ANNE'S ACADEMY
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL - 629004

Common Model Test I (2022 – 23) – Kanyakumari District
CLASS – XI
MATHEMATICS

Time Allowed : 3 Hrs Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) If two sets A and B have 17 elements in common, then the number of elements common to the set $A \times B$ and $B \times A$ is
(1) 2^{17} (2) 17^2 (3) 34 (4) insufficient data

2) The number of constant functions from a set containing m elements to a set containing n elements is
(1) mn (2) m (3) n (4) $m+n$

3) The value of $\log_3 \frac{1}{81}$ is
(1) -2 (2) -8 (3) -4 (4) -9

4) If a and b are the real roots of the equation $x^2 - kx + c = 0$, then the distance between the points $(a, 0)$ and $(b, 0)$ is
(1) $\sqrt{k^2 - 4c}$ (2) $\sqrt{4k^2 - c}$ (3) $\sqrt{4c - k^2}$ (4) $\sqrt{k - 8c}$

5) If $\cos p\theta + \cos q\theta = 0$ and if $p \neq q$, then θ is equal to (n is any integer)
(1) $\frac{\pi(3n+1)}{p-q}$ (2) $\frac{\pi(2n+1)}{p \pm q}$ (3) $\frac{\pi(n \pm 1)}{p \pm q}$ (4) $\frac{\pi(n+2)}{p+q}$

6) If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to
(1) $b^2 - 1$, if $b \leq \sqrt{2}$ (2) $b^2 - 1$, if $b > \sqrt{2}$ (3) $b^2 - 1$, if $b \geq 1$ (4) $b^2 - 1$, if $b \geq \sqrt{2}$

7) In a plane there are 10 points are there out of which 4 points are collinear, then the number of triangles formed is
(1) 110 (2) ${}^{10}C_3$ (3) 120 (4) 116

8) $1 + 3 + 5 + 7 + \cdots + 17$ is equal to
(1) 101 (2) 81 (3) 71 (4) 61

9) The sum up to n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \cdots$ is
(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1.

10) The value of the series $\frac{1}{2} + \frac{7}{4} + \frac{13}{8} + \frac{19}{16} + \cdots$ is
(1) 14 (2) 7 (3) 4 (4) 6.

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11) If a vertex of a square is at the origin and its one side lies along the line $4x + 3y - 20 = 0$, then the area of the square is

(1) 20 sq. units (2) 16 sq. units (3) 25 sq. units (4) 4 sq.units

12) If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$,, then c equals to

(1) -3 (2) -1 (3) 3 (4) 1

13) If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of $\lambda, A^2 = O$?

(1) 0 (2) ± 1 (3) -1 (4) 1

14) The value of x, for which the matrix $A = \begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular

(1) 9 (2) 8 (3) 7 (4) 6

15) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + x\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\vec{a} \cdot (\vec{b} \times \vec{c}) = 70$, then x is equal to

(1) 5 (2) 7 (3) 26 (4) 10

16) If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 2 and inclined at an angle $60°$, then the angle between $\vec{a}$ and $\vec{a} + \vec{b}$ is

(1) 30° (2) 60° (3) 45° (4) 90°

17) $\lim_{n\to\infty}\left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n}{n^2}\right)$ is

(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) ∞

18) If $g(x) = (x^2 + 2x + 1)\, f(x)$ and $f(0) = 5$ and $\lim_{x\to 0} \frac{f(x) - 5}{x} = 4$, then $g'(0)$ is

(1) 20 (2) 14 (3) 18 (4) 12

19) If $\int \frac{3^{\frac{1}{x}}}{x^2} dx = k\,(3^{\frac{1}{x}}) + c$, then the value of k is

(1) $\log 3$ (2) $-\log 3$ (3) $-\frac{1}{\log 3}$ (4) $\frac{1}{\log 3}$

20) Ten coins are tossed. The probability of getting at least 8 heads is

(1) $\frac{7}{64}$ (2) $\frac{7}{32}$ (3) $\frac{7}{16}$ (4) $\frac{7}{128}$

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PART – II

II. Answer any SEVEN questions. Question 30 is compulsory **7x2 = 14**

21) Let $A = \{a, b, c\}$. What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A?

22) Find the complete set of values of a for which the quadratic $x^2 - ax + a + 2 = 0$ has equal roots.

23) If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.

24) Prove that $32(\sqrt{3}) \sin\frac{\pi}{48} \cos\frac{\pi}{48} \cos\frac{\pi}{24} \cos\frac{\pi}{12} \cos\frac{\pi}{6} = 3$.

25) Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time?

26) Write the n^{th} term of the sequence $\frac{3}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2}, \ldots$ as a difference of two terms.

27) If A and B are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$.

28) Integrate the following function with respect to x:

$$\sqrt{3x+2}$$

29) Show that

$$\lim_{n\to\infty} \frac{1+2+3+\ldots+n}{3n^2+7n+2} = \frac{1}{6}$$

30) For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2 \ |\vec{b}|^2$

PART – III

III. Answer any SEVEN questions. Question 40 is compulsory **7x3 = 21**

31) Expand $\frac{1}{(3+2x)^2}$ in powers of x. Find a condition on x for which the expansion is valid.

32) Find the equations of the straight lines, making the y- intercept of 7 and angle between the line and the y-axis is 30°

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33) Solve $\sin x + \sin 5x = \sin 3x$

34) If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots, then prove that $ae = 2(b + f)$.

35) A number of four different digits is formed with the use of the digits 1,2,3,4 and 5 in all possible ways. Find the following

(i) How many such numbers can be formed?
(ii) How many of these are even?
(iii) How many of these are exactly divisible by 4?

36) Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.

37) Show that the vectors $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$ are coplanar.

38) Find $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.

39) Two cards are drawn from a pack of 52 cards in succession. Find the probability that both are Jack when the first drawn card is (i) replaced (ii) not replaced

40) Differentiate the following :

$$h(t) = \left(t - \frac{1}{t}\right)^{\frac{3}{2}}$$

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) If $y = e^{\tan^{-1}x}$, show that $(1+x^2)y'' + (2x-1)y' = 0$.

OR

b) There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

42) a) Integrate the following with respect to x:

$$e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)$$

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OR

b) Find the largest possible domain of the real valued function $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.

43) a) Resolve the following rational expressions into partial fractions.

$$\frac{x+12}{(x+1)^2(x-2)}$$

OR

b) Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)\,(y-z)\,(z-x)\,(xy+yz+zx)$.

44) a) If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) GARDEN (ii) DANGER.

OR

b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

45) a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

OR

b) **Prove by vector method that, The medians of a triangle are concurrent.**

46) a) Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$.

OR

b) Solve $\log_{5-x}(x^2 - 6x + 65) = 2$.

47) a) By the principle of Mathematical induction, prove that, for $n \geq 1$

$$1.2 + 2.3 + 3.4 + \cdots + n.(n+1) = \frac{n(n+1)(n+2)}{3}.$$

OR

b) Urn-I contains 8 red and 4 blue balls and urn-II contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red.