Tsi12M

Tenkasi District Common Examinations
First Revision Examination - January 2023

06-01-2023

Standard 12
MATHEMATICS
Part - I

Time: 3.00 hrs Marks: 90

Note: i) All questions are compulsory.
ii) Choose the correct or most suitable answer from the given four alternatives. Write the options code and the corresponding answer. **20×1=20**

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?
a) adj A = $|A|A^{-1}$ b) adj (AB) = (adj A) (adj B)
c) det A^{-1} = $(\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2) If $z = x+iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is
a) real axis b) imaginary axis c) ellipse d) circle

3) A zero of x^3+64 is
a) 0 b) 4 c) 4i d) -4

4) The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
a) -2 b) -1 c) 1 d) 2

5) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then
a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$

6) Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?
a) $(p \wedge q) \rightarrow (p \vee q)$ b) $\neg(p \vee q) \rightarrow (p \wedge q)$
c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

7) If $x+y=k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
a) 3 b) -1 c) 1 d) 9

8) Distance from the origin to the lane $3x-6y+2z+7 = 0$ is
a) 0 b) 1 c) 2 d) 3

9) Evaluate: $\int_0^1 [2x]\,dx$ where [.] is the greatest integer function.
a) 0 b) 2 c) $\frac{1}{2}$ d) 1

10) The minimum value of the function $|3-x|+9$ is
a) 0 b) 3 c) 6 d) 9

11) If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

12) The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters is
a) $\frac{d^2y}{dx^2} - y = 0$ b) $\frac{d^2y}{dx^2} + y = 0$ c) $\frac{d^2y}{dx^2} = 0$ d) $\frac{d^2x}{dy^2} = 0$

13) A baloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 meters above the ground.
a) $\frac{3}{25}$ radians/sec b) $\frac{4}{25}$ radians/sec c) $\frac{1}{5}$ radians/sec d) $\frac{1}{3}$ radians/sec

14) The area between $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

15) If $2\vec{i} - \vec{j} + 3\vec{k}$, $3\vec{i} + 2\vec{j} + \vec{k}$, $\vec{i} + m\vec{j} + 4\vec{k}$ are coplanar, find the vlue of m.

a) 3 b) $-\frac{1}{3}$ c) -3 d) $\frac{1}{3}$

16) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

a) $y+\sin^{-1}x = c$ b) $x+\sin^{-1}y = 0$ c) $y^2+2\sin^{-1}x = c$ d) $x^2+2\sin^{-1}y=0$

17) If $y = 4x+c$ is a tangent to the circle $x^2+y^2 = 9$ find c

a) $\pm 3\sqrt{17}$ b) $\pm 17\sqrt{4}$ c) $\pm 17\sqrt{3}$ d) $\pm 4\sqrt{17}$

18) If A^TA^{-1} is symmetric, then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

19) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

a) $\frac{11}{243}$ b) $\frac{3}{8}$ c) $\frac{1}{243}$ d) $\frac{5}{243}$

20) Subtraction is not a binary operation in

a) $\mathbb{R}$ b) $\mathbb{Z}$ c) $\mathbb{N}$ d) $\mathbb{Q}$

Part - II

Note: Answer any seven questions. Question No. 30 is compulsory **7×2=14**

21) Prove that $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is orthogonal.

22) Find the square root of $-6+8i$

23) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

24) Find the angle between the planes $\vec{r}.(\vec{i} + \vec{j} - 2\vec{k}) = 3$ and $2x-2y+z = 2$.

25) If $f(x, y) = x^3-3x^2+y^2+5x+6$ then find f_x at $(1, -2)$

26) Evaluate: $\int_0^{\pi/2} (\sin^2 x + \cos^4 x)\,dx$

27) Determine the order and degree (if exists) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$$

28) Find the mean of the distribution $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$

29) Find the equation of the parabola whose vertex is $(-2, 5)$ and focus $(-2, 2)$

30) On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m,n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

Tsi12M

Part - III

7×3=21

Note: Answer any seven questions. Question No. 40 is compulsory.

31) Solve by matrix inversion method: $5x+2y=4$, $7x+3y=5$.

32) Solve the equation $2x^3+11x^2-9x-18=0$

33) State and prove that triangle inequality.

34) Evaluate: $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\,dx$

35) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

36) Solve the differential equation $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

37) If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$

38) Verify whether the compound proposition $(p\to q)\leftrightarrow(\neg p\to q)$ is a tautology or contradiction or contingency.

39) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X=0)$

40) If $\vec{a},\vec{b},\vec{c}$ are three vectors prove that $[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}]=2[\vec{a},\vec{b},\vec{c}]$

Part - IV

7×5=35

Note: Answer all the questions:

41) a] Investigate for what values of λ and μ the system of linear equations $x+2y+z=7$, $x+y+\lambda z=\mu$, $x+3y-5z=5$ has (i) no solution (ii) a unique solution.

(OR)

b] If $z=x+iy$ and $\arg\left(\frac{z-1}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$

42) a] If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

(OR)

b] At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. The flow is from the origin and the path of water is a parabola open upwards, find the height of water at a horizontal distance of 0.75 m from the point of origin.

43) a] If $\vec{a}=\vec{i}-\vec{j}$, $\vec{b}=\vec{i}-\vec{j}-4\vec{k}$, $\vec{c}=3\vec{j}-\vec{k}$ and $\vec{d}=2\vec{i}+5\vec{j}+\vec{k}$ verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

(OR)

b] The equation of electromotive force for an electric circuit containing resistance an self-inductance is $E=Ri+L\frac{di}{dt}$, where E is the electromotive force given to the circuit, the resistance and L, the coefficient of induction. Find the current i at time t when $E=0$.

44) a] Find intervals of concavity and points of inflexion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property, (iv) existence of identity and (v) existence of inverse for the operation $\times_{11}$ on a subset A = {1, 3, 4, 5, 9} of the set of remainder {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

45) a] Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$

(OR)

b] Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\vec{i} - \vec{j} - \vec{k}) + s(-\vec{i} + 2\vec{j} + \vec{k}) + t(-5\vec{i} - 4\vec{j} - 5\vec{k})$

46) a] For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices, foci and the length of latus rectum.

(OR)

b] If $U = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2 \cot u$

47) a] The cumulative distribution function of a discrete random variable is given by $f(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$ Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \ge 2)$

(OR)

b] If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2+y^2+z^2+2xyz = 1$.

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