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COMMON FIRST REVISION TEST – 2023

Reg.No. :

Standard XII

MATHEMATICS

Time: 3.00 hrs. Part - I Marks: 90

I. Choose the correct answer: $20 \times 1 = 20$

1. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 19 b) 17 c) 21 d) 14

2. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

3. If $A^T A^{-1}$ is symmetric, then $A^2 = ?$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

4. $\cos^{-1}\left(\sqrt{3}/2\right)$ - The principal value

a) $\pi/2$ b) $\pi/3$ c) $5\pi/6$ d) $\pi/6$

5. If α, β and γ are the zero's of $x^3 + px^2 + qx + r$ then $\sum \frac{1}{\alpha}$ is

a) $-q/r$ b) $-p/r$ c) q/r d) $-q/p$

6. If $(1 + i)(1 + 2i)(1 + 3i) \ldots\ldots\ldots (1 + ni) = x + iy$ then the value of $2.5.10 \ldots. (1 + n^2)$ is

a) 1 b) i c) $x^2 + y^2$ d) $1 + n^2$

7. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

8. The general equation of a circle with centre $(-3,-4)$ and radius 3 units is

a) $x^2 + y^2 - 6x + 8y + 16 = 0$ b) $x^2 + y^2 - 6x - 8y + 16 = 0$

c) $x^2 + y^2 + 6x - 8y + 16 = 0$ d) $x^2 + y^2 + 6x + 8y + 16 = 0$

9. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$

a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) -1

10. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{10010}$ b) $\frac{1}{11000}$ c) $\frac{1}{10001}$ d) $\frac{1}{10100}$

11. The minimum value of the function $|3x - x| + 9$ is

a) 0 b) 3 c) 6 d) 9

12. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$ $z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

13. If $f(x) = \frac{x}{x+1}$, then its differential is

a) $\frac{1}{x+1}dx$ b) $\frac{-1}{(x+1)^2}dx$ c) $\frac{-1}{x+1}dx$ d) $\frac{-1}{(x+1)^2}dx$

14. The value of $\int_0^\infty e^{-3x}x^2\,dx$ is

a) $7/27$ b) $5/27$ c) $4/27$ d) $2/27$

15. The solution of $\frac{dy}{dx} + P(x)y = 0$ is

a) $y = ce^{\int Pdx}$ b) $y = ce^{-\int Pdx}$ c) $x = ce^{-\int Pdy}$ d) $x = ce^{\int Pdy}$

16. Angle between $y^2 = x$ and $x^2 = y$ at the orgin is

a) $\tan^{-1}(3/4)$ b) $\tan^{-1}(4/3)$ c) $\pi/2$ d) $\pi/4$

17. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = ce^{x^2}$ b) $y = 2x^2 + c$ c) $x = ce^{-x^2}$ d) $x = x^2 + c$

18. If $P(X = 0) = 1 - P(X = 1)$ if $E(X) = 3\,var(X)$, then $P(X = 0)$ is

a) $2/3$ b) $2/5$ c) $1/5$ d) $1/3$

19. The operation * defined by $a * b = ab/7$ is not a binary operation on

a) Q^+ b) Z c) R d) C

20. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

a) 0 b) 1 c) 2 d) ∞

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $\text{adj}A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22. Find a polynomial equation of minimum degree with rational co-efficients, having $2 - \sqrt{3}$ as a root.

23. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

24. Find the principal value of $\tan^{-1}(\sqrt{3})$

25. Obtain the equation of the circle for which (3,4) and (2,−7) are the ends of a diameter.

26. If $\vec{a}, \vec{b}, \vec{c}$ are the three vectors, prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

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27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

29. Write the Maclaurin series expansion of the following function : e^x

30. Show that $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Solve the following system of linear equation using Matrix inversion method : $5x + 2y = 3$, $3x + 2y = 5$

32. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

33. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

34. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$, $-1 < x < 1$

35. The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. there is heating tube located at the focus of each parabola : How high is this tube located above the vertex of the parabola?

36. Evaluate : $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x^2 - 4x + 3}\right)$

37. Use the linear approximation to find approximate value of $(123)^{2/3}$

38. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type, find (i) $A \vee B$ ii) $A \wedge B$

39. Find the mean and variance of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

40. Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Solve the following systems of linear equations by Cramer's rule :

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \quad \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \quad \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$$

(OR)

b) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

42. a) If $z = x + iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

(OR)

b) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

43 a) If $2 + i$ and $3 - \sqrt{2}$ are roots of equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

(OR)

b) Find the vertex, focus, equation of directrix and length of the latus rectum of the following and draw the graph $y^2 - 4y - 8x + 12 = 0$

44. a) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.

(OR)

b) Show that the angle between the curves $y = x^2$ and $x = y^2$ at (0,0) and (1,1) is $\pi/2$ and $\tan^{-1}(3/4)$

45. a) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$

(OR)

b) Find the parametric vector, non parametric vector and cartesian form of the equations of the plane passing through the three non-collinear points (3,6,–2) (–1,–2,6) and (6,4,–2)

46. a) Prove that $g(x, y) = x\log\left(y/x\right)$ is homogeneous, what is the degree? Verify Euler's theorem for g.

(OR)

b) The growth of population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

47. a) Find the area of the region bounded between the parables $y^2 = 4x$ and $x^2 = 4y$

(OR)

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M closed under *. If so, examine the commutative, associate, identity and inverse properties for the operation * on M.

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