta - cosec\theta)$ is equal to
(1) 0 (2) 1 (3) 2 (4) -1
(2)

9) $acot\theta + bcosec\theta = p$ and $bcot\theta + acosec\theta = q$ then $p^2 - q^2$ is equal to
(1) $a^2 - b^2$ (2) $b^2 - a^2$ (3) $a^2 + b^2$ (4) $b - a$

10) If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}: 1$, then the angle of elevation of the sun has measure
(1) 45^o (2) 30^o (3) 90^o (4) 60^o

11) The electric pole subtends an angle of 30^o at a point on the same level as its foot. At a second point $'b'$ metres above the first, the depression of the foot of the tower is 60^o. The height of the tower (in metres) is equal to
(1) $\sqrt{3}b$ (2) $\frac{b}{3}$ (3) $\frac{b}{2}$ (4) $\frac{b}{\sqrt{3}}$

12) A tower is 60 m height. Its shadow is x metres shorter when the sun's altitude is 45^o than when it has been 30^o, then x is equal to
(1) 41.92 m (2) 43.92 m (3) 43 m (4) 45.6 m

13) The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30^o and 60^o respectively. The height of the multistoried building and the distance between two buildings (in metres) is
(1) $20, 10\sqrt{3}$ (2) $30, 5\sqrt{3}$ (3) $20, 10$ (4) $30, 10\sqrt{3}$

14) Two persons are standing $'x'$ metres apart fronm each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is
(1) $\sqrt{2}x$ (2) $\frac{x}{2\sqrt{2}}$ (3) $\frac{x}{\sqrt{2}}$ (4) $2x$

15) The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45^o. The height of location of the cloud from the lake is
(1) $\frac{h(1+tan\beta)}{1-tan\beta}$ (2) $\frac{h(1-tan\beta)}{1+tan\beta}$ (3) $h\ \tan\ (45^o - \beta)$ (4) None of these

UNIT – 7 : MENSURATION

1) The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(1) $60\pi\ cm^2$ (2) $68\pi\ cm^2$ (3) $120\pi\ cm^2$ (4) $136\pi\ cm^2$

2) If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
(1) $4\pi r^2\ cm^2$ (2) $6\pi r^2\ cm^2$ (3) $3\pi r^2\ cm^2$ (4) $8\pi r^2\ cm^2$

3) The height of a right circular cone whose radius 5 cm and slant height is 13 cm will be
(1) 12 cm (2) 10 cm (3) 13 cm (4) 5 cm

4) If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(1) 1 : 2 **(2) 1 : 4** **(3) 1 : 6** **(4) 1 : 8**

5) The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is
(1) $\frac{9\pi h^2}{8}$ sq.units (2) $24\pi h^2$ sq.units (3) $\frac{8\pi h^2}{9}$ sq.units (4) $\frac{56\pi h^2}{9}$ sq.units

6) In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is
(1) $5600\pi\ cm^3$ (2) $1120\pi\ cm^3$ (3) $56\pi\ cm^3$ (4) $3600\pi\ cm^3$

7) If the radius of the base of a cone is tripled and the height is doubled then the volume is
(1) made 6 times (2) made 18 times (3) made 12 times (4) unchanged

8) The total surface area of hemi-sphere is how much times the square of its radius.
(1) π (2) 4π (3) 3π (4) 2π

9) A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is
(1) $3x$ cm (2) x cm (3) $4x$ cm (4) $2x$ cm

10) A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. Then, the volume of the frustum is
(1) $3328\pi\ cm^3$ (2) $3228\pi\ cm^3$ (3) $3240\pi\ cm^3$ (4) $3240\pi\ cm^3$

11) A shuttle cock used for playing badminton has the shape of the combination of
(1) a cylinder and a sphere (2) a hemisphere and a cone
(3) a sphere and a cone (4) frustum of a cone and a hemisphere

12) A spherical ball of radius $\boldsymbol{r_1}$ units is melted to make 8 new identical balls each of radius $\boldsymbol{r_2}$ units. Then $\boldsymbol{r_1} : \boldsymbol{r_2}$ is
(1) 2 : 1 **(2) 1 : 2** **(3) 4 : 1** **(4) 1 : 4**

13) The volue (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is
(1) $\frac{4}{3}\pi$ (2) $\frac{10}{3}\pi$ (3) 5π (4) $\frac{20}{3}\pi$

14) The height and radius of the cone of which the frustum is a part are $\boldsymbol{h_1}$ units and $\boldsymbol{r_1}$ units respectively. Height of the frustum is $\boldsymbol{h_2}$ units and radius $\boldsymbol{r_2}$ units. If $\boldsymbol{h_2 : h_1 = 1 : 2}$ then, $\boldsymbol{r_1 : r_2}$ -is
(1) 1 : 3 **(2) 1 : 2** **(3) 2 : 1** **(4) 3 : 1**

15) The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is
(1) 1 : 2 : 3 **(2) 2 : 1 : 3** **(3) 1 : 3 : 2** **(4) 3 : 1 : 2**

UNIT – 8 : STATISTICS AND PROBABILITY

1) Which of the following is not a measure of dispersion?
(1) Range **(2)** Standard Deviation **(3)** Arithmetic Mean **(4)** Variance

2) The range of the data 8, 8, 8, 8, 8, . . . , 8 is
(1) 0 **(2) 1** **(3) 8** **(4) 3**

3) The sum of all deviations of the data from its mean is
(1) Always positive (2) Always negative
(3) zero (4) non-zero integer

4) The mean of 100 observations is 40 and their standard deviations is 3. The sum of squares of all deviations is
(1) 40000 **(2) 160900** **(3) 160000** **(4) 30000**

5) Variance of first 20 natural numbers is
(1) 32.25 **(2) 44.25** **(3) 33.25** **(4) 30**

6) The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
(1) 3 **(2) 15** **(3) 5** **(4) 225**

7) If the standard deviation of x, y, z is $p$then the standard deviation of $3x + 5, 3y + 5, 3z + 5$ is
(1) $3p + 5$ **(2)** $3p$ **(3)** $p + 5$ **(4)** $9p + 15$

8) If the mean and coefficient of variation of a data are **4** and **87.5%** then the standard deviation is
(1) 3.5 **(2) 3** **(3) 4.5** **(4) 2.5**

9) Which of the following is incorrect?
(1) $P(A) > 1$ **(2)** $0 \leq P(A) \leq 1$ **(3)** $P(\varphi) = 0$ **(4)** $P(A) + P(\bar{A}) = 1$

10) The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is
(1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$ (3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$

11) A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
(1) $\frac{3}{10}$ (2) $\frac{7}{10}$ (3) $\frac{3}{9}$ (4) $\frac{7}{9}$

12) The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is
(1) 2 **(2) 1** **(3) 3** **(4) 1.5**

13) Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is
(1) 5 **(2) 10** **(3) 15** **(4) 20**

14) If a letter is chosen at random from the English alphabets $\{a, b, c, \dots, z\}$, then the probability that the letter chosen precedes x
(1) $\frac{12}{13}$ (2) $\frac{1}{13}$ (3) $\frac{23}{26}$ (4) $\frac{3}{26}$

15) A purse contains 10 notes of Rs.2000, 15 notes of Rs.500 and 25 notes of Rs.200. One note is drawn at random. What is the probability that the note is either a Rs.500 note of Rs.200 note?
(1) $\frac{1}{5}$ (2) $\frac{3}{10}$ (3) $\frac{2}{3}$ (4) $\frac{4}{5}$