V12M

Virudhunagar District Common Examinations

First Revision Examination - January 2023

Standard 12

Time allowed: 3 hours **MATHEMATICS** Maximum Marks: 90

PART - A

Answer all the questions. Choose the correct or most suitable answer: **20×1=20**

1) If A^+A^{-1} is symmetric, then A^2 =
   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

2) If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is
   a) $-4/5$ b) $-3/5$ c) $4/5$ d) $3/5$

3) If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is
   a) $|z|$ b) $\bar{z}$ c) $1/z$ d) 1

4) If z is a complex number such that $z \in C\backslash R$ and $z+\frac{1}{z} \in R$, then $|z|$ is
   a) 0 b) 1 c) 2 d) 3

5) The polynomial x^3-kx^2+9x has three zeros if and only if, k satisfies
   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

6) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z = 3\pi/z$ then the value of $x^{2017}+y^{2018}+z^{2019}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is
   a) 0 b) 1 c) 2 d) 3

7) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to
   a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

8) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
   a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

9) The focus of the parabola $y^2-8x-2y+17 = 0$ is
   a) (1, 4) b) (3, 1) c) (4, 1) d) (1, 3)

10) Which of the complex number is nearer to origin?
   a) 1+4i b) −3+2i c) 4−3i d) 1+2i

11) The tangent to the curve $y^2-xy+9 = 0$ is vertical when
   a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = 1/2$ d) $y = \pm 3$

12) The maximum product of two positive numbers when their sum of the squares is 200, is
   a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13) If $u = x^y y^x$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) $(x+y)u$ b) $(x+y+\log u)u$ c) $x+y+\log u$ d) $u(x+y+\log u)u$

14) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

a) 0.4 cu.cm b) 0.45 cu. cm c) 2 cu. cm d) 4.8 cu. cm

15) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$

a) 3/2 b) 1/2 c) 0 d) 2/3

16) If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

a) 4 b) 1 c) 3 c) 2

17) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ then $A \cap B$

a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

18) The solution of the differential equation $\frac{\partial y}{\partial x} = 2xy$ is

a) $y = ce^{x^2}$ b) $y = 2x^2+c$ c) $y = ce^{-x^2} + c$ d) $y = x^2+c$

19) Which of the following is a discrete random variable?

I] The number of cars crossing a particular signal in a day

II] The number of customers in a queue to buy train tickets at a moment

III] The time taken to complete a telephone call

a) I and II b) II only c) III only d) II and III

20) The operation * defined by $a*b = ab/7$ is not a binary operation

a) Q^+ b) Z c) R d) C

Part - B

i) Answer any seven questions. ii) Q.No. 30 is compulsory. 7×2=14

21) Find rank of the matrix by using minor method $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

22) Construct the cubic equation with roots 2, $\frac{1}{2}$ and 1.

23) Find the domain of the $\tan^{-1}\sqrt{a-x^2}$

24) If $|\vec{a}+\vec{b}| = 60$, $|\vec{a}-\vec{b}| = 40$ and $|\vec{a}| = 22$ then find $|\vec{b}|$

25) Suppose of (x) is differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of f(7)?

26) If $w(x,y) = x^3-3xy+2y^2$, $x, y \in R$ find the linear approximation for w at (1, −1)

27) Evaluate: $\int_0^\infty e^{-ax} x^n \, dx$

28) Find the differential equation for the family of all straight lines passing through the origin.

29) Four coins are tossed once find the probability mass function for number of heads.

30) Prove De Morgan's law by using Truth table.

Part - C

Note: i) Answer any seven questions only. ii) Q.No. 40 is compulsory. 7×3=21

31) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, Prove that $A^{-1} = A^T$

32) Prove by Vector method that the area of the quadrilateral ABCD having diagonal AC and BD is $\frac{1}{2}|\overline{AC} \times \overline{BD}|$

33) Represent the complex number $1 + i\sqrt{3}$ in polar form.

34) If p and q are the roots of the equation $x^2+nx+n = 0$, show that $\sqrt{p/q} + \sqrt{q/p} + \sqrt{n/\ell} = 0$

35) Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

36) If the equation $3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

37) If the radius of the sphere, with radius 10 cm, has to decrease by 0.1 cm approximately how much with it's volume decrease?

38) Evaluate: $\int_0^{\pi/2} \frac{dx}{1+5\cos^2 x}$

39) Find the mean and variance of random variable, x whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

40) Find the local extrema of the function $f(x) = x^4+32x$.

Part - D

Note: Answer all the questions: **7×5=35**

41) a] Solve by Cramer's rule, the system of equations $x_1-x_2 = 3$, $2x_1+3x_2+4x_3 = 17$, $x_2+2x_3 = 7$.

(OR)

b] If $z = (\cos\theta + i\sin\theta)$ show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

42) a] Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

(OR)

b] Evaluate: $\int_0^4 \frac{1}{\sin x + \cos x}\, dx$

43) a] Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabola path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below due line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

b] Find the non parametric form of vector equation and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

44) a] If the roots of $x^3+px^2+qx+r=0$ are in H.P. prove that $9pqr = 27rq + 2p$

(OR)

b] Evaluate $\sin\left(\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{4}\right)\right)$

45) a] For the function $f(x) = 4x^3+3x^2-6x+1$ find point of inflection.

(OR)

b] Find the volume of a sphere when rotating a circle with radius a.

46) a] Find the area of the region bounded by the curve $y = \sin x$ and $y = \cos x$ between $x = \pi/4$ and $x = 5\pi/4$

(OR)

b] Solve: $\frac{\partial y}{\partial x}=\sqrt{\frac{1-y^2}{1-x^2}}$

47) a] If the probability that a fluorescent light has a useful life of atleast 600 hours is 0.9, find the probabilities that among 12 such lights.

(i) exactly 10 will have a useful life of atleast 600 hours.

(ii) at least 11 will have a useful life of at least 600 hours.

(iii) at least 2 will not have a useful life of atleast 600 hours.

(OR)

b] Using the equivalence property, show that $p \leftrightarrow q \equiv (p \cap q) \cup (\neg p \cap \neg q)$

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