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COMMON HALFYEARLY EXAMINATION DECEMBER 2022

Standard-12

Reg No

PART-III-MATHEMATICS

Time Allowed:2.30 Hours Maximum Marks: 90

Instructions: 1. Check the question paper for fairness of printing. If there is any Lack of fairness,inform the Hall Supervisor Immediately

2.Use Blue or Black Ink to write and underline and pencil to draw Diagrams.

PART-I

20x1=20

Note: I) Answer all the questions.

II) Choose the most appropriate answer from the given four alternatives And write the option code and the corresponding answer.

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

2. If $A=\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

(1) $(1,0)$ (2) $(-1,1)$ (3) $(0,1)$ (4) $(1,1)$

4. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(1) -2 (2) -1 (3) 1 (4) 2

5. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(1) 2 (2) 4 (3) 1 (4) ∞

6. If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$, then $p^2+q^2+r^2+2pqr =$

(1) 0 (2) 1 (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

7. If $a \le \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \le b$, then the values of a and b is

(1) $a=0,\ b=\frac{\pi}{4}$ (2) $a=\frac{\pi}{4},\ b=\pi$ (3) $a=0,\ b=\pi$ (4) $a=\frac{\pi}{2},\ b=\pi$

8. For the hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$. Which of the following remains constant when α varies

(1) eccentricity (2) directrix
(3) abscissae of vertices (4) abscissae of foci

9. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(1) $15<m<65$ (2) $35<m<85$ (3) $-85<m<-35$ (4) $-35<m<15$

10. The coordinates of the point where the line $\vec{r}=(6\hat{i}-\hat{j}-3\hat{k})+t(-\hat{i}+4\hat{k})$ meets the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=3$ are

(1) $(2,1,0)$ (2) $(7,-1,-7)$ (3) $(1,2,-6)$ (4) $(5,-1,1)$

11. If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},-2$ (2) $-\frac{1}{2},2$ (3) $-\frac{1}{2},-2$ (4) $\frac{1}{2},2$

12. The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

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13. The value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

14. The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

15. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

16. The number of arbitrary constants in the general solutions of order n and $n+1$ are respectively

(1) $n-1,\ n$ (2) $n,\ n+1$ (3) $n+1,\ n+2$ (4) $n+1,\ n$

17. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is (1) 2 (2) -2 (3) 1 (4) -1

18. Suppose that X takes on one of the values 0, 1, and 2. If for some constant k, $P(X=i) = k\,P(X=i-1)$ for $i = 1,2$ and $P(X=0) = \frac{1}{7}$, then the value of k is

(1) 1 (2) 2 (3) 3 (4) 4

19. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are (1) 1 (2) 2 (3) 3 (4) 4

20. If $x = r\cos\theta, y = r\sin\theta$ then $\frac{\partial r}{\partial x} =$

(1) $\sec\theta$ (2) $\sin\theta$ (3) $\cos\theta$ (4) $\operatorname{cosec}\theta$

PART-II

Answer any seven questions. Question no. 30 is compulsory **7x2=14**

21. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_2$.

22. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

23. Identify the type of conic section for $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

24. Find the acute angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are parallel or perpendicular.

25. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \le 29$ and $f(2) = 17$. What is the maximum value of $f(7)$?

26. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

27. Evaluate $\int_0^1 x^3(1-x)^4\,dx$.

28. Provethat $Var(aX+b) = a^2 Var(X)$ where a and b are constants

29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

30. If $|z - 2 + i| \le 2$, then find the greatest value of $|z|$

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PART-III

Answer any seven questions. Question no. 40 is compulsory 7x3=21

31. Find the rank of $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$.

32. If $\omega \neq 1$ is a cube root of unity, show that $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

33. If α,β,γ, and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

34. Is $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$ true? Justify your answer.

35. Prove by vector method that an angle in a semi-circle is a right angle.

36. Using the l'Hôpital Rule, prove that $\lim\limits_{x\to 0^+}(1+x)^{\frac{1}{x}}=e$.

37. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.

38. Show that $y=e^{-x}+mx+n$ is a solution of the differential equation $e^x\left(\dfrac{d^2y}{dx^2}\right)-1=0$.

39. On $\mathbb{Z}$, define $*$ by $(m*n)=m^n+n^m : \forall m,n\in\mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?

40. The line PP' is a focal chord of the parabola $y^2=8x$ and if the coordinates of P are (18,12) then find the coordinates of P'

PART-IV

Answer all the questions: 7x5=35

41a) Investigate for what values of λ and μ
$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) Find the volume of the spherical cap of height h a sphere of radius r.

42 a) If $z=x+iy$ and $\arg\left(\dfrac{z-i}{z+2}\right)=\dfrac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$.

(OR)

b) The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

43 a) Solve $\cos\left(\sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left\{\cot^{-1}\left(\dfrac{3}{4}\right)\right\}$.

(OR)

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b) A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form.

44 a) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

(OR)

b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,

(i) how fast is the top of the ladder moving down the wall?

(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

45 a) A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

(OR)

b) Let $w(x,y,z)=\dfrac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z)\neq(0,0,0)$. Show that $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2 w}{\partial z^2}=0$.

46a) Determine the intervals of concavity of the curve $y=3+\sin x$.

(OR)

b) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=8$.

47 a) Prove that $p\rightarrow(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table.

(OR)

b) If X is the random variable with probability density function $f(x)$ given by,

$$f(x)=\begin{cases} x-1, & 1\leq x<2 \\ -x+3, & 2\leq x<3 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5\leq X\leq 2.5)$

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