Tsi12Mat **Tenkasi District Common Examinations**

Second Revision Test - January 2023

27-01-2023 **Standard - 12**

Time Allowed:3.00 Hours **MATHEMATICS** Maximum Marks:90

PART - I

Note: **1. All questions are compulsory.** **20×1=20**

2. Choose the correct or most suitable answer from the given four alternatives. Write the option code and the corresponding answer.

1. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^AA^T$ then BB^T =
   a) A b) B c) I_3 d) B^T
2. If z = x + iy is a complex number such that |z+2| = |z−2| then the locus of z is
   a) real axis b) imaginary axis
   c) ellipse d) circle
3. The polynomial $x^3 + 2x + 3$ has
   a) one negative and two imaginary zeros
   b) one positive and two imaginary zeros
   c) three real zeros
   d) no zeros
4. Identify the type of conic section forthe equations $x^2 + y^2 + x - y = 0$.
   a) Circle b) Parabola
   c) Ellipse d) Hyperbola
5. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then cos 2u is equal to
   a) $\tan^2\alpha$ b) 0 c) -1 d) $\tan 2\alpha$
6. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
   a) 1 b) $-\frac{1}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{\sqrt{3}}{2}$
7. The locus of a point whose distance from (-2, 0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
   a) a parabola b) a hyperbola
   c) an ellipse d) a circle
8. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is
   a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{4}$
9. Var (4X + 3) is
   a) 7 b) 16 Var (X) c) 19 d) 0
10. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
    a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$
11. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to
    a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u
12. The circle passing through (1,−2) and touching the axis of x at (3,0) passing through the point
    a) (-5,2) b) (2,-5) c) (5,-2) d) (-2,5)

13. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

14. Which of the following are statements?
(i) May God bless you
(ii) Rose is a flower
(iii) Milk is White
(iv) 1 is a prime number
a) (i), (ii), (iii) b) (i), (ii), (iv)
c) (i), (iii), (iv) d) (ii), (iii), (iv)

15. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals
a) (1, 0) b) (-1, 1)
c) (0, 1) d) (1, 1)

16. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

a) $\frac{x}{e^x}$ b) $\frac{e^x}{x}$ c) λe^x d) e^x

17. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent and let x equal the total number of heads that result. The value of E(X) is
a) 0.11 b) 1.1 c) 11 d) 1

18. If $u = \frac{1}{\sqrt{x^2 + y^2}}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

a) $\frac{1}{2}u$ b) u c) $\frac{3}{2}u$ d) -u

19. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then k =
a) 0 b) $\sin\theta$
c) $\cos\theta$ d) 1

20. In the set R of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on R?
a) $a*b = \min(a-b)$ b) $a*b = \max(a,b)$
c) $a*b = a$ d) $a*b = a^b$

PART - II

Note: Answer any seven questions. 7×2=14
Question Number 30 is compulsory.

21. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it in a row-echelon form.

22. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

23. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

24. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

25. If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?
26. For any vector $\bar{a}$, prove that $\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(\bar{a}\times\hat{j})+\hat{k}\times(\bar{a}\times\hat{k})=2\bar{a}$.
27. Suppose f(x) is a differentiable function for all x with $f'(x)\leq 29$, and $f(2)=17$. What is the maximum value of f(7)?
28. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.
29. Suppose that f(x) given below represents a probability mass function.

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find the Value of C.

30. Form the differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is

PART - III

Note: Answer any seven questions. **7×3=21**
Question Number 40 is compulsory.

31. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)
32. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1+z_2+z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.
33. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
34. If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.
35. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.
36. Find the magnitude and the direction cosines of the torque about the point (2, 0, −1) of a force $2\hat{i}+\hat{j}-\hat{k}$ whose line of action passes through the origin.
37. Find the local extremum of the function $f(x) = x^4 + 32x$.
38. If $v(x, y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$.
39. Prove that $p\rightarrow(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table.
40. Evaluate: $\int_0^3\frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{3-x}}$.

PART - IV

Note: Answer all the questions. **7×5=35**

41. a) By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_6 + O_2 \longrightarrow H_2O + CO_2$

(OR)

b) If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that

(i) $\frac{x}{y}+\frac{y}{x} = 2\cos(\alpha-\beta)$

(ii) $\frac{x^m}{y^n}+\frac{y^n}{x^m} = 2i\sin(m\alpha-n\beta)$

42. a) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$. If it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

(OR)

b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

43. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

44. a) Sketch the curve $y = f(x) = x^3 - 6x - 9$.

(OR)

b) Let $f(x,y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x,y)\in R^2$. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y\partial x}$ and $\frac{\partial^2 f}{\partial x\partial y}$.

45. a) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B and C are (−1,1), (3,2) and (0,5) respectively.

(OR)

b) Find the non-parametric form of Vector equation and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.

46. a) Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C. Find
(i) The temperature of water after 20 minutes
(ii) The time when the temperature is 40°C

$\left[\log_e\frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$

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(OR)

b) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

47. a) Find the angle between the curves $y = x^2$ and $y = (x-2)^2$ at the Point of Intersection.

(OR)

b) i) Let A be Q\{1} Define * on A by $x * y = x + y - xy$. Is * binary on A? If so examine the commutative and associative properties satisfied by * on A.
ii) Let A be Q\{1} Define * on A by $x * y = x + y - xy$. Is * binary on A? If so, examine the existence of identity, existence of inverse properties for the operation * on A.
