V10M

Virudhunagar District Common Examinations
Second Revision Test - February 2023

Standard 10

MATHEMATICS

Time: 3.00 Hrs. Marks: 100

PART - I

Note: i) Answer ALL the questions. **14×1=14**

ii) Choose the correct answer from the four alternatives and write the option code and the corresponding answer.

1) $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is

a) 8 b) 12 c) 20 d) 16

2) If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the value of α and β are

a) (−1, 2) b) (2, −1) c) (−1, −2) d) (1, 2)

3) The sum of the exponents of the prime factors in the prime factorization of 1729 is

a) 1 b) 2 c) 3 d) 4

4) The first term of an arithmetic progression is unity and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?

a) 6 b) 7 c) 8 d) 9

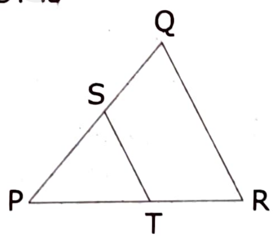
5) If $(x-6)$ is the HCF of $x^2-2x-24$ and x^2-kx-6 then the value of k is

a) 3 b) 5 c) 6 d) 8

6) Which of the following must be added to make x^4+64 a perfect square?

a) $16x^2$ b) $-16x^2$ c) $16x$ d) $-16x$

7) In the figure ST || QR, PS = 2 cm and SQ = 3 cm then the ratio of the area of ΔPQR to the area of ΔPST is



a) 25 : 4 b) 25:9 c) 4:9 d) 9:4

8) A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the y-axis. The path travelled by the man is

a) $x = 10$ b) $y = 10$ c) $x = 0$ d) $y = 0$

9) (2, 1) is the point of intersection of the straight lines

a) $x-y-3 = 0$; $3x-y-7 = 0$ b) $x+y = 3$; $3x+y = 7$

c) $3x+y = 3$; $x+y = 7$ d) $x+3y-3 = 0$; $x-y-7 = 0$

10) A tower is 60m high. Its shadow is x metres shorter when the sun's altitude is 60° than when it has been 45° then x is equal to

a) 34.64 m b) 25.36 m

c) 64.34 m d) 36.25 m

11) If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

a) 1:2 b) 1:4 c) 1:6 d) 1:8

12) The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical 10g of wood of base radius 1 cm and height 5 cm is

a) $\frac{4}{3}\pi$ b) $\frac{10}{3}\pi$ c) 5π d) $\frac{20}{3}\pi$

13) If the standard deviation of x, y, z is p then the standard deviation of 3x+5, 3y+5, 3z+5 is

a) 3p+5 b) 3p c) p+5 d) 9p+15

14) Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is 1/9, then the number of tickets bought by Kamalam is

a) 5 b) 10 c) 15 d) 20

PART - II

Note: i) Answer any TEN questions. **10×2=20**

ii) Question No. 28 is compulsory.

15) Let A = {1, 2, 3} and B = {x/x is a prime number less than 10} find A×B and B×A.

16) A Relation R is given by the set {(x, y) / y = x+3, x ∈ {0, 1, 2, 3, 4, 5}. Determine its domain and range.

17) Find the HCF of 340 and 412 using Euclids Division Algorithm.

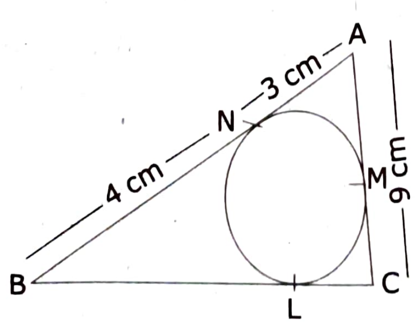
18) Solve: $8x \equiv 1 \pmod{11}$

19) Find the excluded value of the polynomial $\frac{t}{t^2 - 5t + 6}$.

20) If α and β are roots of the quadratic equation $x^2+7x+10 = 0$ then find the value of $\alpha^3-\beta^3$.

21) If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ then find B−5A.

22) In the figure, ΔABC is circumscribing a circle. Find the length of BC.



23) The line through the points (−2, a) and (9, 3) has slope −1/2. Find the value of a.

24) Prove: $\dfrac{\sin A}{1+\cos A}+\dfrac{\sin A}{1-\cos A}=2\operatorname{cosec} A$

25) The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

26) Find the range and co-efficient of range for the data 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

27) A coin is tossed thrice. What is the probability of getting two consecutive tails?

28) The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

PART - III

Note: i) Answer any TEN questions only. **10×5=50**

ii) Question No. 42 is compulsory.

29) A function f is defined by f(x) = 2x−3. (i) find $\dfrac{f(0)+f(1)}{2}$ (ii) find x such that f(x) = 0 (iii) find x such that f(x) = x (iv) find x such that f(x) = f(1−x).

30) Prove (fog)oh = fo(goh) for the functions f(x) = x−4, g(x) = x^2 and h(x) = 3x−5.

31) The 13^{th} term of an A.P is 3 and the sum of first 13 terms is 234. Find the sum of first 21 terms.

32) If a, b, c are three consecutive terms of an A.P and x, y, z are three consecutive terms of a G.P, then prove that $x^{b-c}\times y^{c-a}\times z^{a-b}=1$.

33) If $A=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$ then verify $(AB)^T=B^TA^T$.

34) State and prove Angle bisector theorem.

35) Find the equation of a straight line passing through (−3, 8) and whose sum of the intercepts on the co-ordinate axes is 7.

36) Find the equation of a straight line through the point of intersection of the lines 8x+3y = 18, 4x+5y = 9 and bisecting the line segment joining the points (5, −4) and (−7, 6).

37) A TV Tower stands vertically on the bank of a canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the tower is 60°. From the another point 20m away from this point on the line joining this point to the foot of the tower, the angle elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

38) Calculate the weight of the hollow brass sphere if the inner diameter is 14 cm and thickness is 1mm, and whose density is 17.3 g/cm^3.

39) A right circular cylindrical container of base radius 6 cm and 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

40) The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23.

41) From a well shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

42) Find the GCD of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$.

PART - IV

Note: Answer ALL the questions. **2×8=16**

43) Draw a circle of diameter 6 cm. At a point L on it draw a tangent to the circle using the alternate segment theorem.

(OR)

Draw the ΔABC of base 1 BC = 8 cm, $\angle A = 60^\circ$ and the bisector of $\angle A$ meets BC at D such that BD = 6 cm.

44) A bus is travelling at a uniform speed of 50 km/hr. Draw the distance - time graph and hence find

i) the constant of variation.

ii) how far will it travel in 1½ hours.

iii) the time required to cover a distance of 300 km from the graph.

(OR)

Discuss the nature of solutions of the quadratic equation $x^2+2x+5 = 0$ using graph.

Second Revision VNR - District MATHS - Key. 2023

1) b) 12

2) b) (2, -1)

3) c) 3

4) c) 8

5) b) 5

6) a) $16x^2$

7) a) 25 : 4

8) a) $x = 10$

9) b) $x+y=3;\ 3x+y=7$

10) b) 25 . 36

11) b) 1 : 4

12) a) $\frac{4}{3}\pi$

13) b) 3p

14) c) 15

2 Mark

15) $A = \{1, 2, 3\}$

$B = \{2, 3, 5, 7\}$

$A \times B = \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (5,1), (5,2), (5,3), (7,1), (7,2), (7,3)\}$

16) $y = x + 3$

$R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{3, 4, 5, 6, 7, 8\}$

17) 340, 412

$412 = 340(1) + 72 \neq 0$

$340 = 72 \times 4 + 52 \neq 0$

$72 = 52 \times 1 + 20 \neq 0$

$52 = 20 \times 2 + 12 \neq 0$

$20 = 12 \times 1 + 8 \neq 0$

$12 = 8 \times 1 + 4 \neq 0$

$8 = 4(2) + 0$

$\therefore$ HCF of 340 and 412 is 4

18) $8x \equiv 1 \pmod{11}$

$8x - 1 = 11k$ for some k

$x = \frac{11k+1}{8}$

$k = 5,\ x = 7$

$k = 13,\ x = 18$

The solutions are 7, 18, 29, 40, ...

19) The expression is undefined when $t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t = 3,\ t = 2$

The excluded values are 2, 3

20) $a = 1,\ b = 7,\ c = 10$

$\alpha + \beta = \frac{-b}{a} = -7$

$\alpha\beta = c/a = 10$

$\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

$= (3)^3 + 3(10)(3)$

$= 27 + 90 = 117$

$[\because \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{(-7)^2 - 4 \times 10}$

$= \sqrt{49-40} = \sqrt{9} = 3$

21) $B - 5A = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix}$

$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$

22) $AN = AM = 3cm$

$BN = BL = 4cm$

$CM = AC - AM = 9 - 3 = 6cm$

$\therefore BC = BL + CL = 4 + 6 = 10cm$

23) $Slope = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{1}{2} = \frac{3-a}{9+2}$

$\Rightarrow -\frac{1}{2} = \frac{3-a}{11}$

$-11 = 6 - 2a$

$\therefore 2a = 17$

$\boxed{a = \frac{17}{2}}$

24) $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$

$= \frac{\sin A(1-\cos A) + \sin(A)(1+\cos A)}{(1+\cos A)(1-\cos A)}$

$= \frac{\sin A - \sin A\cos A + \sin A + \sin A\cos A}{1-\cos^2 A}$

$= \frac{2\sin A}{\sin^2 A} = \frac{2}{\sin A} = 2\operatorname{cosec} A$

25) Given $r_1/r_2 = 12/16$
$= 3/4$

$\therefore C.S.A = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$

$= \left(\frac{3}{4}\right)^2$

$= 9/16$

$\therefore$ ratio of C.S.A of ballons $= 9:16$

26) $L = 61.4$

$S = 13.6$

Range $= L - S$
$= 61.4 - 13.6 = 47.8$

coeff of Range $= \frac{L-S}{L+S}$

$= \frac{47.8}{75} = 0.64$

27) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

Let A be the event of getting two consecutive tails

$A = \{HTT, TTH, TTT\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = 3/8$

28) $l = 5cm$, $R = 4cm$, $r = 1cm$

C.S.A of frustum

$= \pi(R+r)l$

$= \frac{22}{7}(4+1)\times 5$

$= \frac{22}{7} \times 5 \times 5$

$= 78.57\ cm^2$

5 Mark

29) $f(x) = 2x - 3$

$f(0) = -3$

$f(1) = -1$

i) $\frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2}$

$= -2$

ii) $f(x) = 0 \Rightarrow 2x - 3 = 0$

$\Rightarrow \boxed{x = 3/2}$

iii) $f(x) = x$

$2x - 3 = x$

$\boxed{x = 3}$

iv) $f(1-x) = 2(1-x) - 3$
$= -2x - 1$

$f(x) = f(1-x)$

$2x - 3 = -2x - 1$

$4x = 2 \Rightarrow \boxed{x = 1/2}$

30) $f(x) = x-4$

$g(x) = x^2$

$h(x) = 3x-5$

LHS $(f \circ g) \circ h$

$f \circ g = f(x^2)$

$= x^2 - 4$

$(f \circ g) \circ h = (3x-5)^2 - 4$

$= 9x^2 + 25 - 30x - 4$

$= 9x^2 - 30x + 21$

RHS $f \circ (g \circ h)$

$g \circ h = g(3x-5)$

$= (3x-5)^2$

$f \circ (g \circ h) = f(3x-5)^2$

$= (3x-5)^2 - 4$

$= 9x^2 - 30x + 21$

$(f \circ g) \circ h = f \circ (g \circ h)$

31) $t_{13} = 3$

$a + 12d = 3 \rightarrow (1)$

Sum of first 13 term $= 234$

$S_{13} = \frac{13}{2}[2a + 12d] = 234$

$2a + 12d = 36 \rightarrow (2)$

Solve (1) & (2)

$a = 33, d = -5/2$

$S_{21} = \frac{21}{2}\left[2(33) + 20\left(-\frac{5}{2}\right)\right]$

$= \frac{21}{2}[66 - 50] = 168$

32) Let a, b, c are three consecutive terms of A.P

$\therefore a = a, b = a+d, c = a+2d$

x, y, z are three cons. terms of G.P

$x = x, y = xr, z = xr^2$

LHS $x^{b-c} \times y^{c-a} \times z^{a-b}$

$= x^{(a+d)-(a+2d)} \times y^{(a+2d-a)} \times (z)^{(a-(a+d))}$

$= x^{-d} \times (xr)^{2d} \times (xr^2)^{-d}$

$= x^{-d} \times x^{2d} \times r^{2d} \times x^{-d} \times r^{-2d}$

$= x^{-d+2d-d} \times r^{2d-2d}$

$= x^0 \times r^0 = 1 \times 1 = 1 = \text{RHS}$

33) $(AB)^T = B^T A^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$

34) Angle bisector Theorem

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

35) $a + b = 7 \Rightarrow b = 7 - a$

$\frac{x}{a} + \frac{y}{7-a} = 1$

$(3, -8) \Rightarrow a^2 + 4a - 21 = 0$

$\boxed{a = 3}$ $a = -7$

$b = 4 \Rightarrow x/3 + y/4 = 1$

$4x + 3y - 12 = 0$

36) $8x+3y=18 \rightarrow (1)$

$4x+5y=9 \rightarrow (2)$

Solving (1) & (2)

$x = \frac{63}{28} = \frac{9}{4}$

$y = 0$

$\therefore (x,y) = \left(\frac{9}{4}, 0\right)$

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right)$

$= (-1, 1)$

Eqn of a St. line

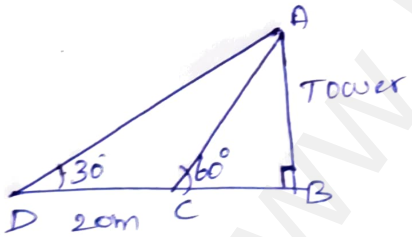
$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-0}{1-0} = \frac{x-9/4}{-1-9/4}$

$\Rightarrow y = \frac{4x-9}{-13}$

$\Rightarrow 4x+13y-9=0$

37)



AB → height of the tower

CD = 20m

In $\triangle ACB$,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC} \rightarrow (1)$

In $\triangle ABD$, $\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BC+20} \rightarrow (2)$

Divide (1) by (2)

$\frac{\sqrt{3}}{1/\sqrt{3}} = \frac{BC+20}{BC}$

$\sqrt{3} \times \sqrt{3} = \frac{BC}{BC} + \frac{20}{BC}$

$3 = 1 + \frac{20}{BC}$

$2 = \frac{20}{BC} \Rightarrow BC = \frac{20}{2}$

$= 10m$

$\boxed{BC = 10m}$

$(1) \Rightarrow \sqrt{3} = AB/BC$

$\sqrt{3} = AB/10$

$AB = 10\sqrt{3}$

$= 10 \times 1.732$

$\boxed{AB = 17.32\ m}$

38) $d = 14$ cm $\Rightarrow r = 7$cm

thickness $= 1mm = \frac{1}{10}$ cm

$R = r + thi = 7+0.1 = 7.1$

Volume $= \frac{4}{3}\pi (R^3 - r^3)$

$= 62.48 cm^3$

Weight base in $1cm^3 = 17.3gm$

Total $= 17.3 \times 62.48$

$= 1080.90gm$

39) $h = 15cm, r = 6$ cm

Volume of the container $= \pi r^2 h$

$= 22/7 \times 6 \times 6 \times 15$

$r_1 = 3, h_1 = 9$

Volume of one icecream = Volume of cone + vol. of hemi-spherical cap

$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$

$= \frac{22}{7} \times 45$

∴ Number of ice cream cone needed,

$= \frac{\frac{22}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45}$

$= 12$ cones

40) $n = 15,\ \bar{x} = 10$

$\sigma = 5$

$\Sigma x = n\bar{x} = 150$

correct total $= 150 - 8 + 23$

$= 165$

correct mean $\bar{x} = \frac{165}{15} = 11$

$S.D = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$(5)^2 = \frac{\Sigma x^2}{n} - (10)^2$

$\frac{\Sigma x^2}{15} = 25 + 100$

$= 125$

$\Sigma x^2 = 125 \times 15 = 1875$

corrected $\Sigma x^2 = \Big\} = 1875 - 8^2 + 23^2$

$= 2340$

∴ S.D $\sigma = \sqrt{\frac{2340}{15} - (11)^2}$

$= \sqrt{156 - 121} = \sqrt{35}$

$= 5.9$

41) $n(S) = 52$

Let A be the event of getting a red king

$n(A) = 2$

$P(A) = 2/52$

Let B be the event of getting a black queen

$n(B) = 2$

$P(B) = 2/52$

$P(A \cup B) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13}$

42) $f(x) = 6x^3 - 30x^2 + 60x - 48$

$= 6(x^3 - 5x^2 + 10x - 8)$

$g(x) = 3x^3 - 12x^2 + 21x - 18$

$= 3(x^3 - 4x^2 + 7x - 6)$

$GCD = 3(x - 2)$

44) a) i) $k = 5/6$

Direct Variation

ii) $x = 90,\ y = \frac{5}{6} \times 90 = 75$ km

iii) $y = 300 \Rightarrow x = 360$ minutes

$= 6$ hrs

b) No - real roots

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