ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Model Exam (2022 – 23)
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) If $P(X=0) = 1 - P(X=1)$. If $E(X) = 3\text{Var}(X)$, then $P(X=0)$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

2) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(1) $[1, 2]$ (2) $[-1, 1]$ (3) $[0, 1]$ (4) $[-1, 0]$

3) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (4) 18

4) If $\text{adj } A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\text{adj } B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $\text{adj}(AB)$ is

(1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

5) The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

(1) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (2) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

(3) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (4) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

6) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2) 2.5 (3) 3 (4) 3.5

7) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

8) The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

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9) If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

10) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

11) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3x dx\, m^3$ (2) $0.03x\, m^3$ (3) $0.03x^2 m^3$ (4) $0.03x^3 m^3$

12) If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

(1) $2x+1=0$ (2) $x=-1$ (3) $2x-1=0$ (4) $x=1$

13) The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

(1) $P = Ce^{kt}$ (2) $P = Ce^{-kt}$ (3) $P = Ckt$ (4) $P = C$

14) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7, \mu \neq -5$ (2) $\lambda = -7, \mu = 5$ (3) $\lambda \neq 7, \mu \neq -5$ (4) $\lambda = 7, \mu = -5$

15) The value of $\int_0^1 \left(\sin^{-1} x\right)^2 dx$ is

(1) $\frac{\pi^2}{4} - 1$ (2) $\frac{\pi^2}{4} + 2$ (3) $\frac{\pi^2}{4} + 1$ (4) $\frac{\pi^2}{4} - 2$

16) If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

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17) The function $\sin^4 x + \cos^4 x$ is increasing in the interval

(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

18) In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(1) 1 (2) 2 (3) 3 (4) 4

19) The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively

(1) 2, 3 (2) 3, 3 (3) 2, 6 (4) 2, 4

20) The area between $y^2 = 4x$ and its latus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

PART – II

II. Answer any SEVEN questions. Question 30 is compulsory **7x2 = 14**

21) If $v(x, y) = x^2 - xy + \frac{1}{4}y^2 + 7, x, y \in R$, find the differential dv

22) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form

23) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

24) The temperature T in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

25) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

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26) If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

27) Establish the equivalence property using truth table: $p \to q \equiv \neg p \vee q$

28) Find the value of $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

29) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Find the expected life of this electronic equipment.

30) If $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

PART – III

III. Answer any SEVEN questions. Question 40 is compulsory **7x3 = 21**

31) If $\int_0^\infty e^{-\alpha x^2} x^3 dx = 32$, $\alpha > 0$, find α

32) Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

33) In a competitive examination, one mark is awarded for every correct answer while 1/4 mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? Use matrix inversion method to solve.

34) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) at least 11 will have a useful life of at least 600 hours;

(ii) at least 2 will *not* have a useful life of at least 600 hours.

35) On $\mathbb{Z}$, define $*$ by $(m * n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?

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36) Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

37) Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

38) Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2 = \frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

39) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

40) If $\lim_{\theta\to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$.

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) The probability density function of X is given by $f(x) = \begin{cases} k\,e^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$.

OR

b) Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the Fig 41 (b).

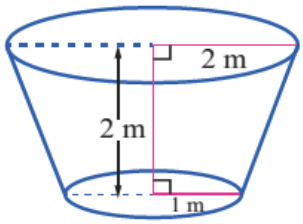

Fig. 41 (b)

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42) a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

OR

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$

43) a) Investigate for what values of λ and μ the system of linear equations

$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

OR

b) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, thedetective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

$\left[\log(2.43)=0.88789;\quad \log(0.5)=-0.69315\right]$

44) a) If $w(x,y,z)=\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$.

OR

b) Find all cube roots of $\sqrt{3}+i$.

45) a) Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$.

OR

b) Evaluate the following definite integrals :

$$\int_0^{\frac{\pi}{2}} e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

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46) a) Find the domain of the following

$$f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$$

OR

b) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,-2),(-1,-2,6)$, and $(6, 4, -2)$.

47) a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following

$$\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$$

OR

b) Let A be $\mathbb{Q}\setminus\{1\}$. Define $*$ on A by $x*y = x+y-xy$. Is $*$ binary on A? If so, examine the commutative and associative properties satisfied by $*$ on A. Also, examine the existence of identity, existence of inverse properties for the operation $*$ on A.

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