SUN TUITION CENTER

2022-10th-2023

MATHEMATICS

Public Exam

Revision Going On.....

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1m, 2m, 5m, 8 Marks

Important Question with Solution

'Life is a Good Circle, You Choose the Best Radius'

10th

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ONE MARK QUESTIONS

BOOKBACK

UNIT.I RELATIONS AND FUNCTIONS

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
   (A) 1 (B) 2 **(C) 3** (D) 6

2. $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is
   (A) 8 (B) 20 **(C) 12** (D) 16

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.
   (A) $(A \times C) \subset (B \times D)$ (B) $(B \times D) \subset (A \times C)$
   (C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is (A) 3 **(B) 2** (C) 4 (D) 8

5. The range of the relation $R = \{(x, x^2) \mid x \text{ is a prime number less than } 13\}$ is
   (A) $\{2, 3, 5, 7\}$ (B) $\{2, 3, 5, 7, 11\}$ **(C) $\{4, 9, 25, 49, 121\}$** (D) $\{1, 4, 9, 25, 49, 121\}$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
   (A) (2, –2) (B) (5, 1) (C) (2, 3) **(D) (3, –2)**

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is (A) m^n (B) n^m **(C) $2^{mn} - 1$** (D) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively
   (A) (8, 6) (B) (8, 8) (C) (6, 8) (D) (6, 6)

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f : A \to B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a
   (A) Many-one function (B) Identity function
   (C) One-to-one function (D) Into function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
    (A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ **(C) $\frac{2}{9x^2}$** (D) $\frac{1}{6x^2}$

11. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
    (A) 7 (B) 49 (C) 1 (D) 14

12. Let f and g be two functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$ $g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)\}$ then the range of $f \circ g$ is
    (A) $\{0, 2, 3, 4, 5\}$ (B) $\{-4, 1, 0, 2, 7\}$ (C) $\{1, 2, 3, 4, 5\}$ **(D) $\{0, 1, 2\}$**

13. Let $f(x) = \sqrt{1 + x^2}$ then (A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
    (C) $f(xy) \leq f(x).f(y)$ (D) None of these

14. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are (A) (–1, 2) **(B) (2, –1)** (C) (–1, –2) (D) (1, 2)

15. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is
(A) linear (B) cubic (C) reciprocal **(D) quadratic**

UNIT.II NUMBERS AND SEQUENCES

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
(A) $1 < r < b$ (B) $0 < r < b$ **(C) $0 \leq r < b$** (D) $0 < r \leq b$

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are **(A) 0, 1, 8** (B) 1, 4, 8 (C) 0, 1, 3 (D) 1, 3, 5

3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
(A) 4 **(B) 2** (C) 1 (D) 3

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
(A) 1 (B) 2 **(C) 3** (D) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
(A) 2025 (B) 5220 (C) 5025 **(D) 2520**

6. $7^{4k} \equiv$ ________ (mod 100) **(A) 1** (B) 2 (C) 3 (D) 4

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
(A) 3 (B) 5 (C) 8 **(D) 11**

8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P. (A) 4551 (B) 10091 **(C) 7881** (D) 13531

9. If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is
(A) 0 (B) 6 (C) 7 (D) 13

10. An A.P. consists of 31 terms. If its 16th term is m, then the sum of all the terms of this A.P. is
(A) $16\,m$ (B) $62\,m$ **(C) $31\,m$** (D) $\frac{31}{2}m$

11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?
(A) 6 (B) 7 **(C) 8** (D) 9

12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \cdots + 2^0$ which of the following is true?
(A) B is 2^{64} more than A (B) A and B are equal
(C) B is larger than A by 1 **(D) A is larger than B by 1**

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots$ is
(A) $\frac{1}{24}$ **(B) $\frac{1}{27}$** (C) $\frac{2}{3}$ (D) $\frac{1}{81}$

14. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, \ldots$ is
(A) a Geometric Progression **(B) an Arithmetic Progression**
(C) neither an A.P. nor a G.P. (D) a constant sequence

15. The value of $(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \ldots + 15)$ is
(A) 14400 (B) 14200 **(C) 14280** (D) 14520

UNIT.III ALGEBRA

1. A system of three linear equations in three variables is inconsistent if their planes
   (A) intersect only at a point (B) intersect in a line
   (C) coincides with each other **(D) do not intersect**

2. The solution of the system $x + y - 3z = -6$, $-7y + 7z = 7$, $3z = 9$ is
   (A) $\boldsymbol{x = 1,\ y = 2,\ z = 3}$ (B) $x = -1,\ y = 2,\ z = 3$
   (C) $x = -1,\ y = -2,\ z = 3$ (D) $x = 1,\ y = -2,\ z = 3$

3. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
   (A) 3 **(B) 5** (C) 6 (D) 8

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is **(A)** $\boldsymbol{\frac{9y}{7}}$ (B) $\frac{9y^3}{(21y-21)}$ (C) $\frac{21y^2 - 42y + 21}{3y^3}$ (D) $\frac{7(y^2 - 2y + 1)}{y^2}$

5. $y^2 + \frac{1}{y^2}$ is not equal to (A) $\frac{y^4+1}{y^2}$ **(B)** $\boldsymbol{\left[y + \frac{1}{y}\right]^2}$ (C) $\left[y - \frac{1}{y}\right]^2 + 2$ (D) $\left[y + \frac{1}{y}\right]^2 - 2$

6. $\frac{x}{x^2 - 25} - \frac{8}{x^2 + 6x + 5}$ gives (A) $\frac{x^2-7x+40}{(x-5)(x+5)}$ (B) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$
   (C) $\boldsymbol{\frac{x^2-7x+40}{(x^2-25)(x+1)}}$ (D) $\frac{x^2+10}{(x^2-25)(x+1)}$

7. The square root of $\frac{256\, x^8 y^4 z^{10}}{25\, x^6 y^6 z^6}$ is equal to
   (A) $\frac{16}{5}\left|\frac{x^2 z^4}{y^2}\right|$ (B) $16\left|\frac{y^2}{x^2 z^4}\right|$ (C) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ **(D)** $\boldsymbol{\frac{16}{5}\left|\frac{xz^2}{y}\right|}$

8. Which of the following should be added to make $x^4 + 64$ a perfect square
   (A) $4x^2$ **(B)** $\boldsymbol{16x^2}$ (C) $8x^2$ (D) $-8x^2$

9. The solution of $(2x - 1)^2 = 9$ is equal to
   (A) -1 (B) 2 **(C) – 1, 2** (D) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
    (A) 100, 120 (B) 10, 12 **(C) −120, 100** (D) 12, 10

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in ___________
    (A) $A.P$ **(B)** $\boldsymbol{G.P}$ (C) Both $A.P$ and $G.P$ (D) none of these

12. Graph of a linear polynomial is a
    (A) straight line (B) circle (C) parabola (D) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is
    (A) 0 **(B) 1** (C) 0 or 1 (D) 2

14. For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is
    (A) 2×3 (B) 3×2 (C) 3×4 **(D)** $\boldsymbol{4 \times 3}$

15. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have
    (A) 3 **(B) 4** (C) 2 (D) 5

'To achieve your target plan well'

16. If number of columns and rows are not equal in a matrix then it is said to be a

(A) diagonal matrix **(B) rectangular matrix**

(C) square matrix (D) identity matrix

17. Transpose of a column matrix is

(A) unit matrix (B) diagonal matrix (C) column matrix **(D) row matrix**

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$

(A) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ **(B)** $\begin{pmatrix} \mathbf{2} & \mathbf{2} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. Which of the following can be calculated from the given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$,

(i) A^2 (ii) B^2 (iii) AB (iv) BA

(A) (i) and (ii) only (B) (ii) and (iii) only **(C) (ii) and (iv) only** (D) all of these

20. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?

(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$ (iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

(A) (i) and (ii) only (B) (ii) and (iii) only (C) (iii) and (iv) only (D) all of these

UNIT.IV GEOMETRY

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ **(C) $\angle B = \angle D$** (D) $\angle A = \angle F$

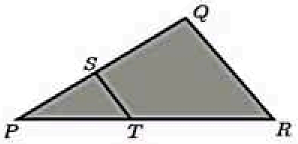
2. In ΔLMN, $\angle L = 60°$, $\angle M = 50°$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is

(A) 40° **(B) 70°** (C) 30° (D) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90°$ and $AC = 5\ cm$, then AB is

(A) $2.5\ cm$ (B) $5\ cm$ (C) $10\ cm$ **(D) $5\sqrt{2}\ cm$**

4. In a given figure $ST \parallel QR$, $PS = 2\ cm$ and $SQ = 3\ cm$. Then the ratio of the area of ΔPQR to the area of ΔPST is



(A) 25 : 4 (B) 25 : 7 (C) 25 : 11 (D) 25 : 13

5. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If $PQ = 10\ cm$, then the length of AB is

(A) $6\frac{2}{3}\ cm$ (B) $\frac{10\sqrt{6}}{3}\ cm$ (C) $66\frac{2}{3}\ cm$ **(D) $15\ cm$**

6. If in ΔABC, DE ∥ BC. $AB = 3.6\ cm$, $AC = 2.4\ cm$ and $AD = 2.1\ cm$ then the length of AE is

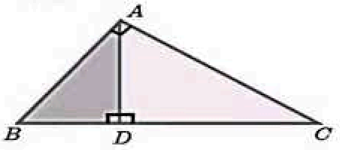
(A) $1.4\ cm$ (B) $1.8\ cm$ (C) $1.2\ cm$ (D) $1.05\ cm$

7. In a ΔABC, AD is the bisector of $\angle BAC$. If $AB = 8\ cm$, $BD = 6\ cm$ and $DC = 3\ cm$. The length of the side AC is (A) $6\ cm$ **(B) $4\ cm$** (C) $3\ cm$ (D) $8\ cm$

8. In the adjacent figure $\angle BAC = 90°$ and $AD \perp BC$ then

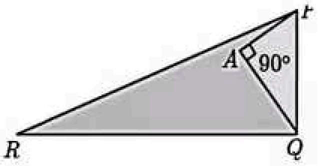
(A) $BD \cdot CD = BC^2$ (B) $AB \cdot AC = BC^2$

(C) $BD \cdot CD = AD^2$ (D) $AB \cdot AC = AD^2$



9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?
(A) 13 m (B) 14 m (C) 15 m (D) 12.8 m

10. In the given figure, $PR = 26$ cm, $QR = 24\ cm$, $\angle PAQ = 90°$, $PA = 6$ cm and $QA = 8\ cm$. Find $\angle PQR$
(A) 80° (B) 85° (C) 75° **(D) 90°**

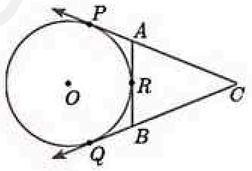


11. A tangent is perpendicular to the radius at the
(A) centre **(B) point of contact** (C) infinity (D) chord

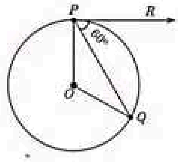
12. How many tangents can be drawn to the circle from an exterior point?
(A) one **(B) two** (C) infinite (D) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is
(A) 100° **(B) 110°** (C) 120° (D) 130°

14. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11\ cm$ and $BC = 7\ cm$, then the length of BR is
(A) 6 cm (B) 5 cm (C) 8 cm **(D) 4 cm**



15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is
(A) 120° (B) 100° (C) 110° (D) 90°



UNIT.V COORDINATE GEOMETRY

1. The area of triangle formed by the points $(-5, 0)$, $(0, -5)$ and $(5, 0)$ is
(A) 0 sq.units **(B) 25 sq.units** (C) 5 sq.units (D) none of these

2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is
(A) $x = 10$ (B) $y = 10$ (C) $x = 0$ (D) $y = 0$

3. The straight line given by the equation $x = 11$ is
(A) parallel to X axis **(B) parallel to Y axis**
(C) passing through the origin (D) passing through the point $(0, 11)$

4. If $(5, 7), (3, p)$ and $(6, 6)$ are collinear, then the value of p is
(A) 3 (B) 6 **(C) 9** (D) 12

5. The point of intersection of $3x - y = 4$ and $x + y = 8$ is
(A) (5, 3) (B) (2, 4) **(C) (3, 5)** (D) (4, 4)

6. The slope of the line joining $(12, 3)$, $(4, a)$ is $\frac{1}{8}$. The value of 'a' is
(A) 1 (B) 4 (C) -5 **(D) 2**

7. The slope of the line which is perpendicular to line joining the points $(0, 0)$ and $(-8, 8)$ is
(A) -1 **(B) 1** (C) $\frac{1}{3}$ (D) -8

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of the perpendicular bisector of PQ is
(A) $\sqrt{3}$ **(B) $-\sqrt{3}$** (C) $\frac{1}{\sqrt{3}}$ (D) 0

A Little Progress each day adds up to our Result

9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is

(A) $8x + 5y = 40$ (B) $8x - 5y = 40$ (C) $x = 8$ (D) $y = 5$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is

(A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$ **(C) $3x + 7y = 0$** (D) $7x - 3y = 0$

11. Consider four straight lines

(i) l_1: $3y = 4x + 5$ (ii) l_2: $4y = 3x - 1$ (iii) l_3: $4y + 3x = 7$ (iv) l_4: $4x + 3y = 2$

Which of the following statement is true ?

(A) l_1 and l_2 are perpendicular (B) l_1 and l_4 are parallel

(C) l_2 and l_4 are perpendicular (D) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21$. Which of the following is true

(A) The slope is 0.5 and the y intercept is 2.6 (B) The slope is 5 and the y intercept is 1.6

(C) The slope is 0.5 and the y intercept is 1.6 (D) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium, it is necessary to show

(A) Two sides are parallel **(B) Two parallel and two non-parallel sides.**

(C) Opposite sides are parallel (D) All sides are of equal length

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

(A) The slopes of two sides **(B) The slopes of two pair of opposite sides**

(C) The lengths of all sides (D) Both the lengths and slopes of two sides

15. $(2, 1)$ is the point of intersection of two lines

(A) $x - y - 3 = 0$; $3x - y - 7 = 0$ **(B) $x + y = 3$; $3x + y = 7$**

(C) $3x + y = 3$; $x + y = 7$ (D) $x + 3y - 3 = 0$; $x - y - 7 = 0$

UNIT.VI TRIGONOMETRY

1. The value of $sin^2\theta + \frac{1}{1+tan^2\theta}$ is equal to

(A) $tan^2\theta$ **(B) 1** (C) $cot^2\theta$ (D) 0

2. $tan\theta\ cosec^2\theta - tan\theta$ is equal to

(A) $sec\theta$ (B) $cot^2\theta$ (C) $sin\theta$ **(D) $cot\theta$**

3. If $(sin\alpha + cosec\alpha)^2 + (cos\alpha + sec\alpha)^2 = k + tan^2\alpha + cot^2\alpha$, then the value of k is equal to

(A) 9 **(B) 7** (C) 5 (D) 3

4. If $sin\theta + cos\theta = a$ and $sec\theta + cosec\theta = b$, then the value of $b(a^2 - 1)$ is equal to

(A) $2a$ (B) $3a$ (C) 0 (D) $2ab$

5. If $5x = sec\theta$ and $\frac{5}{x} = tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to

(A) 25 **(B) $\frac{1}{25}$** (C) 5 (D) 1

6. If $sin\theta = cos\theta$, then $2tan^2\theta + sin^2\theta - 1$ is equal to

(A) $\frac{-3}{2}$ **(B) $\frac{3}{2}$** (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

7. If $x = atan\theta$ and $y = bsec\theta$ then

(A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + tan\ \theta + sec\ \theta)(1 + cot\theta - cosec\theta)$ is equal to

(A) 0 (B) 1 **(C) 2** (D) -1

9. $a\ cot\theta + b\ cosec\theta = p$ and $b\ cot\theta + a\ cosec\ \theta = q$ then $p^2 - q^2$ is equal to

(A) $a^2 - b^2$ **(B) $b^2 - a^2$** (C) $a^2 + b^2$ (D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure
(A) 45° (B) 30° (C) 90° **(D) 60°**

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to (A) $\sqrt{3}\,b$ **(B)** $\frac{b}{3}$ (C) $\frac{b}{2}$ (D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to
(A) 41.92 m **(B) 43.92 m** (C) 43 m (D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is
(A) 20, $10\sqrt{3}$ (B) 30, $5\sqrt{3}$ (C) 20, 10 **(D) 30, $10\sqrt{3}$**

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is
(A) $\sqrt{2}\,x$ **(B)** $\frac{x}{2\sqrt{2}}$ (C) $\frac{x}{\sqrt{2}}$ (D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is
(A) $\frac{h(1+tan\beta)}{1-tan\beta}$ (B) $\frac{h(1-tan\beta)}{1+tan\beta}$ (C) $h\,tan(45°-\beta)$ (D) none of these

UNIT.VII MENSURATION

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(A) $60\pi\ cm^2$ (B) $68\pi\ cm^2$ (C) $120\pi\ cm^2$ **(D) $136\pi\ cm^2$**

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
(A) $4\pi r^2$ sq. units (B) $6\pi r^2$ sq. units (C) $3\pi r^2$ sq. units (D) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be
(A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(A) 1 : 2 **(B) 1 : 4** (C) 1 : 6 (D) 1 : 8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is
(A) $\frac{9\pi h^2}{8}$ sq. units (B) $24\pi h^2$ sq. units **(C) $\frac{8\pi h^2}{9}$ sq. units** (D) $\frac{56\pi h^2}{9}$ sq. units

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is
(A) $5600\pi\ cm^3$ **(B) $1120\pi\ cm^3$** (C) $56\pi\ cm^3$ (D) $3600\pi\ cm^3$

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is
(A) made 6 times **(B) made 18 times** (C) made 12 times (D) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.
(A) π (B) 4π **(C) 3π** (D) 2π

9. A solid sphere of radius $x\ cm$ is melted and cast into a shape of a solid cone of same radius. The height of the cone is
(A) $3x\ cm$ (B) $x\ cm$ **(C) $4x\ cm$** (D) $2x\ cm$

10. A frustum of a right circular cone is of height $16cm$ with radii of its ends as $8cm$ and $20cm$. Then, the volume of the frustum is
(A) $3328\ \pi\ cm^3$ (B) $3228\ \pi\ cm^3$ (C) $3240\ \pi\ cm^3$ (D) $3340\ \pi\ cm^3$

11. A shuttle cock used for playing badminton has the shape of the combination of
(A) a cylinder and a sphere (B) a hemisphere and a cone
(C) a sphere and a cone **(D) frustum of a cone and a hemisphere**

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is **(A) 2 : 1** (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius $1\ cm$ and height $5\ cm$ is **(A) $\frac{4}{3}\pi$** (B) $\frac{10}{3}\pi$ (C) $5\ \pi$ (D) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is (A) 1 : 3 **(B) 1 : 2** (C) 2 : 1 (D) 3 : 1

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is (A) 1 : 2 : 3 (B) 2 : 1 : 3 (C) 1 : 3 : 2 **(D) 3 : 1 : 2**

UNIT.VIII STATISTICS AND PROBABILITY

1. Which of the following is not a measure of dispersion?
(A) Range (B) Standard deviation **(C) Arithmetic mean** (D) Variance

2. The range of the data 8, 8, 8, 8, 8...8 is
(A) 0 (B) 1 (C) 8 (D) 3

3. The sum of all deviations of the data from its mean is
(A) Always positive (B) always negative **(C) zero** (D) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all observations is (A) 40000 **(B) 160900** (C) 160000 (D) 30000

5. Variance of first 20 natural numbers is (A) 32.25 (B) 44.25 **(C) 33.25** (D) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
(A) 3 (B) 15 (C) 5 **(D) 225**

7. If the standard deviation of x, y, z is p then the standard deviation of $3x+5$, $3y+5$, $3z+5$ is
(A) $3p+5$ **(B) $3p$** (C) $p+5$ (D) $9p+15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
(A) 3.5 (B) 3 (C) 4.5 (D) 2.5

9. Which of the following is incorrect?
(A) $P(A) > 1$ (B) $0 \leq P(A) \leq 1$ (C) $P(\emptyset) = 0$ (D) $P(A) + P(\bar{A}) = 1$

10. The probability of a red marble selected at random from a jar containing p red, q blue and r green marbles is (A) $\frac{q}{p+q+r}$ **(B) $\frac{p}{p+q+r}$** (C) $\frac{p+q}{p+q+r}$ (D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is (A) $\frac{3}{10}$ **(B) $\frac{7}{10}$** (C) $\frac{3}{9}$ (D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is (A) 2 **(B) 1** (C) 3 (D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is
(A) 5 (B) 10 **(C) 15** (D) 20

14. If a letter is chosen at random from the English alphabets $\{a, b, \ldots, z\}$, then the probability that the letter chosen precedes x (A) $\frac{12}{13}$ (B) $\frac{1}{13}$ **(C)** $\frac{23}{26}$ (D) $\frac{3}{26}$

15. A purse contains 10 notes of Rs.2000, 15 notes of Rs.500, and 25 notes of Rs.200. One note is drawn at random. What is the probability that the note is either a Rs.500 note or Rs.200 note?
(A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{3}$ **(D)** $\frac{4}{5}$

' Failing to Plan is Planning to Fail '

GEOMETRY & GRAPH

QUESTION BANK-2022

GEOMETRY – Constructions

I. SIMILAR TRIANGLES :- (Big to Small)

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)

2. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$)

3. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5}$)

II. SIMILAR TRIANGLES :- (Small to Big)

4. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)

5. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5}$)

6. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$)

III. TRIANGLES :- (When **MEDIAN** is given)

7. Construct a ΔPQR in which $PQ = 8$ cm, $\angle R = 60°$ and the **median** RG from R to PQ is 5.8 cm. Find the length of the ***altitude*** from R to PQ.

8. Construct a ΔPQR in which $QR = 5$ cm, $\angle P = 40°$ and the **median** PG from P to QR is 4.4 cm. Find the length of the ***altitude*** from P to QR.

9. Construct a ΔPQR in which the base $PQ = 4.5$ cm, $\angle R = 35°$ and the **median** from R to PQ is 6 cm.

'Life is like riding a bicycle to keep your balance , you must keep moving'

IV. TRIANGLES :- (When **ALTITUDE** is given)

10. Construct a triangle ΔPQR such that $QR = 5\,cm$, $\angle P = 30°$ and the **altitude** from P to QR is of length 4.2 cm.

11. Construct a ΔPQR such that $QR = 6.5\,cm$, $\angle P = 60°$ and the **altitude** from P to QR is of length 4.5 cm.

12. Construct a triangle ΔABC such that $AB = 5.5\,cm$, $\angle C = 25°$ and the **altitude** from C to AB is 4 cm.

V. TRIANGLES :- (When the point of **ANGLE BISECTOR** is given)

13. Draw a triangle ABC of base $BC = 8$ cm, $\angle A = 60°$ and the **bisector** of $\angle A$ meets BC at D such that $BD = 6$ cm.

14. Draw a triangle ABC of base $BC = 5.6$ cm, $\angle A = 40°$ and the **bisector** of $\angle A$ meets BC at D such that $CD = 4$ cm.

15. Draw ΔPQR such that $PQ = 6.8$ cm, vertical angle 50° and the **bisector** of the vertical angle meets the base at D where $PD = 5.2$ cm.

VI. TANGENTS TO A CIRCLE: (Using the Centre)

16. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

17. Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P ?

VII. TANGENTS TO A CIRCLE: (Using Alternate Segment Theorem)

18. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the **alternate–segment theorem**.

19. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the **alternate – segment theorem**.

VIII. TANGENTS TO A CIRCLE: (Pair of Tangents or Two Tangents)

20. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. **Draw the two tangents** PA and PB to the circle and measure their lengths.

21. **Draw the two tangents** from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

22. **Draw the two tangents** from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

23. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and **draw the two tangents** to the circle from the point.

24. **Draw a tangent** to the circle from the point P having radius 3.6 cm, and centre at O point P is at a distance 7.2 cm from the centre.

GRAPH

I. GRAPH of VARIATION :- (Direct Variation)

1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship betweem the diameter and circumference of each circle (approximately) as shown in the table and use it to find the circumference of a circle when its diameter is 6 *cm*.

Diameter (x) **cm**	1	2	3	4	5
Circumference (y) **cm**	3.1	6.2	9.3	12.4	15.5

2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find
   (i) the constant of variation
   (ii) how far will it travel in 90 minutes
   (iii) the time required to cover a distance of 300 km from the graph.

3. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find (i) the marked price when a customer gets a discount of Rs.3250 (from Graph)
   (ii) the discount when the marked price is Rs.2500

4. Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also, (i) find y when $x = 9$ (ii) find x when $y = 7.5$

5. A two wheeler parking zone near bus stand charges as below:

Time (in hours) (x)	4	8	12	24
Amount Rs. (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also, (i) find the amount to be paid when parking time is 6 hrs; (ii) find the parking duration when the amount paid is Rs.150.

II. GRAPH of VARIATION :- (Inverse Variation)

6. A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below:

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation.
(ii) From the graph, find the number of days required to complete the work if the company decided to opt for 120 workers?
(iii) If the work has to be completed by 200 days, how many workers are required?

7. Nishanth is the winner in a Marathan race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/kr. And, they have covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hrs respectively. Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

8. Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find, (i) y when $x = 3$ and (ii) find x when $y = 6$.

9. The following table shows the data about the number of pipes and the time taken to fill the same tank

No. of pipes (x)	2	3	6	9
Time taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence
(i) Find the time taken to fill the tank when five pipes are used
(ii) Find the number of pipes when the time is 9 minutes

10. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as shown below

No. of participants (x)	2	4	6	8	10
Amount for each participant in Rs. (y)	180	90	60	45	36

(i) Find the constant of variation.
(ii) Graph the above data. Hence, find how much will each participant get if the number of participants are 12.

III. NATURE of the SOLUTIONS :- (Graphically)

Discuss the **nature of solutions** of the following **quadratic equations**

11. $x^2 + x - 12 = 0$
12. $x^2 - 8x + 16 = 0$
13. $x^2 + 2x + 5 = 0$

Graph the following **quadratic equations** and state its **nature of solutions:**

14. $x^2 - 9x + 20 = 0$
15. $x^2 - 4x + 4 = 0$
16. $x^2 + x + 7 = 0$
17. $x^2 - 9 = 0$
18. $x^2 - 6x + 9 = 0$
19. $(2x - 3)(x + 2) = 0$

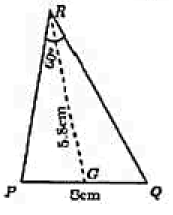
IV. Solving QUADRATIC EQUATIONS :- (Through intersection of lines)

20. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$.
21. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$.
22. Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$.
23. Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.
24. Draw the graph of $y = x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$.
25. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.
26. Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$.
27. Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$.
28. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.
29. Draw the graph of $y = 2x^2 - 3x - 5$ and hence use it to solve $2x^2 - 4x - 6 = 0$
30. Draw the graph of $y = (x - 1)(x + 3)$ and hence use it to solve $x^2 - x - 6 = 0$

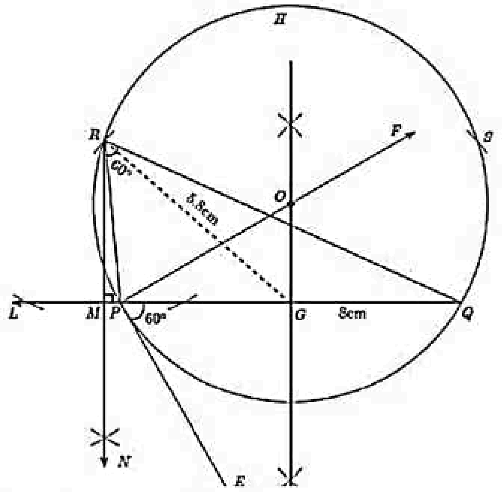
GEOMETRY

1. Construct a ΔPQR in which PQ = 8 cm, ∠R = 60^0 and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.

Solution :



Rough diagram



Construction

Step 1 : Draw a line segment PQ = 8cm.

Step 2 : At P, draw PE such that ∠QPE = 60^0.

Step 3 : At P, draw PF such that ∠EPF = 90^0.

Step 4 : Draw the perpendicular bisector to PQ, which intersects PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

Step 6 : From G mark arcs of radius 5.8 cm on the circle. Mark them as R and S.

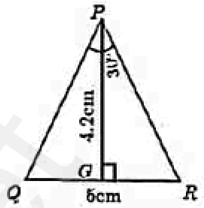
Step 7 : Join PR and RQ. Then ΔPQR is the required triangle .

Step 8 : From R draw a line RN perpendicular to LQ. LQ meets RN at M

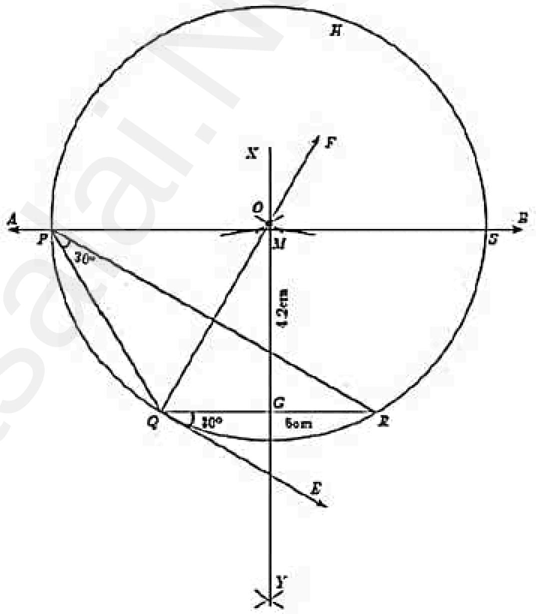
Step 9 : The length of the altitude is RM = 3.5 cm.

2. Construct a triangle ΔPQR such that QR = 5 cm, ∠P = 30^0 and the altitude from P to QR is of length 4.2 cm.

Solution :



Rough diagram



Construction

Step 1 : Draw a line segment QR = 5cm.

Step 2 : At Q, draw QE such that ∠RQE = 30^0.

Step 3 : At Q, draw QF such that ∠EQF = 90^0.

Step 4 : Draw the perpendicular bisector XY to QR, which intersects QF at O and QR at G.

Step 5 : With O as centre and OQ as radius draw a circle.

Step 6 : From G mark an arc in the line XY at M, such that GM = 4.2 cm.

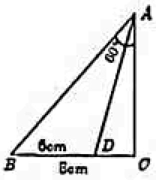
Step 7 : Draw AB through M which is parallel to QR.

Step 8 : AB meets the circle at P and S.

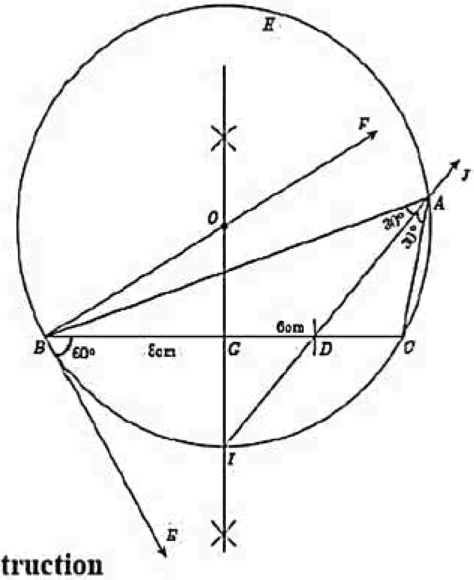
Step 9 : Join QP and RP. Then ΔPQR is the required triangle.

3. Draw a triangle ABC of base BC = 8 cm, ∠A = 60^0 and the bisector of ∠A meets BC at D such that BD = 6 cm.

Solution :



Rough diagram



Construction

Step 1 : Draw a line segment BC = 8cm.

Step 2 : At B, draw BE such that ∠CBE = 60^0.

Step 3 : At B, draw BF such that ∠EBF = 90^0.

Step 4 : Draw the perpendicular bisector to BC, which intersects BF at O and BC at G.

Step 5 : With O as centre and OB as radius draw a circle.

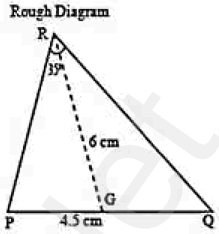
Step 6 : From B mark an arcs of 6 cm on BC at D.

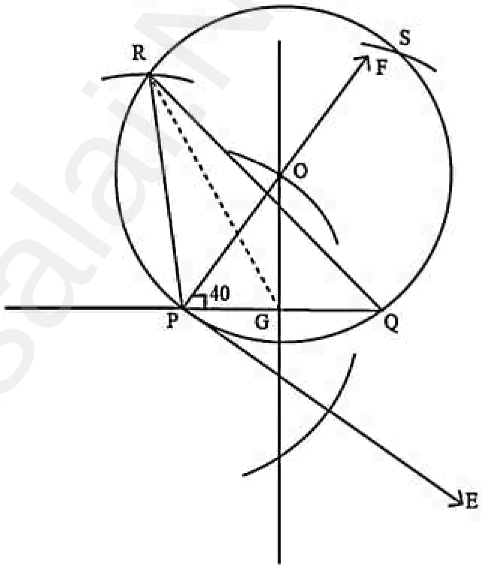
Step 7 : The perpendicular bisector intersects the circle at I. Join ID.

Step 8 : ID produced meets the circle at A. Now join AB and AC. Then ΔABC is the required triangle.

4. **Construct a ΔPQR which the base PQ = 4.5 cm, ∠R = 35^0 and the median from R to PQ is 6 cm.**

Solution :





Construction

Step 1 : Draw a line segment PQ = 4.5cm.

Step 2 : At P, draw PE such that ∠QPE = 35^0.

Step 3 : At P, draw PF such that ∠EPF = 90^0.

Step 4 : Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

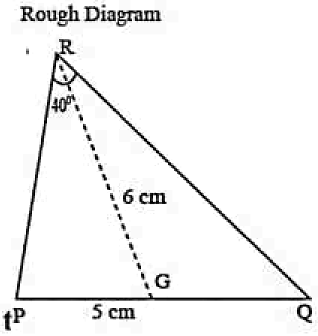
Step 6 : From G mark arcs of 6 cm on the circle at RAS.

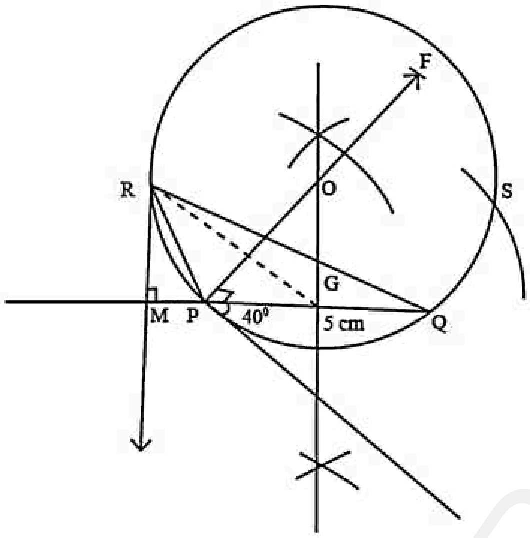
Step 7 : Join PR, RQ. Then ΔPQR is the required Δ.

Step 8 : Join RG, which is the median.

5. **Construct a ΔPQR in which $PQ = 5$ cm, $\angle P = 40^0$ and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.**

Solution :





Construction

Step 1 : Draw a line segment $PQ = 5$ cm.

Step 2 : At P, draw PE such that $\angle QPE = 40^0$.

Step 3 : At P, draw PF such that $\angle EPF = 90^0$.

Step 4 : Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

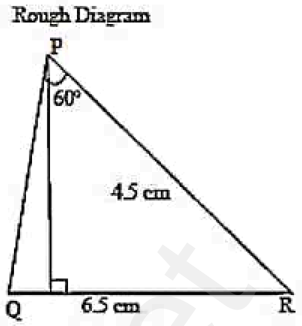
Step 5 : With O as centre and OP as radius draw a circle.

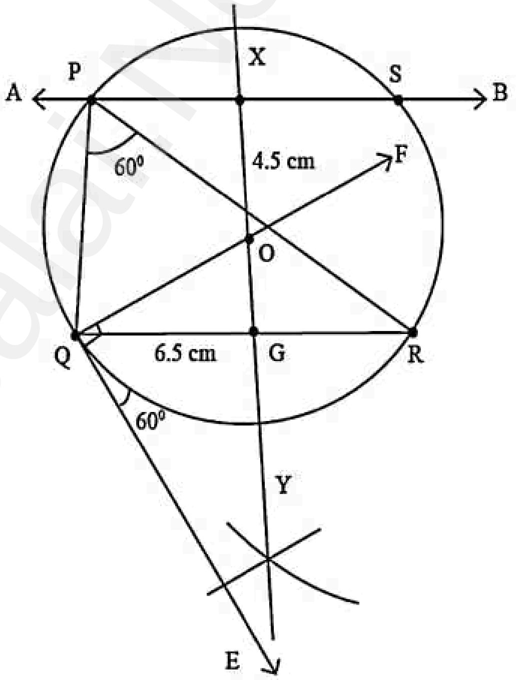
Step 6 : From G mark arcs of 4.4 cm on the circle radius 4.4m.

Step 7 : Join PR, RQ. Then ΔPQR is the required Δ.

Step 8 : Length of altitude is $RM = 3$ cm

6. **Construct a ΔPQR such that $QR = 6.5$ cm, $\angle P = 60^0$ and the altitude from P to QR is of length 4.5 cm.**





Construction

Step 1 : Draw a line segment $QR = 6.5$ cm.

Step 2 : At Q, draw QE such that $\angle RQE = 60^0$.

Step 3 : At Q, draw QF such that $\angle EQF = 90^0$.

Step 4 : Draw the perpendicular bisector XY to QR intersects QF at O & QR at G.

Step 5 : With O as centre and OQ as radius draw a circle.

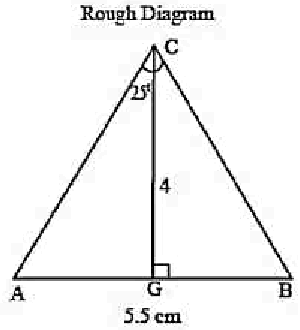
Step 6 : XY intersects QR at G. On XY, from G, mark arc M such that $GM = 4.5$ cm.

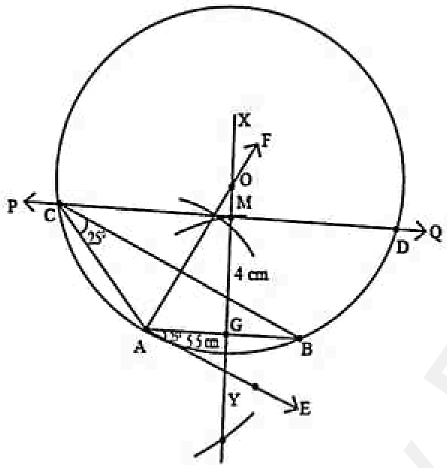
Step 7 : Draw AB, through M which is parallel to QR.

Step 8 : AB meets the circle at P and S.

Step 9 : Join QP, RP. Then ΔPQR is the required Δ.

7. **Construct a ΔABC such that $AB = 5.5$ cm, $\angle C = 25^0$ and the altitude from C to AB is 4 cm.**





Construction

Step 1 : Draw a line segment $AB = 5.5$ cm.

Step 2 : At A, draw AE such that $\angle BAE = 25^0$.

Step 3 : At A, draw AF such that $\angle EAF = 90^0$.

Step 4 : Draw the perpendicular bisector XY to AB intersects AF at O & AB at G.

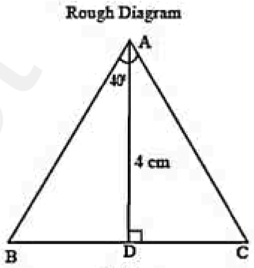
Step 5 : With O as centre and OA as radius draw a circle.

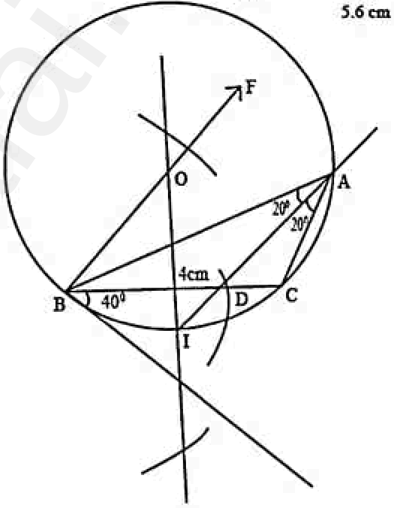
Step 6 : XY intersects AB at G. On XY, from G, mark arc M such that $GM = 4$ cm.

Step 7 : Draw PQ, through M parallel to AB meets the circle at C and D.

Step 8 : Join AC, BC. Then ΔABC is the required Δ.

8. **Draw a triangle ABC of base $BC = 5.6$ cm, $\angle A = 40^0$ and the bisector of $\angle A$ meets BC at D such that $CD = 4$ cm.**





Construction

Step 1 : Draw a line segment $BC = 5.6$ cm.

Step 2 : At B, draw BE such that $\angle CBE = 40^0$.

Step 3 : At B, draw BF such that $\angle CBF = 90^0$.

Step 4 : Draw the perpendicular bisector to BC meets BF at O & BC at G.

Step 5 : With O as centre and OB as radius draw a circle.

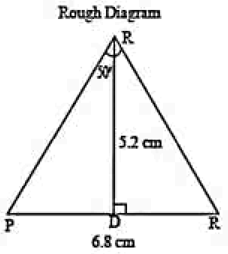
Step 6 : From B, mark an arc of 4 cm on BC at D.

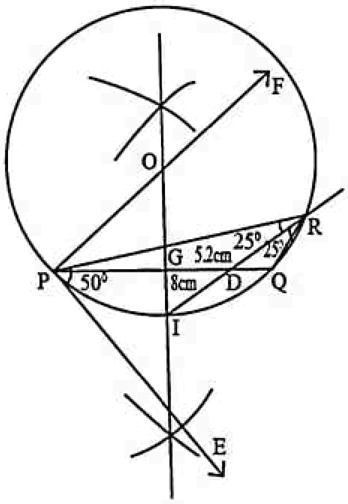
Step 7 : The ⊥r bisector meets the circle at I & Join ID.

Step 8 : ID produced meets the circle at A. Join AB & AC.

Step 9 : Then ΔABC is the required triangle.

9. **Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.**





Construction

Step 1 : Draw a line segment PQ = 6.8 cm.

Step 2 : At P, draw PE such that ∠QPE = 50^0.

Step 3 : At P, draw PF such that ∠QPF = 90^0.

Step 4 : Draw the perpendicular bisector to PQ meets PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

Step 6 : From P mark an arc of 5.2 cm on PQ at D.

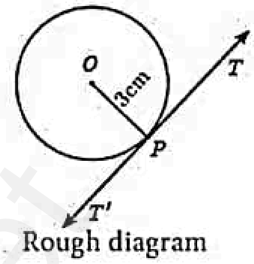
Step 7 : The perpendicular bisector meets the circle at R. Join PR and QR.

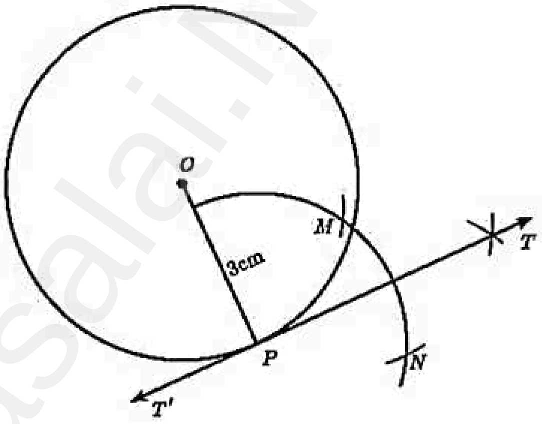
Step 8 : Then ΔPQR is the required triangle.

10. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution :

Given, radius r = 3 cm





Construction

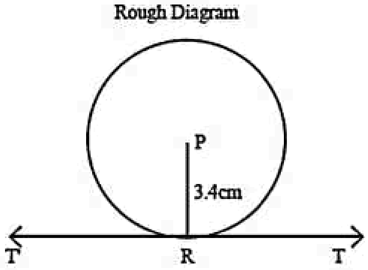
Step 1 : Draw a circle with centre at O of radius 3 cm.

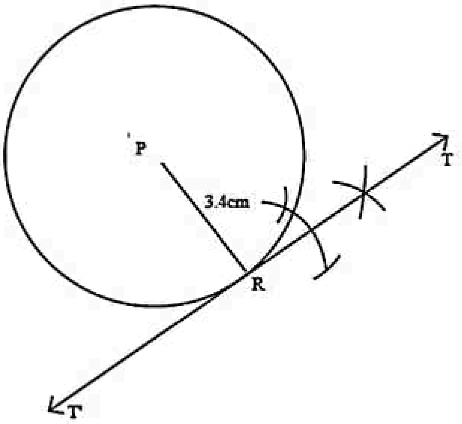
Step 2 : Take a point P on the circle. Join OP.

Step 3 : Draw perpendicular line TT' to OP which passes through P.

Step 4 : TT' is the required tangent.

11. **Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P ?**





Construction

Step 1 : Draw a circle with centre at P of radius 3.4 cm.

Step 2 : Take a point R on the circle and Join PR.

Step 3 : Draw perpendicular line TT' to PR which passes through R.

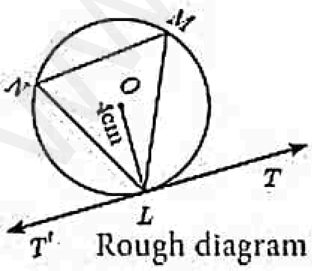
Step 4 : TT' is the required tangent.

---

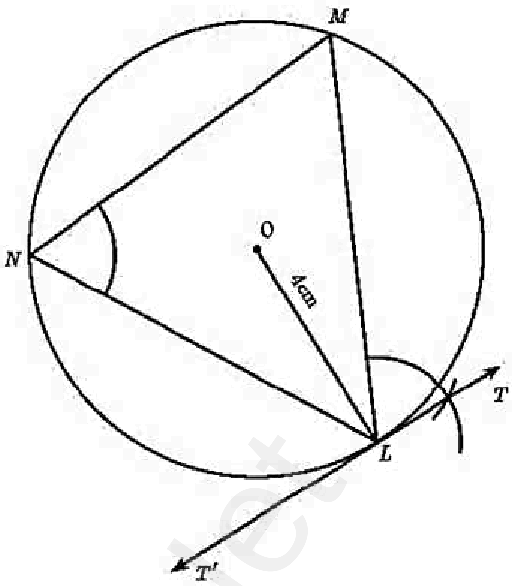
12 Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution :

Given, radius=4 cm



Rough diagram



Construction

Step 1 : With O as the centre, draw a circle of radius 4 cm.

Step 2 : Take a point L on the circle. Through L draw any chord LM.

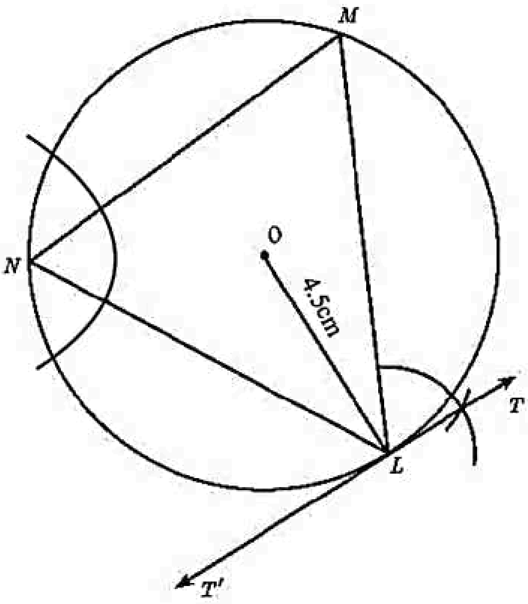
Step 3 : Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

Step 4 : Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.

Step 5 : TT' is the required tangent.

---

13. **Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.**



Construction

Step 1 : With O as the centre, draw a circle of radius 4 cm.

Step 2 : Take a point L on the circle. Through L draw any chord LM.

Step 3 : Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

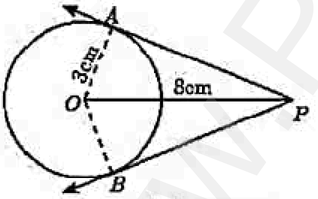
Step 4 : Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.

Step 5 : TT' is the required tangent.

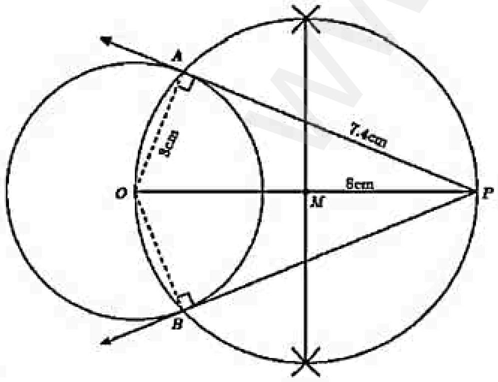
14. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Solution :

Given, diameter (d) = 6 cm, we find radius (r) = $\frac{6}{2}$ = 3 cm.



Rough diagram



Step 1 : With centre at O, draw a circle of radius 3 cm.

Step 2 : Draw a line OP of length 8 cm.

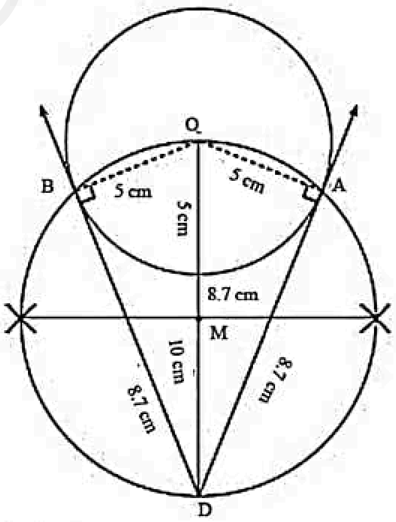
Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm.

15. **Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 5 cm.

Step 2 : Draw a line OP = 10 cm.

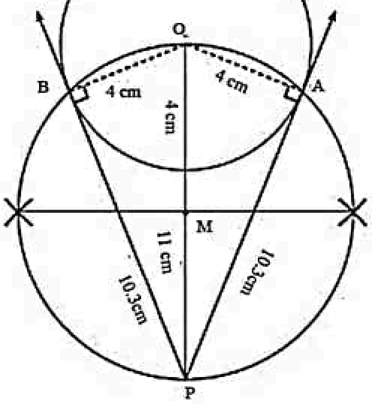
Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm.

16. **Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 4 cm.

Step 2 : Draw a line OP = 11 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. They are the required tangents AP = BP = 10.3 cm.

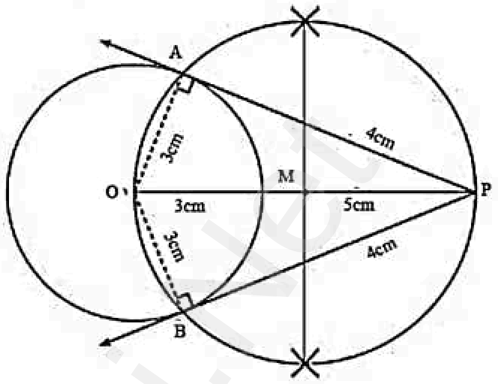
Verification : In the right angle triangle ΔOAP ,

$$AP = \sqrt{OP^2 - OA^2}$$

$$= \sqrt{121 - 16} = \sqrt{105} = 10.3 \text{ cm}$$

17. **Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 3 cm. with centre at O.

Step 2 : Draw a line OP = 5 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step 5 : Join AP and BP. They are the required tangents AP = BP = 4 cm.

Verification :

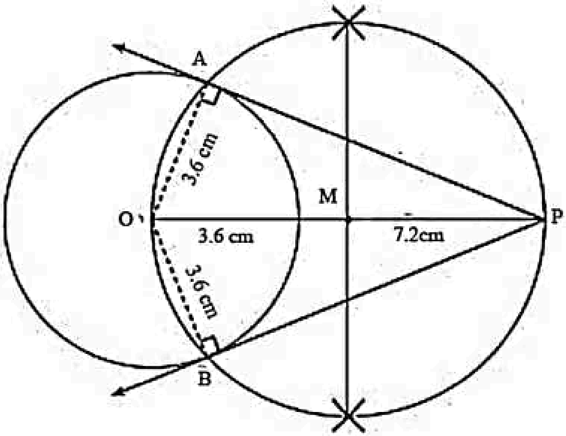
$$AP = \sqrt{OP^2 - OA^2}$$

$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4 \text{ cm}$$

18. **Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.**



Construction

Step 1 : Draw a circle of radius 3.6 cm. with centre at O.

Step 2 : Draw a line OP = 7.2 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts it M.

Step 4 : With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. They are the required tangents AP = BP = 0.3 cm.

Verification :

$$AP = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{(7.2)^2 - (3.6)^2}$$
$$= \sqrt{51.84 - 12.96}$$
$$= \sqrt{38.88} = 6.3 \text{ (approx)}$$

Construction of similar triangles

Example 4.10

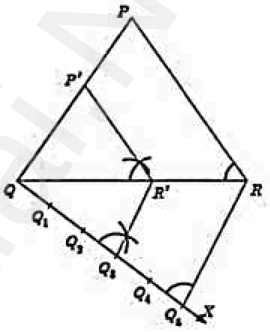
Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5}<1$)

Solution :

Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.

Steps of construction

1. Construct a ΔPQR with any measurement.

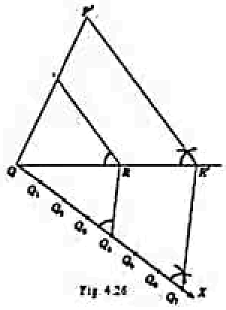


2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) points.

   Q_1, Q_2, Q_3, Q_4, and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$
4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q_5R to intersect QR at R'.
5. Draw line through R' parallel to the line RP to intersect QP at P'. Then, ΔP'QR' is the required triangle each of whose sides is three-fifths of the corresponding sides of ΔPQR.

20. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4}>1$)

Solution :



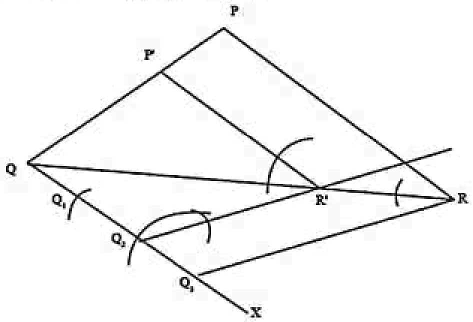
Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the triangle PQR.

Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 7 (the greater of 4 and 7 in $\frac{7}{4}$) points. Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 and Q_7 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 =$
4. Join Q_4 (the 4th point, 4 being smaller of 4 and 7 in $\frac{7}{4}$) to R and draw a line through Q_7 parallel to Q_4R, intersecting the extended line segment QR at R'.
5. Draw a line through R' parallel to RP intersecting the extended line segment QP at P' Then ΔP'QR' is the required triangle each of whose sides is seven-fourths of the corresponding sides of ΔPQR.

---

21. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$).



Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 3 points (greater of 2 and 3 in $\frac{2}{3}$) points.

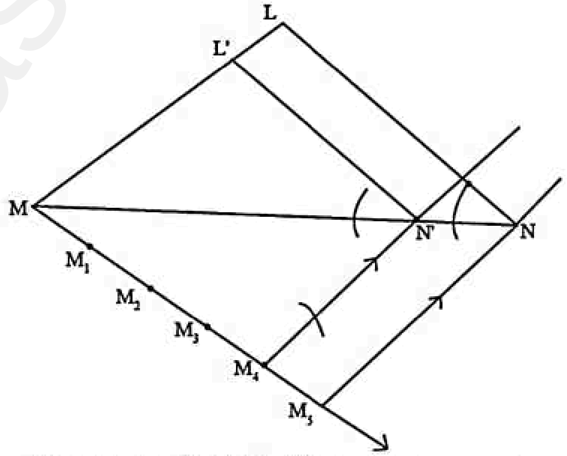
   Q_1, Q_2, Q_3 on QX so that

   $QQ_1 = Q_1Q_2 = Q_2Q_3$
4. Join Q_3R and draw a line through Q_2 (3 being smaller of 2 and 3 in $\frac{2}{3}$) parallel to Q_3R to intersect QR at R'.
5. Draw line through R' parallel to the line RP intersecting the QP at P'. Then, ΔP'QR' is the required Δ.

---

22. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$

Solution :



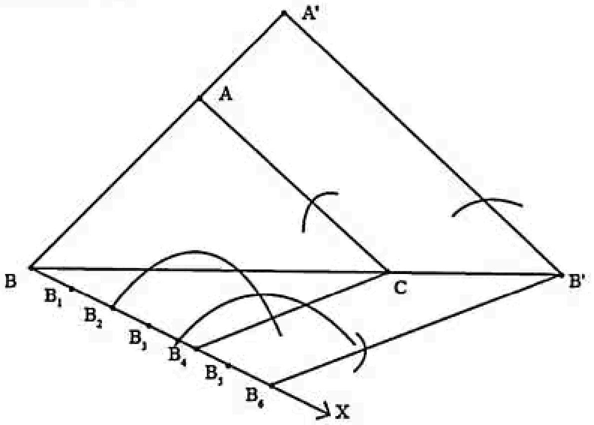
1. Construct a ΔLMN with any measurement.
2. Draw a ray MX making an acute angle with MN on the side opposite to vertex L.
3. Locate 5 points (greater of 4 and 5 in $\frac{4}{5}$) points.

   M_1, M_2, M_3, M_4 & M_5 so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$,
4. Join M_5 to N and draw a line through M_4 (4 being smaller of 4 and 5 in $\frac{4}{5}$) parallel to M_5N to intersect MN at N'.
5. Draw line through N' parallel to the line LN intersecting line segment ML to L'.

   Then, L'MN' is the required Δ.

23. **Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5}$).**

Solution :



Steps of construction

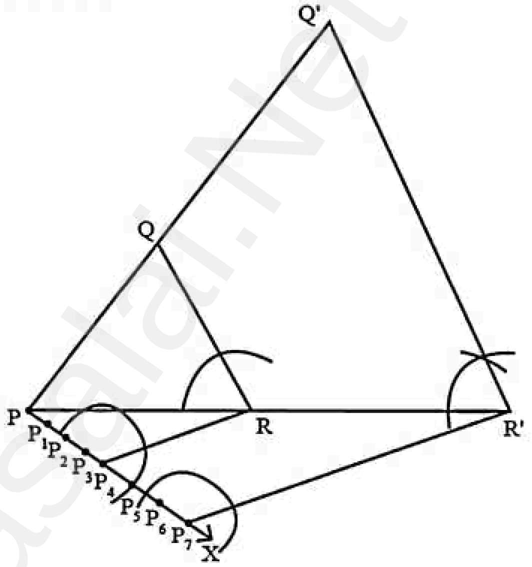
1. Construct a ΔABC with any measurement.
2. Draw a ray BX making an acute angle with BC on the side opposite to vertex A.
3. Locate 6 points (greater of 6 and 5 in $\frac{6}{5}$) points.

   B_1, B_2, ... B_6 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_3B_4 = B_4B_5 = B_5B_6$.
4. Join B_4 (4 being smaller of 4 and 6 in $\frac{6}{4}$) to C and draw a line through B_6 parallel to B_4C to intersecting the extended line segment BC at C'.
5. Draw line through C' parallel to CA intersect the extended line segment BA to A'.

   Then, ΔA'B'C' is the required Δ.

24. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$).**

Solution :



Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray PX making an acute angle with PR on the side opposite to vertex Q.
3. Locate 7 points (greater of 3 and 7 in $\frac{7}{3}$) points.

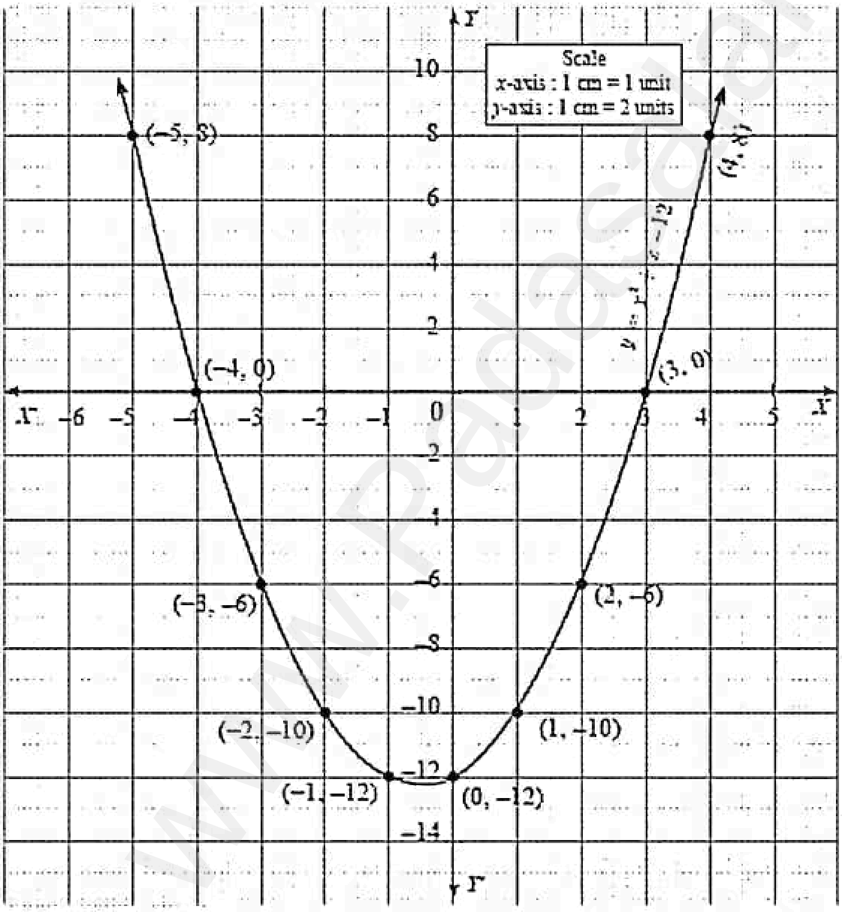
   P_1, P_2, ... P_7 on PX so that $PP_1 = P_1P_2 = P_2P_3$ .... $= P_6P_7$.
4. Join P_3R (3 being smaller of 3 and 7 in $\frac{7}{3}$) and draw a line through P_7 parallel to P_3R to intersecting the extended line segment PR at R'.
5. Draw line through R' parallel to QR intersect the extended line segment PQ to Q'.

   Then, ΔP'Q'R' is the required Δ.

GRAPH

1. Discuss the nature of solution of the following quadratic equation $X^2 + X - 12 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
+ -	25 -17	26 -26	9 -15	4 -16	1 -13	0 -12	2 -12	6 -12	12 -12	20 -12	30 -12
Y	8	0	-6	-10	-12	-12	-10	-6	0	8	18

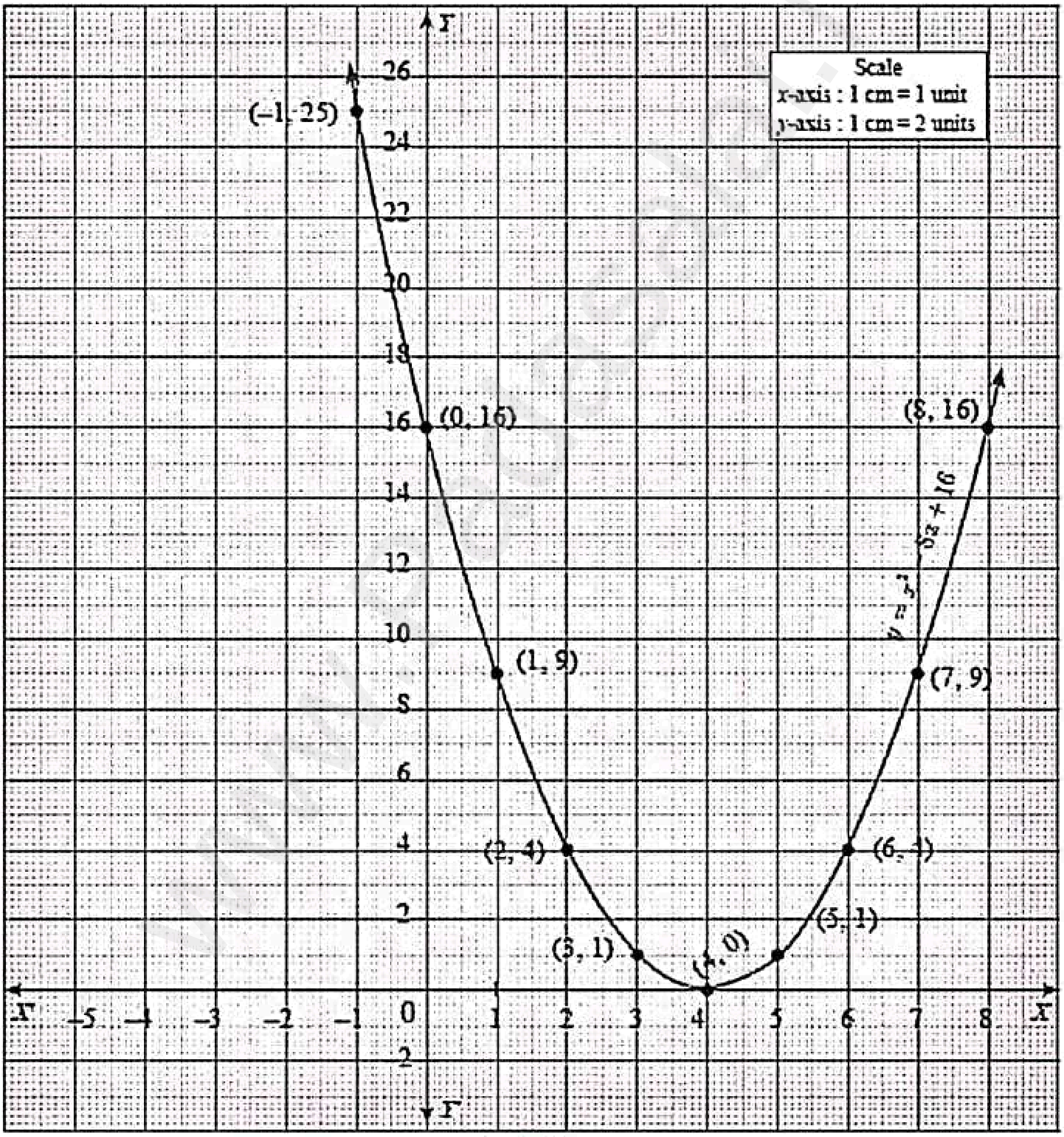


Solution set = { -4, 3 }

Therefore the roots are real and unequal.

2. ***Discuss the nature of solution of the following quadratic equation*** $X^2 - 8X + 16 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
-8X	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56
16	16	16	16	16	16	16	16	16	16	16	16	16	16
+ -	81 0	64 0	49 0	36 0	25 0	16 0	17 -8	20 -16	26 -24	32 -32	41 -40	52 -48	65 -56
Y	81	64	49	36	25	16	9	4	1	0	1	4	9

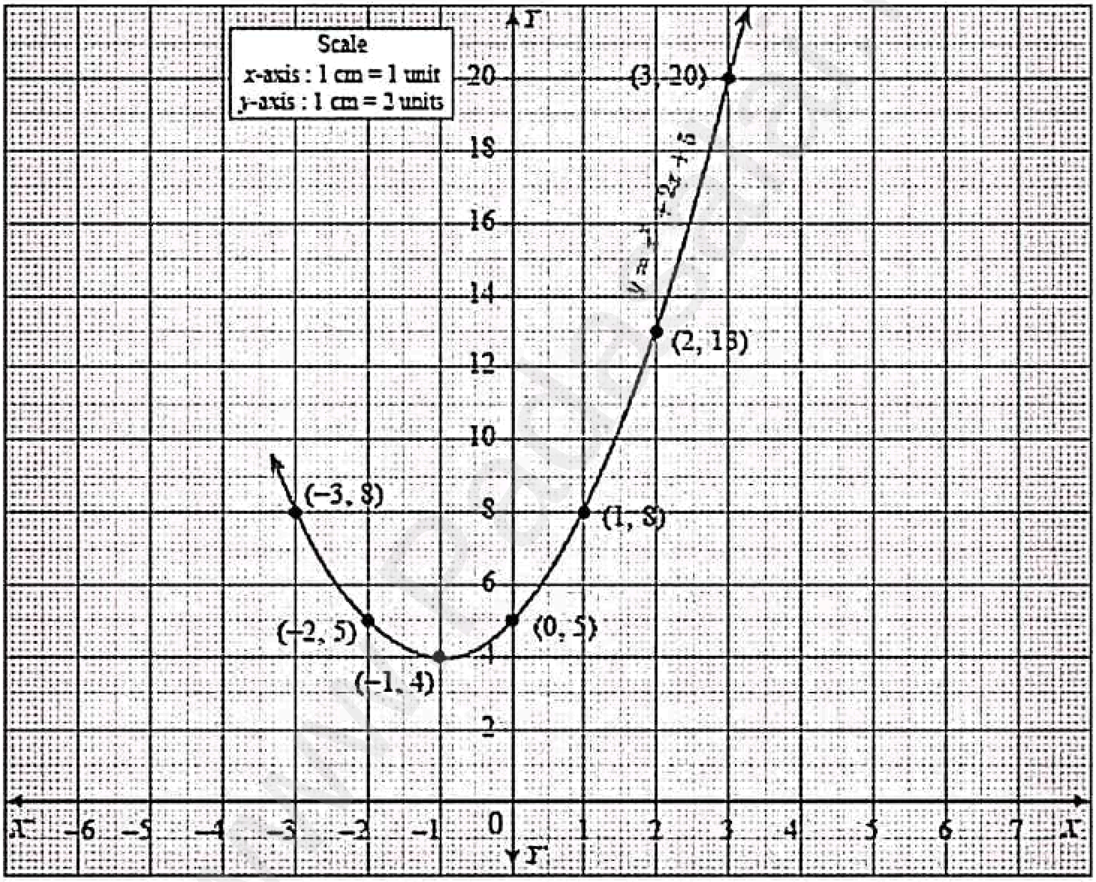


Solution set = {4, 4 }

Therefore the roots are real and equal.

3. *Discuss the nature of solution of the following quadratic equation* $X^2 + 2X + 5 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
2X	-10	-8	-6	-4	-2	0	2	4	6	8	10
5	5	5	5	5	5	5	5	5	5	5	5
+ -	30 -10	21 -8	15 -6	9 -4	6 -2	5 0	8 0	13 0	20 0	29 0	40 0
Y	20	13	8	5	4	5	8	13	20	29	40



No solution

Therefore the roots are unreal.

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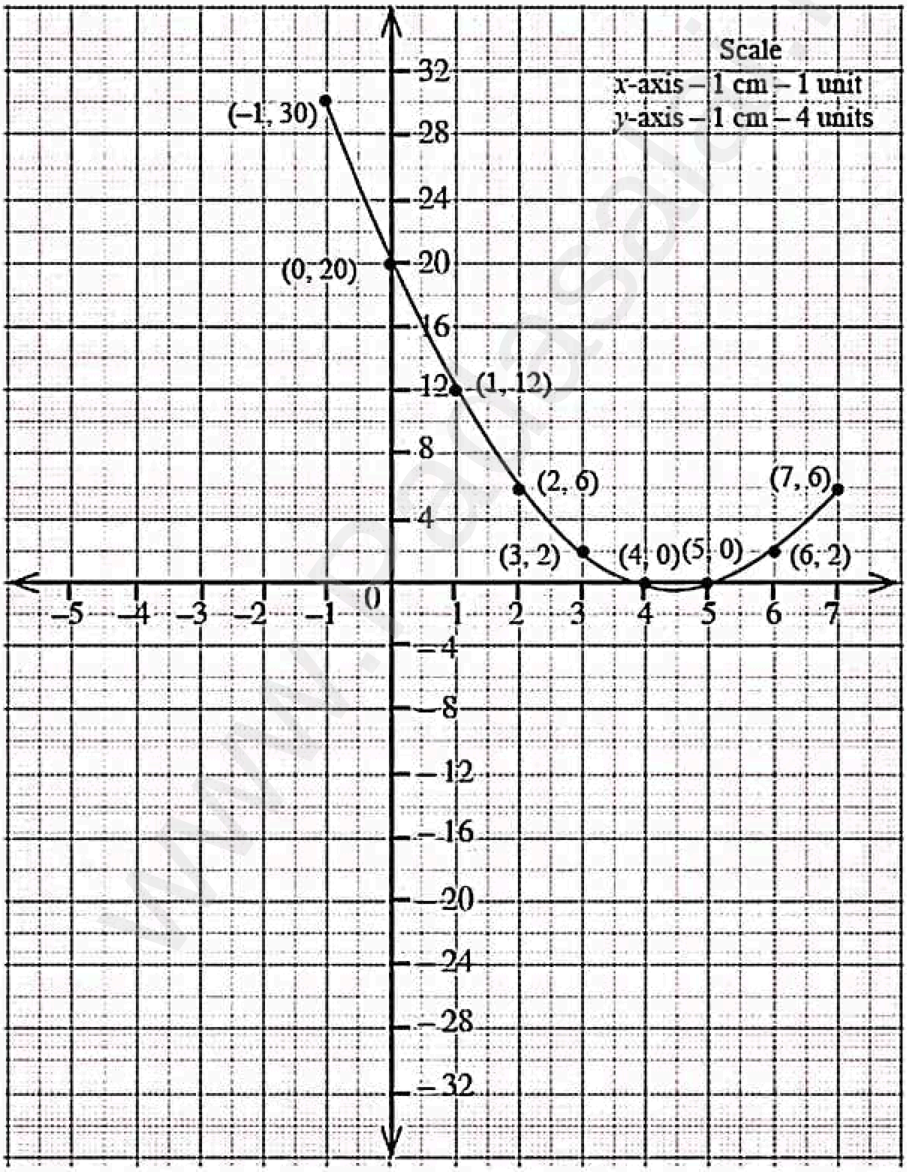
4. *Discuss the nature of solution of the following quadratic equation* $X^2 - 9X + 20 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
$-9X$	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63
20	20	20	20	20	20	20	20	20	20	20	20	20	20
+ -	90 0	72 0	56 0	42 0	30 0	20 0	21 -9	24 -18	29 -27	36 -36	45 -45	56 -54	69 -63
Y	90	72	56	42	30	20	12	6	2	0	0	2	6

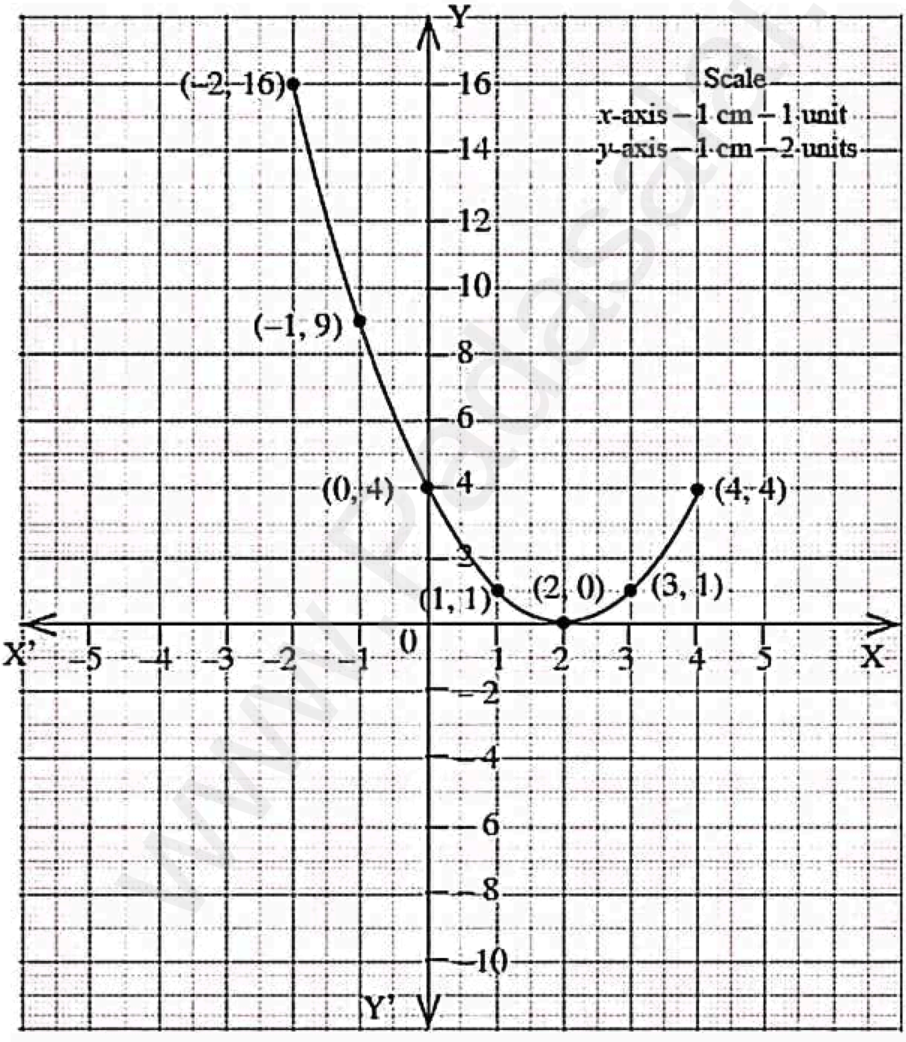


Solution : { 4, 5 }

Therefore the roots are real and unequal.

5. Discuss the nature of solution of the following quadratic equation $X^2 - 4X + 4 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-4X	20	16	12	8	4	0	-4	-8	-12	-16	-20
4	4	4	4	4	4	4	4	4	4	4	4
+ -	49 0	36 0	25 0	16 0	9 0	4 0	5 -1	8 -8	13 -12	20 -16	29 -25
Y	49	36	25	16	9	4	1	0	1	4	9

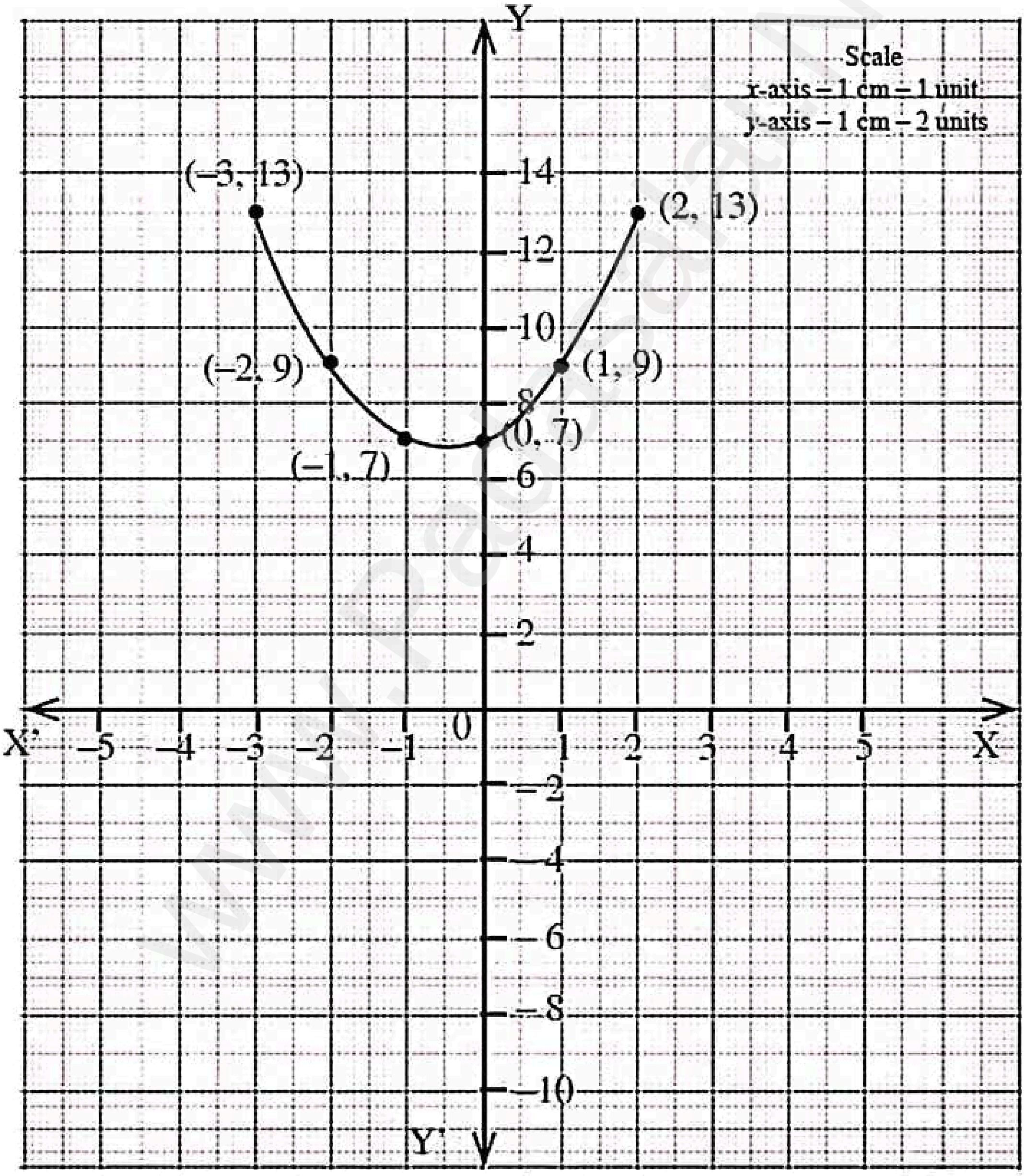


Solution : { 2, 2 }

Therefore the roots are real and equal

6. *Discuss the nature of solution of the following quadratic equation* $X^2 + X + 7 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
7	7	7	7	7	7	7	7	7	7	7	7
+ -	32 -5	23 -4	16 -3	11 -2	8 -1	7 0	9 0	13 0	19 0	27 0	37 0
Y	27	19	13	9	7	7	9	13	19	27	37

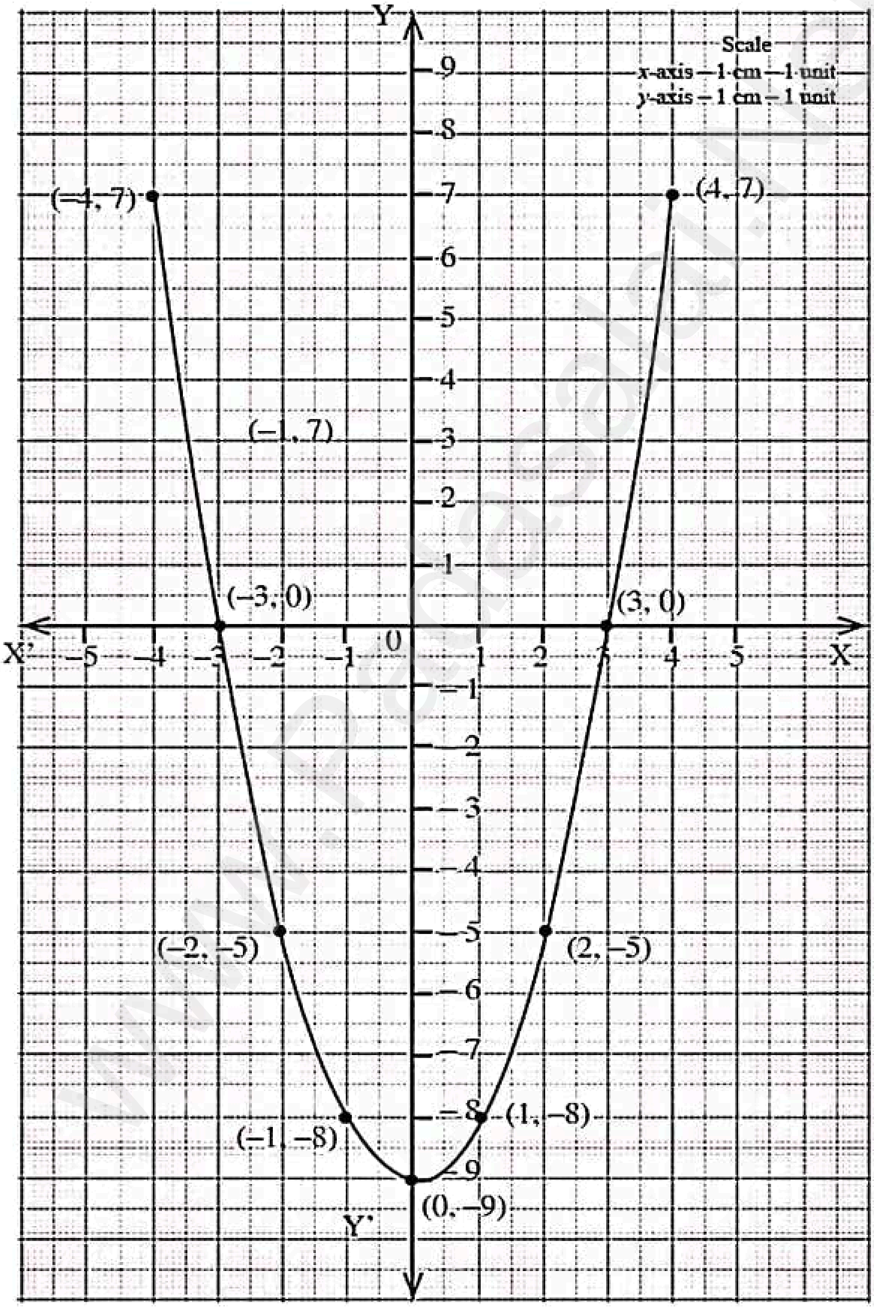


No Solution

Therefore the roots are unreal.

7. *Discuss the nature of solution of the following quadratic equation* $X^2 - 9 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
Y	16	7	0	-5	-8	-9	-8	-5	0	7	16



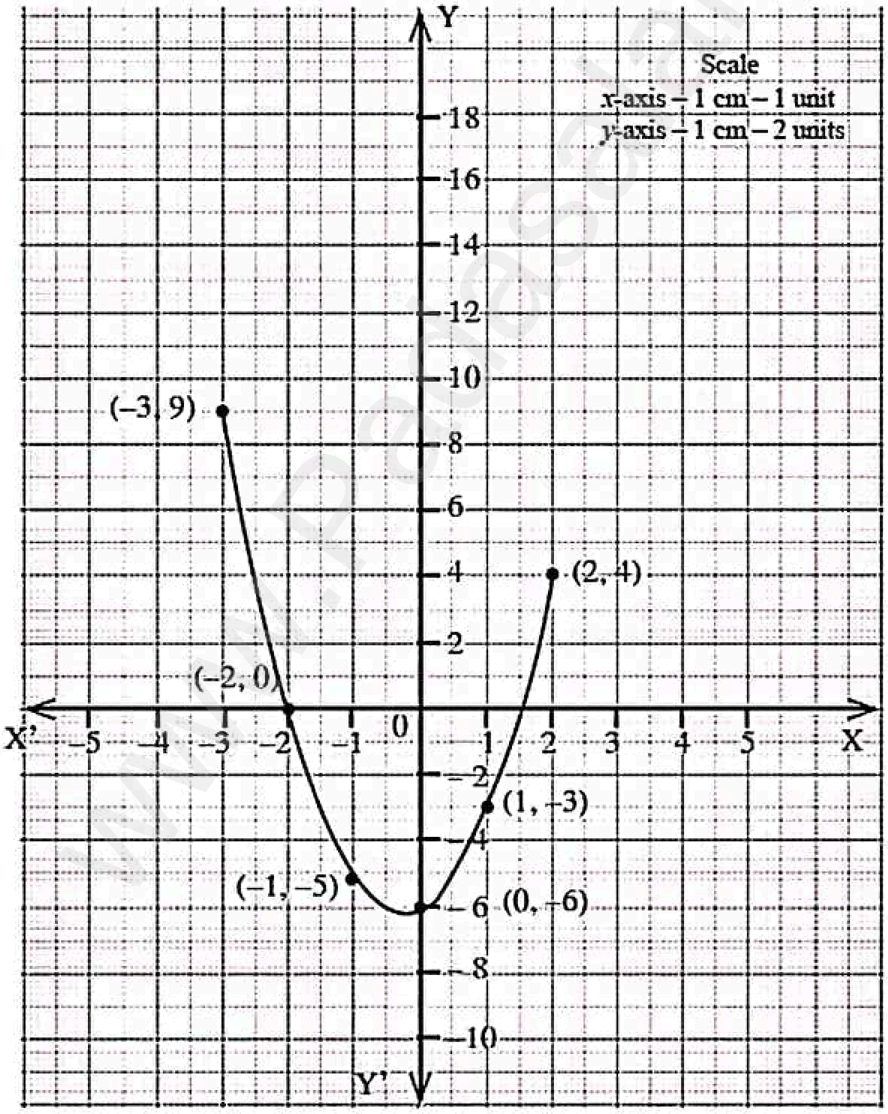
Solution : { -3, 3 }

Therefore the roots are real and unequal.

8. *Discuss the nature of solution of the following quadratic equation $(2x - 3)(x + 2) = 0$*

$(2x - 3)(x + 2) = 0 \qquad \Rightarrow 2x^2 + x - 6 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
$2X^2$	50	32	18	8	2	0	2	8	18	32	50
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
+ -	50 -11	32 -10	18 -9	8 -8	2 -7	0 -6	3 -6	10 -6	21 -6	36 -6	
Y	39	22	9	0	-5	-6	-3	4	15	30	49



Solution **:** ***{ -2, 1.5 }***

Therefore the roots are real and unequal.

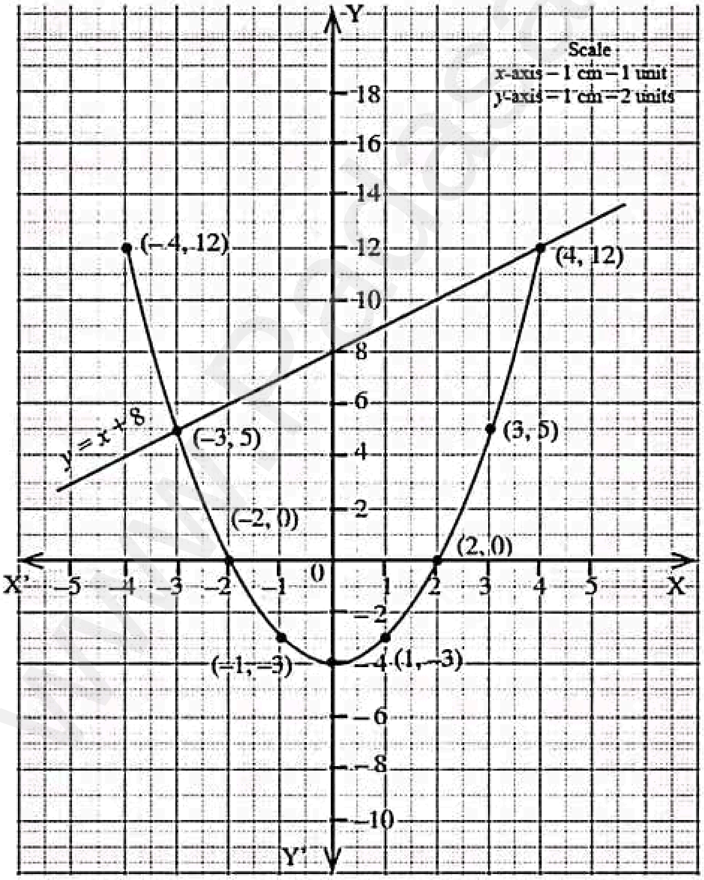
9. Draw the graph of $Y = X^2 - 4$ and hence solve $X^2 - X - 12 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
Y	21	12	5	0	-3	-4	-3	0	5	10	21

To solve $x^2 - x - 12 = 0$, subtract $x^2 - x - 12 = 0$ from $y = x^2 - 4$.

$$\begin{array}{rl} \text{from} & y = x^2 - 4 \\ & y = x^2 + 0x - 4 \\ & 0 = x^2 - x - 12 \\ \hline & y = x + 8 \\ \hline \end{array}$$

x	-4	-3	-2	-1	0	1	2	3	4
y	4	5	6	7	8	9	10	11	12



Solution : { -3, 4 }

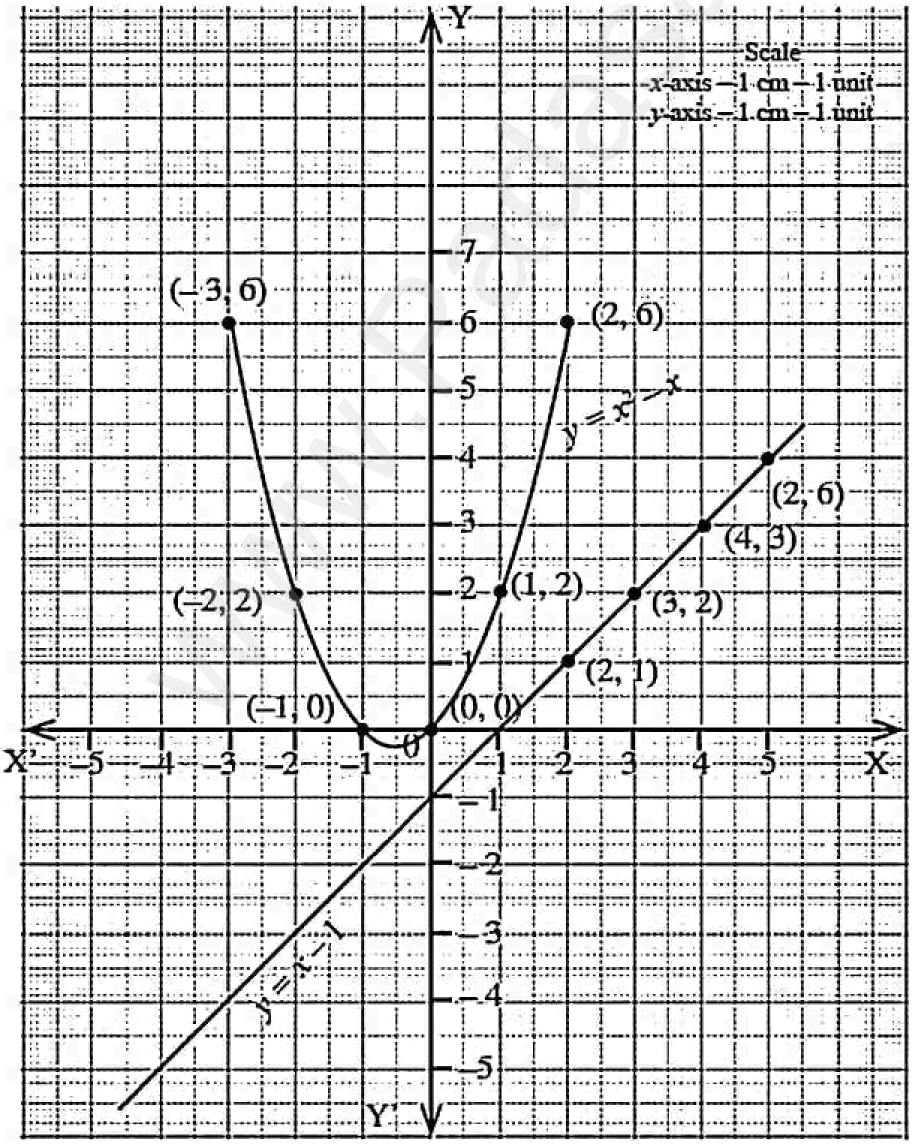
10. ***Draw the graph of $Y = X^2 + X$ and hence solve $X^2 + 1 = 0$***

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y	20	12	6	2	0	0	2	6	12	20	30

To solve $x^2 + 1 = 0$, subtract $x^2 + 1 = 0$ from $y = x^2 + x$.

$$\begin{aligned} y &= x^2 + x \\ 0 &= x^2 - 0x + 1 \\ \hline y &= x - 1 \end{aligned}$$

x	-4	-3	-2	-1	0	1	2	3	4	5
y	-5	-4	-3	-2	-1	0	1	2	3	4



No Solution

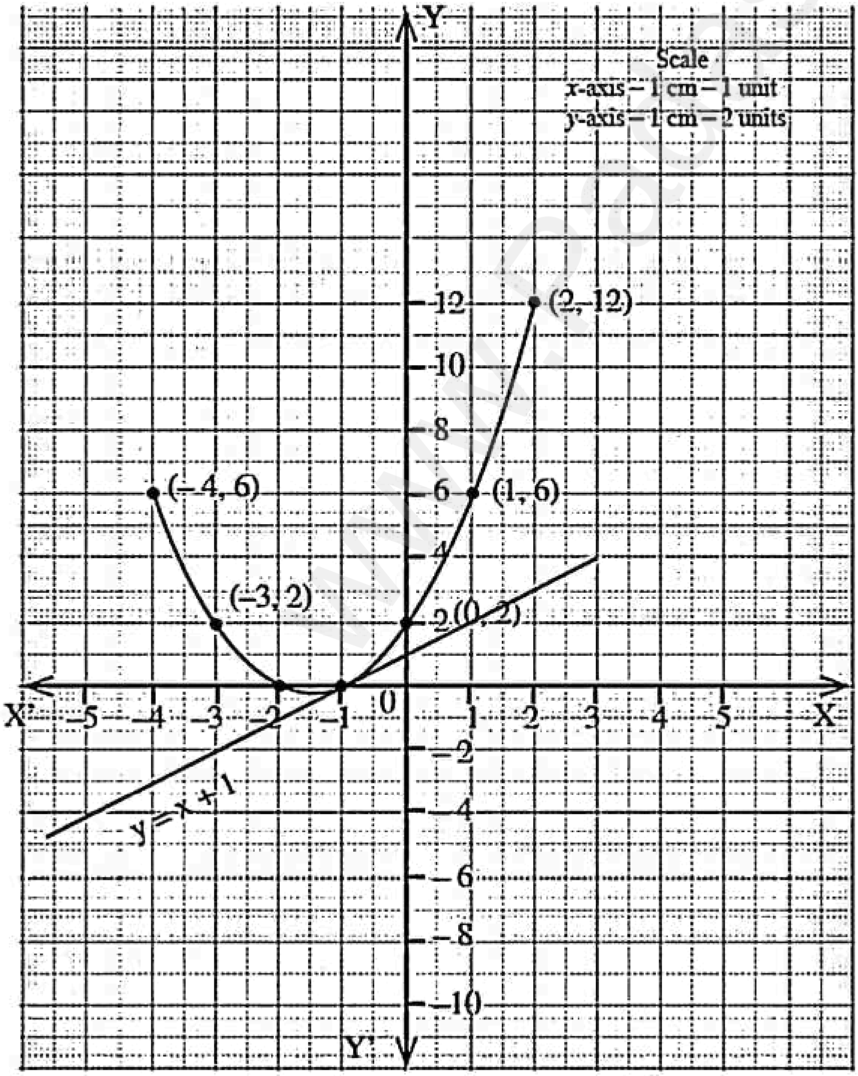
11. ***Draw the graph of $Y = X^2 + 3x + 2$ and use it to solve $X^2 + 2x + 1 = 0$***

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
3X	-15	-12	-9	-6	-3	0	3	6	9	12	15
2	2	2	2	2	2	2	2	2	2	2	2
+ -	27 -15	18 -12	11 -9	6 -6	3 -3	2 0	6 0	12 0	20 0	30 0	42 0
Y	12	6	2	0	0	2	6	12	20	30	42

To solve $x^2 + 2x + 1 = 0$, subtract $x^2 + 2x + 1 = 0$ from $y = x^2 + 3x + 2$.

$$\begin{array}{l} y = x^2 + 3x + 2 \\ 0 = x^2 + 2x + 1 \\ \hline y = \qquad x + 1 \end{array}$$

x	-4	-3	-2	-1	0	1	2	3	4
y	-3	-2	-1	0	1	2	3	4	5



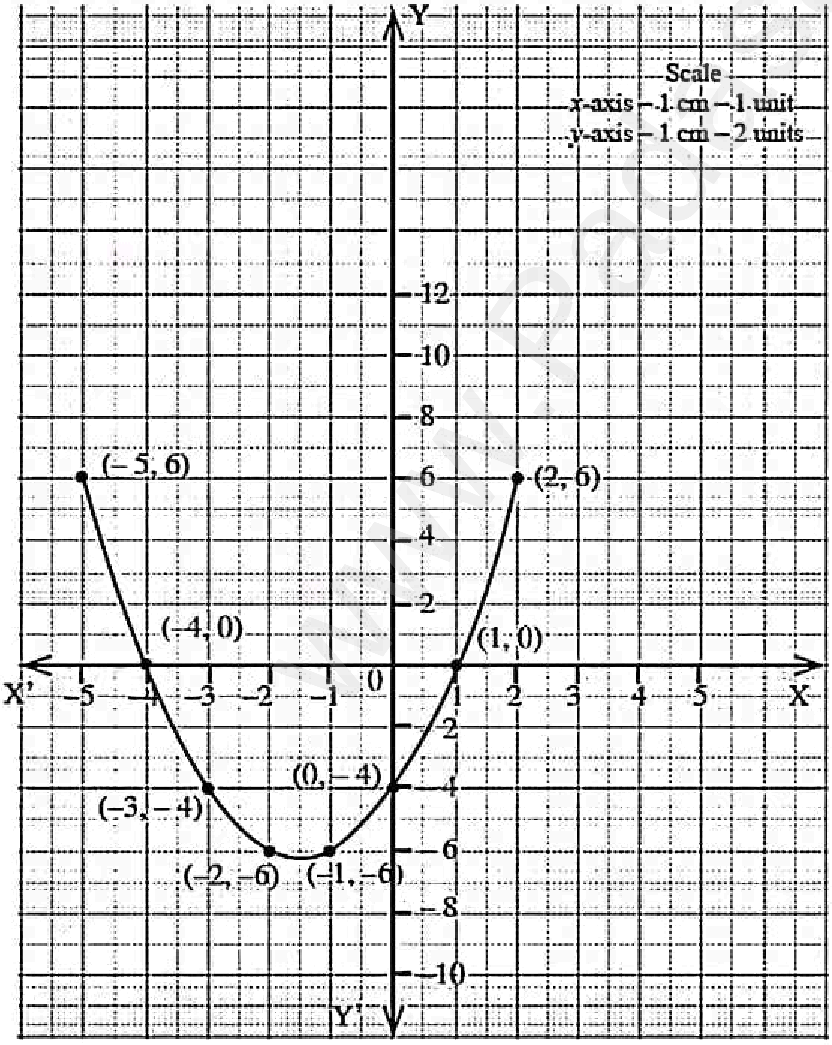
Solution **:** ***{ -1, -1 }***

12. ***Draw the graph of $Y = X^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 - 0$***

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
3X	-15	-12	-9	-6	-3	0	3	6	9	12	15
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
+ -	25 -19	16 -16	9 -13	4 -10	1 -7	0 -4	4 -4	10 -4	18 -4	28 -4	30 -4
Y	6	0	-4	-6	-6	-4	0	6	14	24	26

To solve $x^2 + 3x - 4 = 0$, subtract $x^2 + 3x - 4 = 0$ from $y = x^2 + 3x - 4$.

$$\begin{array}{l} y = x^2 + 3x - 4 \\ 0 = x^2 + 3x - 4 \\ \hline y = \quad 0 \\ \hline \end{array}$$



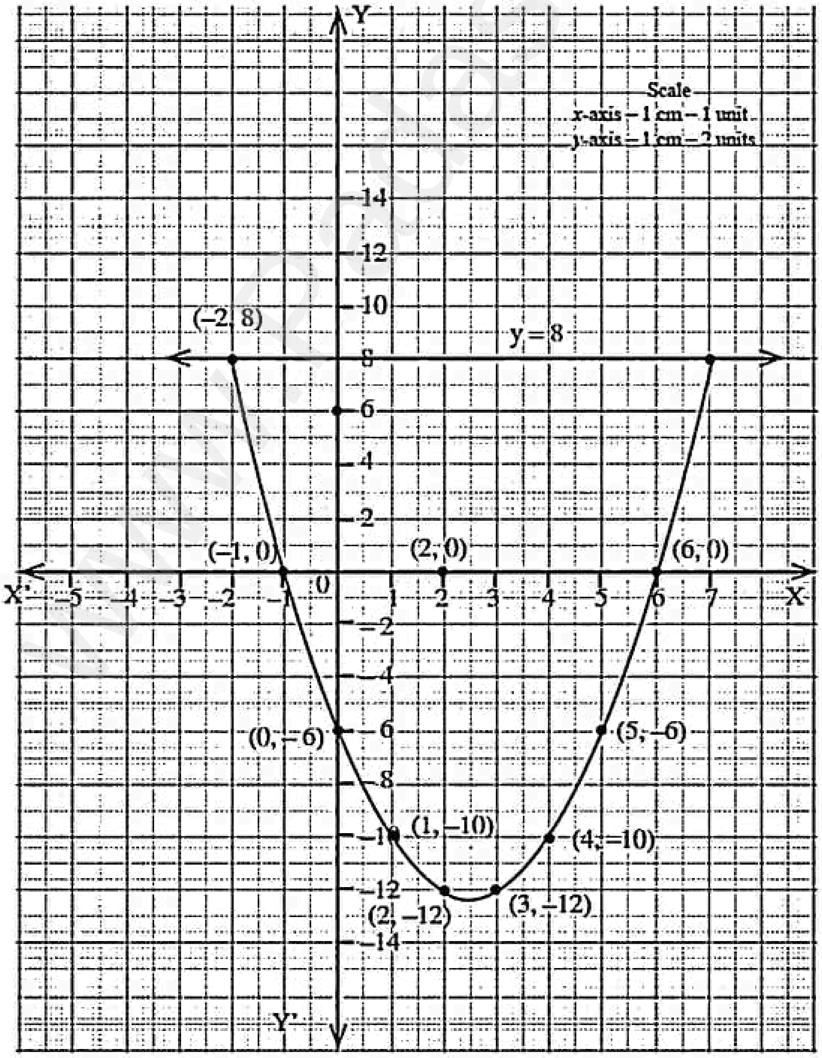
Solution : { -4, 1 }

13. Draw the graph of $Y = X^2 - 5X - 6$ and hence solve $X^2 - 5X - 14 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
-5X	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
+ -	50 -6	36 -6	24 -6	14 -6	6 -6	0 -6	1 -11	4 -16	9 -21	16 -26	25 -31	36 -36	49 -41
Y	44	30	18	8	0	-6	-10	-12	-12	-10	-6	0	8

To solve $x^2 - 5x - 14 = 0$, subtract $x^2 - 5x - 14 = 0$ from $y = x^2 - 5x - 6$.

$$\begin{array}{l} y = x^2 - 5x - \ \ 6 \\ \underline{0 = x^2 - 5x - 14} \\ \underline{y = \qquad\quad 8} \end{array}$$



Solution : { -2, 7 }

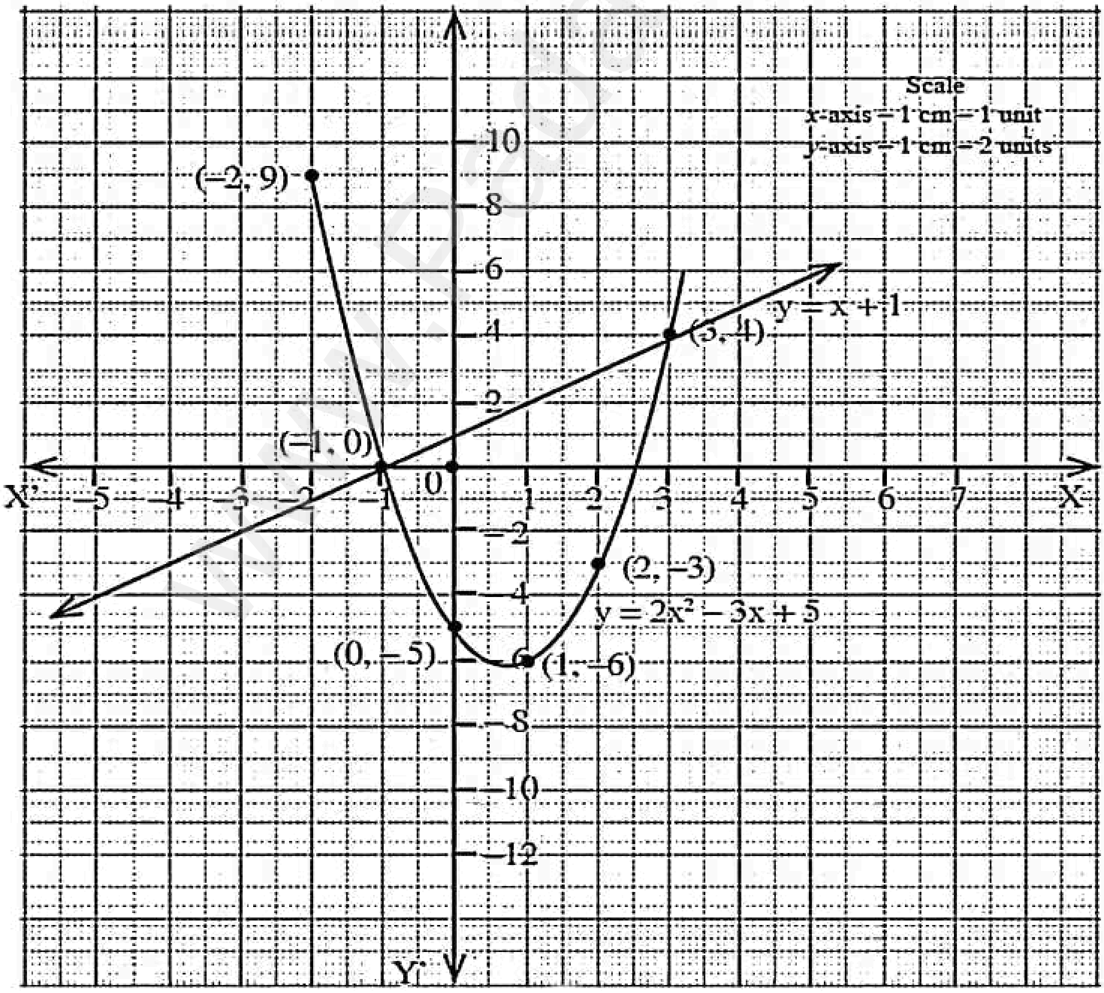
14. ***Draw the graph of Y = $2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$***

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
$2X^2$	50	32	18	8	2	0	2	8	18	32	50
-3x	15	12	9	6	3	0	-3	-6	-9	-12	-15
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
+ -	65 -5	44 -5	27 -5	14 -5	5 -5	0 -5	2 -8	8 -11	18 -14	32 -17	50 -20
Y	60	39	22	9	0	-5	-6	-3	4	15	30

To solve $2x^2 - 4x - 6 = 0$, subtract it from $y = 2x^2 - 3x - 5$.

$$\begin{array}{l} y = 2x^2 - 3x - 5 \\ 0 = 2x^2 - 4x - 6 \\ \hline y = \qquad x + 1 \\ \hline \end{array}$$

X	0	1	2	-1
y	1	2	3	0



Solution ***: { -1, 3 }***

15. **Draw the graph of $Y = (X-1)(X+3)$ and hence solve $X^2 - X - 6 = 0$**

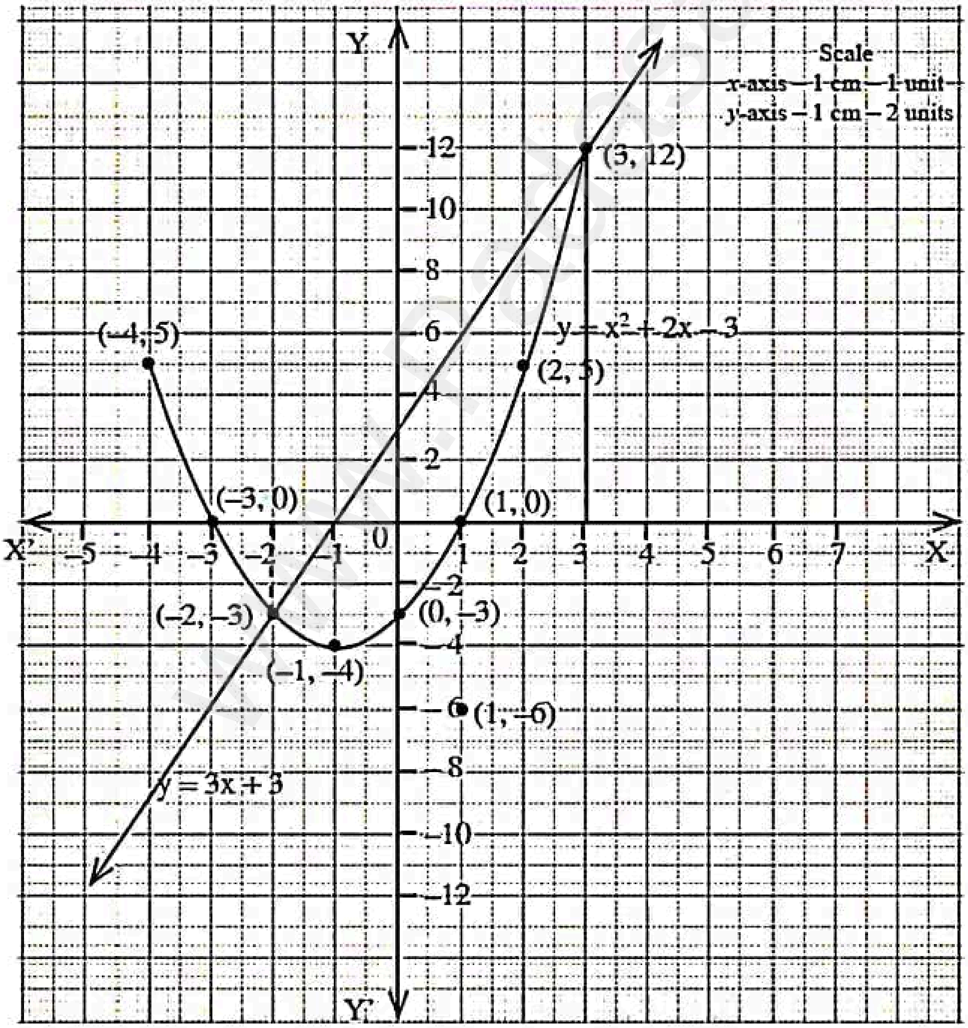
$Y = x^2 + 2x - 3$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
2X	-10	-8	-6	-4	-2	0	2	4	6	8	10
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
+ -	25 -13	16 -11	9 -9	4 -7	1 -5	0 -3	3 -3	8 -3	15 -3	24 -3	35 -3
Y	12	5	0	-3	-4	-3	0	5	12	21	32

To solve $x^2 - x - 6 = 0$, subtract it from $y = x^2 + 2x - 3$.

$$\begin{aligned} y &= x^2 + 2x - 3 \\ 0 &= x^2 - \ \ x - 6 \\ \hline y &= \qquad 3x + 3 \end{aligned}$$

X	0	1	2	-1
y	3	6	9	0



Public Exam Revision Going On..... contact 9629216361

Solution : { -2, 3 }

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2 MARKS MIMIMUM MATERIAL

STANDARD TEN 2 – Marks

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution : Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

2. Find the range of the following distribution.

Age (in years)	16- 18	18- 20	20- 22	22- 24	24- 26	26- 28
Number of students	0	4	6	8	2	2

Solution : Here Largest value L = 28 Smallest value S = 18

Range R = L − S = 28 − 18 = 10Years.

3. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution : Range R = 13.67 Largest value L = 70.08 Range R = L − S

13.67 = 70.08 − S S = 70.08 − 13.67 = 56.41

4. **Find the range and coefficient of range of the following data. 63, 89, 98, 125, 79, 108, 117, 68**

Solution: Range = L − S = 125 − 63 = 62

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{185} = 0.33$$

5. **Find the range and coefficient of range of the following data. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8**

Solution: Range = L − S = 61.4 − 13.6 = 47.8

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{61.4-13.6}{61.4+13.6} = \frac{47.8}{75} = 0.64$$

6. **If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.**

Solution : Given, S.D of a data = 4.5 each value is decreased by 5, then the new SD = 4.5

7. **If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.**

Solution : Given, S.D of a data = 3.6 each value is divided by 3 then the new $S.D = \frac{3.6}{3} = 1.2$

New Variance = $(S.D)^2 = (1.2)^2 = 1.44$

8. **Calculate the range of the following data.**

Income	**400-450**	**450-500**	**500-550**	**550-600**	**600-650**
Number of workers	**8**	**12**	**30**	**21**	**6**

Solution : Here, Largest value = L = 650 Smallest value = S = 400

∴ Range = L − S = 650 − 400 = 250

9. **If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.**

Solution : Given ; range = 36.8 Smallest value = 13.4 ∴ R = L − S 36.8 = L − 13.4

10. **Find the standard deviation of first 21 natural numbers.**

Solution :

$$\text{SD of first 21 natural numbers} = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}} = 6.05$$

11. **The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?**

Solution : Given $\sigma_c = 5$ $F = \frac{9c}{5} + 32 \Rightarrow \sigma_F = \frac{9}{5}\sigma_c = \frac{9}{5}\times 5 = 9$ $\therefore \sigma_F{}^2 = 9^2 = 81.$

12. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution : $n(S) = 5 + 4 = 9$ (i) Let A blue ball. $n(A) = 5$, $P(A) = \frac{5}{9}$

(ii) B will be the event of not getting a blue ball. $n(B) = 4$, $P(B) = \frac{4}{9}$

13. Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution : S = {HH,HT,TH,TT}   Let A be the different faces on the coins.

$n(S) = 4$ A = {HT,TH}; $n(A) = 2$, $P(A) = \frac{2}{4} = \frac{1}{2}$

14. What is the probability that a leap year selected at random will contain 53 saturdays. (Hint: $366 = 52\times7 + 2$)

Solution : A leap year has 366 days.   52 weeks and 2 days.

S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)} ; $n(S) = 7$

Let A 53rd Saturday.   A = {Fri-Sat, Sat-Sun}; $n(A) = 2$, $P(A) = \frac{2}{7}$

15. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution : S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T};   n (S) = 12

Let A odd number and a head.   A = {1H, 3H, 5H}; n (A)= 3 , $P(A) = \frac{3}{12} = \frac{1}{4}$

16. A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution : Number of green balls is n (G) = 6   Let number of red balls is n (R) = x

Therefore, number of black balls is n (B) = $2x$ Total number of balls $n(S) = 6 + x + 2x = 6 + 3x$

It is given that, P(G) = 3 × P(R) $\Rightarrow \frac{6}{6+3x} = 3\times\frac{x}{6+3x} \Rightarrow 3x = 6 \Rightarrow x = \frac{6}{3} = 2 \Rightarrow x = 2.$

(i) Number of black balls = 2×2 = 4   (ii) Total number of balls = 6 +(3×2) = 12

17. **If A is an event of a random experiment such that P(A) : P $(\bar{A})$=17:15 and n(S) = 640 then find (i) P $(\bar{A})$ (ii) n(A).**

Solution : Given $P(A) : P(\bar{A}) = 17 : 15$ Total event = 17+15 = 32

(i) $P(\bar{A}) = \frac{15}{32}$ (ii) $n(A) = \frac{17}{32}\times 640 = 340$

18. **A coin is tossed thrice. What is the probability of getting two consecutive tails?**

Solution : **A coin is tossed thrice.** S = {(HHH), (HHT), (HTH), (HTT), (THH),(THT), (TTH), (TTT)} ,

$n(S) = 8$

Let A two consecutive tails   A = {(HTT), (TTH), (TTT)} , $n(A) = 3$, $P(A) = \frac{3}{8}$

19. **At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?**

i) Let A perfect squares between 500 and 1000 $A = \{23^2, 24^2, 25^2, 26^2 \ldots\ldots 31^2\}$, $n(A) = 9$, $P(A) = \frac{9}{1000}$

(ii) Let A the second player wins a prize, if the first has won $n(S) = 999, n(B) = 8$, $P(B) = \frac{8}{999}$

20 . Find the diameter of a sphere whose surface area is 154 m^2.

Solution : surface area of sphere = 154 m^2 $\Rightarrow$ $4\pi r^2 = 154$

$$4\times\frac{22}{7}\times r^2 = 154 \Rightarrow r^2 = 154\times\frac{1}{4}\times\frac{7}{22} \Rightarrow r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}$$

---

21 .The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Solution : C.S.A. of the cylinder =88 sq. cm diameter = $2r$

Given that, $2\pi rh = 88 \Rightarrow 2\times\frac{22}{7}\times r\times 14 = 88 \Rightarrow 2r = \frac{88\times 7}{22\times 14} = 2$

Therefore, diameter = 2 cm

---

22. If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution : r = 7 cm T.S.A. of cone $= \pi r(l + r)$ sq. units

T.S.A. = 704 cm^2 $\Rightarrow$ $704 = \frac{22}{7}\times 7\,(l+7)$

$$(l+7) = \frac{\overset{32}{\cancel{704}}\times\cancel{7}}{\cancel{7}\times\cancel{22}} \Rightarrow l+7 = 32 \Rightarrow l = 32-7 = 25 \text{ cm}$$

slant height of the cone is 25 cm.

---

23. The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution : Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Now, ratio of C.S.A. of balloons = $\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{3^2}{4^2} = \frac{9}{16}$ Therefore, ratio of C.S.A. of balloons is 9:16.

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24. The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Solution : r =7 m and h = 24 m $\Rightarrow$ $l = \sqrt{r^2+h^2} = \sqrt{49+576} = \sqrt{625} = 25\text{m}$

C.S.A. of the conical tent = πrl sq. units

Area of the canvas = $\frac{22}{7}\times 7\times 25 = 550\ m^2$

length of the canvas = $\frac{\text{Area of canvas}}{\text{Width}} = \frac{550}{4} = 137.5\ m$

---

25. A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden . How much area will it cover in 8 revolutions?

Solution : d = 2.8 m
r = 1.4 m and height = 3 m

Area covered in one revolution = curved surface area of the cylinder = $2\pi rh$ sq. units

$$= 2\times\frac{22}{7}\times 1.4\times 3 = 26.4$$

Area covered in 1 revolution = 26.4 m^2 Area covered in 8 revolutions = 8 × 26.4 = 211.2

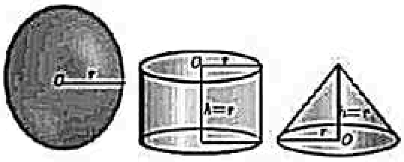
---

26. A sphere, a cylinder and a cone are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.

Solution : Required Ratio = C.S.A. of the sphere: C.S.A. of the cylinder : C.S.A. of the cone

$$= 4\pi r^2 : 2\pi rh : \pi rl$$

$$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$$



26. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution : $l = 5$ cm, R = 4 cm, r = 1 cm

$$\text{C.S.A. of the frustum } = \pi(R+r)l = \frac{22}{7}\times(4+1)\times 5 = \frac{550}{7} = 78.57 \text{ cm}^2$$

27. **4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.**

Solution : Given slant height of the cone $l = 19$ cm

$$\text{Total floor area of 4 persons} = 88 \text{ cm}^2 \Rightarrow \pi r^2 = 88 \Rightarrow \frac{22}{7}\times r^2 = 88 \Rightarrow r^2 = 28$$

$$\therefore h = \sqrt{l^2 - r^2} = \sqrt{19^2 - 28} = \sqrt{361 - 28} = \sqrt{333} \simeq 18.25 \text{ cm.}$$

28. **From a solid Cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm³.**

Solution: Volume of the remaining solid = Vol. of Cylinder – Vol. of Cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3}\times\frac{22}{7}\times 0.7\times 0.7\times 2.4 = 2.46 \text{ cm}^3$$

29. The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.

Solution : Volume of the cone = 11088 cm³ $\Rightarrow \frac{1}{3}\pi r^2 h = 11088 \Rightarrow \frac{1}{3}\times\frac{22}{7}\times r^2\times 24 = 11088$

$$r^2 = 441$$

Therefore, radius of the cone r = 21 cm

30. The ratio of the volumes of two cones is 2 : 3. Find the ratio of their radii if the height of second cone is double the height of the first..

Solution : Given . $h_2 = 2h_1$ and $\frac{\text{Volume of the cone I}}{\text{Volume of the cone II}} = \frac{2}{3}$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3} \Rightarrow \frac{r_1^2}{r_2^2}\times\frac{h_1}{2h_1} = \frac{2}{3} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{4}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

ratio of their radii = 2 : $\sqrt{3}$

31. **The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights.**

Solution : Given volumes of 2 cones = 3600 cm³ & 5040 cm³ & base radius are equal

$$\therefore \text{Ratio of volumes} = \frac{V_1}{V_2} = \frac{3600}{5040} \Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040} \Rightarrow \frac{h_1}{h_2} = \frac{40}{56} = \frac{5}{7}$$

$$\therefore h_1 : h_2 = 5 : 7$$

32. **A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4.**

Solution : Given TSA of a solid sphere = TSA of a solid hemisphere

$$\Rightarrow 4\pi R^2 = 3\pi r^2 \Rightarrow \therefore \frac{R^2}{r^2} = \frac{3}{4} \qquad \therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{Ratio of their volumes} = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left(\frac{\sqrt{3}}{2}\right)^3 = 2\times\frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$$

$$\therefore \text{Ratio of the volumes} = 3\sqrt{3} : 4$$

33. **Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of 10224 and 9648**

Solution : HCF of 10224 and 9648

$10224 = 9648 \times 1 + 576$

$9648 = 576 \times 16 + 432$

$576 = 432 \times 1 + 144$ ∴ The last division "144" is the HCF.

$432 = (144) \times 3 + 0$

---

34. **Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.**

Solution : HCF of 1230 - 12 and 1926 - 12

i.e., HCF of 1218 and 1914

$1914 = 1218 \times 1 + 696$

$1218 = 696 \times 1 + 522$

$696 = 522 \times 1 + 174$

$522 = (174) \times 3 + 0$

∴ HCF = 174 ∴ The required largest number = 174.

---

35. **When the positive integers *a, b* and *c* are divided by 13, the respective remainders are 9,7 and 10. Show that *a* + *b* + *c* is divisible by 13.**

Solution : When a is divided by 13, remainder is 9 i.e., $a = 13q + 9$ .... (1)

When b is divided by 13, remainder is 7 i.e., $b = 13q + 7$ .... (2)

When c is divided by 13, remainder is 11 i.e., $c = 13q + 11$ .... (3)

Adding (1), (2) & (3) $a + b + c = 39q + 26 = 13\,(2q + 2)$

$a + b + c$ is divisible by 13

---

36. **Find the HCF of 252525 and 363636.**

Solution :

5	252525
5	50505
3	10101
7	3367
	481

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
	481

$\therefore 252525 = 5 \times 5 \times \underline{3} \times \underline{7} \times \underline{481}$

$363636 = 2 \times 2 \times \underline{3} \times 3 \times 3 \times \underline{7} \times \underline{481}$

∴ HCF $= 3 \times 7 \times 481 = 10{,}101$

---

37. **Find the least number that is divisible by the first ten natural numbers.**

Solution : The required number is the LCM of (1, 2, 3, ... 10)

$2 = \underline{2} \times 1$ $4 = \underline{2} \times 2$ $6 = 3 \times \underline{2}$ $8 = 2 \times 2 \times \underline{2}$ $9 = 3 \times 3$

$10 = 5 \times \underline{2}$ and 1, 3, 5, 7

L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

---

38. **If $13824 = 2^a \times 3^b$ then find a and b.**

Solution : Given

$2^a \times 3^b = 13824$

$2^a \times 3^b = 2^9 \times 3^2$

$\therefore a = 9, b = 2$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
	3

39. **Find the standard deviation of first 10 natural numbers.**

Solution : SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{100-1}{12}} = \sqrt{\frac{99}{12}} = 2.87$

---

40. **Find the standard deviation of first 13 natural numbers.**

Solution : SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{169-1}{12}} = \sqrt{\frac{168}{12}} = 3.74$

---

41. **A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.**

Solution : A clock strikes bell at 1 o' clock once twice at 2 o' clock, 3 times at 3 o' clock ....

$\therefore$ Number of times it strikes in a particular day $= 2\,(1+2+3+\ldots\ldots\ 12) = 2\left(\frac{12\times13}{2}\right) = 156$ times

S.D of 2 (1, 2, 3, ...12) $= 2\left[\sqrt{\frac{n^2-1}{12}}\right] = 2\left[\sqrt{\frac{144-1}{12}}\right] = 2\sqrt{\frac{143}{12}} = 6.9$

---

42. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution : Mean $\bar{x} = 25.6$, C.V. $= 18.75$ C.V. $= \frac{\sigma}{\bar{x}}\times100$

$$18.75 = \frac{\sigma}{25.6}\times100 = \frac{18.75\times25.6}{100} = 4.8$$

---

43. **The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.**

Solution : Given $\sigma = 6.5, \bar{x} = 12.5$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times100 = \frac{6.5}{12.5}\times100 = 52$

---

44. **The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.**

Solution : Given $\sigma = 1.2$, C.V $= 25.6$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times100$

$$25.6 = \frac{1.2}{\bar{x}}\times100 \Rightarrow \bar{x} = \frac{1.2\times100}{25.6} \Rightarrow \bar{x} = \frac{120}{25.6} = 4.69$$

---

45. **If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.**

Solution : Given $\bar{x} = 15$, CV $= 48$, $\sigma = ?$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times100$

$$48 = \frac{\sigma}{15}\times100 \Rightarrow \sigma = \frac{15\times48}{100} = \frac{720}{100} = 7.2$$

---

46. **If $n = 5, \bar{x} = 6, \Sigma x^2 = 765$, then calculate the coefficient of variation.**

Solution : $n = 5,\ \bar{x} = 6,\ \Sigma x^2 = 765$, CV = ?

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2} = \sqrt{117} = 10.82$$

$$\therefore \text{C.V} = \frac{\sigma}{\bar{x}}\times100 = \frac{10.82}{6}\times100 = 180.33$$

---

47. **If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.**

Solution : Given range = 20, Co.eff. of range = 0.2

$\Rightarrow L - S = 20$ ...(1) $\frac{L-S}{L+S} = 0.2 \Rightarrow \frac{20}{L+S} = 0.2 \Rightarrow L+S = 100$ ...(2)

Solviong (1) and (2) L = 60, S = 40

48. **Find the 12th term from the last term of the A.P – 2, – 4, – 6, ... –100.**

Solution : Given A.P is – 2, – 4, – 6, ... – 100

12th term from the last term $a = -100,\ d = 2$

$t_{12} = a + 11d = -100 + 11(2) = -100 + 22 = -78$

---

49. **If $1 + 2 + 3 + ... + k = 325$, then find $1^3 + 2^3 + 3^3 + ... + k^3$**

Solution : $1 + 2 + 3 + ... + k = 325 \quad \Rightarrow \dfrac{k(k+1)}{2} = 325$

$$\therefore 1^3 + 2^3 + 3^3 + ... + k^3 = \left(\frac{k(k+1)}{2}\right)^2 = (325)^2 = 105625$$

---

50. **Find the G.P. in which the 2nd term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$.**

Solution : Given $t_2 = \sqrt{6},\ t_6 = 9\sqrt{6}$ in G.P.

$a.r = \sqrt{6}$ .... (1)

$a.r^5 = 9\sqrt{6}$ .... (2)

(2) divide (1) $\Rightarrow \dfrac{\cancel{a}.r^{\cancel{5}^{4}}}{\cancel{a.r}} = \dfrac{9\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}}$ $\quad \therefore r^4 = 9 \quad \Rightarrow r = \sqrt{3}$

$\therefore a \times \sqrt{3} = \sqrt{6} \qquad \therefore a = \sqrt{2}$

$\therefore$ The G.P is $\sqrt{2}, \sqrt{6}, \sqrt{18}, ...$

---

51. **When the positive integers *a, b* and *c* are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when $a + 2b + 3c$ is divided by 13.**

Solution : Let $a = 13q + 9 \qquad b = 13q + 7 \Rightarrow 2b = 26q + 14 \qquad c = 13q + 10 \Rightarrow 3c = 39q + 30$

$a + 2b + c = (13q + 9) + (26q + 14) + (39q + 30) = 78q + 53 = 13\ (6q) + 13(4) + 1$

$\therefore$ When $a + 2b + 3c$ is divided by 13, the remainder is 1.

---

52. **The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000 ?**

Solution : P = ₹ 45000, n = 3, r = 15% (depreciation)

$$A = P\left(1 - \frac{r}{100}\right)^n = 45{,}000\left(1 - \frac{15}{100}\right)^3 = 27636$$

---

53. Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Solution : Let $x = 2k + 1$ be any odd integer.

The square of an odd integer $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k\ (k + 1) + 1 = 4q + 1$

---

54. **A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over**

Solution : No. of flower pots = 532 each row to contain 21 flower pots.

$$\begin{array}{r|r} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

$\Rightarrow 532 = 21 \times 25 + 7$

$\therefore$ Number of completed rows = 25

Number of flower pots left out = 7

---

55. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution : $800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$

Hence, $a^b \times b^a = 2^5 \times 5^2$ This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

---

56. **Prove that two consecutive positive integers are always coprime.**

Solution : Let $x,\ x + 1$ be two consecutive integers.

G.C.D. of $(x, x + 1) = 1 \quad \Rightarrow \quad x\ \&\ x + 1$ are Co-prime.

57. Find the number of terms in the A.P. 3, 6, 9, 12, ... 111.

Solution : $a = 3$; $d = 6-3 = 3$; $l = 111$ $\quad n = \left(\frac{l-a}{d}\right)+1 = \left(\frac{111-3}{3}\right)+1 = \left(\frac{108}{3}\right)+1 = 37$

the number of terms in the A.P. 37

---

58. **Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.**

Solution : When n = 1, 2n + 6 × 9n = 2 + (6 × 9) = 56, divisible by 7.

What is the time 100 hours after 7 a.m.?

Solution : Formula : $t + n = f \pmod{24}$ $\quad 100 + 7 = f \pmod{24}$

$\Rightarrow 107 - f$ is divisible by 24

$\therefore$ $f = 11$ so that $107 - 11 = 96$ is divisible by 24.

$\therefore$ The time is 11 A.M.

---

59. Kala and Vani are friends, Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

Solution :

Let 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from Sunday to Saturday respecitvely.

$-74 \pmod 7 \equiv -4 \pmod 7 \equiv 7-4 \pmod 7 \equiv 3 \pmod 7$

The day for the number 3 is Wednesday. Vani's birthday must be on Wednesday.

---

60. **Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?**

Solution : Today is Tuesday Day after 45 days = ?

When we divide 45 by 7, remainder is 3. $\therefore$ The 3rd day from Tuesday is Friday

---

A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi ?

Solution : Starting time 22.30, Travelling time 32 hours. Here we use modulo 24.

The reaching time is $22.30 + 32 \pmod{24} \equiv 54.30 \pmod{24} \equiv 6.30 \pmod{24}$

---

61. **What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?**

Solution : The required number is the LCM of (35, 56, 91) + remainder 7

$35 = 7 \times 5$

$56 = 7 \times 2 \times 2 \times 2$

$91 = 13 \times 7$

$\therefore$ L.C.M $= 7 \times 5 \times 13 \times 8 = 3640$

$\therefore$ The required number is $3640 + 7 = 3647$

---

62. Find the first five terms of the following sequence. $a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2}+3};\ n \geq 3,\ n \in N$

Solution :

$$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3} = \frac{1}{4}$$

$$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{\frac{1}{16}}{\frac{1}{4}+3} = \frac{1}{16} \times \frac{4}{13} = \frac{1}{52}$$

The first five terms of the sequence are 1, 1, $\frac{1}{4}, \frac{1}{16}, \frac{1}{25}$

63. **The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.**

Solution : R = 16 cm h = 13 cm thickness = 4 cm $\therefore$ r = R – w = 16 – 4 = 12

$\therefore$ TSA of hollow cylinder $= 2\pi\,(R + r)\,(R - r + h) = 2 \times \frac{22}{7}(28)\,(4 + 13) = 44 \times 4 \times 17 = 2992\text{ cm}^2$

---

64. Find the volume of a cylinder whose height is 2 m and whose base area is 250 m².

Solution : height h = 2 m,

base area = 250 m²

volume of a cylinder $= \pi r^2 h$ cu. units

$= \text{base area} \times h = 250 \times 2 = 500\text{ m}^3$

Therefore, volume of the cylinder = 500 m³

---

65. **The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.**

Solution : Given $r_1 : r_2 = 1 : 3$

$r_1 = 1$, $r_2 = 3$ | $h_1 = 3r_1 = 3$, $h_2 = 3r_1 = 3$

$l_1 = \sqrt{h_1^2 + r_1^2} = \sqrt{9+1} = \sqrt{10}$ | $l_2 = \sqrt{h_2^2 + r_2^2} = \sqrt{9+9} = 3\sqrt{2}$

$\therefore$ Ratio of their CSA $= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{1}{3} \times \frac{\sqrt{10}}{3\sqrt{2}} = \frac{\sqrt{5}}{9} = \sqrt{5} : 9$

---

66. Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

Solution : Given that, r = 21cm, R = 28 cm, h = 9 cm

volume of hollow cylinder $= \pi(R^2 - r^2)\,h = \frac{22}{7}(28^2 - 21^2) \times 9 = \frac{22}{7}(784 - 441) \times 9 = 9702\text{ cm}^3$.

---

67. If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Solution : A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} Thus A = {3, 5} and B = {2, 4}.

---

68. **Find A × B, A × A and B × A If A = B = {*p, q*}**

Solution: A × B = {(*p, p*), (*p, q*), (*q, p*), (*q, q*)} ,

A × A = {(*p, p*), (*p, q*), (*q, p*), (*q, q*)} , B × A = {(*p, p*), (*p, q*), (*q, p*), (*q, q*)}

---

69. **Find A × B, A × A and B × A If A = {*m, n*} ; B = ϕ**

Solution: A = {m, n}, B = ϕ

If A = ϕ (or) B = ϕ, then A × B = ϕ. and B × A = ϕ A × B = ϕ and B × A = ϕ

A × A = {(*m, m*), (*m, n*), (*n, m*), (*n, n*)}

---

70. **Let A = {1, 2, 3} and B = {*x* | *x* is a prime number less than 10}. Find A × B and B × A.**

Solution : A = {1, 2, 3}, B = {*x* | *x* is a prime number less than 10}. $\therefore$ B = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

---

71. **If B × A = {(– 2, 3), (– 2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.**

Solution : B × A = {(–2, 3), (– 2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} $\therefore$ A = {3, 4}, B = {– 2, 0, 3}

72. **Find the 19th term of an A.P. $-11, -15, -19, \ldots$**

Solution : A.P is $-11, -15, -19, \ldots\ldots.$ $\quad a=-11, \quad d=-15-(-11)=-15+11=-4$

$$t_n = a+(n-1)\,d$$

$$t_{19} = a+18\,d = (-11)+18\,(-4) = -11-72 = -83$$

---

73. **Which term of an A.P. 16, 11, 6, 1.... is -54?**

Solution : A.P. is $16, 11, 6, 1, \ldots\ldots\ldots -54$

$a=16, d=-5, t_n=-54$

$\Rightarrow \quad a+(n-1)\,d=-54 \quad \Rightarrow 16+(n-1)(-5)=-54 \quad \Rightarrow \quad 16-5n+5=-54$

$\Rightarrow \quad -5n+21=-54$

$\Rightarrow \quad -5n+21=-54$

$\Rightarrow \quad -5n=-54-21$

$\Rightarrow \quad -5n=-75$

$\therefore n=15$

$\therefore$ 15th term of A.P. is -54

---

74. **If $a_1=1, a_2=1$ and $a_n=2a_{n-1}+a_{n-2}, n\geq 3, \ n\in$ N, then find the first six terms of the sequence.**

Solution : Given $\quad a_1=1, \quad a_2=1 \quad a_3=2a_2+a_1 = 2\,(1)+1 = 3 \quad a_4=2a_3+a_2 = 2\,(3)+1 = 7$

$a_5=2a_4+a_3=2\,(7)+3=17 \quad a_6=2a_5+a_4=2\,(17)+7=41$

$\therefore$ The first 6 terms are 1, 1, 3, 7, 17, 41

---

75. **Find the middle term(s) of an A.P. 9, 15, 21, 27,...,183.**

Solution : Given A.P is 9, 15, 21, 27, ... 183 $\quad a=9, \ d=6, \ l=183$

$$n=\frac{l-a}{d}+1 = \frac{183-9}{6}+1 = \frac{174}{6}+1 = 29+1 = 30$$

$\therefore$ Middle terms are $30/2, \ 30/2+1 = 15^{th}, \ 16^{th}$

$t_{15} = a+14d$
$= 9+14(6)$
$= 9+84$
$= 93$

$t_{16} = a+15d$
$= 9+15(6)$
$= 9+90$
$= 99$

---

76. **A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Claculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.**

Solution : Cow's milk = 175 lrs. $\quad$ Buffalow's milk = 105 lrs.

i) Capacity of a can = HCF of 175 and 105 = 35 litres

ii) Number of cans of Cow's milk $= \dfrac{175}{35} = 5$ $\quad$ iii) Number of cans of buffalow's milk $= \dfrac{105}{35} = 3$

---

77. **If $3+k, 18-k, 5k+1$ are in A.P. then find k.**

Solution : a, b, c are in A.P. $\Rightarrow 2b = a+c$

$\Rightarrow 2\,(18-k) = (3+k)+(5k+1)$

$36-2k = 6k+4$

$8k = 32 \Rightarrow k=4$

---

78. **Find x, y and z, given that the numbers $x, 10, y, 24, z$ are in A.P.**

Solution : Given that $x, 10, y, 24, z$ are in A.P. $\quad \therefore y$ is the arithmetic mean of 10 & 24

$$2y = 10+24 \Rightarrow y = \frac{10+24}{2} = \frac{34}{2} = 17 \qquad \text{Clearly } d=7$$

$\therefore x, 10, y, 24, z$ are in A.P. $\quad \therefore x=10-7=3 \quad \& \ z=24+7=31$

$\therefore x=3, y=17, z=31$

79. Prove that $\frac{\sin A}{1+\cos A}=\frac{1-\cos A}{\sin A}$

Solution : $\sin A\times\sin A = (1+\cos A)\times(1-\cos A) \Rightarrow \sin^2 A = 1-\cos^2 A \Rightarrow \sin^2 A = \sin^2 A$

80. Prove that $1+\frac{\cot^2\theta}{1+\text{cosec}\,\theta}=\text{cosec}\,\theta$

Solution : $\frac{\cot^2\theta}{1+\text{cosec}\theta} = (\text{cosec}\theta-1) \Rightarrow \frac{(\cancel{\text{cosec}\theta+1})(\text{cosec}\theta-1)}{\cancel{\text{cosec}\theta+1}} = (\text{cosec}\theta-1) \Rightarrow (\text{cosec}\theta-1) = (\text{cosec}\theta-1)$

81. Prove that $\sec\theta-\cos\theta=\tan\theta\sin\theta$

Solution : $\sec\theta-\cos\theta=\frac{1}{\cos\theta}-\cos\theta=\frac{1-\cos^2\theta}{\cos\theta}=\frac{\sin^2\theta}{\cos\theta}=\frac{\sin\theta}{\cos\theta}\times\sin\theta=\tan\theta\sin\theta$

82. Prove that $\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\cot\theta$

Solution : $\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\frac{\sec\theta\cos\theta-\sin\theta\sin\theta}{\cos\theta\sin\theta}=\frac{1-\sin^2\theta}{\sin\theta\cos\theta}=\frac{\cos^2\theta}{\sin\theta\cos\theta}=\frac{\cancel{\cos\theta}\cos\theta}{\sin\theta\cancel{\cos\theta}}=\cot\theta$

83. Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Solution : $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2\Rightarrow\left(\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}\right)=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2\Rightarrow\tan^2 A=(-\tan A)^2\Rightarrow\tan^2 A=\tan^2 A$

84. Prove that $(\text{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Solution : $(\text{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$

$$=\frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{\cancel{\cos^2\theta}\cancel{\sin^2\theta}\times1}{\cancel{\sin^2\theta}\cancel{\cos^2\theta}}=1$$

85. Prove that $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=2\text{cosec}\,A$

Solution : $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=\frac{2\sin A}{(1-\cos A)\times(1+\cos A}=\frac{2\sin A}{1-\cos^2 A}=\frac{2\sin A}{\sin^2 A}=\frac{2\cancel{\sin A}}{\sin A\cancel{\sin A}}=2\text{cosec}\,A$

86. Prove that $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B=1$

Solution : $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B$

$$=\sin^2 A\cos^2 B+\sin^2 A\sin^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B$$
$$=\sin^2 A(\cos^2 B+\sin^2 B)+\cos^2 A(\sin^2 B+\cos^2 B)$$
$$=\sin^2 A(1)+\cos^2 A(1)=\sin^2 A+\cos^2 A=1$$

87. **Prove $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$**

Solution Take $a=1$ $b=\tan^2\theta$ $(\because a+b)^3=a^3+b^3+3ab(a+b))$

$$(1+\tan^2\theta)^3=1+\tan^6\theta+3(1)\tan^2\theta)(1+\tan^2\theta)$$
$$\sec^6\theta=1+\tan^6\theta+3\tan^2\theta\,.\,\sec^2\theta$$

88. **Prove $(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\theta)^2=1+(\sec\theta+\text{cosec}\theta)^2$**

Solution : $(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\,\theta)^2=\sin^2\theta+\sec^2\theta+2\sin\theta\,.\,\sec\theta+\cos^2\theta+\text{cosec}^2\theta+2\cos\theta\,.\,\text{cosec}\,\theta$

$$=1+(\sec\theta\,\text{cosec}\,\theta)^2$$

89. Find the sum of 0.40 + 0.43 + 0.46 + ... + 1.

Solution : $a = 0.40$ and $l = 1$, $d = 0.43 - 0.40 = 0.03$. $n = \left(\frac{l-a}{d}\right)+1 = \left(\frac{1-0.40}{0.03}\right)+1 = 21$

$$S_n = \frac{n}{2}[a+l] \qquad S_{21} = \frac{21}{2}[0.40+1] = 14.7$$

---

90. Find the sum of first 15 terms of the A.P. $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, ...$

Solution : $a = 8$, $d = 7\frac{1}{4} - 8 = -\frac{3}{4}$, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{15} = \frac{15}{2}\left[2\times8+(15-1)(-\frac{3}{4})\right] \qquad S_{15} = \frac{15}{2}\left[16-\frac{21}{2}\right] = \frac{165}{4}$$

---

91. **In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?**

Solution : $a = 20, d = 2, n = 30$ $t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

$t_n = a + (n-1)\,d$ ∴ The no. of seats in 30th row = 78

---

92. **Find the sum of all odd positive integers less than 450.**

Solution : $1+3+5+7+.........+449 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{449+1}{2}\right]^2 = \left[\frac{450}{2}\right]^2 = [225]^2 = 50,625$

---

93. **In a G.P. 729, 243, 81,... find t_7.**

Solution : 729, 243, 21, ... $a = 729$, $r = {}^{8}\!/_{243} = {}^{1}\!/_{3}$

$\therefore t_n = a \,.\, r^{n-1}$

$$\Rightarrow t_7 = a\,.\,r^6 = 729\times\left({}^{1}\!/_{3}\right)^6 = 729\times\left({}^{1}\!/_{729}\right) = 1$$

---

94. **Find x so that $x + 6, x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.**

Solution : Given $x + 6, x + 12, x + 15$ are consecutive terms of a G.P.

a, b, c are in G.P. $\Rightarrow b^2 = ac$

$\Rightarrow (x+12)^2 = (x+15)(x+6) \Rightarrow x^2+24x+144 = x^2+21x+90 \Rightarrow 3x = -54 \Rightarrow x = -18$

---

95. **Find the 10th term of a G.P. whose 8th term is 768 and the common ratio is 2.**

Solution : $t_8 = 768$, $r = 2$

$\Rightarrow a\,.\,r^7 = 768 \Rightarrow a\times 2^7 = 768 \Rightarrow a\times128 = 768 \Rightarrow a = 6$

$\therefore t_{10} = a\,.\,r^9 = 6\times2^9 = 6\times512 = 3072$

---

96. **If a, b, c are in A.P. then show that $3a, 3b, 3c$ are in G.P.**

Solution : Given a, b, c are in A.P. $\Rightarrow 2b = a+c$ .....(1)

To Prove : $3^a, 3^b, 3^c$ are in G.P. i.e. TP : $(3^b)^2 = 3^a\,.\,3^c$

LHS : $(3^b)^2 = 3^{2b} = 3^{a+c} = 3^a\,.\,3^c$ = RHS (from (1))

$\therefore 3^a, 3^b, 3^c$ are in G.P.

---

97. Find the sum of $2+4+6+....+80$

Solution : $2+4+6+.....+80 = 2\,(1+2+3+....+40) = 2\times\frac{40\times(40+1)}{2} = 1640$

---

98. Find the sum of $1+3+5+....+55$

Solution : $1+3+5+....+55 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{(55+1)}{2}\right]^2 = \left[\frac{56}{2}\right]^2 = (28)^2 = 784.$

99. **In a box there are 20 non-defective and some defective bulbs. If the probability that a bul selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.**

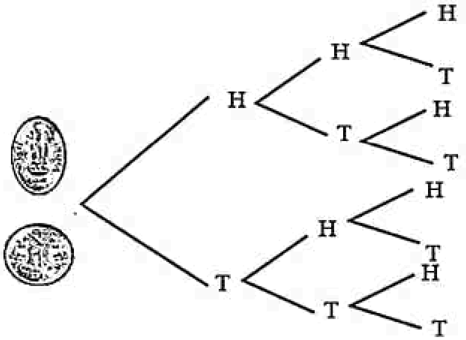
Solution : Let x be the number of defective bulbs. $\therefore n(S) = x + 20$

Let A defective balls $\therefore n(A) = x$ $P(A) = \frac{x}{x+20}$

Given $\frac{x}{x+20} = \frac{3}{8} \Rightarrow 8x = 3x + 60 \Rightarrow 5x = 60 \Rightarrow x = 12$

100. **Write the sample space for tossing three coins using tree diagram**

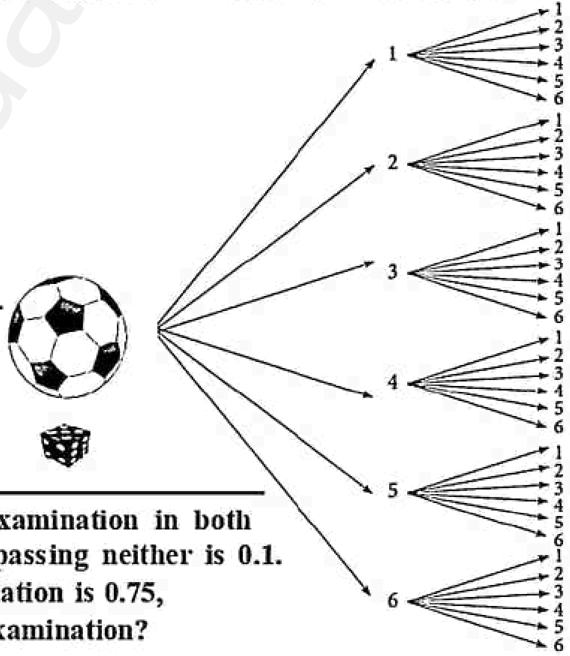
Solution :



Sample space = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

101. **Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).**

Solution : $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$



102. Express the sample space for rolling two dice using tree diagram.

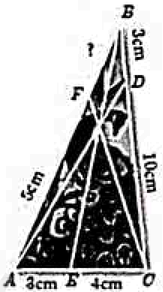
103. **The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?**

Solution : $P(E \cap T) = 0.5$; $P(\overline{E} \cap \overline{T}) = 0.1$ & $P(E) = 0.75 \Rightarrow P(E \cup T) = 1 - 01 = 0.9$

$P(E \cup T) = P(E) + P(T) - P(E \cap T) \Rightarrow 0.9 = 0.75 + P(T) - 0.5$

$P(T) = 0.9 - 0.25 = 0.65 = \frac{65}{100} = \frac{13}{20}$

104. **An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.**



Solution: By applying Ceva's theorem,

$$BD \times CE \times AF = DC \times EA \times FB$$

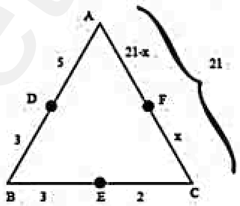
$$\Rightarrow \quad 3 \times 4 \times 5 = 10 \times 3 \times FB \, ,$$

$$\Rightarrow \quad 60 = 30 \times FB$$

$$\therefore \quad FB = 2 \text{ cm}$$

---

105. **Let ABC be a triangle and D,E,F are points on the respective sides AB, BC, AC Let AD : DB = 5 : 3 BE : EC = 3 : 2 and AC = 21 . Find the length of the line segment CF.**



Solution : By Ceva's theorem, $\dfrac{AD}{DB} \times \dfrac{BE}{EC} \times \dfrac{CF}{FA} = 1 \quad \Rightarrow \dfrac{5}{3} \times \dfrac{3}{2} \times \dfrac{x}{21-x} = 1$

$$\Rightarrow \frac{x}{21-x} = \frac{2}{5}$$

$$\Rightarrow 5x = 42 - 2x \quad \Rightarrow 7x = 42 \quad \therefore \; x = 6 \quad \therefore \; CF = 6$$

---

106. Ceva's Theorem : Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\dfrac{BD}{DC} \times \dfrac{CE}{EA} \times \dfrac{AF}{FB} = 1$

---

107. Menelaus Theorem : A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB of a triangle ABC to be collinear is that $\dfrac{BP}{PC} \times \dfrac{CQ}{QA} \times \dfrac{AR}{RB} = 1$

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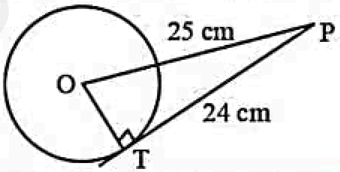
108. **The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?**

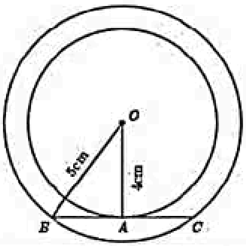


Solution: $\therefore \; OT = \sqrt{25^2 - 24^2} = \sqrt{625 - 576} = \sqrt{49} = 7 \text{ cm}$

$\therefore$ Radius = 7 cm

---

109. If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.



Solution : $OB^2 = OA^2 + AB^2$

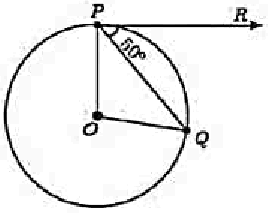
$5^2 = 4^2 + AB^2$ gives $AB^2 = 9$

Therefore AB = 3 cm

BC = 2AB hence BC = 2 × 3 = 6 cm

---

110. In Figure O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$



Solution : $\angle OPQ = 90^0 - 50^0 = 40^0$

OP = OQ (Radii of a circle are equal)

$\angle OPQ = \angle OQP = 40^0$ (ΔOPQ is isosceles)

$\angle POQ = 180^0 - \angle OPQ - \angle OQP$

$\angle POQ = 180^0 - 40^0 - 40^0 = 100^0$

111. **Find the sum of first six terms of the G.P. 5, 15, 45, …**

Solution : Given G.P is 5, 15, 45, ………… $a=5, r=3>1$ $S_n=a.\frac{r^n-1}{r-1}$

$$\therefore\ S_6=5.\frac{3^6-1}{3-1}\quad =\frac{5}{2}\times 728\quad =5\times 364\quad =1820$$

---

112. Find the sum $3+1+-+\ldots\infty$

Solution : Here $a=3,\ r=\frac{t_2}{t_1}=\frac{1}{3}$ Sum of infinite terms $=\frac{a}{1-r}=\frac{3}{1-\frac{1}{3}}=\frac{9}{2}$

---

113. **Find the sum to infinity of $21+14+\frac{28}{3}+\ldots.$**

Solution : $a=21,\ r=\frac{14}{21}=\frac{2}{3}<1$ $\therefore S_\infty=\frac{a}{1-r}=\frac{21}{1-\frac{2}{3}}=\frac{21}{1-\frac{2}{3}}=\frac{21}{\frac{1}{3}}\quad =63$

---

114. **If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.**

Solution : $.\ a=8,\ S_\infty=\frac{32}{3},\ r=?$

$$\Rightarrow\ \frac{a}{1-r}=\frac{32}{3}\quad\Rightarrow\ \frac{\overset{1}{\cancel{8}}}{1-r}=\frac{\overset{4}{\cancel{32}}}{3}\quad\Rightarrow\ 3=4-4r\quad\Rightarrow\quad 4r=1$$

$$\therefore\qquad r=\frac{1}{4}$$

---

115. Find the first term of G.P. in which $S_6=4095$ and $r=4$.

Solution : $S_n=\frac{a(r^n-1)}{r-1}=4095$ $r=4,\ \frac{a(4^6-1)}{4-1}=4095$ gives $a\times\frac{4095}{3}=4095$ $a=3.$

---

116. **Find the rational form of the number $0.\overline{123}$.**

Solution : Let $x\ =0.\overline{123}$ $x\ =0.123123123$

$\Rightarrow 1000x\ =123.123123$

$\Rightarrow 1000x\ =123+0.123123123$

$\Rightarrow 1000x\ =123+x$

$1000x-x=123\Rightarrow 999x=123\ \Rightarrow x=\frac{123}{999}$ $\therefore x=\frac{41}{333}$

---

117. Find the 8th term of the G.P. 9, 3, 1, …

Solution : First term $a=9$, common ratio $r=\frac{t_2}{t_1}=\frac{3}{9}=\frac{1}{3}\ \Rightarrow\ t_8=9\times\left(\frac{1}{3}\right)^{8-1}=9\times\left(\frac{1}{3}\right)^7=\frac{1}{243}$

---

118. **If $1^3+2^3+3^3+\ldots\ldots+k^3=44100$, then find $1+2+3+\ldots\ldots\ldots+k$.**

Solution : Given $1^3+2^3+3^3+\ldots\ldots+k^3=44100$

$$\Rightarrow\ \left(\frac{k(k+1)}{2}\right)^2=44100\quad\Rightarrow\ \frac{k(k+1)}{2}=210\ \Rightarrow\ 1+2+3+\ldots..+k=210$$

---

119. **Find the sum of the following series $3+6+9+\ldots\ldots+96$**

Solution : $3+6+9+\ldots\ldots+96$

$$=3\ (1+2+3+\ldots\ldots+32)=\ 3\left(\frac{32\times 33}{2}\right)\ =\ 3\times 16\times 33\ \ =\ 1584$$

---

120. **Find the sum of the following series $1+4+9+16+\ldots\ldots+225$**

Solution : $1+4+9+16+\ldots\ldots+225\ =1^2+2^2+3^2+\ldots\ldots+15^2\ =\ \frac{15\times 16\times 31}{6}\ =\ 1240$

$$\sum n^2\ =\ \frac{n(n+1)(2n+1)}{6}$$

---

121. **Find the sum of the following series $1+3+5+\ldots\ldots+71$** .

Solution : $1+3+5+\ldots..+71$ $(\because\ 1+3+5+\ldots\ldots+n \text{ terms}=n^2)$

$\therefore\ 1+3+5+\ldots..+71=(36)^2\ =1296$

122. If $A=\begin{pmatrix}1&3&-2\\5&-4&6\\-3&2&9\end{pmatrix}$ $B=\begin{pmatrix}1&8\\3&4\\9&6\end{pmatrix}$ find A+B.

Solution : A is of order 3×3 B is of order 3×2 It is not possible to add A and B because different orders.

123. **Given** $A=\begin{pmatrix}p&0\\0&2\end{pmatrix},B=\begin{pmatrix}0&-q\\1&0\end{pmatrix},C=\begin{pmatrix}2&-2\\2&2\end{pmatrix}$ **and if BA = C², find p and q.**

Solution : Given BA = C² $\Rightarrow \begin{pmatrix}0&-q\\1&0\end{pmatrix}\begin{pmatrix}p&0\\0&2\end{pmatrix}=\begin{pmatrix}2&-2\\2&2\end{pmatrix}\begin{pmatrix}2&-2\\2&2\end{pmatrix}\Rightarrow\begin{pmatrix}0&-2q\\p&0\end{pmatrix}=\begin{pmatrix}0&-8\\8&0\end{pmatrix}$

$\therefore$ p = 8, −2q = −8, q = 4

124. **If** $A=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$ **prove that $AA^T=I$**

Solution : $A.A^T=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta+\sin^2\theta&-\cos\theta\sin\theta+\sin\theta\cos\theta\\-\sin\theta\cos\theta+\cos\theta\sin\theta&\sin^2\theta+\cos^2\theta\end{pmatrix}$

$=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$ Hence proved.

125. **Verify that A² = I when** $A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}$

Solution : $A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}$ $A^2=A.A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}\begin{pmatrix}5&-4\\6&-5\end{pmatrix}=\begin{pmatrix}25-24&-20+20\\30-30&-24+25\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$

126. **If** $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2$ **find x.**

Solution :

$$\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2 \Rightarrow \begin{pmatrix}\cos^2\theta+x\sin\theta & 0\\ 0 & \cos^2\theta+x\sin\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow \cos^2\theta + x\sin\theta = 1 \Rightarrow x\sin\theta = 1-\cos^2\theta \Rightarrow x\sin\theta=\sin^2\theta \Rightarrow x=\sin\theta$$

---

127. **Let** $A=\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ **Show that** $(A-B)^T=A^T-B^T$

Solution : $A=\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\quad (A-B)^T=\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}^T=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix}$ ....(1)

$$A^T-B^T=\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}-\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix} \text{.......(2)}$$

From (1) & (2) $\quad (A-B)^T=A^T-B^T$

---

128. **Show that the matrices** $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ **satisfy commutative property** $AB=BA$

Solution :

$$AB=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}(1\ \ 2)\begin{pmatrix}1\\ -3\end{pmatrix} & (1\ \ 2)\begin{pmatrix}-2\\ 1\end{pmatrix}\\ (3\ \ 1)\begin{pmatrix}1\\ -3\end{pmatrix} & (3\ \ 1)\begin{pmatrix}-2\\ 1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-6 & -2+2\\ 3-3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}=\begin{pmatrix}(1\ \ -2)\begin{pmatrix}1\\ 3\end{pmatrix} & (1\ \ -2)\begin{pmatrix}2\\ 1\end{pmatrix}\\ (-3\ \ 1)\begin{pmatrix}1\\ 3\end{pmatrix} & (-3\ \ 1)\begin{pmatrix}2\\ 1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-6 & 2-2\\ -3+3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$\therefore AB=BA$

$\therefore$ Commutative property is true.

---

129. **If** $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$ **then show that** $A^2+B^2=I$.

Solution : $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$

$$A^2=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta & 0\\ 0 & \cos^2\theta\end{pmatrix}$$

$$B^2=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}=\begin{pmatrix}\sin^2\theta & 0\\ 0 & \sin^2\theta\end{pmatrix}$$

$$\therefore A^2+B^2=\begin{pmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad A^2+B^2=I.$$

---

130. **If A is of order p × q and B is of order q × r what is the order of AB and BA?**

Solution : A is of order p × q $\quad$ B is of order q × r

$\therefore$ Order of AB = p × r $\quad$ Order of BA is not defined

131. **Construct a 3 × 3 matrix whose elements are given by** $a_{ij} = \dfrac{(i+j)^3}{3}$ $a_{11} = 8/3$, $a_{12} = 27/3 = 9$, $a_{13} = 64/3$ $a_{21} = 27/3 = 9$, $a_{22} = 64/3$, $a_{23} = 125/3$ $a_{31} = 64/3$, $a_{32} = 125/3$, $a_{33} = 216/3 = 72$

$$\therefore\ A = \begin{pmatrix} 8/3 & 9 & 64/3 \\ 9 & 64/3 & 125/3 \\ 64/3 & 125/3 & 72 \end{pmatrix}$$

---

132. **If** $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ **then find the transpose of A.** ***Solution :*** $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ $\therefore\ A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

---

133. **If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?**

Solution : Given, a matrix has 18 elements. The possible orders 18 × 1, 1 × 18, 9 × 2, 2 × 9, 6 × 3, 3 × 6 The matrix has 6 elements. The order are 1 × 6, 6 × 1, 3 × 2, 2 × 3

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134. **If** $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ **then find the transpose of-A.**

Solution : $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ $-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$ $\therefore$ Transpose of $-A = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

---

135. **Construct a 3 × 3 matrix whose elements are given by** $a_{ij} = |i - 2j|$

Solution : Given $a_{ij} = |i - 2j|$, 3 × 3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = |1-2| = |-1| = 1$ $a_{12} = |1-4| = |-3| = 3$

$a_{13} = |1-6| = |-5| = 5$ $a_{21} = |2-2| = 0$

$a_{22} = |2-4| = |-2| = 2$ $a_{23} = |2-6| = |-4| = 4$ $\therefore\ A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$a_{31} = |3-2| = |1| = 1$ $a_{32} = |3-4| = |-1| = 1$

$a_{33} = |3-6| = |-3| = 3$

---

136. **If** $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ **then verify** $(A^T)^T = A$

Solution :

$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & 5/2 & 1 \end{pmatrix}$ $(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & 5/2 \\ 8 & 3 & 1 \end{pmatrix} = A$

---

137. **Find the values of x, y and z from the following equations** $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

Solution : $\begin{pmatrix} 12 & 3 \\ x & 3/2 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix} \Rightarrow x = 3,\ y = 12,\ z = 3$

138. **Find the values of x, y and z from the following equations** $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution :

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix} \begin{array}{l} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{array} \begin{array}{r|l|r} x+y+z=0 & \quad x+z=5 & \quad y+z=7 \\ 5+y=9 & \Rightarrow x+3=5 & \Rightarrow 4+z=7 \\ y=4 & \Rightarrow \quad x=2 & \Rightarrow \quad z=3 \end{array}$$

---

139. If $A=\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find $2A+B$

Solution : $$2A+B=2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}+\begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}=\begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix}+\begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}=\begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

---

140. **Find the values of x, y and z from the following equations** $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix}=\begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

Solution : $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix}=\begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$ $\Rightarrow x+y=6,\quad xy=8,\qquad 5+z=5 \quad \Rightarrow z=0$

$x=2$ (or) 4, $\quad y=4$ (or) 2

---

141. If $A=\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$ $B=\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ then Find $4A-3B$

Solution :

$$4A-3B=4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}-3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}=\begin{pmatrix} 20 & 6 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix}+\begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}=\begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

---

142. If $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find $A+B$

Solution :

$$A+B=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}+\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}=\begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix}=\begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

---

143. If $A=\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}$, $B=\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}$, $C=\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute $3A+2B-C$

Solution :

$$3A+2B-C=3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}+2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}-\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$=\begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix}+\begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix}+\begin{pmatrix} -5 & -3 & 0 \\ 1 & 7 & -2 \\ -1 & -4 & -3 \end{pmatrix}=\begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

144. **If** $A=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}, B=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$ **then verify that A + B = B + A**

Solution :

$$\therefore\ A+B=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix} \qquad B+A=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix}$$

$$\therefore\ A+B=B+A$$

145. Find the value of a, b, c, d, x, y from the following matrix equation. $\begin{pmatrix}d & 8\\ 3b & a\end{pmatrix}+\begin{pmatrix}3 & a\\ -2 & -4\end{pmatrix}=\begin{pmatrix}2 & 2a\\ b & 4c\end{pmatrix}+\begin{pmatrix}0 & 1\\ -5 & 0\end{pmatrix}$

Solution : $\begin{pmatrix}d+3 & 8+a\\ 3b-2 & a-4\end{pmatrix}=\begin{pmatrix}2 & 2a+1\\ b-5 & 4c\end{pmatrix}$ $\quad d+3=2 \Rightarrow d=2-3 \quad \Rightarrow \quad d=-1$

$8+a=2a+1 \Rightarrow 8-1=2a-a \Rightarrow a=7$

$3b-2=b-5 \Rightarrow 3b-b=-5+2 \Rightarrow 2b=-3 \quad \Rightarrow \quad b=\dfrac{-3}{2}$

Substituting $a=7$ in $a-4=4c \Rightarrow 7-4=4c \Rightarrow 3=4c \Rightarrow c=\dfrac{3}{4}$

146. **If** $A=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}, B=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$ **then verify that A + (−A) = (−A) + A = O**

Solution :

$$A+(-A)=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}=O$$

$$(-A)+A=\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}=O$$

147. If $A=\begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix}, B=\begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix}, C=\begin{pmatrix}5 & 3 & 0\\ -1 & -7 & 2\\ 1 & 4 & 3\end{pmatrix}$ compute $\frac{1}{2}A-\frac{3}{2}B$

Solution :

$$\frac{1}{2}A-\frac{3}{2}B=\frac{1}{2}(A-3B)=\frac{1}{2}\left(\begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix}-3\begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix}\right)=\begin{pmatrix}-\frac{23}{2} & 13 & \frac{15}{2}\\ -\frac{3}{2} & -14 & \frac{9}{2}\\ 4 & 2 & -\frac{9}{2}\end{pmatrix}$$

148. **If** $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ **find the value of B − 5A**

Solution :

$$B-5A=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-\begin{pmatrix}0 & 20 & 45\\ 40 & 15 & 35\end{pmatrix}=\begin{pmatrix}7 & -17 & -37\\ -39 & -11 & -26\end{pmatrix}$$

149. **If** $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ **find the value of 3A − 9B**

Solution :

$$3A-9B=\begin{pmatrix}0 & 12 & 27\\ 24 & 9 & 21\end{pmatrix}-\begin{pmatrix}63 & 27 & 72\\ 9 & 36 & 81\end{pmatrix}=\begin{pmatrix}-63 & -65 & -45\\ 15 & -27 & -60\end{pmatrix}$$

150. Find the non-zero values of x satisfying the matrix equation $x\begin{pmatrix}2x & 2\\ 3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\ 4 & 4x\end{pmatrix}=2\begin{pmatrix}x^2+8 & 24\\ 10 & 6x\end{pmatrix}$

Solution : $\Rightarrow \begin{pmatrix}2x^2 & 2x\\ 3x & x^2\end{pmatrix}+\begin{pmatrix}16 & 10x\\ 8 & 8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix} \Rightarrow \begin{pmatrix}2x^2+16 & 12x\\ 3x+8 & x^2+8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix}$

$\therefore\ 12x = 48 \Rightarrow x = 4$

---

151. Construct a 3 × 3 matrix whose elements are $a_{ij} = i^2j^2$

Solution : The general 3 × 3 matrix is given by $A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$ $\quad a_{ij}=i^2j^2$

$a_{11} = 1^2\times 1^2 = 1\times 1 = 1$; $a_{12} = 1^2\times 2^2 = 1\times 4 = 4$; $a_{13} = 1^2\times 3^2 = 1\times 9 = 9$

$a_{21} = 2^2\times 1^2 = 4\times 1 = 4$; $a_{22} = 2^2\times 2^2 = 4\times 4 = 16$; $a_{23} = 2^2\times 3^2 = 4\times 9 = 36$

$a_{31} = 3^2\times 1^2 = 9\times 1 = 9$; $a_{32} = 3^2\times 2^2 = 9\times 4 = 36$; $a_{33} = 3^2\times 3^2 = 9\times 9 = 81$

$A=\begin{pmatrix}1 & 4 & 9\\ 4 & 16 & 36\\ 9 & 36 & 81\end{pmatrix}$

---

152. **A has 'a' rows and 'a + 3' columns. B has 'b' rows and '17–b' columns, and if both products AB and BA exist, find a, b?**

Solution : Given Order of A is $a\times(a+3)$ Order of B is $b\times(17-b)$

$\Rightarrow a+3=b \Rightarrow a-b=-3$........ (1) Solving (1) & (2) $2a=14$ $a=7$

$\Rightarrow 17-b=a \Rightarrow a+b=17$ .... (2) Sub $a=7$ in (1) $7-b=-3$ $b=10$

---

153. **In the matrix** $A=\begin{pmatrix}8 & 9 & 4 & 3\\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5\\ 1 & 4 & 3 & 0\\ 6 & 8 & -11 & 1\end{pmatrix}$, **write**

(i) The number of elements

(ii) The order of the matrix

(iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$

Solution : i) A has 4 rows and 4 columns Number of elements = 16

ii) Order of the matrix = 4 × 4

iii) $a_{22}=\sqrt{7}$, $a_{23}=\sqrt{3}/2$ $a_{24}=5$ $a_{34}=0$, $a_{43}=-11$, $a_{44}=1$

---

154. **If cotθ + tanθ = x and secθ – cosθ = y, then prove that** $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

Solution : $\therefore\ x^2y = \sec^3\theta$ and $xy^2 = \tan^3\theta \Rightarrow (x^2y)^{2/3} - (xy^2)^{2/3} = (\sec^3\theta)^{2/3} - (\tan^3\theta)^{2/3} = \sec^2\theta - \tan^2\theta = 1$

---

155. **Prove** $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

Solution : $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}\times\dfrac{1+\sin\theta}{1+\sin\theta}} = \sqrt{\dfrac{(1+\sin\theta)^2}{\cos^2\theta}} = \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

---

156. **Prove** $\dfrac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

Solution :

$$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cancel{\cos\theta}\ \cos\theta} = \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta$$

---

157. Prove that **cot θ + tan θ = sec θ cosec θ**

Solution: $\cot\theta + \tan\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos\theta\ \cos\theta + \sin\theta\ \sin\theta}{\sin\theta.\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta} = \dfrac{1}{\sin\theta.\cos\theta} = \sec\theta.\text{cosec}\,\theta$

---

158. Prove that $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

Solution :

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} = \frac{\sin^2 A\cos^2 B - \sin^2 B\cos^2 A}{\cos^2 A\cos^2 B} = \frac{\sin^2 A\left(1-\sin^2 B\right) - \sin^2 B\left(1-\sin^2 A\right)}{\cos^2 A\cos^2 B}$$
$$= \frac{\sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B}{\cos^2 A\cos^2 B} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$$

---

159. Prove that $\left(\dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$

Solution :

$$\text{LHS} = \left(\frac{(\cos A - \sin A)\,(\cos^2 A + \sin^2 A + \cos A\sin A)}{\cos A - \sin A}\right) - \left(\frac{(\cos A + \sin A)\,(\cos^2 A + \sin^2 A - \cos A\sin A)}{\cos A + \sin A}\right)$$
$$= (1 + \cos A\sin A) - (1 - \cos A\sin A) = 2\cos A\sin A$$

---

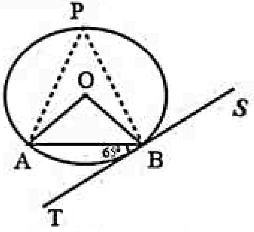
160. **A tangent ST to a circle touches it at B. AB is a chord such that ∠ABT = 65°. Find ∠AOB, where "O" is the centre of the circle.**

Solution:

(angles in altemate segment).

$\angle TBA = 65^0 \Rightarrow \angle APB = 65^0$

$\therefore\ \angle AOB = 2\angle APB = 2(65^0) = 130^0$



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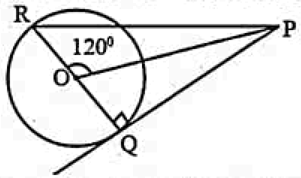
161. **PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120°. Find ∠OPQ .**

Solution: Given $\angle POR = 120^0 \Rightarrow \angle POQ = 60^0$ (linear pair)

$\angle OQP = 90^0$ (Radius ⊥ tangent)

$\therefore\ \angle OPQ = 90^0 - 60^0 = 30^0$



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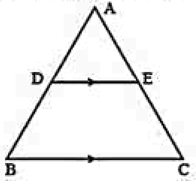
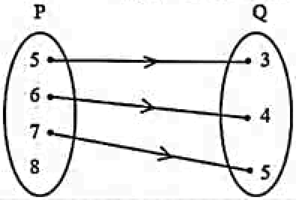
162. **O is any point inside a triangle ABC. The bisector of ∠AOB , ∠BOC and ∠COA meet the sides AB, BC and CA in point D, E and F respectively. Show that AD × BE × CF = DB × EC × FC**

Solution : By using Ceva's Theorem, $\dfrac{AD}{DB}\times\dfrac{BE}{EC}\times\dfrac{AF}{FC} = 1$

$\Rightarrow AD \times BE \times AF = DB \times EC \times FC$ Hence proved.



163. The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution : (i) Set builder form of $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain of $R = \{5, 6, 7\}$ and range of $R = \{3, 4, 5\}$

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164. Let A = {1, 2, 3, 4 ....., 45} and R be the relation defined as "is square of" on A. Write R as a subset of A × A. Also, find the domain and range of R.

Solution : A = {1, 2, 3, 4, ... 45} R : "is square of" R = {1, 4, 9, 16, 25, 36} Clearly R is a subset of A.

∴ Domain = {1, 2, 3, 4, 5, 6} ∴ Range = {1, 4, 9, 16, 25, 36}

---

165. A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution : Given $R = \{(x, y) / y = x + 3,$

$x \in \{0, 1, 2, 3, 4, 5\}\}$

$x = 0 \Rightarrow y = 3$ $x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 5$ $x = 3 \Rightarrow y = 6$

$x = 4 \Rightarrow y = 7$ $x = 5 \Rightarrow y = 8$

∴ R = {(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)}

∴ Domain : {0, 1, 2, 3,4, 5}

Range : {3, 4, 5, 6, 7, 8}

---

166. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid y = x + 3, x, y$ are natural numbers $< 10\}$

Solution : x, y are natural numbers < 10

$y = x + 3$

$x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 5$

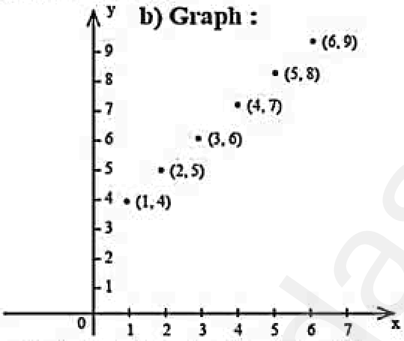
$x = 3 \Rightarrow y = 6$

$x = 4 \Rightarrow y = 7$

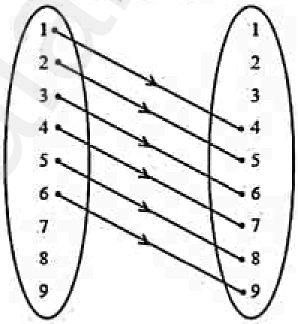
$x = 5 \Rightarrow y = 8$

$x = 6 \Rightarrow y = 9$

b) Graph :



a) Arrow Diagram :



c) a set in roster : {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

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167. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

Solution :

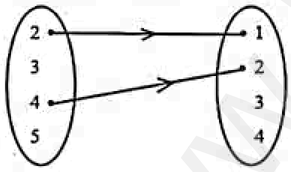
$y = 1 \Rightarrow x = 2$

$y = 2 \Rightarrow x = 4$

$y = 3 \Rightarrow x = 6$

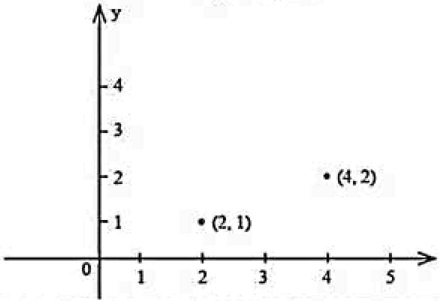
$y = 4 \Rightarrow x = 8$

a) Arrow diagram :



c) a set in roster : {(2, 1), (4, 2)}

b) Graph :



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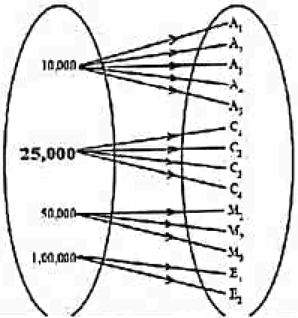
168. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants ; C_1, C_2, C_3, C_4 were Clerks ; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Solution : **a) Ordered Pair :** {(₹10,000, A_1), (₹10,000, A_2), (₹10,000, A_3), (₹10,000, A_4), (₹10,000, A_5), (₹25,000, C_1), (₹25,000, C_2), (₹25,000, C_3), (₹25,000, C_4), (₹50,000, M_1), (₹50,000, M_2), (₹50,000, M_3), (₹1,00,000, E_1), (₹1,00,000, E_2)}

b) arrow diagram.



169. **Find the values of x, y, z if** $\begin{pmatrix} x & y-z & z+3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

Solution : $\begin{pmatrix} x & y-z & z+3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

$\Rightarrow x+y=4$ | $y-z+4=8$ | $z+6=16$

$\Rightarrow x+14=4$ | $y-z=4$ | $z=10$

$\Rightarrow x=-10$ | $y-10=4$

$\therefore y=14$

$\therefore x=-10, y=14, z=10$

170. **Find x and y if** $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Solution :

Given $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ $4x-2y=4$ .... (1)

$-3x+3y=6$ .... (2)

$\Rightarrow 2x-y=2$ .... (1)

$\Rightarrow -x+y=2$ ...... (2)

(1) $\Rightarrow 2x-y=2$

(2) $\Rightarrow -x+y=2$

Adding, $x=4$

Sub $x=4$ in (2)

$-4+y=2 \Rightarrow y=6$

$\therefore x=4, y=6$

171. **Find the values of x, y, z if** $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

Solution : $\Rightarrow x-3=1$ | $3x-z=0$ | $x+y+7=1$

$\therefore x=4$ | $12-z=0 \Rightarrow z=12$ | $\Rightarrow x+y=-6 \Rightarrow 4+y=-6 \Rightarrow y=-10$

172. If a matrix has 16 elements, what are the possible orders it can have?

Solution : The matrix has 16 elements . Hence possible orders are 1 × 16, 16 × 1, 4 × 4, 2 × 8, 8 × 2.

173. If $A=\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B=\begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ find AB.

Solution :

$$AB=\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix} = \begin{pmatrix} (1\ 2\ 0)\begin{pmatrix}8\\2\\5\end{pmatrix} & (1\ 2\ 0)\begin{pmatrix}3\\4\\3\end{pmatrix} & (1\ 2\ 0)\begin{pmatrix}1\\1\\1\end{pmatrix} \\ (3\ 1\ 5)\begin{pmatrix}8\\2\\5\end{pmatrix} & (3\ 1\ 5)\begin{pmatrix}3\\4\\3\end{pmatrix} & (3\ 1\ 5)\begin{pmatrix}1\\1\\1\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix} = \begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$$

174. If $A=\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$ Show that A and B satisfy commutative property with respect to matrix multiplication.

Solution :

$$AB=\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \quad BA=\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

A and B satisfy commutative property

175. Solve $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}4\\ 5\end{pmatrix}$

Solution :

By matrix multiplication $\begin{pmatrix}2x+y\\ x+2y\end{pmatrix}=\begin{pmatrix}4\\ 5\end{pmatrix}$ $2x+y=4$ .... (1)

$x+2y=5$ .... (2)

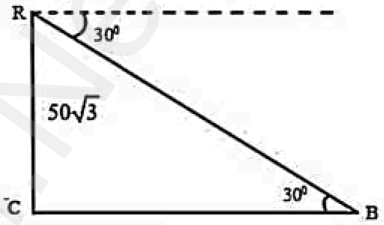
$$(1)-2\times(2) \text{ gives } \begin{array}{rl} 2x+y= & 4 \\ \underline{2x+4y= \;\; 10} & (-) \\ -3y=-6 & \text{gives } y=2 \end{array}$$

Substituting $y=2$ in (1), $2x+2=4$ gives $x=1$ Therefore, $x=1, y=2$.

---

176. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

Solution:



$$\tan 30^0=\frac{RC}{CB} \Rightarrow \frac{1}{\sqrt{3}}=\frac{50\sqrt{3}}{CB}$$

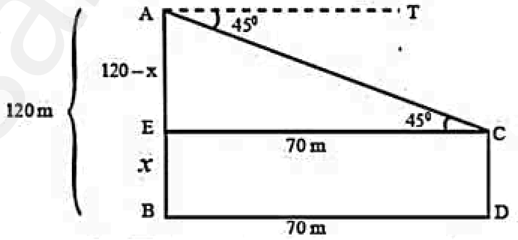
$$\Rightarrow CB = 50\sqrt{3}\sqrt{3} \quad \Rightarrow CB = 150m$$

$\therefore$ Dist. of the car from the rock = 150m

---

177. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.

Solution:



$$\tan 45^0=\frac{AE}{EC} \Rightarrow 1=\frac{120-x}{70}$$

$$\Rightarrow 70=120-x$$

$$\Rightarrow x=120-70$$

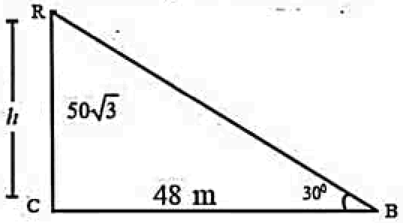
$$\Rightarrow x=50$$

$\therefore$ Height of 1st building = 50 m

---

178. A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is30°. Find the height of the tower.

Solution :



$$\tan 30^0=\frac{h}{48}$$

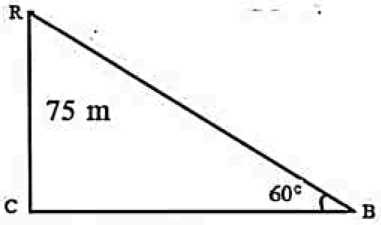
$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{48} \Rightarrow h=16\sqrt{3}$$

The height of the tower is $16\sqrt{3}$ m

---

179. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution :



$$\sin 60^0=\frac{75}{RB} \Rightarrow \frac{\sqrt{3}}{2}=\frac{75}{RB}$$

$$\Rightarrow RB=\frac{150}{\sqrt{3}}=50\sqrt{3}$$

length of the string is $50\sqrt{3}$ m

---

180. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Solution : base area = πr^2 = 1386 sq. m

T.S.A. = $3\pi r^2$ sq.m = $3\times 1386 = 4158$ m².

181. Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

Solution : $f(x) = 2x + 1, g(x) = x^2 - 2$

$$f \circ g = (2x+1)(x^2-2) = 2(x^2-2)+1 = 2x^2-3$$

$$g \circ f = (x^2-2)(2x+1) = (2x+1)^2 - 2 = 4x^2+4x+1-2 = 4x^2+4x-1 \quad \therefore f \circ g \neq g \circ f$$

---

182. Represent the function $f(x) = \sqrt{2x^2-5x+3}$ as a composition of two functions.

Solution : $f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

$$f(x) = \sqrt{2x^2-5x+3} = \sqrt{f_2(x)} = f_1 f_2(x)$$

---

183. Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Solution : $f \circ f(k) = (2k-1)(2k-1) = 2(2k-1) - 1 = 4k - 3$. But, $\because f \circ f(k) = 5$

Therefore $4k - 3 = 5 \Rightarrow 4k = 5+3 \Rightarrow 4k = 8 \Rightarrow k = 2$.

---

184. If $f(x) = x - 6, g(x) = x^2$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = x - 6, g(x) = x^2$

$$(f \circ g) = (x-6)(x^2) = x^2 - 6$$

$$(g \circ f) = (x^2)(x-6) = (x-6)^2 = x^2 - 12x + 36$$

$\therefore f \circ g \neq g \circ f$ $\quad$ $, g(x) = 1 + x$

---

185. If $f(x) = 4x^2 - 1, g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = 4x^2 - 1, g(x) = 1 + x$

$$(f \circ g) = (4x^2-1)(1+x) = 4(1+x^2) - 1 = 4(1+x^2+2x) - 1 = 4x^2 + 8x + 3$$

$$(g \circ f) = (1+x)(4x^2-1) = 1 + 4x^2 - 1 = 4x^2 \quad \therefore f \circ g \neq g \circ f$$

---

186. If $f(x) = 4x^2 - 1, g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$$(f \circ g) = \left(\frac{2}{x}\right)(2x^2-1) = \frac{2}{2x^2-1}$$

$$(g \circ f) = \left(\frac{2}{x}\right)(2x^2-1) = 2\left(\frac{2}{x}\right)^2 - 1 = \frac{8}{x^2} - 1 \quad \therefore f \circ g \neq g \circ f$$

---

187. If $f(x) = \frac{x+6}{3}, g(x) = 3 - x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution :

$$f(x) = \frac{x+6}{3}, g(x) = 3 - x$$

$$(f \circ g) = \left(\frac{x+6}{3}\right)(3-x) = \frac{(3-x)+6}{3} = \frac{9-x}{3}$$

$$(g \circ f)(x) = \left(\frac{x+6}{3}\right)(3-x) = 3 - \frac{x+6}{3} = \frac{9-x-3}{3} = \frac{6-x}{3} \quad \therefore f \circ g \neq g \circ f$$

188 **If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a, if $g \circ f(a) = 1$.**

Solution : $f(x) = x^2 - 1, g(x) = x - 2$

Given $g \circ f(a) = 1 \Rightarrow (x-2)(x^2-1) = 1 \Rightarrow (x-2)(a^2-1) = 1$

$\Rightarrow a^2 - 1 - 2 = 1 \Rightarrow a^2 - 3 = 1 \Rightarrow a^2 = 4 \therefore a = \pm 2$

189 **Find k, if $f(k) = 2k - 1$ and $f \circ f(k) = 5$.**

Solution : $f(k) = 2k - 1 \Rightarrow f \circ f(k) = 5 \Rightarrow (2k-1)(2k-1) = 5 \Rightarrow 2(2k-1) - 1 = 5$

$\Rightarrow 4k - 2 = 6 \Rightarrow 4k = 8 \quad \therefore k = 2$

190 **If $f(x) = 2x - k$, $g(x) = 4x + 5$ Find k, $f \circ g = g \circ f$**

$\Rightarrow (f \circ g) = (g \circ f) \Rightarrow (2x-k)(4x+5) = (4x+5)(2x-k)$

$\Rightarrow 2(4x+5) - k = 4(2x-k) + 5$

$\Rightarrow 8x + 10 - k = 8x - 4k + 5 \Rightarrow 10 - k = -4k + 5 \Rightarrow -k + 4k = 5 - 10 \Rightarrow 3k = -5$

191 **Let A, B, C $\subseteq$ N and a function $f : A \to B$ be defined by $f(x) = 2x + 1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.**

Solution : $f : A \to B$, $g : B \to C$ where $A, B, C \subseteq N$. $f(x) = 2x + 1, g(x) = x^2$

Range of $f \circ g$ $= (2x+1)(x^2) = 2x^2 + 1 \quad \therefore$ Range of $f \circ g = \{y / y = 2x^2 + 1, x \in N\}$.

Range of $g \circ f$ $= (x^2)(2x+1) = (2x+1)^2 \quad \therefore$ Range of $g \circ f = \{y/y = (2x+1)^2, x \in N\}$.

192 **Let $f(x) = x^2 - 1$. Find $f \circ f$,**

Solution :

Given $f(x) = x^2 - 1$

a) $f \circ f = ?$

$(f \circ f) = (x^2-1)(x^2-1) = (x^2-1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$

193 **Let $f(x) = x^2 - 1$. Find $f \circ f \circ f$**

Solution :

$(f \circ f \circ f) = (x^2-1)(x^2-1)(x^2-1) = (x^4 - 2x^2)(x^2-1) = (x^2-1)^4 - 2(x^2-1)^2 = (x^4 - 2x^2)^2 - 1$

194 **If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one ?**

Solution : Let A be the domain. B be the co-domain.

For every element $\in A$, 192. there is a unique image in B. Since f is an odd function $\therefore f$ is $1 - 1$.

But $g(x)$ is an even function.

$\therefore$ Two elements of domain will have the since image in co-domain. $\therefore g$ is not $1 - 1$.

195 **Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from Z into Z. Find $f(x)$.**

Solution : Given $f = \{(-1, 3), (0, -1), (2, -9)\}$ is a linear function from Z into Z.

Let $y = ax + b$ When $x = -1, y = 3 \Rightarrow 3 = -a + b$ —(1)

When $x = 0$, $y = -1 \Rightarrow -1 = 0 + b \quad \therefore b = -1 \quad \therefore (1) \Rightarrow 3 = -a - 1 \quad \Rightarrow a = -4$

$\therefore a = -4, b = -1$

$\therefore y = -4x - 1$ is the required linear function.

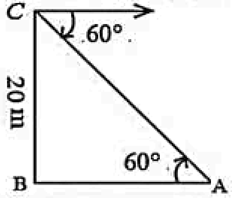
196 **In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit C(t) = 3t is linear.**

Solution : Given C(t) = 3t To Prove : C(t) is linear.

$C(at_1) = 3at_1$, $C(bt_2) = 3bt^2$ Adding,

$C(at_1) + C(bt_2) = 3at_1 + 3bt_2 = 3(at_1 + bt_2) \quad \therefore$ C(t) = 3t is a linear function.

197. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60° . Find the distance between the foot of the tower and the ball. ($\sqrt{3}=1.732$)

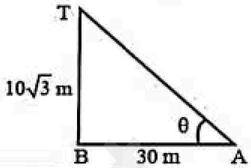


Solution : $\tan 60^0 = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{20}{AB} \Rightarrow AB = \frac{20}{\sqrt{3}} \Rightarrow AB = \frac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{20\times1.732}{3}$

$= 11.54\text{m}$

distance between the foot of the tower and the ball is 11.54 m.

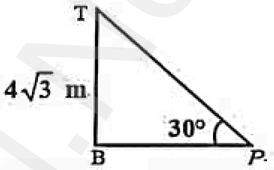
198. **Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.**



Solution :

$\tan\theta = \frac{10\sqrt{3}}{30} \Rightarrow \tan\theta = \frac{\sqrt{3}}{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \quad \therefore \theta = 30^0$

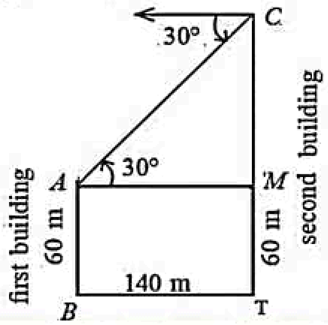
199. **A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.**



Solution : $\tan 30^0 = \frac{4\sqrt{3}}{PB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{PB} \Rightarrow PB = 12\text{ m}$

$\therefore$ Width of the road $= 2\ PB = 2\,(12) = 24$ m

200. The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30° . If the height of the first building is 60 m, find the height of the second building. ($\sqrt{3}=1.732$)



Solution :

$$\tan 30^0 = \frac{CM}{140}$$

$$\frac{1}{\sqrt{3}} = \frac{CM}{140}$$

$$CM = \frac{140}{\sqrt{3}} = \frac{140\sqrt{3}}{3} = \frac{140\times1.732}{3}$$

$$CM = 80.78$$

height of the second building $= 60 + 80.78 = 140.78$ m

201. **Prove** $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = (\sec\theta + \cancel{\tan\theta}) + (\sec\theta - \cancel{\tan\theta}) = \sec\theta + \sec\theta = 2\sec\theta$

202. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \text{cosec}\,\theta + \cot\theta$

Solution : $\sqrt{\frac{1+\cos\theta}{1\quad\cos\theta}} = \sqrt{\frac{(1+\cos\theta)\times(1+\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta} = \text{cosec}\,\theta + \cot\theta$

203. **Prove** $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution : $\tan^2\theta\,(\tan^2\theta + 1) = \sec^2\theta\,.\,(\sec^2\theta - 1) \Rightarrow \tan^2\theta\sec^2\theta = \tan^2\theta\sec^2\theta$

204. **Prove that** $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Solution : Take $1+\sin\theta = a \quad \cos\theta = b \quad \therefore \frac{(a-b)^2}{(a+b)^2} = \frac{a^2+b^2-2ab}{a^2+b^2+2ab} = \frac{2\cancel{(1+\sin\theta)}\,[1-\cos\theta]}{2\cancel{(1+\sin\theta)}\,[1+\cos\theta]} = \frac{1-\cos\theta}{1+\cos\theta}$

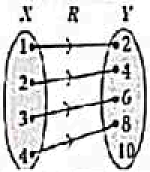
205. **Prove** $\frac{1-\tan^2\theta}{\cot^2\theta-1} = \tan^2\theta$

Solution : $1-\tan^2\theta = \tan^2\theta\,(\cot^2\theta-1) \Rightarrow 1-\tan^2\theta = \tan^2\theta\cot^2\theta - \tan^2\theta \Rightarrow 1-\tan^2\theta = 1-\tan^2\theta$

206. Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

Solution : $\tan^2\theta - \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \sin^2\theta\left(\frac{1}{\cos^2\theta} - 1\right) = \sin^2\theta(\sec^2\theta - 1) = \tan^2\theta\sin^2\theta$

207. Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range ?



Solution : Thus all elements in X have only one image in Y. Therefore R is a function.
Domain X = {1, 2, 3, 4}; Co-domain Y = {2, 3, 6, 8, 10}; Range of f = {2, 4, 6, 8}.

---

208. A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) If f a function ?

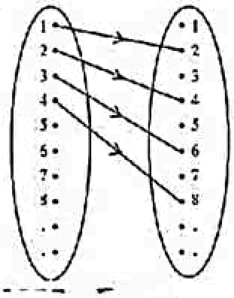
Solution : $f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$ (i) $f(-2) = (-2)^2 - 2 = 2$;
$f(-1) = (-1)^2 - 2 = -1$
$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$

$f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) each element in the domain of f has a unique image. Therefore f is a function.

---

209. **Let $f = \{(x, y) \mid x, y \in N$ and $y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function ?**



Solution : Given $f = \{(x, y) \mid x, y \in N$ and $y = 2x\}$
Domain = {1, 2, 3, 4,....} Co-domain = {1, 2, 3, 4, ...} Range = {2, 4, 6, 8,.....}
Since all the elements has unique element Yes, f is a function.

---

210. **Let f(x) = 2x + 5. If $x \neq 0$ then find $\dfrac{f(x+2)-f(2)}{x}$.**

Solution :

$$f(x) = 2x + 5$$
$$f(x+2) = 2(x+2) + 5 = 2x + 9$$
$$f(2) = 2(2) + 5 = 9$$
$$\therefore \frac{f(x+2) - f(x)}{x} = \frac{2x + 9 - 9}{x} = 2$$

---

211. **Let X = {3, 4, 6, 8}. Determine whether the relation R = {(x, f(x)) | $x \in$ X, $f(x) = x^2 + 1$} is a function from X to N ?**

Solution : X = {3, 4, 6, 8} Given R = {(x, f(x)) | $x \in$ X, $f(x) = x^2 + 1$}

$x = 3 \Rightarrow f(x) = f(3) = 9 + 1 = 10$ $\quad$ $x = 6 \Rightarrow f(x) = f(6) = 36 + 1 = 37$
$x = 4 \Rightarrow f(x) = f(4) = 16 + 1 = 17$ $\quad$ $x = 8 \Rightarrow f(x) = f(8) = 64 + 1 = 65$

R = {(3, 10), (4, 17), (6, 37), (8, 65)} $\therefore$ The relation R : X → N is a function.

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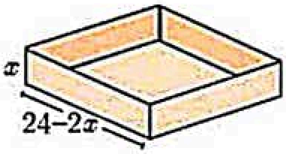
212. **An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in the figure. Express the volume V of the box as a function of x.**



Solution : $l = b = 24 - 2x$ cm, height = x cm.

$$\therefore \text{Volume of the box, } V = lbh = (24 - 2x)(24 - 2x)x = (24 - 2x)^2 x$$
$$= (576 + 4x^2 - 96x)x$$
$$= 4x^3 - 96x^2 + 576x$$

$\therefore$ Volume is expressed as a function of x.

---

213. **A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.**

Solution : $f(x) = 3 - 2x$ and $f(x^2) = (f(x))^2$

$\Rightarrow 3 - 2x^2 = (3 - 2x)^2 \Rightarrow 3 - 2x^2 = 9 + 4x^2 - 12x \Rightarrow 6x^2 - 12x + 6 = 0$
$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0$
$\Rightarrow x = 1$ (twice)

214. **A plane is flying at a speed of 500km per hour. Express the distance *d* travelled by the plane as function of time *t* in hours.**

Solution : Speed of the plane $= 500$ km / h $\quad\therefore$ Distance $=$ Time $\times$ Speed $= 500$t

215. Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

Solution : $3a - 5 = 4 \Rightarrow a = 3$

$3(1) - 5 = b \Rightarrow \mathrm{b} = -2$

216. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \rightarrow B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Solution : $A = \{-2, -1, 0, 1, 2\}$ and $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1 = 3$;

$f(-1) = (-1)^2 + (-1) + 1 = 1$;

$f(0) = 0^2 + 0 + 1 = 1$;

$f(1) = 1^2 + 1 + 1 = 3$

$f(2) = 2^2 + 2 + 1 = 7$

f is an onto function, range of $f = B =$ co-domain of f. Therefore, $B = \{1, 3, 7\}$.

217. **The Cartesian product A × A has 9 elements among which (–1, 0) and (0, 1) are found. Find the set A and the remaining elements of A × A.**

Solution : $n(A \times A) = 9$ and $(-1, 0), (0, 1) \in A \times A \qquad A = \{-1, 0, 1\}$

set A and the remaining elements of A × A $= \{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

218. **Find the domain of the function** $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$.

Solution : If $x > 1$ and $x < -1$, $f(x)$ leads to unreal $\quad\therefore$ The domain of $f(x) = \{-1, 0, 1\}$

219. **Write the domain** $f(x) = \dfrac{2x+1}{x-9}$ **ng real**

Solution :

If $x = 9, f(x) \rightarrow \infty$ The domain is $R - \{9\}$

Write the domain $g(x) = \sqrt{x-2}$

Solution : The function exists only if $x \geq 2$ $\quad\therefore$ The domain is $[2, \infty)$.

220. **Let A = {–1, 1} and B = {0, 2}. If the function $f : A \rightarrow B$ defined by $f(x) = ax + b$ is an onto function ? Find *a* and *b*.**

Solution : $\therefore f(-1) = 0 \Rightarrow -a + b = 0$ — (1)

$f(1) = 2 \Rightarrow a + b = 2$ — (2)

Solving (1) and (2) $\quad 2b = 2 \Rightarrow b = 1$

$\Rightarrow a = 1 \qquad \therefore a = 1, b = 1$

221. **If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y.**

Solution : Given $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$\therefore x^2 - 3x = -2$	$y^2 + 4y = 5$
$\Rightarrow x^2 - 3x + 2 = 0$	$y^2 + 4y - 5 = 0$
$\Rightarrow (x-2)(x-1) = 0$	$(y+5)(y-1) = 0$
$\therefore x = 2, 1$	$y = -5, 1$

222. **Let A = {1, 2} and B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify whether A × C is a subset of B × D =?**

Solution : $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$, $D = \{5, 6, 7, 8\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{\mathbf{(1, 5)}, \mathbf{(1, 6)}, (1, 7), (1, 8), \mathbf{(2, 5)}, \mathbf{(2, 6)}, (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8).\}$

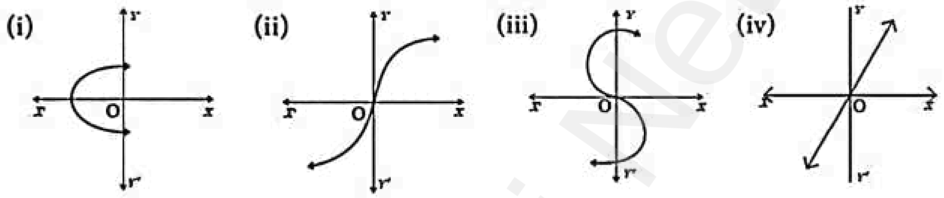
Clearly $A \times C$ is a subset of $B \times D$.

223. **Let A = {9, 10, 11, 12, 13, 14, 15, 16, 17} and let $f: A \to N$ be defined by $f(n)$ = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.**

Solution : $f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$

Range of $f = \{2, 3, 5, 7, 11, 13, 17\}$

224. **Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.**



Solution :

(i) The curve do not represent a function since it meets y-axis at 2 points.

(ii) The curve represents a function as it meets x-axis or y-axis at only one point.

(iii) The curve do not represent a function since it meets y-axis at 2 points.

(iv) The line represents a function as it meets axes at origin.

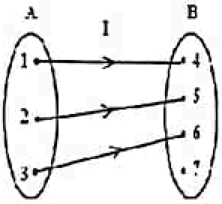
225. **Let A = {1, 2, 3, 4} and B = N. Let $f: A \to B$ be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function**

Solution : A = {1, 2, 3, 4}, B = N $f(x) = x^3$

$x = 1 \Rightarrow f(1) = 1$ $x = 3 \Rightarrow f(3) = 27$ (diff. elements have diff. images)

$x = 2 \Rightarrow f(2) = 8$ $x = 4 \Rightarrow f(4) = 64$ (Range ≠ co-domain)

(i) Range of $f = \{1, 8, 27, 64\}$ (ii) f is one-one and f is into



226. Let A = {1, 2, 3}, B = {4, 5, 6, 7| and $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one but not onto function.

Solution: A = {1, 2, 3}, B = {4, 5, 6, 7} ; $f = \{(1, 4), (2, 5), (3, 6)\}$

different elements in A are different images in B.

Hence f is one-one function. Note that the element 7 does not have any pre-image in A

Hence f is not onto.

227. **Show that the function $f: N \to N$ defined by $f(m) = m^2 + m + 3$ is one-one function.**

Solution :

Given $f: N \to N$ defined by $f(m) = m^2 + m + 3$

$m = 1 \Rightarrow f(1) = 1 + 1 + 3 = 5$ $m = 3 \Rightarrow f(3) = 9 + 3 + 3 = 15$

$m = 2 \Rightarrow f(2) = 4 + 2 + 3 = 9$ $m = 4 \Rightarrow f(4) = 16 + 4 + 3 = 23$ ....

different elements in N are different images in N $\therefore f$ is one-one function.

228. **Show that the function $f: N \to N$ defined $f(x) = 2x - 1$ is one-one but not onto.**

Solution : Given $f: N \to N$ defined by $f(x) = 2x - 1$.

$x = 1 \Rightarrow f(1) = 2 - 1 = 1$ $x = 3 \Rightarrow f(3) = 6 - 1 = 5$

$x = 2 \Rightarrow f(2) = 4 - 1 = 3$ $x = 4 \Rightarrow f(4) = 8 - 1 = 7$ ....

different elements in N are different images in N $\therefore f$ is one-one function.

$\therefore$ Range ≠ Co-domain. $\therefore f$ is not on-to.

229 The line p passes through the points (3, –2), (12, 4) and the line q passes through the points (6, –2) and (12, 2) . Is p parallel to q ?

Solution : The slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ The slope of line p is $m_1 = \dfrac{(4)-(-2)}{(12)-(3)} = \dfrac{4+2}{12-3} = \dfrac{6}{9} = \dfrac{2}{3}$

The slope of line q is $m_2 = \dfrac{(2)-(-2)}{(12)-(6)} = \dfrac{2+2}{12-6} = \dfrac{4}{6} = \dfrac{2}{3}$

Thus, slope of line p = slope of line q. Therefore, line p is parallel to the line q.

---

230 The line r passes through the points (–2, 2) and (5, 8) and the line s passes through the points (–8,7) and (–2, 0). Is the line r perpendicular to s ?

Solution : The slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

The slope of line r is $m_1 = \dfrac{(8)-(2)}{(5)-(-2)} = \dfrac{8-2}{5+2} = \dfrac{6}{7}$

The slope of line s is $m_2 = \dfrac{(0)-(7)}{(-2)-(-8)} = \dfrac{0-7}{-2+8} = \dfrac{-7}{6}$

The product of slopes $= \dfrac{6}{7} \times \dfrac{-7}{6} = -1$ That is, $m_1 m_2 = -1$

---

231 Without using Pythagoras theorem, show that the points (1,–4) , (2, –3) and (4, –7) form a right angled triangle.

Solution : Let the given points be A(1, – 4) , B(2, –3) and C(4, –7) .

The slope of AB $= \dfrac{-3+4}{2-1} = \dfrac{1}{1} = 1$ The slope of BC $= \dfrac{-7+3}{4-2} = \dfrac{-4}{2} = -2$

The slope of AC $= \dfrac{-7+4}{4-1} = \dfrac{-3}{3} = -1$ Slope of AB slope of AC = (1)(–1) = –1

AB is perpendicular to AC. $\angle A = 90°$ Therefore, ΔABC is a right angled triangle.

---

232 **If the three points (3, –1), (*a*, 3) and (1, –3) are collinear, find the value of *a*.**

Solution : $\therefore$ Slope of AB = Slope of BC

$$\Rightarrow \frac{4}{a-3} = \frac{-6}{1-a} \Rightarrow 4-4a = -6a+18 \Rightarrow 2a = 14 \Rightarrow a = 7$$

---

233 **Find the slope of a line joining the points (sin θ, – cos θ) and (– sin θ, cos θ)**

Solution : $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\cos\theta + \cos\theta}{-\sin\theta - \sin\theta} = \dfrac{2\cos\theta}{-2\sin\theta} = -\cot\theta$

Show that the given points are collinear (– 3, – 4), (7, 2) and (12, 5).

Solution : Given points are A (–3, –4), B (7, 2), C (12, 5)

Slope of AB $= \dfrac{2+4}{7+3} = \dfrac{6}{10} = \dfrac{3}{5}$ Slope of BC $= \dfrac{5-2}{12-7} = \dfrac{3}{5}$

$\therefore$ Slope of AB = Slope of BC $\therefore$ A, B, C are collinear.

---

234 **What is the slope of a line perpendicular to the line joining A (5, 1) and P where P is the mid-point of the segment joining (4, 2) and (–6, 4).**

Solution : P is the midpoint of (4, 2), (–6, 4) $\Rightarrow P = \left(\dfrac{4-6}{2}, \dfrac{2+4}{2}\right) = (-1, 3)$

$\therefore$ Slope of the line joining A (5, 1), P (–1, 3) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{-1-5} = \dfrac{2}{-6} = \dfrac{-1}{3}$

$\therefore$ Slope of the line perpendicular = 3

235. **The line through the points (–2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a.**

Solution :

Slope of the line joining (–2, a), (9, 3) = $-\frac{1}{2} \Rightarrow \frac{3-a}{9+2}=\frac{-1}{2} \Rightarrow \frac{3-a}{11}=\frac{-1}{2} \Rightarrow 6-2a=-11$

$\Rightarrow 2a=17 \quad \therefore a=\frac{17}{2}$

236. **The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). FInd the value of x.**

Solution : Slope of line joining (–2, 6), (4, 8) $m_1=\frac{8-6}{4+2}=\frac{2}{6}=\frac{1}{3}$

Slope of line joining (8, 12), (x, 24) $m_2=\frac{24-12}{x-8}=\frac{12}{x-8}$

Since two lines are perpendicular, $\Rightarrow \frac{1}{3}\times\frac{12}{x-8}=-1 \Rightarrow \frac{4}{x-8}=-1 \Rightarrow -x+8=4 \Rightarrow x=4$

237. Find the intercepts made by the line $4x-9y+36=0$ on the coordinate axes.

Solution : put $x=0 \Rightarrow 4x=-36$ x intercept $a=-9$

put $y=0 \Rightarrow -9y+36=0.\ -9y=-36 \Rightarrow y$ intercept $b=4$

238. Show that the straight lines $2x+3y-8=0$ and $4x+6y+18=0$ are parallel.

Solution :

Slope of the straight line $2x+3y-8=0$ is $m_1=\frac{-\text{coefficient of } x}{\text{coefficient of } y}=\frac{-2}{3}$

Slope of the straight line $4x+6y+18=0$ is $m_2=\frac{-4}{6}=\frac{-2}{3}$ Here, $m_1=m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

239. Show that the straight lines $x-2y+3=0$ and $6x+3y+8=0$ are perpendicular.

Solution : Slope of the straight line $x-2y+3=0$ is $m_1=\frac{-1}{-2}=\frac{1}{2}$

Slope of the straight line $6x+3y+8=0$ is $m_2=\frac{-6}{3}=-2$

Now, $m_1\times m_2=\frac{1}{2}\times(-2)=-1$ Hence, the two straight lines are perpendicular.

240. Find the equation of a straight line which is parallel to the line $3x-7y=12$ and passing through the point (6, 4).

Solution : Equation of the straight line, parallel to $3x-7y-12=0$ is $3x-7y+k=0$

$3(6)-7(4)+k=0 \Rightarrow k=28-18=10$

the required straight line is $3x-7y+10=0$.

241. Find the equation of a straight line perpendicular to the line $y=\frac{4}{3}x-7$ and passing through the point (7, – 1).

Solution : The equation $y=\frac{4}{3}x-7$ can be written as $4x-3y-21=0$.

Equation of a straight line perpendicular to $4x-3y-21=0$ is $3x+4y+k=0$

Since it is passes through the point (7, –1), $21-4+k=0$ we get, $k=-17$

the required straight line is $3x+4y-17=0$.

242. Find the equation of a straight line passing through (5,7) and is (i) parallel to X axis (ii) parallel to Y axis.

Solution : (i) The equation of any straight line parallel to X axis is y=b

The required equation of the line is y=7.

(ii) The equation of any straight line parallel toY axis is x=c

The required equation of the line is $x = 5$.

---

243. Find the equation of a straight line whose Slope is 5 and x intercept is -9

Solution : Given, Slope = 5, x intercept, d =-9

The equation of a straight line is $y = m(x - d)$ $\quad$ $y = 5(x+9)$ $\quad$ $y = 5x + 45$

---

244. Find the equation of a line passing through the point $(3, -4)$ and having slope $\frac{-5}{7}$.

Solution : Given slope of the line is $-\frac{5}{7}$ and $(3, -4)$ is a point on the line.

$$y - y_1 = m(x - x_1) \qquad y + 4 = -\frac{5}{7}(x - 3) \qquad 5x + 7y + 13 = 0.$$

---

245. **Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1,2).**

Solution : slope of the line is $\frac{-5}{4}$ and (–1, 2) is a point on the line. $\therefore$ its equation is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = \frac{-5}{4}(x+1) \quad \Rightarrow 4y - 8 = -5x - 5 \quad \Rightarrow 5x + 4y - 3 = 0$$

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246. Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6, 10) to (14, 12), find the equation of the rod joining the buildings ?

Solution : $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y - 10}{12 - 10} = \frac{x - 6}{11 - 6} \Rightarrow \frac{y - 10}{2} = \frac{x - 6}{8} \Rightarrow x - 4y + 34 = 0.$

---

247. **Find the equation of a straight line passing through the mid-point of a line segment joining the points (1,-5), (4,2) and parallel to (i) X axis (ii) Y axis**

Solution : Equation of a Straight line parallel to the Y axis is $x = c$.

Equation of a straight line parallel to X axis is $y = b$

Mid point of the line joining the points $(1, -5), (4, 2)$ is $= \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right] = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$

(i) Parallel to x-axis is $y = -3/2$ $\qquad$ (ii) Parallel to y-axis is $x = 5/2$

---

248. **Determine the sets of points are collinear ? $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$**

Solution : Area of triangle $= \frac{1}{2}\begin{bmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{bmatrix}$

$$= \frac{1}{2}[(a^2 + b^2 + c^2 + ab + bc + ca) - (a^2 + b^2 + c^2 + ab + bc + ca)] = \frac{1}{2}[0] = 0$$

$\therefore$ The 3 points are collinear.

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249. **If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'.**

Solution : $12y = -(p + 3)x + 12$,

$\Rightarrow (p + 3)x + 12y = 12$ and $12x - 7y = 16$ are perpendicular

$$m_1 = \frac{-(p+3)}{12} \quad m_2 = \frac{12}{7} \quad m_1 \times m_2 = -1 \quad \Rightarrow \quad \frac{-(p+3)}{12} \times \frac{12}{7} = -1 \quad \Rightarrow \quad p = 4$$

250. **Determine the sets of points are collinear ?** $\left\{-\frac{1}{2},3\right\}$**, (–5, 6) and (–8, 8)**

Solution : Area of triangle $=\frac{1}{2}\begin{bmatrix} -1/2 & -5 & -8 & -1/2 \\ 3 & 6 & 8 & 3 \end{bmatrix}=\frac{1}{2}[(-3-40-24)-(-15-48-4)]$

$$=\frac{1}{2}[-67-(67)]=\frac{1}{2}(0)=0$$

∴ The 3 points are collinear.

251. **Find the value of ‘p’.**

Vertices	Area (sq. units)
(p, p), (5, 6), (5, –2)	32

Solution : Area of triangle $=\frac{1}{2}\begin{bmatrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{bmatrix}=32$

$\Rightarrow (6p-10+5p)-(5p+30-2p)=64 \quad \Rightarrow (11p-10)-(3p+30)=64$

$\Rightarrow 8p=104 \Rightarrow p=\frac{104}{8} \Rightarrow p=13$

252. If the points **(2, 3), (4, *a*)** and **(6, –3) are collinear**, then **find the value of ‘*a*’**

Solution : $\frac{1}{2}\begin{bmatrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{bmatrix}=0$

$\Rightarrow (2a-12+18)-(12+6a-6)=0$

$\Rightarrow (2a+6)-(6a+6)=0 \Rightarrow -4a=0 \Rightarrow a=0$

253. If the points **(–a+1, 2*a*)** and **(– 4 – *a*, 6 – 2*a*) are collinear**, then **find the value of ‘*a*’**

Solution : $\frac{1}{2}\begin{bmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{bmatrix}$

$\Rightarrow 8a^2+4a-4=0 \quad \Rightarrow 2a^2+a-1=0 \quad \Rightarrow a=-1,\ \frac{1}{2}$

254. Find the slope of a line joining the given points $\left(-\frac{1}{3},\frac{1}{2}\right)$ and $\left(\frac{2}{7},\frac{3}{7}\right)$

Solution : The slope $=\frac{y_2-y_1}{x_2-x_1}=\frac{\left(\frac{3}{7}\right)-\left(\frac{1}{2}\right)}{\left(\frac{2}{7}\right)-\left(-\frac{1}{3}\right)}=\frac{\frac{3}{7}-\frac{1}{2}}{\frac{2}{7}+\frac{1}{3}}=\frac{\frac{6-7}{14}}{\frac{6+7}{21}}=-\frac{1}{14}\times\frac{21}{13}=-\frac{3}{26}.$

255. Find the slope of a line joining the given points (14, 10) and (14, – 6)

Solution : The slope $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-6)-(10)}{(14)-(14)}=\frac{-6-10}{14-14}=\frac{-16}{0}.$ The slope is undefined.

256. Show that the points (–2, 5), (6, –1) and (2, 2) are collinear.

Solution : The vertices are A (–2,5), B (6, –1) and C (2, 2). The slope $=\frac{y_2-y_1}{x_2-x_1}$

Slope of AB $=\frac{(-1)-(5)}{(6)-(-2)}=\frac{-1-5}{6+2}=\frac{-6}{8}=\frac{-3}{4}$

Slope of BC $=\frac{(2)-(-1)}{(2)-(6)}=\frac{2+1}{2-6}=\frac{3}{-4}=\frac{-3}{4}$

We get, Slope of AB = Slope of BC.

Hence the points A, B and C are collinear.

257. **Find the slope and y intercept of $\sqrt{3}x+(1-\sqrt{3})y=3$.**

Solution : $a=\sqrt{3}\quad b=(1-\sqrt{3})\quad c=-3$

$$\text{Slope of the line}=\frac{-a}{b}=\frac{-\sqrt{3}}{(1-\sqrt{3})}=\frac{3+\sqrt{3}}{2}$$

$$\text{Interecept on } y\text{-axis}=\frac{-c}{b}=\frac{-(-3)}{1-\sqrt{3}}=\frac{3+3\sqrt{3}}{-2}$$

---

258. **Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.**

Solution : Given $\theta=30^\circ \Rightarrow \text{m}=\tan 30^\circ=\frac{1}{\sqrt{3}}$ and y - intercept $=-3$

The required equation of the line is $y=mx+c \Rightarrow y=\frac{1}{\sqrt{3}}x-3 \Rightarrow \sqrt{3}y=x-3\sqrt{3}$

$\Rightarrow x-\sqrt{3}y-3\sqrt{3}=0$

---

259. **Find the equation of a line through the given pair of points $\left(2,\frac{2}{3}\right)$ and $\left(\frac{-1}{2},-2\right)$**

Solution : Given points are $\left(2,\frac{2}{3}\right),\left(\frac{-1}{2},-2\right)$ two-point form $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}}=\frac{x-2}{\frac{-1}{2}-2}\Rightarrow\frac{3y-2}{-8}=\frac{x-2}{\frac{-5}{2}}\Rightarrow\frac{3y-2}{-8}=\frac{2x-4}{-5}\Rightarrow 15y-10=16x-32\Rightarrow 16x-15y-22=0$$

---

260. **The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on the Y axis.**

Solution : $2(x-y)+5=0 \Rightarrow 2x-2y+5=0$

i) Slope of the line $=\frac{-a}{b}=\frac{-2}{-2}=1$

ii) The slope of the straight line is $\text{m}=\tan\theta$

Slope of the line $=1$ $\therefore \tan\theta=1$ $\therefore \theta=45^\circ$.

iii) Interecept on y-axis $=\frac{-c}{b}=\frac{-5}{-2}$ $\therefore y$ - intercept $=5/2$

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261. **The hill in the form of a right triangle has its foot at (19, 3). The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.**

Solution : $\therefore$ Equation of slope $\text{m}=\tan 45^\circ=1$ and passing through C(19, 3)

$\Rightarrow y-y_1=m(x-x_1) \Rightarrow y-3=1(x-19) \Rightarrow x-y-16=0.$

---

262. **Find the value of 'a', if the line through (–2, 3) and (8, 5) is perpendicular to $y=ax+2$.**

Solution :

Slope of the line joining (– 2, 3), (8, 5). $\Rightarrow \frac{y_2-y_1}{x_2-x_1}=\frac{5-3}{8+2}=\frac{2}{10}\Rightarrow m_1=\frac{1}{5}$

Slope of the line $y=ax+2 \Rightarrow ax-y+2=0$ Slope of the line $=\frac{-a}{b}=\frac{-a}{-1}\Rightarrow m_2=a.$

$m_1m_2=-1 \Rightarrow \frac{1}{5}\times a=-1 \Rightarrow a=-5$

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263. **Find the equation of a straight line passing through (5, –3) and (7, – 4).**

Solution : $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\Rightarrow\frac{y+3}{-4+3}=\frac{x-5}{7-5}\Rightarrow 2y+6=-x+5\Rightarrow x+2y+1=0.$

The equation of the required straight line is $x+2y+1=0.$

IMPORTANT 5 MARKS MINIMUM MATERIAL

1. If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $f(x) = 2x + 3$, $g(x) = 1 - 2x$, $h(x) = 3x$

$$(f \circ g) = (2x + 3)(1 - 2x) = 2(1 - 2x) + 3 = 2 - 4x + 3 = 5 - 4x$$

$$(f \circ g) \circ h = (5 - 4x)(3x) = 5 - 4(3x) = 5 - 12x \quad \text{....(1)}$$

$$(g \circ h) = (1 - 2x)(3x) = 1 - 2(3x) = 1 - 6x$$

$$f \circ (g \circ h) = (2x + 3)(1 - 6x) = 2(1 - 6x) + 3 = 2 - 12x + 3 = 5 - 12x \quad \text{....(2)}$$

From (1) and (2), we get $f \circ (g \circ h) = (f \circ g) \circ h$.

---

2. **If $f(x) = 4x^2 - 1$ and $g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.**

Solution : $(f \circ g) = (4x^2 - 1)(1 + x) = 4(1 + x)^2 - 1 = 4(1 + x^2 + 2x) - 1 = 4x^2 + 8x + 3$

$(g \circ f) = (1 + x)(4x^2 - 1) = 1 + 4x^2 - 1 = 4x^2 \qquad \therefore\ f \circ g \neq g \circ f.$

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3. **If $f(x) = \dfrac{x+6}{3}$ and $g(x) = 3 - x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.**

Solution :

$(f \circ g) = \left(\dfrac{x+6}{3}\right)(3 - x) = \dfrac{(3-x)+6}{3} = \dfrac{9-x}{3}$ | $(g \circ f) = (3 - x)\left(\dfrac{x+6}{3}\right) = 3 - \dfrac{x+6}{3} = \dfrac{9-x-3}{3} = \dfrac{6-x}{3}$

From (1) and (2), we get $\therefore\ f \circ g \neq g \circ f$.

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4. **If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$,** Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $(f \circ g) = (x - 4)(x^2) = x^2 - 4$

$$\therefore ((f \circ g) \circ h) = (x^2 - 4)(3x - 5) = (3x - 5)^2 - 4 \quad \text{...(1)}$$

$$(g \circ h) = (x^2)(3x - 5) = (3x - 5)^2$$

$$\therefore (f \circ (g \circ h) = (x - 4)(3x - 5)^2 = (3x - 5)^2 - 4 \quad \text{...(2)}$$

From (1) and (2), we get $, (f \circ g) \circ h = f \circ (g \circ h)$

---

5. **If $f(x) = x^2$, $g(x) = 2x$ and $h(x) = 3x - 5$,** Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $(f \circ g) = (x^2)(2x) = (2x)^2 = 4x^2$

$$\therefore ((f \circ g) \circ h) = (4x^2)(x + 4) = 4(x + 4)^2 \quad \text{...(1)}$$

$$(g \circ h) = (2x)(x + 4) = 2(x + 4) = 2x + 8$$

$$\therefore (f \circ (g \circ h) = (x^2)(2x + 8) = (2x + 8)^2 = (2(x + 4))^2 = 4(x + 4)^2 \quad \text{...(2)}$$

From (1) and (2), we get $, (f \circ g) \circ h = f \circ (g \circ h)$

---

6. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

Solution : $f \circ g = (3x - 2)(2x + k) = 3(2x + k) - 2 = 6x + 3k - 2$

$$g \circ f = (2x + k)(3x - 2) = 2(3x - 2) + k = 6x - 4 + k$$

$$f \circ g = g \circ f \Rightarrow 6x + 3k - 2 = 6x - 4 + k$$

$$6x - 6x + 3k - k = -4 + 2$$

$$2k = -2$$

$$k = -1$$

---

7. **If $f(x) = x^2 - 1$, find $f \circ f \circ f$**

Solution : $(f \circ f \circ f)(x) = (x^2 - 1)(x^2 - 1)(x^2 - 1) = (x^2 - 1)((x^2 - 1)^2 - 1) = (x^2 - 1)(x^4 - 2x^2) = (x^4 - 2x^2)^2 - 1$

If $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f \circ g = (2x + 1)(x^2 - 2) = 2(x^2 - 2) + 1 = 2x^2 - 3$

$g \circ f = (x^2 - 2)(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x - 1 \qquad \therefore\ f \circ g \neq g \circ f.$

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8. **Given the function $f : x \to x^2 - 5x + 6$, evaluate (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x - 1)$**

Solution : $f : x \to x^2 - 5x + 6 \Rightarrow f(x) = x^2 - 5x + 6$

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6 = x^2 - 7x + 12$

9. If the function $f : R \to R$ defined by $f(x) = \begin{cases} 2x+7, x<-2 \\ x^2-2, -2 \le x < 3, \\ 3x-2, x \ge 3 \end{cases}$ then find the values of (i) f(4) (ii) f(– 2) (iii) f(4) + 2f(1) (iv) $\frac{f(1)-3f(4)}{f(-3)}$

Solution : (i) $f(4) = 3(4) - 2$
$= 12 - 2 = 10$

(ii) $f(-2) = (-2)^2 - 2 = 2 = 4 - 2 = 2$

(iii) $f(1) = (1)^2 - 2 = 1 - 2 = -1$

$\therefore\ f(4) + 2f(1) = 10 + 2(-1) = 8$

(iv) $f(-3) = 2(-3) + 7 = 1$

$\therefore\ \frac{f(1)-3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = -31$

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10. **A function $f : [-5, 9] \to$ R is defined as follows :**

$$f(x) = \begin{cases} 6x+1 & if\ -5 \le x < 2 \\ 5x^2-1 & if\ \ 2 \le x < 6 \\ 3x-4 & if\ \ 6 \le x < 6 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ (iii) $2f(4) + f(8)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Solution : $f(-3) = [6(-3) + 1] = -18 + 1 = -17$ $\quad f(1) = [6(1) + 1] = 6 + 1 = 7$

$f(2) = [5(4) - 1] = 20 - 1 = 19$ $\quad f(7) = [3(7) - 4] = 21 - 4 = 17$

$f(4) = [5(16) - 1] = 80 - 1 = 79$ $\quad f(8) = [3(8) - 4] = 24 - 4 = 20$

$f(-2) = [6(-2) + 1] = -12 + 1 = -11$ $\quad f(6) = [3(6) - 4] = 18 - 4 = 14$

(i) $f(-3) + f(2) = -17 + 19 = 2$ $\quad$ (ii) $f(7) - f(1) = 17 - 7 = 10$

(iii) $2f(4) + f(8) = 2[79] + 20 = 158 + 20 = 178$ $\quad$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{79-11} = \frac{-9}{17}$

---

11. **If the function f is defined by $f(x) = \begin{cases} x+2 & if\ x>1 \\ 2 & if\ -1 \le x \le 1 \\ x-1 & if\ -3 < x < -1 \end{cases}$; find the values of**

(i) $f(3)$ (ii) $f(0)$ (iii) $f(-1, 5)$
(iv) $f(2) + f(-2)$

Solution : (i) $f(3) = 3 + 2 = 5$ $\quad$ (ii) $f(0) = 2$ $\quad$ (iii) $f(-1.5) = -1.5 - 1 = -2.5$

(iv) $f(2) + f(-2) = (2 + 2) + (-2 - 1) = 4 - 3 = 1$

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12. **The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C) = F where F = $\frac{9}{5}$ C + 32, Find, (i) t(0) (ii) t(28) (iii) t(– 10) (iv) the value of C when t(C) = 212 (v) the temperature when the Celsius value is equal to the Farenheit value.**

Solution :

(*i*) $t(0) = \frac{9(0)}{5} + 32 = 32^\circ F$ $\quad$ (*ii*) $t(28) = \frac{9(28)}{5} + 32 = 50.4 + 32 = 82.4^\circ F$

(*iii*) $t(-10) = \frac{9(-10)}{5} + 32 = -18 + 32 = 14^\circ F$ $\quad$ (*iv*) When $t(c) = 212$, $212 = \frac{9C}{5} + 32 \Rightarrow C = 100^\circ C$

(v) When Celsius value = Farenheit value, $C = \frac{9C}{5} + 32 \Rightarrow 5C = 9C + 160 \Rightarrow -4C = 160 \Rightarrow C = -40^\circ$

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13. Let f be a function $f : N \to N$ be defined by $f(x) = 3x + 2$, $x \in N$ (i) Find the images of 1, 2, 3 (i i) Find the pre-images of 29, 53 (iii) Identify the type of function

Solution :

$f(x) = 3x + 2, x \in N$

(i) If $x = 1$, f(1) $= 3(1) + 2 = 5$
If $x = 2$, f(2) $= 3(2) + 2 = 8$
If $x = 3$, f(3) $= 3(3) + 2 = 11$
The images of 1, 2, 3 are 5, 8, 11

(ii) $3x + 2 = 29 \Rightarrow 3x = 27 \Rightarrow x = 9.$
9 is the pre-image of 53
$3x + 2 = 53 \Rightarrow 3x = 51 \Rightarrow x = 17.$
17 is the pre-image of 53

(iii) Since different elements of N have different images in N f is one-one and into function.

14. **The functions f and g are defined by $f(x) = 6x+8$; $g(x) = \frac{x-2}{3}$ (i) Calculate the value of $gg\left(\frac{1}{2}\right)$ (ii) Write an expression for $gf(x)$ in its simplest form.**

Solution :

$$i)\ gg(x) = \left(\frac{x-2}{3}\right)\left(\frac{x-2}{3}\right) = \left(\frac{\frac{x-2}{3}-2}{3}\right) = \left(\frac{\frac{x-2-6}{3}}{3}\right) = \left(\frac{x-8}{9}\right) \quad \therefore gg\left(\frac{1}{2}\right) = \left(\frac{\frac{1}{2}-8}{9}\right) = \frac{-15}{18} = \frac{-5}{6}$$

$$\text{(ii)}\ gf(x) = \left(\frac{x-2}{3}\right)(6x+8) = \frac{6x+8-2}{3} = \frac{6x+6}{3} = 2x+2 = 2(x+1)$$

15. **If $f(x) = 2x-1$, $g(x) = \frac{x+1}{2}$, show that $f \circ g = g \circ f = x$.**

Solution :

$$f \circ g = (2x-1)\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) - 1 = x+1-1 = x$$

$$g \circ f = \left(\frac{x+1}{2}\right)(2x-1) = \frac{2x-1+1}{2} = x \qquad \therefore f \circ g = g \circ f = x$$

16. **If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x-2$. Prove that $(f \text{o}\, g) \text{ o } h = f \text{o}\,(g \text{ o } h)$.**

Solution : $f(x) = x^2$, $g(x) = 3x$, $h(x) = x-2$

$(f \text{o}\, g) = (x^2)(3x) = (3x)^2 = 9x^2$ $\qquad$ $(g \text{ o } h) = (3x)(x-2) = 3\,(x-2) = 3x-6$

$(f \text{o}\, g) \text{ o } h) = (9x^2)(x-2) = 9\,(x-2)^2$ — (1) $\qquad$ $f \text{o}\,(g \text{ o } h) = (x^2)(3x-6) = (3x-6)^2 = (3(x-2))^2$

$\therefore$ From (1) and (2), $(f \text{o} g) \text{ o } h = f \text{o}\,(g \text{ o } h)$. $\qquad$ $= 9(x-2)^2$ — (2)

17. **If $f(x) = 2x-k$, $g(x) = 4x+5$ such that $f \text{ o } g = g \text{ o} f$. Find the value of k**

Solution : $(f \text{o}\, g) = (g \text{ o} f) \Rightarrow (2x-k)(4x+5) = (4x+5)(2x-k)$

$$\begin{aligned} 2\,(4x+5)-k &= 4\,(2x-k)+5 \\ 8x+10-k &= 8x-4k+5 \\ 10-k &= -4k+5 \\ -k+4k &= 5-10 \\ 3k &= -5 \Rightarrow k = \frac{-5}{3} \end{aligned}$$

18. **If $f(x) = 3x+2$, $g(x) = 6x-k$ such that $f \text{ o } g = g \text{ o} f$. Find the value of k**

Solution : $(f \text{o}\, g) = (g \text{ o} f) \Rightarrow (3x+2)(6x-k) = (6x-k)(3x+2)$

$$\begin{aligned} 3(6x-k)+2 &= 6(3x+2)-k \\ 18x-3k+2 &= 18x+12-k \\ -3k+2 &= 12-k \\ -2k &= 10 \quad \Rightarrow k = \frac{-10}{2} = -5 \end{aligned}$$

19. Find x if $gff(x) = fgg(x)$, given $f(x) = 3x+1$ and $g(x) = x+3$.

Solution :

$gff(x) = (x+3)(3x+1)(3x+1) = (x+3)[\,3(3x+1)+1] = (x+3)(9x+4) = [(9x+4)+3] = 9x+7$

$fgg(x) = (3x+1)(x+3)(x+3) = (3x+1)[(x+3)+3] = (3x+1)(x+6) = [3(x+6)+1\,] = 3x+19$

$gff(x) = fgg(x) \Rightarrow 9x+7 = 3x+19 \Rightarrow 9x-3x = 19-7 \Rightarrow 6x = 12 \Rightarrow x = 2.$

20. **$f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = 2x+1$ whether the function is bijective or not. Justify your answer.**

Solution : $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = 2x+1$ $\qquad$ Let $f(x_1) = f(x_2)$

$\Rightarrow 2x_1+1 = 2x_2+1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2 \quad \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\therefore f$ is 1 – 1 function.

$y = 2x+1 \Rightarrow \therefore 2x = y-1 \Rightarrow x = \frac{y-1}{2} \quad \therefore f(x) = 2\left(\frac{y-1}{2}\right)+1 = y \quad \therefore f$ is onto.

$\therefore f$ is one-one and onto $\quad \Rightarrow f$ is bijective.

21. **Given that $f(x)=\begin{cases}\sqrt{x-1} & x\geq 1\\ 4 & x<1\end{cases}$. Find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a+1)$ in terms of a. (Given that $a\geq 0$)**

Solution : Given $f(x)=\begin{cases}\sqrt{x-1} & x\geq 1\\ 4 & x<1\end{cases}$ $i)$ $f(0)=4$ $ii)$ $f(3)=\sqrt{3-1}=\sqrt{2}$ $iii)$ $f(a+1)=\sqrt{a+1-1}=\sqrt{a}$

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22. **Let $A=\{x\in W \mid x<2\}$, $B=\{x\in N \mid 1<x\leq 4\}$ and $C=\{3,5\}$. Verify that $A\times(B\cup C)=(A\times B)\cup(A\times C)$**

Solution : $A=\{x\in W \mid x<2\}\Rightarrow A=\{0,1\}$ $B=\{x\in N \mid 1<x\leq 4\}\Rightarrow B=\{2,3,4\}$ $C=\{3,5\}$

$B\cup C=\{2,3,4,5\}$

$\therefore A\times(B\cup C)=\{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$ ...(1)

$A\times B=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A\times C=\{(0,3),(0,5),(1,3),(1,5)\}$

$\therefore (A\times B)\cup(A\times C)=\{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$ ...(2)

$\therefore$ From (1) and (2) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

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23. **Let $A=\{x\in W \mid x<2\}$, $B=\{x\in N \mid 1<x\leq 4\}$ and $C=\{3,5\}$. Verify that $A\times(B\cap C)=(A\times B)\cap(A\times C)$**

Solution : $A=\{x\in W \mid x<2\}\Rightarrow A=\{0,1\}$ $B=\{x\in N \mid 1<x\leq 4\}\Rightarrow B=\{2,3,4\}$ $C=\{3,5\}$

$B\cap C=\{3\}$

$\therefore A\times(B\cap C)=\{(0,3),(1,3)\}$ ...(1)

$A\times B=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A\times C=\{(0,3),(0,5),(1,3),(1,5)\}$

$\therefore (A\times B)\cap(A\times C)=\{(0,3),(1,3)\}$ ...(2)

$\therefore$ From (1) and (2), $A\times(B\cap C)=(A\times B)\cap(A\times C)$

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24. **Let $A=\{x\in W \mid x<2\}$, $B=\{x\in N \mid 1<x\leq 4\}$ and $C=\{3,5\}$. Verify that $(A\cup B)\times C=(A\times C)\cup(B\times C)$**

Solution : $A=\{x\in W \mid x<2\}\Rightarrow A=\{0,1\}$ $B=\{x\in N \mid 1<x\leq 4\}\Rightarrow B=\{2,3,4\}$ $C=\{3,5\}$

$A\cup B=\{0,1,2,3,4\}$

$\therefore (A\cup B)\times C=\{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$ ...(1)

$A\times C=\{(0,3),(0,5),(1,3),(1,5)\}$

$B\times C=\{(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$

$\therefore (A\times C)\cup(B\times C)=\{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5)\ (3,3),(3,5),(4,3),(4,5)\}$

$\therefore$ From (1) and (2) $(A\cup B)\times C=(A\times C)\cup(B\times C)$

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25. Let $A=\{x\in N \mid 1<x<4\}$, $B=\{x\in W \mid 0\leq x<2\}$ and $C=\{x\in N \mid x<3\}$.
verify that $A\times(B\cup C)=(A\times B)\cup(A\times C)$

Solution : $A=\{x\in N \mid 1<x<4\}=\{2,3\}$, $B=\{x\in W \mid 0\leq x<2\}=\{0,1\}$. $C=\{x\in N \mid x<3\}=\{1,2\}$

$B\cup C=\{0,1\}\cup\{1,2\}=\{0,1,2\}$

$A\times(B\cup C)=\{2,3\}\times\{0,1,2\}=\{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ ... (1)

$A\times B=\{2,3\}\times\{0,1\}=\{(2,0),(2,1),(3,0),(3,1)\}$

$A\times C=\{2,3\}\times\{1,2\}=\{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cup(A\times C)=\{(2,0),(2,1),(3,0),(3,1)\}\cup\{(2,1),(2,2),(3,1),(3,2)\}$

$=\{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ ... (2)

From (1) and (2), $A\times(B\cup C)=(A\times B)\cup(A\times C)$ is verified.

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26. Let $A=\{x\in N \mid 1<x<4\}$, $B=\{x\in W \mid 0\leq x<2\}$ and $C=\{x\in N \mid x<3\}$.
verify that $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Solution : $A=\{x\in N \mid 1<x<4\}=\{2,3\}$, $B=\{x\in W \mid 0\leq x<2\}=\{0,1\}$. $C=\{x\in N \mid x<3\}=\{1,2\}$

$B\cap C=\{0,1\}\cap\{1,2\}=\{1\}$

$A\times(B\cap C)=\{2,3\}\times\{1\}=\{(2,1),(3,1)\}$ ... (1)

$A\times B=\{2,3\}\times\{0,1\}=\{(2,0),(2,1),(3,0),(3,1)\}$

$A\times C=\{2,3\}\times\{1,2\}=\{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cap(A\times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\} = \{(2, 1), (3, 1)\}$ ... (2)

From (1) and (2), $A\times(B\cap C) = (A\times B)\cap(A\times C)$ is verified.

27. **A function f is defined by $f(x) = 2x - 3$**

(i) find $\dfrac{f(0)+f(1)}{2}$ (ii) find x such that $f(x) = 0$ (iii) find x such that $f(x) = x$

(iv) find x such that $f(x) = f(1 - x)$.

Solution : Given $f(x) = 2x - 3$

(i) $f(0) = 2(0) - 3 = 0 - 3 = -3$

$f(1) = 2(1) - 3 = 2 - 3 = -1$

$$\frac{f(0)+f(1)}{2} = \frac{(-3)+(-1)}{2} = \frac{-4}{2} = -2$$

(iii) $f(x) = x \Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3$

$\Rightarrow x = 3$

(ii) $f(x) = 0 \Rightarrow 2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = \dfrac{3}{2}$

(iv) $f(x) = f(1 - x) \Rightarrow 2x - 3 = 1 - x \Rightarrow 2x + x = 1 + 3$

$\Rightarrow 3x = 4 \Rightarrow x = \dfrac{4}{3}$

28. **Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $(A\cap B)\times C = (A\times C)\cap(B\times C)$**

Solution : $A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{1, 3, 5, 7\}$ $C = \{2\}$

$A\cap B = \{1, 3, 5, 7\}$

$\therefore (A\cap B)\times C = \{1, 3, 5, 7\}\times\{2\} = \{(1, 2), (3, 2), (5, 2), (7, 2)\}$ ... (1)

$A\times C = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$B\times C = \{(1, 2), (3, 2), (5, 2), (7, 2)\}$

$(A\times C)\cap(B\times C) = \{(1, 2), (3, 2), (5, 2), (7, 2)\}$ ... (2)

$\therefore$ From (1) and (2), $(A\cap B)\times C = (A\times C)\cap(B\times C)$

29. **If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$. Show that $A\times A = (B\times B)\cap(C\times C)$.**

Solution : $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$

$A\times A = \{5, 6\}\times\{5, 6\} = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B\times B = \{4, 5, 6\}\times\{4, 5, 6\} = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C\times C = \{5, 6, 7\}\times\{5, 6, 7\} = \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B\times B)\cap(C\times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(2)

$\therefore$ From (1) and (2). **$A\times A = (B\times B)\cap(C\times C)$.**

30. **Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)$ is true ?**

Solution : $A = \{1, 2, 3\}, B = \{2, 3, 5\}, C = \{3, 4\}, D = \{1, 3, 5\}$ $A\cap C = \{3\}$, $B\cap D = \{3, 5\}$

$\therefore (A\cap C)\times(B\cap D) = \{(3, 3), (3, 5)\}$ ... (1)

$A\times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)\}$

$C\times D = \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$\therefore (A\times B)\cap(C\times D) = \{(3, 3), (3, 5)\}$ ...(2)

$\therefore$ From (1) and (2) **$(A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)$**

31. If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then (i) find $A\times B$ and $B\times A$. (ii) Is $A\times B = B\times A$? If not why ? (iii) Show that $n(A\times B) = n(B\times A) = n(A)\times n(B)$.

Solution : Given that $A = \{1, 3, 5\}$ and $B = \{2, 3\}$

(i) $A\times B = \{1, 3, 5\}\times\{2, 3\} = \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$

$B\times A = \{2, 3\}\times\{1, 3, 5\} = \{(2, 1), (2, 3), (2, 5), (3, 1), (3. 3), (3, 5)\}$

(ii) $(1, 2)\neq(2, 1) \Rightarrow A\times B\neq B\times A$

(iii) $n(A\times B) = n(B\times A) = 6$; $n(B)\times n(A) = 2\times 3 = 6$

$\therefore n(A\times B) = n(B\times A) = n(A)\times n(B)$.

32. **Let $f : A \to B$ be a function define by $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}. Represent f by (i) set of ordered pairs ; (ii) a table ; (iii) an arrow diagram ; (iv) a graph**

Solution : Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0 \quad x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2 \quad x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

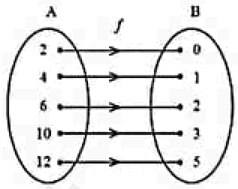
(i) Set of order pairs :

f = {(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

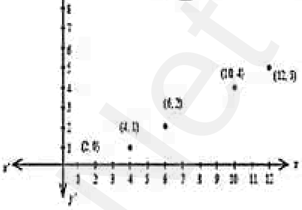
(ii) Table :

x	2	4	6	10	12
f(x)	0	1	2	4	5

(iii) Arrow diagram :

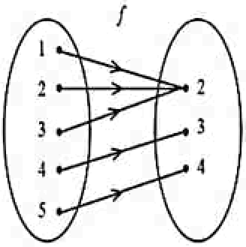


(iv) Graph



33. **Represent the function f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through (i) an arrow diagram (ii) a table form (iii) a graph**

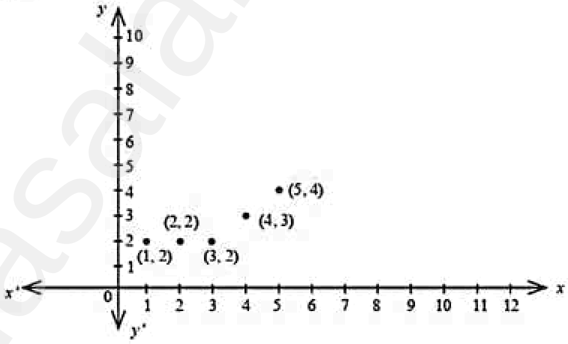
Solution : **(i) Arrow Diagram :**



(ii) Table Form :

x	1	2	3	4	5
f(x)	2	2	2	3	4

(iii) Graph :



34. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f : A \to B$ be a function given by f(x) = 3x – 1. Represent this function (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical form

Solution :

A = {1, 2, 3, 4} ; B = {2, 5, 8, 11, 14} ;

f (x) = 3x – 1

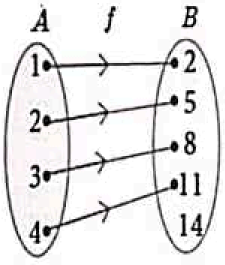
f(1) = 3(1) –1 = 3 – 1 = 2 ;

f(2) = 3(2) – 1 = 6 – 1 = 5

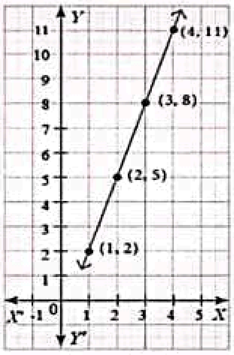
f(3) = 3(3) – 1 = 9 – 1 = 8 ;

f(4) = 4(3) – 1 = 12 – 1 = 11

(i) Arrow diagram



(iv) Graphical form



(ii) Table form

x	1	2	3	4
f(x)	2	5	8	11

(iii) Set of ordered pairs

f = {(1, 2), (2, 5), (3, 8), (4, 11)}

35. Given f(x) = 2x – x^2, find (i) f(1) (ii) f(x+1) (iii) f(x) + f(1)

Solution :

(i) f(1) = 2(1) – $(1)^2$ = 2 – 1 = 1

(ii) $f(x+1) = 2(x+1) - (x+1)^2$

$= 2x + 2 - (x^2 + 2x + 1)$

$= 2x + 2 - x^2 - 2x - 1 = -x^2 + 1$

(iii) f(x) + f(1)

f(1) = 2(1) – $(1)^2$ = 2 – 1 = 1

f(x) + f(1) = $(2x - x^2) + 1 = -x^2 + 2x + 1$

36. **Prove that the product of two consecutive positive integers is divisible by 2.**

Solution : Let the 2 consecutive positive integers be $x, x+1$ $\therefore$ Product of 2 integers $= x(x+1) = x^2 + x$

Case (i) If x is an even number Let $x = 2k$ $\therefore x^2 + x = (2k)^2 + 2k = 2k(2k+1)$ divisible by 2

Case (ii) If x is an odd number, Let $x = 2k+1$ $\therefore x^2 + x = (2k+1)^2 + (2k+1)$

$= 4k^2 + 4k + 1 + 2k + 1$

$= 4k^2 + 6k + 2$

$= 2(2k^2 + 3k + 2)$ divisible by 2

$\therefore$ Product of 2 consecutive positive integers is divisible by 2.

37. **If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$.**

Solution : a, b, c are three consecutive terms of an A.P. $\Rightarrow a, a+d, a+2d, \ldots\ldots.$

x, y, z are three consecutive terms of a G.P. $\Rightarrow x, xr, xr^2, \ldots\ldots$

$x^{b-c} \times y^{c-a} \times z^{a-b} = (x)^{-d} \times (xr)^{2d} \times (xr^2)^{-d} = x^0 \times r^{2d} \times r^{-2d} = x^0 \times r^0 = 1$ Hence proved.

38. **Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36?**

Solution : L.C.M of 24, 15, 36

$$\begin{array}{r|lll} 3 & 24, & 15, & 36 \\ \hline 2 & 8, & 5, & 12 \\ \hline 2 & 4, & 5, & 6 \\ \hline & 2, & 5, & 3 \end{array}$$

L.C.M $= 5 \times 3^2 \times 2^3 = 5 \times 9 \times 8 = 360$

The greatest 6 digit no. is 999999

$$\begin{array}{r|l} 360 & 999999 \;(277 \\ & 720 \\ \hline & 2799 \\ & 2520 \\ \hline & \;\;2799 \\ & \;\;2520 \\ \hline & \;\;\;279 \\ \hline \end{array}$$

$\therefore$ Required greatest number $= 999999 - 279 = 999720$

39. **Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.**

Solution :

$$\begin{array}{r|l} 2 & 408 \\ \hline 2 & 204 \\ \hline 2 & 102 \\ \hline 3 & 51 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

$\therefore 408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

$\therefore$ H.C.F $= 2 \times 17 = 34$

L.C.M $= 2^3 \times 17 \times 5 \times 3 = 2040$

40. **If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.**

Solution : $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

$$\begin{array}{r|l} 2 & 113400 \\ \hline 2 & 56700 \\ \hline 2 & 28350 \\ \hline 3 & 14175 \\ \hline 3 & 4725 \\ \hline 3 & 1575 \\ \hline 3 & 525 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\therefore 113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7$ and

$x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1$

41. Find the HCF of 396, 504, 636.

Solution : Using Euclid's division algorithm

$504 = 396 \times 1 + 108$

$396 = 108 \times 3 + 72$

$108 = 72 \times 1 + 36$

$72 = 36 \times 2 + 0$

To find the HCF of 636 and 36. $636 = 36 \times 17 + 24$

$36 = 24 \times 1 + 12$

$24 = 12 \times 2 + 0$ remainder is zero.

HCF of 636, 36 = 12

Highest Common Factor of 396, 504 and 636 is 12.

42. **The ratio of 6th and 8th term of an A.P. is 7 : 9. Find the ratio of 9th term to 13th term.**

Solution : **The ratio of 6th and 8th term of an A.P. is 7 : 9** $\Rightarrow \frac{t_6}{t_8} = \frac{7}{9}$

$$\Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9} \Rightarrow 9a+45d = 7a+49d \Rightarrow 2a = 4d \Rightarrow a = 2d \ .....(1)$$

$$\therefore \frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{2d+8d}{2d+12d} = \frac{10d}{14d} = \frac{5}{7} \text{ (from (1))} \qquad \therefore t_9 : t_{13} = 5 : 7$$

43. **The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.**

Solution : Let the 3 consecutive terms in an A.P. be $a-d,\ a,\ a+d$

Sum of 3 terms = 27 $\Rightarrow a-d+a+a+d = 27 \Rightarrow 3a = 27 \Rightarrow a = 9$

Product of 3 terms = 288 $\Rightarrow (a-d)\,.\,a\,.\,(a+d) = 288 \Rightarrow a^2(a^2-d^2) = 288 \Rightarrow 9(81-d^2) = 288$

$\Rightarrow 81 - d^2 = 32$

$\Rightarrow d^2 = 49$

$\Rightarrow d = \pm 7$

a = 9, d = 7 $\Rightarrow$ the 3 terms are 2, 9, 16

a = 9, d = –7 $\Rightarrow$ the 2 terms are 16, 9, 2

44. **Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872.**

Solution : Given $r = 5,\ S_6 = 46872$

$$S_n = a\,.\frac{r^n-1}{r-1} \Rightarrow a \times \frac{5^6-1}{4} = 46872$$

$$\Rightarrow a(5^6-1) = 46872 \times 4 \Rightarrow a(15624) = 46872 \times 4 \qquad \therefore a = \frac{46872 \times 4}{15624} = 3 \times 4 = 12$$

45. A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child. ***Solution :*** Let the amount form of A.P. $a-d,\ a,\ a+d.$

$(a-d) + a + (a+d) = 207 \Rightarrow 3a = 207 \Rightarrow a = 69$

It is given that product of the two least amounts is 4623 $\quad (a-d)\,a = 4623$

$(69-d)\,69 = 4623 \Rightarrow d = 2$

Amount given by the mother to her three children are

₹(69 –2), ₹69, ₹(69 + 2). That is, ₹67, ₹69 and ₹71.

46. In an A.P., sum of four consective terms is 28 and their sum of their squares is 276. Find the four numbers.

Solution : four consective terms A.P. $(a-3d), (a-d), (a+d)$ and $(a+3d)$.

sum of the four terms is 28.$\Rightarrow a-3d+a-d+a+d+a+3d = 28 \Rightarrow 4a = 28 \Rightarrow a = 7$

sum of their squares is 276, $(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276.$

$$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$$

$$4a^2 + 20d^2 = 276 \Rightarrow 4(7)^2 + 20d^2 = 276.$$

$$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

If $a = 7$, d = 2 then the four numbers are 1, 5, 9 and 13

If $a = 7$, d = –2 then the four numbers are 13, 9, 5 and 1

47. Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Solution : The required number is the HCF of the number 445 – 4 = 441, 572 – 5 = 567.

Using Euclid's Division Algorithm, 567 = 441 × 1 + 126

441 = 125 × 3 + 63

126 = 63 × 2 + 0 The remainder is zero.

HCF of 441,567 = 63 and the required number is 63.

48. **The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the value of n.**

Solution : $\frac{n(n+1)(2n+1)}{6} = 285$ ....(1)

$$\left(\frac{n(n+1)}{2}\right)^2 = 2025 \Rightarrow \left(\frac{n(n+1)}{2}\right) = 45 \text{......(2)}$$

$$(1) \Rightarrow \frac{n(n+1)}{2} \times \frac{2n+1}{3} = 285 \Rightarrow 45 \times \frac{2n+1}{3} = 285 \Rightarrow 2n+1 = \frac{285}{15} = 19 \Rightarrow 2n = 19-1$$

$$\Rightarrow 2n = 18 \quad \therefore n = 9$$

49. If $1+2+3+\ldots.+n = 666$ then find n.

Solution : $1+2+3+\ldots.+n = 666$

$$\frac{n(n+1)}{2} = 666 \Rightarrow n^2+n-1332 = 0 \Rightarrow n = -37 \text{ or } n = 36$$

$n \neq -37$ (Since n is a natural number) ; Hence $n = 36$.

50. **If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.**

Solution : Given $9(t_9) = 15(t_{15})$

To Prove : $6(t_{24}) = 0 \Rightarrow 9(t_9) = 15(t_{15}) \Rightarrow 9(a+8d) = 15(a+14d)$

$$\Rightarrow 3(a+8d) = 5(a+14d) \Rightarrow 3a+24d = 5a+70d$$

$$\Rightarrow 2a+46d = 0$$

$$\Rightarrow 2(a+23d) = 0$$

Multiplying 3 on both sides, $\Rightarrow 6(a+23d) = 0 \Rightarrow 6(t_{24}) = 0$

51. The sum of first, n, 2n and 3n terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution : $S_1 = t_1 = a$, $S_2 = t_1 + t_2 = a + a + d = 2a + d$, $S_3 = t_1 + t_2 + t_3 = a + a + d + a + 2d = 3a + 3d$

$$S_2 - S_1 = 2a + d - a = a + d$$

$$3(S_2 - S_1) = 3a + 3d = S_3$$

52. **The sum of first n terms of a certain series is given as $2n^2 - 3n$. Show that the series is an A.P.**

Solution : Given $S_n = 2n^2 - 3n$ $\quad n = 1 \Rightarrow S_1 = 2 - 3 = -1$

$$n = 2 \Rightarrow S_2 = 2(4) - 3(2) = 8 - 6 = 2$$

$$\therefore S_1 = t_1 = a = -1, \quad S_2 = 2 \Rightarrow t_2 + t_1 = 2 \Rightarrow t_2 - 1 = 2 \Rightarrow t_2 = 3$$

$$\therefore \quad a = -1, d = 3 - (-1) = 3 + 1 = 4$$

$\therefore$ The series is $-1 + 3 + 7 + \ldots\ldots\ldots$ is an A.P.

53. Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Solution : $301 + 308 + 315 + \ldots + 595.$ $\quad a = 301$; $d = 7$; $l = 595.$

$$n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{595-301}{7}\right) + 1 = 43 \qquad \left[\therefore S_n = {}^{n}\!/_{2}[a+l]\right]$$

$$S_{57} = \frac{43}{2}[301+595] = 19264.$$

54. **How many consecutive odd integers beginning with 5 will sum to 480?**

Solution : $5 + 7 + 9 + \ldots\ldots. n = 480$ $\quad \therefore \quad a = 5, d = 2, S_n = 480$

$$\Rightarrow {}^{n}\!/_{2}\,[2a+(n-1)d] = 480 \Rightarrow {}^{n}\!/_{2}\,[10+(n-1)d] = 480 \Rightarrow {}^{n}\!/_{2}\,[5+(n-1)] = 480$$

$$\Rightarrow n[n+4] = 480 \Rightarrow n^2 + 4n - 480 = 0$$

$$\Rightarrow (n+24)(n-20) = 0 \Rightarrow n = -24, \; n = 20$$

$$\therefore n = 20$$

55. **In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.**

Solution : $a, a+d, a+2d, a+3d, a+4d$ **are temperature of Ooty from Monday to Friday to be in A.P.**

Given $a + (a+d) + (a+2d) = 0 \Rightarrow 3a+3d = 0 \Rightarrow a+d=0 \Rightarrow a = -d$

Given $(a+2d) + (a+3d) + (a+4d) = 18 \Rightarrow 3a+9d = 18 \Rightarrow -3d+9d = 18 \Rightarrow 6d = 18$

$\Rightarrow d = 3 \quad \therefore a = -3$

The temperature of each of the 5 days -3^0 C, 0^0 C, 3^0 C, 6^0 C, 9^0 C

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56. The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number?

Solution : Let Senthil's house number be x.

$$1+2+3+....+(x-1) = (x-1)+(x+2)+....+49$$

$$1+2+3+......+(x-1) = [1+2+3+....+49]-[1+2+3+....+x]$$

$$\frac{x-1}{2}[1+(x-1)] = \frac{49}{2}[1+49]-\frac{x}{2}[1+x] \qquad \left[\therefore S_n = {}^{n}\!/_{2}[a+l]\right]$$

$$\frac{x(x-1)}{2} = \frac{49\times 50}{2} - \frac{x(x+1)}{2}$$

$$x^2 - x = 2450 - x^2 - x \Rightarrow 2x^2 = 2450 \quad x^2 = 1225 \text{ gives } x = 35$$

Senthil's hosue number is 35.

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57. **A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.**

(i) How many bricks are required for the top most step?

(ii) How many bricks are required to build the stair case?

Solution : $\therefore$ 100, 98, 96, 94, ... for 30 steps form an A.P. $\quad [t_n = a+(n-1)\,d]$

$a = 100, d = -2, n = 30$

i) No. of bricks used in the top most step $t_{30} = a+29d = 100+29(-2) = 100-58 = 42$

ii) Total no. of bricks used to build the stair case $S_{30} = \frac{30}{2}(100+42) = 15\times 142 = 2130 \quad \left[\therefore S_n = {}^{n}\!/_{2}[a+l]\right]$

---

58. **Find the sum** $\left[\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+\text{....... to 12 terms}\right]$

Solution : $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+\text{......... to 12 terms} \qquad a = \frac{a-b}{a+b},\ d = \frac{2a-b}{a+b}$

$$S_{12} = \frac{12}{2}\left[2\left(\frac{a-b}{a+b}\right)+11\left(\frac{2a-b}{a+b}\right)\right] = \frac{6}{a+b}[24a-13b] \qquad (\because S_n = \frac{n}{2}[2a+(n-1)d])$$

---

59. **If (m + 1)th term of an A.P. is twice the (n + 1)th term, then prove that (3m + 1)th term is twice the (m + n + 1)th term.**

Solution : Given $t_{m+1} = 2(t_{n+1})$

$$a+(m+1-1)\,d = 2\,(a+(n+1-1)\,d)$$

$$a+md = 2\,(a+nd) \qquad [t_n = a+(n-1)\,d]$$

$$a+md = 2a+2nd \quad —(1)$$

To Prove : $t_{3m+1} = 2\,(t_{m+n+1})$

$$t_{3m+1} = a+(3m+1-1)d = a+3md = (a+md)+2md = 2a+2nd+2md \quad \text{(from (1))}$$

$$= 2\,[a+(m+n)d]$$

$$= 2\,[t_{m+n+1}]$$

60. Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution : $5+55+555+....+n$ terms $=5\,[1+11+111+....+\text{n terms}]$

$$=\frac{5}{9}[9+99+999+....+n\text{ terms}]$$

$$=\frac{5}{9}[(10-1)+(100-1)+(1000-1)+....+n\text{ terms}]$$

$$=\frac{5}{9}[(10+100+1000+...+n\text{ terms})-n] \qquad \left[\therefore S_n=a\,.\frac{r^n-1}{r-1}\right]$$

$$=\frac{5}{9}\left[\frac{10(10^n-1)}{(10-1)}-n\right]=\frac{50(10^n-1)}{81}-\frac{5n}{9}$$

---

61. **In a G.P. the 9th term is 32805 and 6th term is 1215. Find the 12th term.**

Solution : Given $t_9=32805,\ t_6=1215,\ t_{12}=?$

$a\,.\,r^8=32805$ .... (1)

$a\,.\,r^5=1215$ .... (2)

$(1)\div(2)\Rightarrow\ \frac{\cancel{a}\,.\,\cancel{r^8}^3}{\cancel{a}\,.\,\cancel{r^5}}=\frac{32805}{1215}\ \Rightarrow\ r^3=\frac{32805}{1215}\Rightarrow r^3=27\ \Rightarrow\ r^3=3^3\Rightarrow r=3$

Sub. $r=3$ in (2) $\quad a\times3^5\ =1215\Rightarrow a\times243=1215\ \Rightarrow\ a=\frac{1215}{243}\ \Rightarrow\ a=5$

$\Rightarrow\ \therefore t_{12}=a\,.r^{11}\ =5\times3^{11}\qquad [t_n=a+(n-1)\,d]$

---

62. The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Solution : 3 consecutive terms $\frac{a}{r}$, a, ar.

Product of the terms = 343

$\frac{a}{r}\times a\times ar=343$

$a^3=7^3$

$a=7$

Sum of the terms $=\frac{91}{3}$

$\cancel{7}\left(\frac{1+r+r^2}{r}\right)=\frac{\cancel{91}^{13}}{3}\ \Rightarrow\ 3+3r+3r^2=13r$

$\Rightarrow 3r^2-10r+3=0$

$\Rightarrow(3r-1)\,(r-3)=0$

$r=3$ or $r=\frac{1}{3}$

If $a=7$, r = 3 then the three terms are $\frac{7}{3}$, 7, 21.

If $a=7$, $r=\frac{1}{3}$ then the three terms are 21, 7, $\frac{7}{3}$.

---

63. **Find the sum to n terms of the series 0.4 + 0.44 + 0.444 + ... to n terms**

Solution :

0.4 + 0.44 + 0.444 + ... to n terms $=\frac{4}{10}+\frac{44}{100}+\frac{444}{1000}+........$ to n terms

$$=4\left[\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+......n\text{ terms}\right] \qquad \left[\therefore S_n=a\,.\frac{r^n-1}{r-1}\right]$$

$$=\frac{4}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+......n\text{ terms}\right]$$

$$=\frac{4}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+......n\text{ terms}\right]$$

$$= \frac{4}{9}\left[(1+1+1+\ldots\ldots n \text{ terms}) - \left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots\ldots n \text{ terms}\right)\right] = \frac{4}{9}\left[n - \frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right] = \frac{4}{9}\left[n - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right]$$

---

64. **Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they con-tinue the process similarly. Assuming that the process is unaltered and it costs ₹ 2 to mail one letter, spent on find the amount postage when 8th set of letters is mailed.**

Solution : ∴ The total cost $= (4\times 2)+(16\times 2)+(64\times 2)+\ldots\ldots\ldots\ 8^{\text{th}}$ set

$= 8+32+28+\ldots\ldots\ldots\ 8^{\text{th}}$ set

$$S_8 = 8.\frac{4^8-1}{3} = 8\times\frac{65535}{3} \qquad \therefore a=8, r=4, n=8 \qquad \left[\therefore S_n = a.\frac{r^n-1}{r-1}\right]$$

$$= 8\times 21845 = ₹174760$$

---

65. **If $S_n = (x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\ldots\ldots\ n$ terms then prove that**

$$(x-y)\,S_n = \left|\frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1}\right|$$

Solution : $(x-y)\,S_n = (x^2-y^2)+(x^3+y^3)+(x^4-y^4)+\ldots\ldots n$ terms

$= (x^2+x^3+x^4+\ldots\ldots n \text{ terms}) - (y^2+y^3+y^4+\ldots\ldots n \text{ terms})$

$$= \frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1} \qquad \left(a=x^2, r=x \;\&\; a=y^2, r=y \quad \because S_n = a.\frac{r^n-1}{r-1}\right)$$

---

66. **Find the sum to n terms of the series $3+33+333+\ldots\ldots$ to n terms**

Solution $3(1+11+111+\ldots\ldots+n \text{ terms}) = \frac{3}{9}(9+99+999+\ldots\ldots+n \text{ terms})$

$$= \frac{3}{9}[(10-1)+(100-1)+(1000-1)+\ldots\ldots n \text{ terms}] \qquad \left[S_n = a.\frac{r^n-1}{r-1}\right]$$

$$= \frac{3}{9}[(10+100+1000+\ldots\ldots n \text{ terms}] - [(1+1+1+\ldots\ldots n \text{ terms}]$$

$$= \frac{3}{9}\left[10.\left(\frac{10^n-1}{n}\right)-n\right] = \frac{30}{81}(10^n-1)-\frac{3n}{9} = \frac{10}{27}(10^n-1)-\frac{n}{3}$$

---

67. Find the sum of $10^3+11^3+12^3+\ldots\ldots+20^3$

$10^3+11^3+12^3+\ldots\ldots+20^3 = (1^3+2^3+\ldots\ldots+20^3)-(1^3+2^3+\ldots\ldots+9^3)$

$$= \left(\frac{20\times 21}{2}\right)^2 - \left(\frac{9\times 10}{2}\right)^2 = (210)^2-(45)^2 \qquad \left(\sum_{k=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2\right)$$

$$= (210+45)(210-45) = (255)\times(165) = 42075$$

---

68. **Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.**

Solution :

Case (i) $a=603, d=1, l=901 \Rightarrow n = \frac{901-603}{1}+1 = 298+1 = 299$

$$S_{299} = \frac{299}{2}\times 1504 = 299\times 752 = 224848 \qquad \left[\therefore n = \frac{l-a}{d}+1 \;\; \therefore S_n = \frac{n}{2}[a+l]\right]$$

Case (ii) $a=604, \quad d=4, \quad l=900 \Rightarrow n = \frac{900-604}{4}+1 = \frac{296}{4}+1 = 74+1 = 75$

$$S_{75} = \frac{75}{2}\times 1504 = 75\times 752 = 56{,}400$$

∴ **Sum of all natural numbers between 602 and 902 which are not divisible by 4**

$= 224848 - 56400 = 168448$

69. Find the sum of $9^3 + 10^3 + ... + 21^3$

Solution : $9^3+10^3+....+21^3=(1^3+2^3+3^3+.....+21^3)-(1^3+2^3+3^3+....+8^3)$

$$=\left[\frac{21\times(21+1)}{2}\right]^2=\left[\frac{8\times(8+1)}{2}\right]^2=(231)^2-(36)^2=52065 \quad \left[\therefore \sum_{k=1}^{n} K^3=\left(\frac{n(n+1)}{2}\right)^2\right]$$

---

70. Find the sum of $5^2 + 10^2 + 15^2 + ... + 105^2$

Solution : $5^2+10^2+15^2+....+105^2 = 5^2(1^2+2^2+3^2+....+21^2)$

$$=25\times\frac{25\times(21+1)\,(2\times21+1)}{6}=\frac{25\times21\times22\times43}{6}=82775 \quad \left[\therefore \sum_{k=1}^{n} K^2=\frac{n(n+1)\,(2n+1)}{6}\right]$$

---

71. Find the sum of $15^2 + 16^2 + 17^2 + ... 28^2$

Solution : $15^2+16^2+17^2+....28^2 = (1^2+2^2+3^2+....+28^2)-(1^2+2^2+3^2+.....+14^2)$

$$=\frac{28\times29\times57}{6}-\frac{14\times15\times29}{6}=7714-1015=6699 \quad \left[\therefore \sum_{k=1}^{n} K^2=\frac{n(n+1)\,(2n+1)}{6}\right]$$

---

72. **Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,..., 24 cm. How much area can be decorated with these colour papers?**

Solution : 10 cm, 11 cm, 12 cm, ... 24 cm

$10^2 + 11^2 + 12^2 + ... + 24^2 = (1^2 + 2^2 + 3^2 + ... + 24^2) - (1^2 + 2^2 + ... + 9^2)$ $\left[\therefore \sum_{k=1}^{n} K^2=\frac{n(n+1)\,(2n+1)}{6}\right]$

$$=\frac{24\times25\times49}{6}-\frac{9\times10\times19}{6}=4900-285=4615\text{ cm}^2$$

---

73. **Find the sum of the series to $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) +$ .... to (i) n terms (ii) 8 terms**

Solution : (i) $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) +$ .... n terms

$= (2^3 + 4^3 + 6^3 +$ .... n terms$) - (1^3 + 3^3 + 5^3 +$ .... n terms$)$

$$=12\sum n^2-6\sum n+n=\overset{2}{\cancel{12}}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right]-\overset{3}{\cancel{6}}\left[\frac{n(n+1)}{\cancel{2}}\right]+n=4n^3+3n^2$$

(ii) $S_8 = 4(8^3) + 3(8^2) = 4(512) + 3(64) = 2048 + 192 = 2240$

---

74. **How many terms of the series $1^3 + 2^3 + 3^3 +$ .... should be taken to get the sum 14400?**

Solution : $1^3 + 2^3 + 3^3 + ... + k^3 = 14400$

$$\left(\frac{k(k+1)}{2}\right)^2=14400 \Rightarrow \frac{k(k+1)}{2}=120 \Rightarrow k^2+k-240=0$$

$$\Rightarrow (k+16)\,(k-15)=0 \qquad \therefore k=15$$

15 terms of the series $1^3 + 2^3 + 3^3 +$ .... should be taken to get the sum 14400.

---

75. How many terms of the series 1 + 4 + 16 + ······ make the sum 1365 ?

Solution : $a=1, r=4$ $\quad \therefore S_n=1365$

$$\therefore \frac{a(r^n-1)}{r-1}=1365$$

$$\frac{1(4^n-1)}{4-1}=1365$$

$$\frac{(4^n-1)}{3}=1365 \Rightarrow (4^n-1)=4095 \Rightarrow 4^n=4096 \Rightarrow 4^n=4^6 \Rightarrow n=6$$

$\therefore$ 6 terms of the series 1 + 4 + 16 + ······ make the sum 1365

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76. If $4x^4 - 12x^3 + 37x^2 + bx + a$ is perfect square, Find the values of a and b

Solution :

$$\begin{array}{r|lllll}
 & 2 & -3 & 7 & & \\
\hline
2 & 4 & -12 & 37 & b & a \\
 & 4 & & & & \\
 & (-) & & & & \\
\hline
4 \quad -3 & & -12 & 37 & & \\
 & & -12 & 9 & & \\
 & & (+) & (-) & & \\
\hline
4 \quad -6 \quad 7 & & & 28 & b & a \\
 & & & 28 & -42 & 49 \\
\hline
 & & & 0 & & \\
\hline
\end{array}$$

$\therefore \; a = 49 \quad , \quad b = -42$

77. If $ax^4 + bx^3 + 361x^2 + 220x + 100$ is perfect square, Find the values of a and b

Solution :

$$\begin{array}{r|lllll}
 & 10 & 11 & 12 & & \\
\hline
10 & 100 & 220 & 361 & b & \\
 & 100 & & & & \\
 & (-) & & & & \\
\hline
20 \quad 11 & & 220 & 361 & & \\
 & & 220 & 121 & & \\
 & & (-) & (-) & & \\
\hline
20 \quad 22 \quad 12 & & & 240 & b & a \\
 & & & 240 & 264 & 144 \\
\hline
 & & & 0 & & \\
\hline
\end{array}$$

$\therefore \; a = 144 \quad , \quad b = 264$

78. If $x^4 - 8x^3 + mx^2 + nx + 16$ is perfect square, Find the values of m and n

Solution :

$$\begin{array}{r|lllll}
 & 1 & -4 & 4 & & \\
\hline
1 & 1 & -8 & m & n & 16 \\
 & 1 & & & & \\
 & (-) & & & & \\
\hline
2 \quad -4 & & -8 & m & & \\
 & & -8 & 16 & & \\
 & & (+) & (-) & & \\
\hline
2 \quad -8 \quad 4 & & & (m-16) & n & 16 \\
 & & & 8 & -32 & 16 \\
\hline
 & & & 0 & & \\
\hline
\end{array}$$

$\therefore \; m = 24 \quad , \quad n = -32$

79. Find $\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1}$

Solution :

$$\begin{array}{r|lllll}
 & 8 & -1 & 1 & & \\
\hline
8 & 64 & -16 & 17 & -2 & 1 \\
 & 64 & & & & \\
 & (-) & & & & \\
\hline
16 \quad -1 & & -16 & 17 & & \\
 & & -16 & 1 & & \\
 & & (+) & (-) & & \\
\hline
16 \quad -2 \quad 1 & & & 16 & -2 & 1 \\
 & & & 16 & -2 & 1 \\
 & & & (-) & (+) & (-) \\
\hline
 & & & 0 & & \\
\hline
\end{array}$$

$$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$$

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80. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution :

$$\begin{array}{r|rrrrr}
 & 3 & 2 & 4 & & \\ \hline
3 & 9 & 12 & 28 & a & b \\
 & 9 & & & & \\
 & (-) & & & & \\ \hline
6 \quad 2 & & 12 & 28 & & \\
 & & 12 & 4 & & \\
 & & (-) & (-) & & \\ \hline
6 \quad 4 \quad 4 & & & 24 & a & b \\
 & & & 24 & 16 & 16 \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$\therefore \quad a = 16, b = 16.$

81. **Find the square root by division method $121x^4 - 198x^3 - 183x^2 + 216x + 144$**

Solution :

$$\begin{array}{r|rrrrr}
 & 11 & -9 & -12 & & \\ \hline
11 & 121 & -198 & -183 & 216 & 144 \\
 & 121 & & & & \\
 & (-) & & & & \\ \hline
22 \quad -9 & & -198 & -183 & & \\
 & & -198 & 81 & & \\
 & & (+) & (-) & & \\ \hline
22 \quad -18 \quad -12 & & & -264 & 216 & 144 \\
 & & & -264 & 216 & 144 \\
 & & & (+) & (-) & \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144}$$
$$= \left| 11x^2 - 9x - 12 \right|$$

82. **Find the square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ by division method**

Solution :

$$\begin{array}{r|rrrrr}
 & 1 & -6 & 3 & & \\ \hline
1 & 1 & -12 & 42 & -36 & 9 \\
 & 1 & & & & \\
 & (-) & & & & \\ \hline
2 \quad -6 & & -12 & 42 & & \\
 & & -12 & 36 & & \\
 & & (+) & (-) & & \\ \hline
2 \quad -12 \quad 3 & & & 6 & -36 & 9 \\
 & & & 6 & -36 & 9 \\
 & & & (-) & (+) & (-) \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = \left| x^2 - 6x + 3 \right|$$

83. **Find the square root of $37x^2 - 28x^3 + 4x^4 + 42x + 9$ by division method**

Solution :

$$\begin{array}{r|rrrrr}
 & 2 & -7 & -3 & & \\ \hline
2 & 4 & -28 & 37 & 42 & 9 \\
 & 4 & & & & \\
 & (-) & & & & \\ \hline
4 \quad -7 & & -28 & 37 & & \\
 & & -28 & 49 & & \\
 & & (+) & (-) & (+) & \\ \hline
4 \quad -14 \quad -3 & & & -12 & 42 & 9 \\
 & & & -12 & 42 & 9 \\
 & & & (+) & (-) & (-) \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$$\therefore \sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = \left| 2x^2 - 7x - 3 \right|$$

84. **Find the square root of $289x^4 - 612x^3 + 970x^2 - 684x + 361$.**

Solution :

$$\begin{array}{r|rrrrr}
 & 17 & -18 & 19 & & \\ \hline
17 & 289 & -612 & 970 & -684 & 361 \\
 & 289 & & & & \\
 & (-) & & & & \\ \hline
34 \quad -18 & & -612 & 970 & & \\
 & & -612 & 324 & & \\
 & & (+) & (-) & & \\ \hline
34 \quad -36 \quad 19 & & & 646 & -684 & 361 \\
 & & & 646 & -684 & 361 \\
 & & & (-) & (+) & (-) \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$$\therefore \sqrt{289x^4 - 612x^3 + 970x^2 - 684x + 361}$$
$$= \left| 17x^2 - 18x + 19 \right|$$

85. **Find the GCD of the following by division algorithm $3x^4+6x^3-12x^2-24x,\ 4x^4+14x^3+8x^2-8x$**

Solution : Let $f(x)=4x^4+14x^3+8x^2-8x = 2x\,(2x^3+7x^2+4x-4)$

$g(x)=3x^4+6x^3-12x^2-24x = 3x\,(1x^3+2x^2-4x-8)$ GCD of 2x , 3x = x

$$\begin{array}{r|rrrr} & 2 & & & \\ \hline 1\quad 2\quad -4\quad -8 & 2 & 7 & 4 & -4 \\ & 2 & 4 & -8 & -16 \\ & (-) & (-) & (+) & (+) \\ \hline & & 3 & 12 & 12 \end{array}$$

$\therefore 3\,(x^2+4x+4)\neq 0$

3 is not a divisor of f(x)

$$\begin{array}{r|rrrr} & 1 & -2 & & \\ \hline 1\quad 4\quad 4 & 1 & 2 & -4 & -8 \\ & (-)1 & (-)4 & (-)4 & \\ \hline & & -2 & -8 & -8 \\ & & (+)-2 & (+)-8 & (+)-8 \\ \hline & & & 0 & \end{array}$$

$\therefore$ G.C.D $= x\,(x^2+4x+4)$

86. **Find the GCD of $x^4+3x^3-x-3,\ x^3+x^2-5x+3$**

Solution : $f(x)=x^4+3x^3-x-3$ $g(x)=x^3+x^2-5x+3$

$$\begin{array}{r|rrrrr} & 1 & 2 & & & \\ \hline 1\quad 1\quad -5\quad 3 & 1 & 3 & 0 & -1 & -3 \\ & (-)1 & (-)1 & (+)-5 & (-)3 & \\ \hline & & 2 & 5 & -4 & -3 \\ & & (-)2 & (-)2 & (+)-10 & (-)6 \\ \hline & & & 3 & 6 & -9 \end{array}$$

$3\,(x^2+2x-3)\neq 0$

3 is not a divisor of f(x) , g(x)

$$\begin{array}{r|rrrr} & 1 & -1 & & \\ \hline 1\quad 2\quad -3 & 1 & 1 & -5 & 3 \\ & (-)1 & (-)2 & (+)-3 & \\ \hline & & -1 & -2 & 3 \\ & & (+)-1 & (+)-2 & (-)3 \\ \hline & & & 0 & \end{array}$$

$\therefore$ **GCD** $=(x^2+2x-3)$

87. **Find the GCD of the following by division algorithm $3x^3+3x^2+3x+3,\ 6x^3+12x^3+6x+12$**

Solution : Let $f(x)=6x^3+12x^2+6x+12 = 6\,(x^3+2x^2+x+2)$

$g(x)=3x^3+3x^2+3x+3 = 3\,(x^3+x^2+x+1)$ GCD of 6, 3 = 3

$$\begin{array}{r|rrrr} & 1 & & & \\ \hline 1\quad 1\quad 1\quad 1 & 1 & 2 & 1 & 2 \\ & (-)1 & (-)1 & (-)1 & (-)1 \\ \hline & & 1 & 0 & 1 \end{array}$$

$\therefore x^2+1\neq 0$

$$\begin{array}{r|rrrr} & 1 & 1 & & \\ \hline 1\quad 0\quad 1 & 1 & 1 & 1 & 1 \\ & 1 & (-)0 & (-)1 & 0 \\ \hline & & 1 & 0 & 1 \\ & & (-)1 & (-)0 & (-)1 \\ \hline & & & 0 & \end{array}$$

$\therefore$ G.C.D $=3\,(x^2+1)$

88. **Find the GCD of the following by division algorithm $2x^4+13x^3+27x^2+23x+7,\ x^3+3x^2+3x+1,\ x^2+2x+1$**

Solution :

$$\begin{array}{r|rrrr} & 1 & 1 & & \\ \hline 1\quad 2\quad 1 & 1 & 3 & 3 & 1 \\ & (-)1 & (-)2 & (-)1 & \\ \hline & & 1 & 2 & 1 \\ & & (-)1 & (-)2 & (-)1 \\ \hline & & & 0 & \end{array}$$

$$\begin{array}{r|rrrrr} & 2 & 9 & 7 & & \\ \hline 1\quad 2\quad 1 & 2 & 13 & 27 & 23 & 7 \\ & (-)2 & (-)4 & (-)2 & & \\ \hline & & 9 & 25 & 23 & \\ & & (-)9 & (-)18 & (-)9 & \\ \hline & & & 7 & 14 & 7 \\ & & & (-)7 & (-)14 & (-)7 \\ \hline & & & 0 & & \end{array}$$

$\therefore$ G.C.D. $=x^2+2x+1$

89. Find the GCD of $6x^3 - 30x^2 + 60x - 48$ and $3x^3 - 12x^2 + 21x - 18$

Solution : Let $f(x) = 6x^3 - 30x^2 + 60x - 48 = 6\,(x^3 - 5x^2 + 10x - 8)$

$g(x) = 3x^3 - 12x^2 + 21x - 18 = 3\,(x^3 - 4x^2 + 7x - 6)$ GCD of 3 and 6 is 3.

$$\begin{array}{r|rrrr} & 1 & & & \\ \hline 1\ \ -5\ \ \ 10\ \ -8 & 1 & -4 & 7 & -6 \\ & 1 & -5 & 10 & -8 \\ & (-) & (+) & (-) & (+) \\ \hline & & 1 & -3 & 2 \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -2 & & \\ \hline 1\ -\ 3\ \ \ 2 & 1 & -5 & 10 & -8 \\ & 1 & -3 & 2 & \\ & (-) & (+) & (-) & \\ \hline & & -2 & 8 & -8 \\ & & -2 & 6 & -4 \\ & & (+) & (-) & (+) \\ \hline & & & 2 & -4 \end{array}$$

$2\,(x - 2) \neq 0$

2 is not a divisor of g(x)

$$\begin{array}{r|rrr} & 1. & -1 & \\ \hline 1\ -2 & 1 & -3 & 2 \\ & 1 & -2 & \\ & (-) & (+) & \\ \hline & & -1 & 2 \\ & & -1 & 2 \\ & & (+) & (-) \\ \hline & & 0 & \end{array}$$

GCD $= 3(x - 2)$

---

90. Find the GCD of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

Solution : Let $f(x) = 2x^3 - 5x^2 + 5x - 3$ and $g(x) = x^3 + x^2 - x + 2$

$$\begin{array}{r|rrrr} & 2 & & & \\ \hline 1\ \ \ 1\ \ -1\ \ \ 2 & 2 & -5 & 5 & -3 \\ & 2 & 2 & -2 & 4 \\ & (-) & (-) & (+) & (-) \\ \hline & & -7 & 7 & -7 \\ \hline \end{array}$$

$-7\,(x^2 - x + 1) \neq 0$

-7 is not a divisor of g(x)

$$\begin{array}{r|rrrr} & 1 & 2 & & \\ \hline 1\ -1\ \ \ 1 & 1 & 1 & -1 & 2 \\ & 1 & -1 & 1 & \\ & (-) & (+) & (-) & \\ \hline & & 2 & -2 & 2 \\ & & 2 & -2 & 2 \\ & & (-) & (+) & (-) \\ \hline & & & 0 & \\ \hline \end{array}$$

GCD $= x^2 - x + 1$

---

91. **Find the GCD of the following by division algorithm** $x^3 - 11x^2 + x - 11$, $x^4 - 1$

Solution : Let $f(x) = x^4 - 1$ $g(x) = x^3 - 11x^2 + x - 11$

$$\begin{array}{r|rrrrr} & 1 & 11 & & & \\ \hline 1\ -11\ \ \ 1\ -11 & 1 & 0 & 0 & 0 & -1 \\ & 1 & -11 & 1 & -11 & \\ & (-) & (+) & (-) & (+) & \\ \hline & & 11 & -1 & 11 & -1 \\ & & 11 & -121 & 11 & -121 \\ & & (-) & (+) & (-) & (+) \\ \hline & & & 120 & 0 & 120 \\ \hline \end{array}$$

$120(x^2 + 0x + 1) \neq 0$

120 is not a divisor of f(x) , g(x)

$$\begin{array}{r|rrrr} & 1 & 0 & -11 & \\ \hline 1\ \ \ 0\ \ \ 1 & 1 & -11 & 1 & -11 \\ & 1 & 0 & 1 & \\ & (-) & (-) & (-) & \\ \hline & & -11 & 0 & -11 \\ & & -11 & 0 & -11 \\ & & (+) & (-) & (+) \\ \hline & & & 0 & \\ \hline \end{array}$$

GCD $= x^2 + 1$

92. Find the square root of $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$

Solution : $\sqrt{(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)} = \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$

$= |(3x-1)(2x+1)(x+1)|$

93. Simplify $\dfrac{1}{x^2-5x+6}+\dfrac{1}{x^2-3x+2}-\dfrac{1}{x^2-8x+15}$

Solution :

$$\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15} = \frac{1}{(x-2)(x-3)}+\frac{1}{(x-2)(x-1)}-\frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x^2-6x+5)+(x^2-8x+15)-(x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+8}{(x-1)(x-2)(x-3)(x-5)} = \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)} = \frac{x-9}{(x-1)(x-3)(x-5)}$$

94. **Find the square root of** $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

Solution : $\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)} = \sqrt{(4x-1)(x-2)(7x+1)(x-2).(7x+1)(4x-1)}$

$= |(7x+1)(4x-1)(x-2)|$

95. **Find the square root of the following** $\left(2x^2+\dfrac{17}{6}x+1\right)\left(\dfrac{3}{2}x^2+4x+2\right)\left(\dfrac{4}{3}x^2+\dfrac{11}{3}x+2\right)$

Solution :

$$\sqrt{(2x^2+{}^{17}\!/_6\,x+1)\,({}^{3}\!/_2\,x^2+4x+2)\,\;({}^{4}\!/_3\,x^2+{}^{11}\!/_3\,x+2)} = \sqrt{\frac{(12x^2+17x+6)}{6}.\frac{(3x^2+8x+4)}{2}.\frac{(4x^2+11x+6)}{3}}$$

$$= \sqrt{\frac{(4x+3)(3x+2).(3x+2)(x+2).(4x+3)(x+2)}{36}}$$

$$= {}^{1}\!/_6\sqrt{(4x+3)^2.(3x+2)^2.(x+2)^2}$$

$$= {}^{1}\!/_6\,|(4x+3)(3x+2)(x+2)|$$

---

96. **Find X and Y if** $X+Y=\begin{pmatrix}7&0\\3&5\end{pmatrix}$ **and** $X-Y=\begin{pmatrix}3&0\\0&4\end{pmatrix}$

Solution :

$$X+Y=\begin{pmatrix}7&0\\3&5\end{pmatrix}\text{ .... (1)}\qquad X-Y=\begin{pmatrix}3&0\\0&4\end{pmatrix}\text{ .... (2)}$$

$$(1)+(2)\Rightarrow 2X=\begin{pmatrix}10&0\\3&9\end{pmatrix}\Rightarrow X=\begin{pmatrix}5&0\\3/2&9/2\end{pmatrix}\qquad (1)-(2)\Rightarrow 2Y=\begin{pmatrix}4&0\\3&1\end{pmatrix}\Rightarrow Y=\begin{pmatrix}2&0\\3/2&1/2\end{pmatrix}$$

---

97. If $A=\begin{pmatrix}2&1\\1&3\end{pmatrix}, B=\begin{pmatrix}2&0\\1&3\end{pmatrix}$ find AB and BA. Check if AB = BA.

Solution :

$$AB=\begin{pmatrix}2&1\\1&3\end{pmatrix}\times\begin{pmatrix}2&0\\1&3\end{pmatrix}=\begin{pmatrix}(2\ \ 1)\begin{pmatrix}2\\1\end{pmatrix}&(2\ \ 1)\begin{pmatrix}0\\3\end{pmatrix}\\(1\ \ 3)\begin{pmatrix}2\\1\end{pmatrix}&(1\ \ 3)\begin{pmatrix}0\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}4+1&0+3\\2+3&0+9\end{pmatrix}=\begin{pmatrix}5&3\\5&9\end{pmatrix}$$

$$BA=\begin{pmatrix}2&0\\1&3\end{pmatrix}\times\begin{pmatrix}2&1\\1&3\end{pmatrix}=\begin{pmatrix}(2\ \ 0)\begin{pmatrix}2\\1\end{pmatrix}&(2\ \ 0)\begin{pmatrix}1\\3\end{pmatrix}\\(1\ \ 3)\begin{pmatrix}2\\1\end{pmatrix}&(1\ \ 3)\begin{pmatrix}1\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}4+0&2+0\\2+3&1+9\end{pmatrix}=\begin{pmatrix}4&2\\5&10\end{pmatrix}\qquad AB\neq BA$$

---

98. If $A=(1\ \ -1\ \ 2),\ B=\begin{pmatrix}1&-1\\2&1\\1&3\end{pmatrix}$ and $C=\begin{pmatrix}1&2\\2&-1\end{pmatrix}$ show that(AB)C = A(BC).

Solution :

$$(AB)=(1\ \ -1\ \ 2)\times\begin{pmatrix}1&-1\\2&1\\1&3\end{pmatrix}=\begin{pmatrix}(1\ -1\ 2)\begin{pmatrix}1\\2\\1\end{pmatrix}&(1\ -1\ 2)\begin{pmatrix}-1\\1\\3\end{pmatrix}\end{pmatrix}=(1-2+2-1-1+6)\ =(1\ \ 4)$$

$$(AB)\,C=(1\ \ 4)\times\begin{pmatrix}1&2\\2&-1\end{pmatrix}=\begin{pmatrix}(1\ 4)\begin{pmatrix}1\\2\end{pmatrix}&(1\ 4)\begin{pmatrix}2\\-1\end{pmatrix}\end{pmatrix}=(1+8\ \ 2-4)=(9\ \ -2)\text{......(1)}$$

$$BC=\begin{pmatrix}1&-1\\2&1\\1&3\end{pmatrix}\times\begin{pmatrix}1&2\\2&-1\end{pmatrix}=\begin{pmatrix}(1\ -1)\begin{pmatrix}1\\2\end{pmatrix}&(1\ -1)\begin{pmatrix}2\\-1\end{pmatrix}\\(2\ \ 1)\begin{pmatrix}1\\2\end{pmatrix}&(2\ \ 1)\begin{pmatrix}2\\-1\end{pmatrix}\\(1\ \ 3)\begin{pmatrix}1\\2\end{pmatrix}&(1\ \ 3)\begin{pmatrix}2\\-1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-2&2+1\\2+2&4-1\\1+6&2-3\end{pmatrix}=\begin{pmatrix}-1&3\\4&3\\7&-1\end{pmatrix}$$

$$A(BC)=(1\ \ -1\ \ 2)\times\begin{pmatrix}-1&3\\4&3\\7&-1\end{pmatrix}=\begin{pmatrix}(1\ -1\ 2)\begin{pmatrix}-1\\4\\7\end{pmatrix}&(1\ -1\ 2)\begin{pmatrix}3\\3\\-1\end{pmatrix}\end{pmatrix}$$

Life is a Good Circle,

You Choose the Best Radius...

$$=(-1-4+14 \quad 3-3-2) \;=\; (9 \quad -2) \text{............} (2)$$

From (1) and (2), (AB)C = A(BC).

---

99. If $A=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}, B=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}, C=\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}$ verify that A (B + C) = AB + AC.

Solution :

$$B+C=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}+\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}=\begin{pmatrix}(1\;\;1)\begin{matrix}-6\\-1\end{matrix} & (1\;\;1)\begin{matrix}8\\4\end{matrix}\\ (-1\;\;3)\begin{matrix}-6\\-1\end{matrix} & (-1\;\;3)\begin{matrix}8\\4\end{matrix}\end{pmatrix}=\begin{pmatrix}-6-1 & 8+4\\ 6-3 & -8+12\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix} \text{...} (1)$$

$$AB=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}=\begin{pmatrix}(1\;\;1)\begin{matrix}1\\-4\end{matrix} & (1\;\;1)\begin{matrix}2\\2\end{matrix}\\ (-1\;\;3)\begin{matrix}1\\-4\end{matrix} & (-1\;\;3)\begin{matrix}2\\2\end{matrix}\end{pmatrix}=\begin{pmatrix}1-4 & 2+2\\ -1-12 & -2+6\end{pmatrix}=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}$$

$$AC=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}(1\;\;1)\begin{matrix}-7\\3\end{matrix} & (1\;\;1)\begin{matrix}6\\2\end{matrix}\\ (-1\;\;3)\begin{matrix}-7\\3\end{matrix} & (-1\;\;3)\begin{matrix}6\\2\end{matrix}\end{pmatrix}=\begin{pmatrix}-7+3 & 6+2\\ 7+9 & -6+6\end{pmatrix}=\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}$$

$$AB+AC=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}+\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix} \text{......} (2)$$

From (1) and (2), A (B + C) = AB + AC. Hence proved.

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100. If $A=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$ show that $(AB)^T=B^T A^T$.

Solution :

$$AB=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}\times\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}=\begin{pmatrix}(1\;2\;1)\begin{matrix}2\\-1\\0\end{matrix} & (1\;2\;1)\begin{matrix}-1\\4\\2\end{matrix}\\ (2\;-1\;1)\begin{matrix}2\\-1\\0\end{matrix} & (2\;-1\;1)\begin{matrix}-1\\4\\2\end{matrix}\end{pmatrix}=\begin{pmatrix}2-2+0 & -1+8+2\\ 4+1+0 & -2-4+2\end{pmatrix}=\begin{pmatrix}0 & 9\\ 5 & -4\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix} \text{..........} (1)$$

$$B^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}, A^T=\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}$$

$$B^T A^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}(2\;-1\;0)\begin{matrix}1\\2\\1\end{matrix} & (2\;-1\;0)\begin{matrix}2\\-1\\1\end{matrix}\\ (-1\;4\;2)\begin{matrix}1\\2\\1\end{matrix} & (-1\;4\;2)\begin{matrix}2\\-1\\1\end{matrix}\end{pmatrix}=\begin{pmatrix}2-2+0 & 4+1+0\\ -1+8+2 & -2-4+2\end{pmatrix}=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix} \text{.....} (2)$$

From (1) and (2), $(AB)^T=B^T A^T$. Hence proved.

101. **If** $A=\begin{pmatrix}2&5\\4&3\end{pmatrix}, B=\begin{pmatrix}1&-3\\2&5\end{pmatrix}$ **find AB, BA and check if AB = BA?**

Solution :

$$AB=\begin{pmatrix}2&5\\4&3\end{pmatrix}\begin{pmatrix}1&-3\\2&5\end{pmatrix}=\begin{pmatrix}(2\ \ 5)\begin{pmatrix}1\\2\end{pmatrix} & (2\ \ 5)\begin{pmatrix}1\\2\end{pmatrix}\\ (4\ \ 3)\begin{pmatrix}-3\\5\end{pmatrix} & (4\ \ 3)\begin{pmatrix}-3\\5\end{pmatrix}\end{pmatrix}=\begin{pmatrix}2+10&-6+25\\4+6&-12+15\end{pmatrix}=\begin{pmatrix}12&19\\10&3\end{pmatrix}$$

$$BA=\begin{pmatrix}1&-3\\2&5\end{pmatrix}\begin{pmatrix}2&5\\4&3\end{pmatrix}=\begin{pmatrix}(1\ \ -3)\begin{pmatrix}2\\4\end{pmatrix} & (1\ \ -3)\begin{pmatrix}5\\3\end{pmatrix}\\ (2\ \ 5)\begin{pmatrix}2\\4\end{pmatrix} & (2\ \ 5)\begin{pmatrix}5\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}2-12&5-9\\4+20&10+15\end{pmatrix}=\begin{pmatrix}-10&-4\\24&25\end{pmatrix}\ \ldots\ldots(2)$$

$\therefore$ From (1) & (2) AB $\neq$ BA

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102. **If** $A=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix}, B=\begin{pmatrix}2&3&4\\1&9&2\\-7&1&-1\end{pmatrix}$ and $C=\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}$ **then verify that A + (B + C) = (A + B) + C**

Solution :

$$B+C=\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}\qquad A+(B+C)=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix}+\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix}\ldots\ldots(1)$$

$$A+B=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}\qquad \therefore (A+B)+C=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}+\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix}\ldots\ldots(2)$$

$\therefore$ From (1) & (2) **A + (B + C) = (A + B) + C**

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103. **Given that** $A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$ **verify that A(B + C) = AB + AC**

Solution :

$$A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

$$(B+C)=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}+\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}=\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}$$

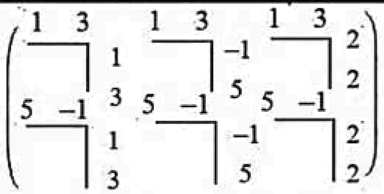
$$A(B+C)=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}=\begin{pmatrix}(1\ \ 3)\begin{pmatrix}2\\-1\end{pmatrix} & (1\ \ 3)\begin{pmatrix}2\\6\end{pmatrix} & (1\ \ 3)\begin{pmatrix}4\\5\end{pmatrix}\\ (5\ \ -1)\begin{pmatrix}2\\-1\end{pmatrix} & (5\ \ -1)\begin{pmatrix}2\\6\end{pmatrix} & (5\ \ -1)\begin{pmatrix}4\\5\end{pmatrix}\end{pmatrix}=\begin{pmatrix}2-3&2+18&4+15\\10+1&10-6&20-5\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix}\ \ldots(1)$$

$$AB=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}=\begin{pmatrix}(1\ \ 3)\begin{pmatrix}1\\3\end{pmatrix} & (1\ \ 3)\begin{pmatrix}-1\\5\end{pmatrix} & (1\ \ 3)\begin{pmatrix}2\\2\end{pmatrix}\\ (5\ \ -1)\begin{pmatrix}1\\3\end{pmatrix} & (5\ \ -1)\begin{pmatrix}-1\\5\end{pmatrix} & (5\ \ -1)\begin{pmatrix}2\\2\end{pmatrix}\end{pmatrix}$$

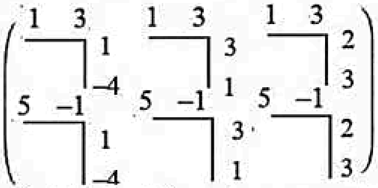
$$=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}$$

$$AB = \begin{pmatrix}1 & 3\\5 & -1\end{pmatrix}\begin{pmatrix}1 & -1 & 2\\3 & 5 & 2\end{pmatrix} =$$



$$= \begin{pmatrix}1+9 & -1+15 & 2+6\\5-3 & -5-5 & 10-2\end{pmatrix} = \begin{pmatrix}10 & 14 & 8\\2 & -10 & 8\end{pmatrix}$$

$$AC = \begin{pmatrix}1 & 3\\5 & -1\end{pmatrix}\begin{pmatrix}1 & 3 & 2\\-4 & 1 & 3\end{pmatrix} =$$



$$= \begin{pmatrix}1-12 & 3+3 & 2+9\\5+4 & 15-1 & 10-3\end{pmatrix}$$

$$AB + AC = \begin{pmatrix}10 & 14 & 8\\2 & -10 & 8\end{pmatrix}+\begin{pmatrix}-11 & 6 & 11\\9 & 14 & 7\end{pmatrix} = \begin{pmatrix}-1 & 20 & 19\\11 & 4 & 15\end{pmatrix} \text{ .... (2)}$$

∴ From (1) & (2) **A(B + C) = AB + AC**

104. **If A = $\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$**

Solution : **If** $A = \begin{pmatrix}a & b\\c & d\end{pmatrix}$ and $I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **$A^2 - (a+d)A = (bc - ad)I_2$.**

$A = \begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$ ∴ **$A^2 - (3+2)A = ((1)(-1) - (3)(2))I_2$** ∴ $A^2 - 5A + 7I_2$

105. **Let** $A = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}$, $C = \begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$ **Show that A(BC) = (AB)C**

Solution : $A = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}$, $C = \begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$

$$BC = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix} = \begin{pmatrix}8+0 & 0+0\\2+5 & 0+10\end{pmatrix} = \begin{pmatrix}8 & 0\\7 & 10\end{pmatrix}$$

$$A(BC) = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}\begin{pmatrix}8 & 0\\7 & 10\end{pmatrix} = \begin{pmatrix}8+14 & 0+20\\8+21 & 0+30\end{pmatrix} = \begin{pmatrix}22 & 20\\29 & 30\end{pmatrix} \text{ .... (1)}$$

$$AB = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}\begin{pmatrix}4 & 0\\1 & 5\end{pmatrix} = \begin{pmatrix}4+2 & 0+10\\4+3 & 0+15\end{pmatrix} = \begin{pmatrix}6 & 10\\7 & 15\end{pmatrix}$$

$$(AB)C = \begin{pmatrix}6 & 10\\7 & 15\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix} = \begin{pmatrix}12+10 & 0+20\\14+15 & 0+30\end{pmatrix} = \begin{pmatrix}22 & 20\\29 & 30\end{pmatrix} \text{ .... (2)}$$

∴ From (1) & (2) **A(BC) = (AB)C**

106. **Let** $A = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}$, $C = \begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$ **Show that (A − B) C = AC − BC**

Solution : $A = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}$, $C = \begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$

$$(A - B)C = \begin{pmatrix}-3 & 2\\0 & -2\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix} = \begin{pmatrix}-6+2 & 0+4\\0-2 & 0-4\end{pmatrix} = \begin{pmatrix}-4 & 4\\-2 & -4\end{pmatrix} \text{ .... (1)}$$

$$AC = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix} = \begin{pmatrix}2+2 & 0+4\\2+3 & 0+6\end{pmatrix} = \begin{pmatrix}4 & 4\\5 & 6\end{pmatrix}$$

$$BC = \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix} = \begin{pmatrix}8+0 & 0+0\\2+5 & 0+10\end{pmatrix} = \begin{pmatrix}8 & 0\\7 & 10\end{pmatrix}$$

$\therefore AC-BC=\begin{pmatrix}4 & 4\\5 & 6\end{pmatrix}-\begin{pmatrix}8 & 0\\7 & 10\end{pmatrix}=\begin{pmatrix}-4 & 4\\-2 & -4\end{pmatrix}$ ....(2)   ∴ From (1) & (2), $(A-B)\,C=AC-BC$

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107. **Solve for x, y** $\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$

Solution : $\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$

$\Rightarrow x^2-4x=5$ | $y^2-2y=8$

$\Rightarrow x^2-4x-5=0$ | $\Rightarrow y^2-2y-8=0$

$\Rightarrow (x-5)(x+1)=0$ | $\Rightarrow (y-4)(y+2)=0$

$\Rightarrow x=5,-1$ | $\Rightarrow \therefore y=4, y=-2$

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108. **If** $A=\begin{pmatrix}a & b\\c & d\end{pmatrix}$ **and** $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **show that** $\mathbf{A^2-(a+d)A=(bc-ad)I_2}$.

Solution :

$$A^2=A.A=\begin{pmatrix}a & b\\c & d\end{pmatrix}.\begin{pmatrix}a & b\\c & d\end{pmatrix}=\begin{pmatrix}a^2+bc & ab+bd\\ac+cd & bc+d^2\end{pmatrix}$$

$$(a+d)A=\begin{pmatrix}a^2+ad & ab+bd\\ca+cd & ad+d^2\end{pmatrix}$$

$$A^2-(a+d)A=\begin{pmatrix}a^2+bc & ab+bd\\ac+cd & bc+d^2\end{pmatrix}-\begin{pmatrix}a^2+ad & ab+bd\\ca+cd & ad+d^2\end{pmatrix}=\begin{pmatrix}bc-ad & 0\\0 & bc-ad\end{pmatrix}=(bc-ad)\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$=\mathbf{(bc-ad)\,I_2}$.

---

109. **If** $A=\begin{pmatrix}5 & 2 & 9\\1 & 2 & 8\end{pmatrix}, B=\begin{pmatrix}1 & 7\\1 & 2\\5 & -1\end{pmatrix}$ **verify that** $\mathbf{(AB)^T=B^TA^T}$

Solution :

$$AB=\begin{pmatrix}5 & 2 & 9\\1 & 2 & 8\end{pmatrix}\begin{pmatrix}1 & 7\\1 & 2\\5 & -1\end{pmatrix}=\begin{pmatrix}5+2+45 & 35+4-9\\1+2+40 & 7+4-8\end{pmatrix}=\begin{pmatrix}52 & 30\\43 & 3\end{pmatrix}$$

$$\therefore (AB)^T=\begin{pmatrix}52 & 43\\30 & 3\end{pmatrix} \text{ ....(1)}$$

$$B^TA^T=\begin{pmatrix}1 & 1 & 5\\7 & 2 & -1\end{pmatrix}\begin{pmatrix}5 & 1\\2 & 2\\9 & 8\end{pmatrix}=\begin{pmatrix}5+2+45 & 1+2+40\\35+4-9 & 7+4-8\end{pmatrix}=\begin{pmatrix}52 & 43\\30 & 3\end{pmatrix} \text{ .....(2)}$$

∴ From (1) & (2), $(AB)^T=B^TA^T$ Hence proved.

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110. $A=\begin{pmatrix}3 & 0\\4 & 5\end{pmatrix}, B=\begin{pmatrix}6 & 3\\8 & 5\end{pmatrix}, C=\begin{pmatrix}3 & 6\\1 & 1\end{pmatrix}$ **find the matrix D, such that CD–AB = 0**

Solution : Given $CD-AB=0 \Rightarrow \begin{pmatrix}3 & 6\\1 & 1\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix}=\begin{pmatrix}3 & 0\\4 & 5\end{pmatrix}\begin{pmatrix}6 & 3\\8 & 5\end{pmatrix}\Rightarrow\begin{pmatrix}3a+6c & 3b+6d\\a+c & b+d\end{pmatrix}=\begin{pmatrix}18 & 9\\64 & 37\end{pmatrix}$

$3a+6c=18$ .... (1)

$a+c=64$ .... (2)

$(1)\Rightarrow a+2c=6$

$(3)\Rightarrow a+c=64$

$c=-58$

$a-58=64$

$a=122$

$3b+6d=9$ .... (3)

$b+d=37$ .... (4)

$(3)\Rightarrow b+2d=3$

$(4)\Rightarrow b+d=37$

$d=-34$

$b-34=37$

$b=71$

$\therefore a=122, b=71, c=-58, d=-34$

$$\therefore D=\begin{pmatrix}122 & 71\\-58 & -34\end{pmatrix}$$

111. **The area of a triangle is 5 sq.units. Two of its vertices are (2,1) and (3, –2). The third vertex is (x, y) where $y = x + 3$. Find the coordinates of the third vertex.**

Solution :

$$\therefore \text{Area of } \Delta = \frac{1}{2}\begin{bmatrix} 2 & 3 & x & 2 \\ 1 & -2 & y & 1 \end{bmatrix} = 5 \Rightarrow (-4+3y+x)-(3-2x+2y)=10$$

$$\Rightarrow x+3y-4-3+2x-2y=10 \Rightarrow 3x+y= 17$$

$$\begin{aligned} 3x+y &= 17 \;\ldots\ldots (1) \\ x-y &= -3 \;\ldots\ldots (2) \\ \hline 4x &= 14 \\ x &= 7/2 \end{aligned}$$

Sub, $x = 7/2$ in (2)

$$7/2 - y = -3$$

$$y = 7/2 + 3 = 13/2$$

$\therefore$ Third vertex is $\left(7/2, 13/2\right)$

112. **Find the area of a triangle formed by the lines 3x +y –2 = 0 , 5x +2y –3 = 0 and $2x - y - 3 = 0$.**

Solution :

$$\begin{aligned} 3x+y-2 &= 0 \quad \ldots\ldots (1) \\ 5x+2y-3 &= 0 \quad \ldots\ldots (2) \\ 2x-y-3 &= 0 \quad \ldots\ldots (3) \end{aligned}$$

$$\begin{aligned} (1)\times 2 \;&\Rightarrow\; 6x+2y=4 \\ (2) \;&\Rightarrow\; 5x+2y=3 \\ \hline &\quad\; x \;\;\; =1 \end{aligned}$$

Sub. in (1) $\;3+y-2=0 \Rightarrow y=-1$

$\therefore A(1,-1)$

$$\begin{aligned} 3x+y &= 2 \\ 2x-y &= 3 \\ \hline 5x \;\; &= 5 \\ x &= 1 \quad \therefore y=-1 \end{aligned}$$

$\therefore$ B is $(1,-1)$

$$\begin{aligned} (2) \;&\Rightarrow\; 5x+2y=3 \\ (3)\times 2 \;&\Rightarrow\; 4x-2y=6 \\ \hline &\quad\; 9x \;\; =9 \\ &\quad\quad x=1 \end{aligned}$$

$\therefore (3) \Rightarrow 2-y-3=0$

$\Rightarrow \quad -y=1 \quad \therefore \; y=-1$

$\therefore$ C is $(1,-1)$

$\therefore A(1,-1), B(1,-1), C(1,-1)$ $\quad$ $\therefore$ All point line on the same line $\quad$ $\therefore$ Area of $\Delta = 0$ sq. units

113. **If vertices of a quadrilateral are at A(-5,7) , B(-4,k) , C(-1,-6) and D(4,5) and its area is 72 sq.units. Find the value of k.**

Solution : A (–5, 7), B (–4, k), C (–1, –6), D (4, 5) & its area = 72 sq.units

$$\Rightarrow \frac{1}{2}\begin{bmatrix} -5 & & -4 & & -1 & & 4 & & -5 \\ & \searrow & & \searrow & & \searrow & & \searrow & \\ 7 & & k & & -6 & & 5 & & 7 \end{bmatrix} = 72$$

$$(-5k+24-5+28)-(-28-k-24-25)=144$$

$$(-5k+47)-(-k-77)=144$$

$$-4k+124=144$$

$$-4k=20 \Rightarrow k=-5$$

114. **Without using distance formula, show that the points (-2,-1) , (4, 0) , (3, 3) and (-3,2) are vertices of a parallelogram.**

Solution : Slope of $AB = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0+1}{4+2} = \dfrac{1}{6}$ $\quad$ Slope of $CD = \dfrac{3-2}{3+3} = \dfrac{1}{6}$ $\quad$ $\therefore$ AB & CD are parallel

Slope of $AD = \dfrac{2+1}{-3+2} = \dfrac{3}{-1} = -3$ $\quad$ Slope of $BC = \dfrac{0-3}{4-3} = \dfrac{-3}{1} = -3$ $\quad$ $\therefore$ AD & BC are parallel

$\therefore$ ABCD is a parallelogram

115. Find the area of the triangle whose vertices are (-3,5) , (5,6) and (5,-2).

Solution :
The area of the triangle

$$=\frac{1}{2}\left\{\begin{array}{ccccccc} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{array}\right\}$$

$$=\frac{1}{2}\left\{\begin{array}{ccccccc} -3 & & 5 & & 5 & & -3 \\ & \times & & \times & & \times & \\ 5 & & 6 & & -2 & & 5 \end{array}\right\}$$

$$=\frac{1}{2}\{(-18-10+25)-(25+30+6)\}$$

$$=\frac{1}{2}\{-3-61\}=\frac{1}{2}(-64)=32 \text{ sq. units}$$

116. Show that the points P(–1.5, 3) , Q(6, –2) , R(–3, 4) are collinear.

Solution :
The area of the triangle

$$=\frac{1}{2}\left\{\begin{array}{ccccccc} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{array}\right\}$$

$$=\frac{1}{2}\left\{\begin{array}{ccccccc} -1.5 & & 6 & & -3 & & -3 \\ & \times & & \times & & \times & \\ 3 & & -2 & & 4 & & 5 \end{array}\right\}$$

$$=\frac{1}{2}\{(3+24-9)-(18+6-6)\} \quad =\frac{1}{2}\{18-18\}=0$$

∴ The given points are collinear.

117. If the points P(–1, – 4), Q (b, c) and R(5, –1) are collinear and if 2b + c = 4 , then find the values of b and c.

Solution : Since the three points are collinear,
Area of triangle PQR = 0

$$\frac{1}{2}\left\{\begin{array}{ccccccc} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{array}\right\}=0$$

$$\frac{1}{2}\left\{\begin{array}{ccccccc} -1 & & b & & 5 & & -1 \\ & \times & & \times & & \times & \\ -4 & & c & & -1 & & -4 \end{array}\right\}=0$$

$$\frac{1}{2}\{(-c-b-20)-(-4b+5c+1)\}=0$$

$$-c-b-20+4b-5c-1=0$$

$b-2c=7$ — (1)
$2b+c=4$ — (2)

Solving (1) and (2) we get $b=3$, $c=-2$

118. If the area of the triangle formed by the vertices A(–1, 2) , B(k , –2) and C(7, 4) (taken in order) is 22 sq. units, find the value of *k*.

Solution : Area of triangle ABC is 22 sq.units

$$\frac{1}{2}\left\{\begin{array}{ccccccc} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{array}\right\}=22$$

$$\frac{1}{2}\left\{\begin{array}{ccccccc} -1 & & k & & 7 & & -1 \\ & \times & & \times & & \times & \\ 2 & & -2 & & 4 & & 2 \end{array}\right\}=22$$

$$\frac{1}{2}\{(2+4k+14)-(2k-14-4)\}=22$$

$$\{(2+4k+14)-(2k-14-4)\}=44$$

$$\{2+4k+14-2k+14+4\}=44$$

$$\Rightarrow 2k+34=44 \quad \Rightarrow 2k=10 \Rightarrow k=5$$

119. The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (– 3, 2) , (–1, –1) and (1, 2) . If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Solution :

Area of this tile
$$=\frac{1}{2}\left\{\begin{array}{ccccccc} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{array}\right\}$$

$$=\frac{1}{2}\left\{\begin{array}{ccccccc} -3 & & -1 & & 1 & & -3 \\ & \times & & \times & & \times & \\ 2 & & -1 & & 2 & & 2 \end{array}\right\}$$

$$=\frac{1}{2}\{(3-2+2)-(-2-1-6)\} \text{ sq. units}$$

$$=\frac{1}{2}(12)=6 \text{ sq. units}$$

Area of floor = 110×6 = 660 sq.units

120. Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (–5,12) and (– 4, 3) .

Solution : The area of the quadrilateral

$$=\frac{1}{2}\left[\begin{array}{ccccccccc} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{array}\right]$$

$$=\frac{1}{2}\left[\begin{array}{ccccccccc} 8 & & 5 & & -5 & & -4 & & 8 \\ & \times & & \times & & \times & & \times & \\ 6 & & 11 & & 12 & & 3 & & 6 \end{array}\right]$$

$$=\frac{1}{2}\{(88+60-15-24)-(30-55-48+24)\}$$

$$=\frac{1}{2}\{109+49\} = \frac{1}{2}\{158\}=79 \text{ sq. units}$$

121. The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot?

Solution : Area of parking lot

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{Bmatrix}$$

$$= \frac{1}{2}\{(10+45+28+2)-(10+20+9+14)\}$$

$$= \frac{1}{2}\{85-53\} = \frac{1}{2}(32) = 16 \text{ sq. units.}$$

Total cost for constructing the parking lot

$= 16 \times 1300 = ₹20800$

122. Find the area of the triangle formed by the points (1, –1), (–4, 6) and (–3, –5)

Solution: ∴ Area of triangle

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} 1 & -4 & -3 & 3 \\ -1 & 6 & -5 & -1 \end{Bmatrix}$$

$$= \frac{1}{2}\{(6+20+3)-(4-18-5)]$$

$$= \frac{1}{2}[29+19] = \frac{1}{2}(48) = 24 \text{ sq. units}$$

123. Find the area of the triangle formed by the points (–10, – 4), (–8, –1) and (–3, –5)

Solution: ∴ Area of triangle

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -10 & -3 & -8 & -10 \\ -4 & -5 & -1 & -4 \end{Bmatrix}$$

$$= \frac{1}{2}[(50+3+32)-(12+40+10)]$$

$$= \frac{1}{2}[85-62] = \frac{23}{2} = 11.5 \text{ sq.units}$$

124. **Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)**

Solution : Area of quadrilateral

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$$

$$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$= \frac{1}{2}[58-(-12)] = \frac{1}{2}[70] = 35 \text{ sq. units}$$

125 . **Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)**

Solution : Area of quadrilateral

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$$

$$= \frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$$

$$= \frac{1}{2}[33-(-35)] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

126. **Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3)**

Solution : Area of quadrilateral

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix} = 28$$

$$\Rightarrow (-4k+6+9-4)-(6+3k-4-12) = 56$$

$$\Rightarrow (11-4k)-(3k-10) = 56$$

$$\Rightarrow 21-7k = 56$$

$$\therefore 7k = -35$$

$$k = -5$$

127 **Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)**

Solution : Area of quadrilateral

$$=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$$

$$=\frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$=\frac{1}{2}[58-(-12)]=\frac{1}{2}[70]=35 \text{ sq. units}$$

128 **Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)**

Solution : Area of quadrilateral

$$=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$$

$$=\frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$$

$$=\frac{1}{2}[33-(-35)]=\frac{1}{2}[68]=34 \text{ sq. units}$$

127 **Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3)**

Solution : Area of quadrilateral

$$=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix}=28$$

$$\Rightarrow (-4k+6+9-4)-(6+3k-4-12)=56$$

$$\Rightarrow (11-4k)-(3k-10)=56$$

$$\Rightarrow 21-7k=56$$

$$\therefore 7k=-35$$

$$k=-5$$

128 **If the points A(-3, 9) , B (*a, b*) and C(4,-5) are collinear and if *a* + *b* = 1 , then find *a* and *b*.**

Solution : A (– 3, 9), B (*a*, *b*), C (4, –5) are collinear and $a+b=1$.

$b = 1-a$ —(1) and Area of triangle = 0.

$$\text{ie) }\frac{1}{2}\begin{bmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{bmatrix}=0$$

$$\Rightarrow (-3b-5a+36)-(9a+4b+15)=0$$

$$\Rightarrow -5a-3b+36-9a-4b-15=0$$

$$\Rightarrow -14a-7b+21=0$$

$$\Rightarrow 2a+b-3=0$$

$$\Rightarrow 2a+1-a-3=0 \text{ (from (1))}$$

$$\therefore \Rightarrow a=2 \quad b=-1$$

129\. **A triangular shaped glass with vertices at A(-5,-4) , B(1,6) and C(7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.**

Solution : ∴ Area of triangle

$$=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_1 \\ & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_1 \end{matrix}\right\}$$

$$\text{Area of }\Delta ABC=\frac{1}{2}\begin{bmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{bmatrix}$$

$$=\frac{1}{2}[(-30-4-28)-(-4+42+20)]$$

$$=\frac{1}{2}[-62-(58)]$$

$$=\frac{1}{2}[-120]$$

$= 60$ sq. units (Area can't be –ve).

∴ No. of paint cans needed $=\frac{60}{6}=10$

130. **Find the equation of a line passing through (6,–2) and perpendicular to the line joining the points (6,7) and (2,–3).**

Solution : ∴ Slope of the line joining (6, 7) and (2, –3) $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-3)-(7)}{(2)-(6)}=\frac{-3-7}{2-6}=\frac{-10}{-4}=\frac{5}{2}$

∴ Slope of the line perpendicular to $\frac{5}{2}$ is $\frac{-2}{5}$

∴ Equation of the required line is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{-2}{5}$, $(x_1,y_1)=(6,-2)$

$$\Rightarrow y+2=\frac{-2}{5}(x-6)\Rightarrow 5y+10=-2x+12 \Rightarrow 2x+5y-2=0$$

131. **Find the equation of a straight line passing through the point P(-5,2) and parallel to the line joining the points Q(3,-2) and R(-5, 4) .**

Solution : Slope of QR $=\frac{y_2-y_1}{x_2-x_1}=\frac{(4)-(-2)}{(-5)-(3)}=\frac{4+2}{-5-3}=\frac{6}{-8}=\frac{-3}{4}$

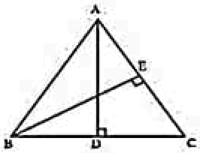
∴ Equation of the required line is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{-3}{4}$, $(x_1,y_1)=(-5,2)$

$$y-2={}^{-3}\!/_{4}(x+5) \Rightarrow 4y-8=-3x-15 \Rightarrow 3x+4y+7=0$$

132 **A(-3, 0) B(10,-2) and C(12, 3) are the vertices of ΔABC . Find the equation of the altitude through A and B.**

Solution : **A(-3, 0) B(10,-2) and C(12, 3) are the vertices of ΔABC .**

Equation of the altitude through A is AD



Slope of BC $=\frac{y_2-y_1}{x_2-x_1}=\frac{(3)-(-2)}{(12)-(10)}=\frac{3+2}{12-10}=\frac{5}{2}$ ∴ Slope of AD is $=\frac{-2}{5}$ (AD ⊥ BC)

∴ Equation of AD is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{-2}{5}$, $(x_1,y_1)=(-3,0)$

$$y-0={}^{-2}\!/_{5}(x+3) \Rightarrow 5y=-2x-6 \Rightarrow 2x+5y+6=0$$

∴ Equation of the altitude through A and B is BE

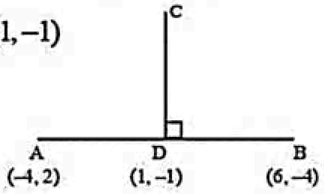
Slope of AC $=\frac{y_2-y_1}{x_2-x_1}=\frac{(3)-(0)}{(12)-(-3)}=\frac{3-0}{12+3}=\frac{3}{15}=\frac{1}{5}$ ∴ Slope of BE $=-5$ (∵ BE ⊥ AC)

∴ Equation of BE is $\Rightarrow y-y_1=m(x-x_1)$ here $m=-5$, $(x_1,y_1)=(10,-2)$

$$y+2=-5(x-10) \Rightarrow y+2=-5x+50 \Rightarrow 5x+y-48=0$$

133. **Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4)**

Solution : D is the midpoint of AB $\therefore D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+6}{2},\frac{2-4}{2}\right)=(1,-1)$



Slope of AB $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-4)-(2)}{(6)-(-4)}=\frac{-4-2}{6+4}=\frac{-6}{10}=\frac{-3}{5}$

∴ Slope of CD $=\frac{5}{3}$ (∵ CD ⊥ AB)

∴ Equation of perpendicular bisector CD is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{5}{3}$, $(x_1,y_1)=(1,-1)$

$$y+1=\frac{5}{3}(x-1) \Rightarrow 3y+3=5x-5 \Rightarrow 5x-3y-8=0$$

144. **If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.**

Solution : Given circumference of a cone = 484 cm and h = 105 cm

$$2\pi r = 484 \Rightarrow 2\times\frac{22}{7}\times r = 484 \Rightarrow r = \frac{484\times 7}{2\times 22} = 77\text{ cm}$$

$$\therefore \text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times 77\times 77\times \overset{5}{\cancel{105}} = 652190\text{ cm}^3$$

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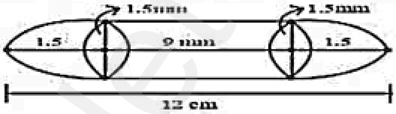
145. **A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?**

Solution: **Cylinder** $\Rightarrow$ H = 9 mm , r = 1.5 mm = $^3/_2$

Hemisphere $\Rightarrow$ r = 1.5 mm = $^3/_2$



$\therefore$ Volume of the Capsule = Vol. of Cylinder + 2 (Vol. of hemisphere)

$$= \pi r^2 H + 2\left(\frac{2}{3}\pi r^3\right) = \frac{22}{7}\left[\frac{9}{4}\times 9 + \frac{4}{3}\times\frac{27}{8}\right] = \frac{22}{7}\left[\frac{81}{4}+\frac{9}{2}\right] = \frac{22}{7}\left[\frac{81+18}{4}\right]$$

$$= \frac{22\times 99}{28} = \frac{11\times 99}{14}$$

$$= 77.78\text{ mm}^3$$

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146. **The outer and the inner surface areas of a spherical copper shell are 576π cm² and 324π cm² respectively. Find the volume of the material required to make the shell.**

Solution: Given $4\pi R^2 = 576\pi$ | $4\pi r^2 = 324\pi$

$R^2 = 144$ | $r^2 = 81$

$R = 12$ cm | $r = 9$ cm

$$\therefore \text{Volume of the material} = \frac{4}{3}\pi\left(R^3 - r^3\right) = \frac{4}{3}\times\frac{22}{7}(1728-729) = \frac{4}{3}\times\frac{22}{7}\times 999 = \frac{88\times 333}{7} = 4186.29\text{ cm}^3$$

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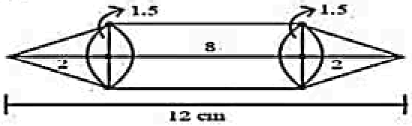
147. **Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.**

Solution:

Cone $\Rightarrow$ h = 2cm , $r = 1.5\text{ cm} = {}^3/_2$

Cylinder $\Rightarrow$ H = 8cm , $r = 1.5\text{ cm} = {}^3/_2$



$\therefore$ Volume of the model = 2 (Vol. of Cone) + Vol. of Cylinder

$$= {}^2/_3\,\pi r^2 h + \pi r^2 H = \pi r^2\left[\frac{2h}{3}+H\right] = \frac{22}{7}\times\frac{9}{4}\left[\frac{4}{3}+8\right] = \frac{11\times 9}{7\times 2}\left[\frac{28}{3}\right] = \frac{11\times 3\times \cancel{14}}{\cancel{7}}$$

$$= 66\text{ cm}^3$$

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148. **The radius of a sphere increases by 25%. Find the percentage increase in its surface area.**

Solution : sphere surface area = $4\pi r^2$ $\quad r = 100\ \%$

old surface area $= 4\pi\left(100\right)^2$

If the radius increases by 25% $\Rightarrow$ New radius = 125 %

New surface area $= 4\pi\left(125\right)^2$

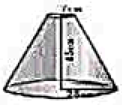
$$\therefore \text{Increment in SA} = 4\pi\left(125\right)^2 - 4\pi\left(100\right)^2 = 4\pi\left(\left(125\right)^2-\left(100\right)^2\right) = 4\pi\left(\left(125\right)^2-\left(100\right)^2\right) = 4\pi\ 225\times 25$$

$\therefore$ Percentage inc. in SA $= \dfrac{4\pi\, 225\times25}{4\pi\left(100\right)^2}\times100 = \dfrac{225}{4} = 56.25\%$

149. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

Solution : Given that, h = 45 cm, R = 28 cm, r = 7 cm

Volume $= \dfrac{1}{3}\pi[R^2+Rr+r^2]h = \dfrac{1}{3}\times\dfrac{22}{7}\times[28^2+(28\times7)+7^2]\times45 = 48510\ \text{cm}^3$



150. **A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm.**

Solution : Given r = 5 cm, h = 12 cm in a cone $\therefore l = \sqrt{h^2+r^2} = \sqrt{144+25} = \sqrt{169} = 13$

$\therefore$ CSA of cone $= \pi rl = \dfrac{22}{7}\times5\times13 = \dfrac{110\times13}{7}\ \text{cm}^2$

Area of sheet of paper = 5720 cm² $\therefore$ Number of caps $= \dfrac{5720\times7}{110\times3} = 28$ caps

151. **Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.**

Solution: **Cylindrical Pipe** ⇒ Speed of water in the pipe = 15 Km/hr ⇒ H = 15000 m

Radius of pipe r = 7 cm $= \dfrac{7}{100}$ m

Rectangular Tank ⇒ $l = 50$ m $b = 44\ m$ $h = 21\text{cm} = \dfrac{21}{100}m$

$\therefore$ Required time $= \dfrac{\text{Volume of tank}}{\text{Volume of pipe}} = \dfrac{lbh}{\pi r^2 H} = \dfrac{50\times44\times{}^{21}\!/_{100}}{\dfrac{22}{7}\times\dfrac{7}{100}\times\dfrac{7}{100}\times15000} = 2$ hrs

152. **An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.**

Solution: Radius of sphere ⇒ R = 12 cm & Radius of cylinder ⇒ r = 8 cm

Volume of sphere = Volume of Cylinder ⇒ $\dfrac{4}{3}\pi R^3 = \pi r^2 h$

$\dfrac{4}{3}\times12\times12\times12 = 8\times8\times h \Rightarrow h = 36$ cm $\therefore$ Height of the cylinder = 36 cm

153. A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Solution : height of the cone $h_1 = 24$ cm; same radius of the cone and cylinder

let h_2 be the height of the cylinder.

Volume of cylinder = Volume of cone ⇒ $\pi r^2 h_2 = \dfrac{1}{3}\pi r^2 h_1 \Rightarrow h_2 = \dfrac{1}{3}\times h_1 \Rightarrow h_2 = \dfrac{1}{3}\times24 = 8$

Height of cylinder is 8 cm.

154. **A conical flask is full of water. The flask has base radius r units and height h units, the water poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask.**

Solution: Volume of Cylindrical Flask = Volume of Conical Flask

$$\Rightarrow \pi(xr)^2 H = \frac{1}{3}\pi r^2 h \quad \Rightarrow x^2 r^2 H = \frac{1}{3} r^2 h \quad \Rightarrow \quad H = \frac{h}{3x^2}$$

$\therefore$ Height of the Cylindrical Flask $= \frac{h}{3x^2}$ cm

155. **A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.**

Solution: **Right Circular Cone** $\Rightarrow$ r = 7 cm and h = 8 cm

Hollow Sphere $\Rightarrow$ R = 5 cm and r = ?

Volume of Hollow Sphere = Vol. of Right Circular Cone $\Rightarrow \frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi r^2 h$

$$\Rightarrow 4(125 - r^3) = 49 \times 8 \quad \Rightarrow 125 - r^3 = 49 \times 2 \quad \Rightarrow r^3 = 125 - 98 \quad \Rightarrow r^3 = 27 \quad \Rightarrow \quad \therefore \quad r = 3$$

$\therefore$ Internal diameter of hollow sphere = 6 cm

156. **A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thick ness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.**

Solution: **Solid Sphere** $\Rightarrow$ r = 6 cm

Hollow Cylinder $\Rightarrow$ R = 5 cm H = 32 cm t = ?

Volume of Hollow Cylinder = Volume of Solid Sphere $\Rightarrow \pi(R^2 - r^2)\,H = \frac{4}{3}\pi r^3$

$$(25 - r^2)32 = \frac{4}{\cancel{3}} \times \overset{2}{\cancel{6}} \times 6 \times 6 \Rightarrow 25 - r^2 = \frac{\cancel{4} \times \cancel{2} \times \overset{3}{\cancel{6}} \times \overset{3}{\cancel{6}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{32}}} \Rightarrow 25 - r^2 = 9 \Rightarrow r^2 = 16 \Rightarrow r = 4$$

$\therefore$ Thickness = R − r = 5 − 4 = 1 cm

157. **Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (under ground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sumpwhich has been full, initially.**

Solution: **Over head tank** (Cylinder) $\Rightarrow$ R = 60 cm H = 105 cm

Sump (Cuboid) $\Rightarrow$ l = 2 m = 200 cm b = 1.5 m = 150 cm h = 1 m = 100 cm

Volume of water left = Volume of Sump − Volume of tank $= lbh - \pi R^2 H$

$$= 200 \times 150 \times 100 - \frac{22}{\cancel{7}} \times 60 \times 60 \times \overset{15}{\cancel{105}} = 3000000 - 1188000 = 2812000 \text{ cm}^3$$

158. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Solution : **Cylinder** $\Rightarrow h_1 = 15$ cm, $r_1 = 6$ cm

cones (**Cone** + hemispherical cap) $\Rightarrow r_2 = 3$ cm, $h_2 = 9$ cm Also, $r_2 = 3$ cm is the radius hemispherical cap

$$\text{Number of ice cream cones needed} = \frac{\text{Volume of the cylinder}}{\text{Volume of the cone} + \text{Volume of the hemispherical cap}}$$

$$\frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2+\frac{2}{3}\pi r_2^3}=\frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{1}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 9+\frac{2}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 3}=\frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{22}{7}\times 9(3+2)}=\frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{22}{7}\times 45}=12$$

---

159 **A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.**

Solution: **Hemisphere** $\Rightarrow$ Radius $= r$

Cylinder $\Rightarrow$ Radius $= r = h+\frac{1}{2}h=\frac{3}{2}h$

$\therefore$ Volume of hemisphere $=\frac{2}{3}\pi r^3=\frac{2}{3}\pi\times\left(\frac{3}{2}h\right)^3=\frac{2}{3}\pi\times\frac{27}{8}h^3=\frac{9}{4}\pi h$

$\therefore$ Volume of Cylinder $=\pi r^2 h=\pi\times\left(\frac{3}{2}h\right)^2 h=\pi\times\frac{9}{4}h^2\,h=\frac{9}{4}\pi h^3$

$\therefore$ Vol. of Hemisphere = Vol. of Cylinder

$\therefore$ % of juice that can be trasnferred to the cylindrical vessel = 100 %

---

160 **A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.**

Solution : **.hemi-spherical hollow bowl has material of volume** $\frac{436\pi}{3}$ $D=14$ cm $\Rightarrow$ $R=7$ cm

$\Rightarrow\frac{2}{3}\pi(R^3-r^3)=\frac{436\pi}{3}\Rightarrow 7^3-r^3=218\Rightarrow 343-r^3=218\quad\therefore\ r^3=125\quad\therefore\ r=5$ cm

$\therefore$ thickness $=R-r=7-5=2$ cm

---

161 **A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.**

Solution : **Solid cylinder.** $h=12$ cm $d=?$

Hollow Cylinder $R=4.3$ cm $r=1.1$ cm $H=4$ cm

Volume of hollow cylinder = Volume of solid cylinder

$\Rightarrow\ \cancel{\pi}H(R^2-r^2)=\cancel{\pi}r^2h\Rightarrow 4[(4.3)^2-(1.1)^2]=r^2\times 12\Rightarrow r^2=\frac{4(17.28)}{12}=5.76$

$\therefore\ r=2.4$ $\therefore$ Diameter of solid cylinder $=2r=4.8$ cm

---

162 **A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.**

Solution : **A solid iron cylinder has total surface area = 1848 sq.m.** & CSA $=\frac{5}{6}$(TSA)

$\Rightarrow 2\pi rh=\frac{5}{6}\times 1848=5\times 308\Rightarrow 2\pi rh=1540$ .... (1)

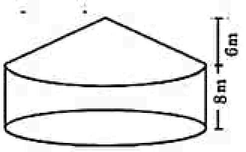
$2\pi r(h+r)=1848\Rightarrow 2\pi rh+2\pi r^2=1848\Rightarrow 1540+2\pi r^2=1848\Rightarrow 2\pi r^2=308\Rightarrow 2\times\frac{22}{7}\times r^2=308$

$\Rightarrow r^2=\frac{\overset{\overset{7}{\cancel{14}}}{\cancel{308}}\times 7}{\cancel{2}\times\underset{1}{\cancel{22}}}\Rightarrow r^2=49\Rightarrow r=7\,m$

Sub $r=7$ in (1) $\Rightarrow 2\times\frac{22}{7}\times 7\times h=1540\Rightarrow h=\frac{1540}{2\times 22}\Rightarrow h=35$ $\therefore$ Radius = 7 m, Height = 35 m.

---

163. Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?.



Solution : Let h_1 and h_2 be the height of cylinder and cone

Area for one person = 4 sq. m and Total number of persons = 150

Total base area = 150 × 4 ⇒ πr^2 = 600

$$r^2 = 600 \times \frac{7}{22} = \frac{2100}{11} \qquad ....(1)$$

Volume of air required for 1 person = 40 m^3

Total Volume of air required for 150 persons = 150 × 40 = 6000 m^3

$$\pi r^2 h_1 + \frac{1}{3}\pi r_2 h_2 = 6000 \Rightarrow \pi r^2\left(h_1 + \frac{1}{3}h_2\right) = 6000$$

$$\frac{22}{7}\times\frac{2100}{11}\left(8 + \frac{1}{3}h_2\right) = 6000 \quad \text{[using (1)]}$$

$$8 + \frac{1}{3}h_2 = \frac{6000\times7\times11}{22\times2100} \Rightarrow \frac{1}{3}h_2 = 10 - 8 = 2$$

Therefore, the height of the conical tent h_2 is 6 m

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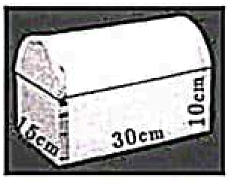
164. A jewel box is in the shape of a cuboid of dimensions 30 cm × 15 cm × 10 cm surmounted by a half part of a cylinder. Find the volume and T.S.A. of the box.



Solution : Let l, b and h_1 be the length, breadth and height of the cuboid.
Let r and h_2 be the radius and height of the cylinder.

Volume of the box = Volume of the cuboid + $\frac{1}{2}$(Volume of cylinder)

$$= (l \times b \times h_1) + \frac{1}{2}(\pi r^2 h_2) \text{ cu. units}$$

$$= (30\times15\times10) + \frac{1}{2}\left(\frac{22}{7}\times\frac{15}{2}\times\frac{15}{2}\times30\right) = 4500 + 2651.79 = 7151.79 \text{ cm}^3$$

T.S.A. of the box = C.S.A. of the cuboid + $\frac{1}{2}$(C.S.A. of the cylinder) $= 2(l+b)\,h_1 + \frac{1}{2}(2\pi r h_2)$

$$= 2\,(45\times10) + \left(\frac{22}{7}\times\frac{15}{2}\times30\right) = 900 + 707.14 = 1607.14 \text{ cm}^2$$

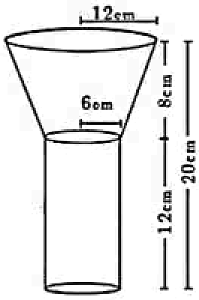
---

165. A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.

Solution : Let R, r be the top and bottom radii of the frustum.
Let h_1, h_2 be the heights of the frustum and cylinder.

R = 12 cm, r = 6 cm, h_2 = 12 cm ;
h_1 = 20 – 12 = 8 cm

Slant height of the frustum $l = \sqrt{(R-r)^2 + h_1^2} = \sqrt{36+64} = 10$
$l = 10$ cm



Outer surface area $= 2\pi r h_2 + \pi(R+r)\,l$ sq. units
$= \pi[2rh_2 + (R+r)\,l]$
$= \pi[(2\times6\times12) + (18\times10)]$
$= \pi[144 + 180]$
$= \frac{22}{7}\times 324 = 1018.28 \text{ cm}^2$

166. **The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.**

Solution : Given volume of a cone = $1005\ {}^{5}\!/_{7}$ cm³ & base area = $201\ {}^{1}\!/_{7}$ cm²

$$\therefore \frac{1}{3}\pi r^2 h = \frac{7040}{7} \ \& \ \pi r^2 = \frac{1408}{7}$$

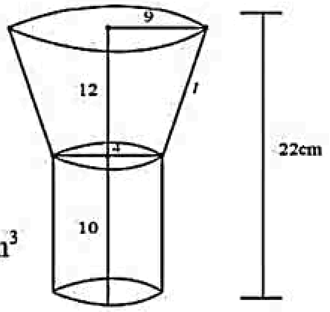
$$\Rightarrow \frac{1}{3}\times\frac{1408}{\cancel{7}}\times h = \frac{7040}{\cancel{7}} \quad \Rightarrow h = \frac{7040}{1408}\times 3 \quad \Rightarrow h = 5\times 3 \quad \Rightarrow h = 15$$

$$\Rightarrow \pi r^2 = \frac{1408}{7} \quad \Rightarrow \frac{22}{7}\times r^2 = \frac{1408}{7} \quad \Rightarrow r^2 = \frac{1408}{7} = 64 \quad \Rightarrow \therefore r = 8$$

$$\therefore l = \sqrt{h^2+r^2} = \sqrt{15^2+8^2} = \sqrt{225+64} = \sqrt{289} = 17\text{ cm}$$

∴ Slant height = 17cm

167. **An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.**



Solution : R = 9 cm r = 4 cm, H = 10 cm

$$l = \sqrt{(R-r)^2+h^2} = \sqrt{25+144} = \sqrt{169} = 13$$

Area of tin sheet required to make the funnel

$$= \text{CSA of Frustum} + \text{CSA of Cylinder}$$

$$= \pi\,(R+r)\,l + 2\pi rH \ = \pi\,[13\times 13 + 2\times 4\times 10]$$

$$= \frac{22}{7}[169+80] \ = \frac{22}{7}\times 249 \ = \frac{5478}{7} = 782.57\text{ cm}^3$$

168. **The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?**

Solution : Let A - electrification contract $\overline{B}$ - not plumbing contract , B - plumbing contract

$$P(A) = \frac{3}{5},\ P(\overline{B}) = \frac{5}{8},\ P(A\cup B) = \frac{5}{7}$$

$$\Rightarrow P(B) = 1-\frac{5}{8} = \frac{3}{8}$$

$$\therefore P(A\cap B) = P(A)+P(B)-P(A\cup B) = \frac{3}{5}+\frac{3}{8}-\frac{5}{7} = \frac{73}{280}$$

169. **In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50years. What is the probability that a chosen individual from the town is either a female or over 50 years?**

Solution : Let A - Female , B - Over 50 years

$$n(S) = 8000,\ n(A) = 3000,\ n(B) = 1300 \qquad n(A\cap B) = \frac{30}{100}\times 3000 = 900$$

$$\therefore P(A) = \frac{3000}{8000},\ P(B) = \frac{1300}{8000},\ P(A\cap B) = \frac{900}{8000}$$

$$P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{3000+1300-900}{8000} = \frac{3400}{8000} = \frac{34}{80} = \frac{17}{40}$$

170. **The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?**

Solution : **Sathya**

$$\sum x_1 = 460 \quad n = 5$$

$$\therefore \overline{x_1} = \frac{460}{5} = 92$$

$$\sigma_1 = 4.6$$

$$\therefore \text{C.V}_1 = \frac{\sigma_1}{\overline{x_1}} \times 100 = \frac{4.6}{92} \times 100 = \frac{460}{92} = 5$$

Vidhya

$$\sum x_2 = 480 \quad n = 5$$

$$\therefore \overline{x_2} = \frac{480}{5} = 96$$

$$\sigma_2 = 2.4$$

$$\therefore \text{C.V}_2 = \frac{\sigma_2}{\overline{x_2}} \times 100 = \frac{2.4}{96} \times 100 = \frac{240}{96} = 2.5$$

$\text{C.V}_2 < \text{C.V}_1$ $\quad\therefore$ Vidhya is more consistent than Sathya.

171. **Find the coefficient of variation of 24, 26, 33, 37, 29, 31.**

Solution : Given data is 24, 26, 33, 37, 29, 31. $\bar{x} = \frac{24+26+33+37+29+31}{6} = \frac{180}{6} = 30$

x	$d = x - 30$	d^2
24	−6	36
26	−4	16
29	−1	1
31	1	1
33	3	9
37	7	49
	0	112

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{112}{6} - \left(\frac{0}{6}\right)^2} = 4.31$$

$$\therefore \text{C.V} = \frac{4.31}{30} \times 10 = 14.36$$

172. The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

Solution :

x	$d = x - 9$	d^2
4	−5	25
7	−2	4
8	−1	1
9	0	0
10	1	1
12	3	9
13	4	16
7	0	56

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{56}{7} - \left(\frac{0}{7}\right)^2} = \sqrt{8} = 2.83$$

173. **The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.**

Solution : Given data is 38, 40, 47, 44, 46, 43, 49, 53.

$$\bar{x} = \frac{38+40+47+44+46+43+49+53}{8} = \frac{360}{8} = 45$$

x	$d = x - 45$	d^2
38	−7	49
40	−5	25
43	−2	4
44	−1	1
46	1	1
47	2	4
49	4	16
53	8	64
8	0	172

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{164}{8} - \left(\frac{0}{8}\right)^2} = 4.53$$

$$\therefore \text{C.V} = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100 = 10.07$$

174. The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Solution :

x	$d = x - 35$	d^2
25	–10	100
29	6	36
30	–5	25
33	–2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
n = 10	9	453

$$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$$

$$= 6.67$$

175. The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Solution : $\text{Mean} = \dfrac{11.4 + 12.5 + 12.8 + 16.3 + 17.8 + 19.2}{6} = \dfrac{90}{6} = 15$

x	$d = x - 15$	d^2
11.4	–3.6	12.96
12.5	–2.5	6.25
12.8	–2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
6	0	51.22

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{51.22}{6} - \left(\frac{0}{6}\right)^2}$$

$$= 2.9$$

176. **The frequency distribution is given below.**

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a variance of 160. Determine the value of *k*.

Solution : Given variance = 160

x	f	$d = \frac{x - A}{k}$	d^2	$f.d$	$f.d^2$
k	2	– 3	9	– 6	18
$2k$	1	– 2	4	– 2	4
$3k$	1	– 1	1	– 1	1
$4k$	1	0	0	0	0
$5k$	1	1	1	1	1
$6k$	1	2	4	2	4
	7			– 6	28

$$\therefore k^2\left(\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right) = 160$$

$$\Rightarrow \quad k^2\left[\frac{28}{7} - \left(\frac{-6}{7}\right)^2\right] = 160 \Rightarrow k^2\left[4 - \frac{36}{49}\right] = 160$$

$$\Rightarrow \quad k^2\left[\frac{160}{49}\right] = 160 \Rightarrow k^2 = \frac{16 \times 40}{16} \Rightarrow k^2 = 49$$

$$\therefore k = 7 \quad (\because k \text{ is positive})$$

177. **The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.**

Solution : Given $n = 7, \bar{x} = 8, \sigma^2 = 16$

5 of the observerations are 2, 4, 10, 12, 14

Let the **remaining** 2 observations be *a*, *b*. $\therefore \bar{x} = 8 \Rightarrow \dfrac{\sum x}{n} = 8$

$$\Rightarrow \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \sigma^2 \quad \Rightarrow \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 16 \quad \Rightarrow \frac{\sum x^2}{7} - 8^2 = 16 \quad \Rightarrow \frac{\sum x^2}{7} - 64 = 16 \quad \Rightarrow \frac{\sum x^2}{7} = 80$$

$$\Rightarrow \sum x^2 = 560$$

$$\Rightarrow 2^2 + 4^2 + 10^2 + 12^2 + 10^2 + a^2 + b^2 = 560$$

$$\Rightarrow 460 + a^2 + b^2 = 560 \quad \Rightarrow a^2 + b^2 = 100 \quad \Rightarrow 8^2 + 6^2 = 100 \quad \therefore \quad a = 8, \quad b = 6$$

178. Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Solution :

x	$d=x-8$	d^2
4	−4	16
7	−1	1
8	0	0
10	2	4
11	3	9
5	0	30

x	$d=x-11$	d^2
7	−4	16
10	−1	1
11	0	0
13	2	4
14	3	9
5	0	30

$$\sigma=\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}=\sqrt{\frac{30}{5}-\left(\frac{0}{5}\right)^2}=\sqrt{6} \qquad \sigma=\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}=\sqrt{\frac{30}{5}-\left(\frac{0}{5}\right)^2}=\sqrt{6}$$

standard deviation will not change

179. Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

Solution :

x	$d=x-5$	d^2
2	−3	9
3	−2	4
5	0	0
7	2	4
8	3	9
5	0	26

x	$d=x-20$	d^2
8	−12	144
12	−8	64
20	0	0
28	8	64
32	12	144
5	0	416

$$\sigma=\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}=\sqrt{\frac{26}{5}-\left(\frac{0}{5}\right)^2}=\sqrt{\frac{26}{5}} \qquad \sigma=\sqrt{\frac{\Sigma d^2}{n}-\left(\frac{\Sigma d}{n}\right)^2}=\sqrt{\frac{416}{5}-\left(\frac{0}{5}\right)^2}=4\sqrt{\frac{26}{5}}$$

standard deviation also multiplied by 4.

180. The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40. Using step deviation method, find the standard deviation of the amount they have spent.

Solution :

x	$d=\frac{x-20}{5}$	d^2
5	−3	9
10	−2	4
15	−1	1
20	0	0
25	1	1
30	2	4
35	3	9
40	4	16
	$\Sigma d=4$	$\Sigma d^2=44$

Standard Deviation

$$\sigma=\sqrt{\frac{\Sigma d_i^2}{n}-\left(\frac{\Sigma d_i}{n}\right)^2}\times c$$

$$=\sqrt{\frac{44}{8}-\left(\frac{4}{8}\right)^2}\times 5=11.45$$

181. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$, $P(A\cup B\cup C)=\frac{9}{10}$, $P(A\cap B\cap C)=\frac{1}{15}$, then find P(A),P(B) and P(C) ?

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STANDARD 9th TO 12th

Solution : $P(B) = 2.P(A)$, $P(C) = 3.P(A)$, $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$,

$P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\frac{9}{10} = P(A) + 2.P(A) + 3.P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$ $\quad \therefore P(A) = \frac{11}{48}$ $\quad \therefore P(B) = 2.P(A) = 2 \times \frac{11}{48} = \frac{11}{24}$

$\therefore P(C) = 3.P(A) = 3 \times \frac{11}{48} = \frac{11}{16}$

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182. **In a class of 35, students are numbered from 1 to 35.The ratio of boys to girls is 4:3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.**

Solution : $n(S) = 35$ and ratio of boys and girls=4:3

No. of girls $= \frac{3}{7} \times 35 = 15$

No. of boys $= \frac{4}{7} \times 35 = 20$

Let A - a boy with prime roll no

$A = \{2, 3, 5, 7, 11, 13, 19\}$, $n(A) = 7 \Rightarrow P(A) = \frac{7}{35}$

Let B - a girl with composite roll no.

$B = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$, $n(B) = 12 \Rightarrow P(B) = \frac{12}{25}$

Let C - even roll no.

$C = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$ $n(C) = 17 \Rightarrow P(C) = \frac{17}{35}$

$A \cap B = \{\ \}$, $n(A \cap B) = 0$, $P(A \cap B) = 0$

$B \cap C = \{22, 24, 26, 28, 30, 32, 34\}$, $n(B \cap C) = 7 \Rightarrow P(B \cap C) = \frac{7}{35}$

$C \cap A = \{2\}$, $n(C \cap A) = 1 \Rightarrow P(C \cap A) = \frac{1}{35}$ $\quad \therefore P(A \cap B \cap C) = 0$

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{7}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0 = \frac{28}{35} = \frac{4}{5}$$

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183. **Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.**

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} $\quad n(S) = 36$

Let A even number on the 1^{st} die.

A = {(2, 1),(2, 2),(2, 3),(2, 4),(2, 5),(2, 6), (4, 1),(4, 2),(4, 3),(4, 4),(4, 5),(4, 6),
(6, 1),(6, 2),(6, 3),(6, 4),(6, 5),(6, 6)} $n(A) = 18 \Rightarrow P(A) = \frac{18}{36}$

Let B - Total of face sum as 8.

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} $n(B) = 5 \Rightarrow P(B) = \frac{5}{36}$

$A \cap B$ = {(2, 6), (4, 4), (6, 2)} , $n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{3}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

184. **The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card**

Solution : By the data given, $n(S) = 52 - 2 - 2 - 2 = 46$

i) Let A clubber card. $n(A) = 13 \Rightarrow P(A) = \frac{13}{46}$

ii) Let B - queen of red card. $n(B) = 0 \Rightarrow P(B) = 0$ (queen diamond and heart are included in S)

iii) Let C - King of black cards $n(C) = 1$ (encluding spade king) $\therefore P(C) = \frac{1}{46}$

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185. **In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.**

Solution : S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (HTT), (TTT)} $n(S) = 8$

i) P (gets double entry fee) = $\frac{1}{8}$ ($\because$ 3 heads) ii) P (just gets for her entry fee) = $\frac{6}{8} = \frac{3}{4}$ ($\because$ 1 (or) 2 heads)

iii) P (loses the entry fee) = $\frac{1}{8}$ ($\because$ 3 no heads (TTT) only)

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186. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} $n(S) = 36$

Let A a doublet
A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)} $n(A) = 6$, $P(A) = \frac{6}{36}$

Let B face sum 4.
B = {(1,3),(2,2),(3,1)} $n(B) = 3$, $P(B) = \frac{3}{36}$

$A \cap B = \{(2,2)\}$, $n(A \cap B) = 1$. $P(A \cap B) = \frac{1}{36}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

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187. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution : Total number of cards = 52; n (S) = 52

Let A king card. Let B heart card. Let C red card.

$P(A) = \frac{4}{52}$, $P(B) = \frac{13}{52}$, $P(C) = \frac{26}{52}$, $P(A \cap C) = \frac{2}{52}$, $P(A \cap B) = \frac{1}{52}$, $P(B \cap C) = \frac{13}{52}$

$P(A \cap B \cap C) = \frac{1}{52}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

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188. **Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.**

Solution : S = {(1,1),(1,1),(1,2),(1,2),(1,3),(1,3), (2,1),(2,1),(2,2),(2,2),(2,3),(2,3)
(3,1),(3,1),(3,2),(3,2),(3,3),(3,3), (4,1),(4,1),(4,2),(4,2),(4,3),(4,3)
(5,1),(5,1),(5,2),(5,2),(5,3),(5,3), (6,1),(6,1),(6,2),(6,2),(6,3),(6,3)} n(S) = 36

i) Let A - Sum of 2 n(A) = 2 $\therefore P(A)=\frac{2}{36}$

ii) Let B - Sum of 3 n(B) = 4 $P(B)=\frac{4}{36}$

iii) Let C - Sum of 4 n(C) = 6 $P(C)=\frac{6}{36}$

iv) Let D - Sum of 5 n(D) = 6 $P(D)\ \frac{6}{36}$

v) Let E - Sum of 6 n(E) = 6 $P(E)=\frac{6}{36}$

vi) Let F - Sum of 7 n(F) = 6 $P(F)=\frac{6}{36}$

vii) Let G - Sum of 8 n(G) = 4 $P(G)=\frac{4}{36}$

viii) Let H - Sum of 9 n(H) = 2 $P(H)=\frac{2}{36}$

189. **A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black**

Solution : S = {5R, 6W, 7G, 8B}

i) Let A - White ball n(A) = 6 $\Rightarrow P(A)=\frac{6}{26}=\frac{3}{13}$

ii) Let B - Black (or) red n(B) = 5 + 8 = 13 $\Rightarrow P(B)=\frac{13}{26}=\frac{1}{2}$

iii) Let C - not white n(C) = 20 $\Rightarrow P(C)=\frac{20}{26}=\frac{10}{13}$

iv) Let D - Neither white nor black n(D) = 12 $\Rightarrow P(D)=\frac{12}{26}=\frac{6}{13}$

190. What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution : Total number of cards = 52

Let A king card $n(A)=4$ $P(A)=\frac{4}{52}$ | Let B queen card $n(B)=4$ $P(B)=\frac{4}{52}$ | $P(A\cap B)=\frac{0}{52}$

$\Rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{4}{52}+\frac{4}{52}-\frac{0}{52}=\frac{2}{13}$

191. **Two unbiased dice are rolled once. Find the probability of getting**
(i) a doublet (equal numbers on both dice) (ii) the product as a prime number
(iii) the sum as a prime number (iv) the sum as 1

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} n(S) = 36

i) Let A a doublet
A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} n(A) = 6 $\therefore P(A)=\frac{6}{36}=\frac{1}{6}$

ii) Let B the product as a prime number.
B = {(1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1)} n(B) = 6 $\therefore P(B)=\frac{6}{36}=\frac{1}{6}$

iii) Let C be the sum of numbers on the dice is prime.
C = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (2, 5),
(3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)} n(C) = 14 $\therefore P(C)=\frac{7}{36}$

iv) Let D be the sum of numbers is 1. n(D) = 0 $\therefore$ P(D) = 0

192. **The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.**

Solution : $n(S) = 52 - 3 = 49$

i) Let A - a diamond card $n(A) = 13$ $\therefore P(A) = \frac{13}{49}$

ii) Let B - a queen card $n(B) = 3$ (except spade queen out of 4) $\therefore P(B) = \frac{3}{49}$

iii) Let C - a spade card $n(C) = 10$ $(13 - 3 = 10)$ $\therefore P(C) = \frac{10}{49}$

iv) Let D - 5 of heart $n(D) = 1$ $\therefore P(D) = \frac{1}{49}$

193. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, …12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

Solution :

$S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$; $n(S) = 12$

(i) Let A be 7. $n(A)=1$, $P(A) = \frac{1}{12}$

(ii) Let B a prime number. $B = \{2,3,5,7,11\}$; $n(B) = 5$, $P(B) = \frac{5}{12}$

(iii) Let C composite number. $C = \{4,6,8,9,10,12\}$; $n(C)=6$, $P(C) = \frac{6}{12} = \frac{1}{2}$

194. **If for a distribution, $\Sigma(x - 5) = 3$, $\Sigma(x-5)^2 = 43$, and total number of observations is 18, find the mean and standard deviation.**

Solution : Given $\Sigma(x-5) = 3$,

$\Rightarrow \Sigma x - \Sigma 5 = 3$

$\Rightarrow \Sigma x - 5.\Sigma 1 = 3$

$\Rightarrow \Sigma x - 5(18) = 3$

$\Rightarrow \Sigma x = 93$

$\Sigma (x-5)^2 = 43, n = 18$

$\Rightarrow \Sigma (x^2 - 10x + 25) = 43$

$\Rightarrow \Sigma x^2 - 10.\Sigma x + 25\,\Sigma 1 = 43$

$\Rightarrow \Sigma x^2 - 10(93) + 25(18) = 43$

$\Rightarrow \Sigma x^2 = 523$

i) **Mean :** $\bar{x} = \frac{\Sigma x}{n} = \frac{93}{18} = 5.17$

ii) **SD :** $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2} = 1.536$

195. **The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.**

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?

Solution : $C.V = \frac{\sigma}{\bar{x}} \times 100$ For Maths, $C.V = \frac{12}{56} \times 100 = 21.428$

For Science, $C.V = \frac{14}{65} \times 100 = 21.538$ For Social Science, $C.V = \frac{10}{60} \times 100 = 16.67$

Highest variation in Science. Lowest variation in Social Science.

196. **If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.**

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} n(S) = 36

Let A - Product of face value is 6.
A = {(1, 6), (2, 3), (3, 2), (6, 1)} n(A) = 4 $P(A) = \frac{4}{36}$

Let B - Difference of face value is 5. B = {(6, 1)} n(B) = 1 $P(B) = \frac{1}{36}$

$A\cap B = \{(6,1)\}$ $n(A\cap B) = 1$ $P(A\cap B) = \frac{1}{36}$

$$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{4}{36} + \frac{1}{36} - \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$$

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197. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
(i) The student opted for NCC but not NSS. (ii) The student opted for NSS but not NCC.
(iii) The student opted for exactly one of them.

Solution: Total number of students n(S)= 50. Let A and B be NCC and NSS
n(A) = 28 , n(B) = 30 , n(A∩B) = 18 $P(A) = \frac{28}{50}$ $P(B) = \frac{30}{50}$ $P(A\cap B) = \frac{18}{50}$

(i) Probability of the students opted for NCC but not NSS $P(A\cap\overline{B}) = P(A) - P(A\cap B) = \frac{28}{50} - \frac{18}{50} = \frac{1}{5}$

(ii) Probability of the students opted for NSS but not NCC. $P(A\cap\overline{B}) = P(B) - P(A\cap B) = \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$

(iii) Probability of the students opted for exactly one of them $P(A\cap\overline{B}) + P(\overline{A}\cap B) = \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$

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198. **A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i) then find *x*.**

Solution : Total number of balls in the bag $n(S) = x + 12$. ($x \to$ red $12 \to$ black)

i) Let A red balls $n(A) = x$, $P(A) = \frac{x}{x+12}$

ii) If 8 more red balls are added in the bag. $n(S) = x + 20$

By the problem, $\frac{x+8}{x+20} = 2\left(\frac{x}{x+12}\right) \Rightarrow (x+8)(x+12) = 2x^2 + 40x \quad \Rightarrow x^2 + 20x - 96 = 0$

$\Rightarrow (x+24)(x-4) = 0 \quad \therefore x = -24, 4$

$\therefore x = 4 \qquad \therefore P(A) = \frac{4}{16} = \frac{1}{4}$

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199. A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Solution: P(A) = 0.5 , P(A∩B) = 0.3

$P(A\cup B) \le 1 \qquad P(A) + P(B) - P(A\cap B) \le 1$

$0.5 + P(B) - 0.3 \le 1$

$P(B) \le 1 - 0.2$

$P(B) \le 0.8$

Therefore, probability of B getting selected is atmost 0.8.

200. **From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.**

Solution : $n(S) = 52$

Let A - Red King $n(A) = 2 \Rightarrow P(A) = \frac{2}{52}$ Let B - Black Queen $n(B) = 2 \Rightarrow P(B) = \frac{2}{52}$

$n(A \cap B) = 0 \Rightarrow P(A \cap B) = \frac{0}{52}$ $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{52} + \frac{2}{52} - \frac{0}{52} = \frac{4}{52} = \frac{1}{13}$

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201. Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution : $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ $n(S) = 36$

(i) Let A the sum of outcome values equal to 4.
$A = \{(1,3),(2,2),(3,1)\}$; $n(A) = 3$. $P(A) = \frac{3}{36} = \frac{1}{12}$

(ii) Let B the sum of outcome values greater than 10.
$B = \{(5,6),(6,5),(6,6)\}$; $n(B) = 3$ $P(B) = \frac{3}{36} = \frac{1}{12}$

(iii) Let C the sum of outcomes less than 13.
$n(C) = n(S) = 36$ $P(C) = \frac{36}{36} = 1$

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202. From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution : $n(S) = 52$

(i) Let A red card. $n(A) = 26 \Rightarrow P(A) = \frac{26}{52} = \frac{1}{2}$

(ii) Let B heart card. $n(B) = 13 \Rightarrow P(B) = \frac{13}{52} = \frac{1}{4}$

(iii) Let C red king card. $n(C) = 2 \Rightarrow P(C) = \frac{2}{52} = \frac{1}{26}$

(iv) Let D face card.
The face cards are Jack (J), Queen (Q), and King (K). $n(D) = 4 \times 3 = 12 \Rightarrow P(D) = \frac{12}{52} = \frac{3}{13}$

(v) Let E a number card.
The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10. $n(E) = 4 \times 9 = 36 \Rightarrow P(E) = \frac{36}{52} = \frac{9}{13}$

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203. **Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails**

Solution : When 3 fair coins are tossed,
$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$ $n(S) = 8$

i) Let A all heads. $A = \{(HHH)\}$ $n(A) = 1$ $\therefore P(A) = \frac{1}{8}$

ii) Let B atleast one tail.
$B = \{(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$ $n(B) = 7 \Rightarrow P(B) = \frac{7}{8}$

iii) Let C at most one head.
$C = \{(HTT), (THT), (TTH), (TTT)\}$ $n(C) = 4 \Rightarrow P(C) = \frac{4}{8} = \frac{1}{2}$

iv) Let D - atmost 2 tails
$D = \{(HHH), (HHT), (HTT), (HTH), (THH), (THT), (TTH)\}$ $n(D) = 7 \Rightarrow P(D) = \frac{7}{8}$

204. **A box contains cards numbered 3, 5, 7, 9, ... 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.**

Solution : $S = \{3, 5, 7, 9, \ldots\ldots\ldots.. 35, 37\}$, $n(S) = 18$

Let A - multiple of 7. $A = \{7, 14, 21, 28, 35\}$ $n(A) = 5 \Rightarrow P(A) = \frac{5}{18}$

Let B - a prime number

$B = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$ $n(B) = 11 \Rightarrow P(B) = \frac{11}{18}$

$A \cap B = \{7\}$, $n(A \cap B) = 1$, $P(A \cap B) = \frac{1}{18}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cup B) = \frac{5}{18} + \frac{11}{18} - \frac{1}{18} = \frac{15}{18} = \frac{5}{6}$

205. **Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.**

Solution : $S = \{(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)\}$ $n(S) = 8$

Let A - at most 2 tails

$A = \{(HHT), (HTH), (THH), (HTT), (THT), (TTH), (HHH)\}$ $n(A) = 7 \Rightarrow P(A) = \frac{7}{8}$

Let B - atleast 2 heads

$B = \{(HHH), (HHT), (HTH), (THH)\}$ $n(B) = 4 \Rightarrow P(B) = \frac{4}{8}$

$\therefore A \cap B = \{(HHH), (HHT), (HTH), (THH)\}$ $n(A \cap B) = 4 \Rightarrow P(A \cap B) = \frac{4}{8}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

206. **A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.**

Solution : $S = \{(HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)\}$ $n(S) = 8$

Let A - exactly 2 heads , $A = \{(HHT), (HTH), (THH)\}$ $n(A) = 3 \Rightarrow P(A) = \frac{3}{8}$

Let B - atleast one tail $B = \{(HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)\}$ $n(B) = 7 \Rightarrow P(B) = \frac{7}{8}$

Let C - Consecutively 2 heads , $C = \{(HHH), (HHT), (THH)\}$ $n(C) = 3 \Rightarrow P(C) = \frac{3}{8}$

$A \cap B = \{(HHT), (HTH), (THH)\}$ $n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{3}{8}$

$B \cap C = \{(HHT), (THH)\}$ $n(B \cap C) = 2 \Rightarrow P(B \cap C) = \frac{2}{8}$

$C \cap A = \{(HHT), (THH)\}$, $n(C \cap A) = 2 \Rightarrow P(C \cap A) = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8} = \frac{8}{8} = 1$

207. **The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\overline{A}) + P(\overline{B})$.**

Solution : Given $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow 0.6 + 0.2 = P(A) + P(B)$

$\therefore P(A) + P(B) = 0.8$

$\therefore P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B) = 2 - (P(A) + P(B)) = 2 - 0.8 = 1.2$

208. Find the mean and variance of the first n natural numbers.

Solution :

$$\text{Mean } \bar{x} = \frac{\Sigma x_i}{n} = \frac{\cancel{n}(n+1)}{2\times\cancel{n}} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\cancel{n}(n+1)(2n+1)}{6\times\cancel{n}} - \left[\frac{\cancel{n}(n+1)}{2\times\cancel{n}}\right]^2 = \frac{(n+1)(2n+1)}{6} - \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2-1}{12}$$

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209. 48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Solution :

x	f	$d = x - 9$	d^2	$f.d$	$f.d^2$
6	3	–3	9	– 9	27
7	6	–2	4	–12	24
8	9	–1	1	– 9	9
9	13	0	0	0	0
10	8	1	1	8	8
11	5	2	4	10	20
12	4	3	9	12	36
	48			0	124

$$\sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{124}{48}}$$

$$= 1.6$$

---

210. The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Solution :

x	f	$d = x - 8$	d^2	$f.d$	$f.d^2$
4	7	– 4	16	–28	112
6	3	– 2	4	– 6	12
8	5	0	0	0	0
10	9	2	4	18	36
12	5	4	16	20	80
	7			4	240

$$\sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = 2.87$$

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211. **Find the variance and standard deviation of the wages of 9 workers given below:**
₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Solution :

x	$d = x - 300$	d^2
280	–20	400
280	–20	400
290	–10	100
290	–10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	$\Sigma d - 0$	$\Sigma d^2 = 2000$

$$\text{variance } \sigma^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2$$

$$= \frac{2000}{9} - \left(\frac{0}{9}\right)^2$$

$$= \frac{2000}{9} = 222.2$$

$$\text{S.D} = \sqrt{222.2} = 14.91$$

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