TWO MARK QUESTIONS

1) $A = \{1, 3, 5\}$ $B = \{2,3\}$ then find (i) $A \times B$ and $B \times A$.(ii) Is $\times B = B \times A$? If not why? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

1) **[S-21]**

Solution:-

(i) $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

$B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

(ii) $A \times B \neq B \times A \therefore (1,2) \neq (2,1)$

(iii) $n(A \times B) = 6, n(B \times A) = 6$

$n(A) = 3, n(B) = 2$

$n(A) \times n(B) = 3 \times 2 = 6$

$\therefore n(A \times B) = n(B \times A) = n(A) \times n(B)$

2) Let $A = \{1,2,3\}$ and $B = \{x | x \text{is a prime number less than } 10\}$. Find $A \times B$ and $B \times A$. **[M-22]**

Solution:-

$A = \{1,2,3\}, B = \{2,3,5,7\}$

$A \times B = \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{(2,1), (3,1), (5,1), (7,1), (2,2), (3,2), (5,2), (7,2), (2,3), (3,3), (5,3), (7,3)\}$

3) $A = \{m, n\}$ and $B = \emptyset$ then find (i)A X B and (ii) A X A. காண்க. **[PTA-1]**

Solution:-

(i) $A X B = \{m, n\} X \emptyset = \emptyset$

(ii) $A X A = \{m, n\} X \{m, n\}$

$= \{(m, m), (m, n), (n, m), (n, n)\}$

4) $If\ A\ X\ B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B - **S-20 , A-22]**

Soluation:-

$A = \{3, 5\}$

$B = \{2, 4\}$

5) $If\ B\ X\ A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ then find A and B

Solution:-

$A = \{3,4\}$

$B = \{-2,0,3\}$

6) $If\ A = \{5, 6\},\ B = \{4, 5, 6\},\ C = \{5, 6, 7\}$ then verify that $A\ X\ A = \{(B\ X\ B) \cap (C\ X\ C)$ **[A-22]**

Soluation

$A = \{5,6\}, B = \{4,5,6\}$, C= $\{5,6,7\}$

LHS:-

$\boldsymbol{A \times A} = \{\mathbf{5, 6}\} \times \{\mathbf{5, 6}\}$

$= \{(\mathbf{5, 5}), (\mathbf{5, 6}), (\mathbf{6, 5}), (\mathbf{6, 6})\} \rightarrow (1)$

RHS:-

$B \times B = \{4,5,6\} \times \{4,5,6\}$

$= \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$C \times C = \{5,6,7\} \times \{5,6,7\}$

$= \{(5,5), (5,6), (5,7), (6,5), (6,6,), (6,7), (7,5), (7,6), (7,7)\}$

$(\boldsymbol{B \times B}) \cap (\boldsymbol{C \times C}) = \{(\mathbf{5, 5}), (\mathbf{5, 6}), (\mathbf{6, 5}), (\mathbf{6, 6})\} \rightarrow (2)$

∴ From (1) and (2) $\boldsymbol{A \times A} = (\boldsymbol{B \times B}) \cap (\boldsymbol{C \times C})$

7) A relation R is given by the set $\{(x, y) | y = x + 3, x \in \{0,1,2,3,4.5\}\}$. Determine its domain and range. **[PTA-5]**

Soluation:-

$$y = 0 + 3 = 3 \quad y = 3 + 3 = 6$$
$$y = 1 + 3 = 4 \quad y = 4 + 3 = 7$$
$$y = 2 + 3 = 5 \quad y = 5 + 3 = 8$$

$\therefore R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain= $\{0,1,2,3,4,5\}$

Range $= \{3,4,5,6,7,8\}$

8) A relation R is given by the set $\{(x,y)/\, y = x^2 + 3, x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range.

Soluation-

$$y = f(x) = x^2 + 3$$

$$f(0) = (0)^2 + 3 = 0 + 3 = 3$$
$$f(1) = (1)^2 + 3 = 1 + 3 = 4$$
$$f(2) = (2)^2 + 3 = 4 + 3 = 7$$
$$f(3) = (3)^2 + 3 = 9 + 3 = 12$$
$$f(4) = (4)^2 + 3 = 16 + 3 = 19$$
$$f(5) = (5)^2 + 3 = 25 + 3 = 28$$

Domain $= \{0,1,2,3,4,5\}$

Range $= \{3,4,7,12,19,28\}$

9) Let $A=\{1,2,3,4,...,45\}$ and R be the relation defined as "is square of a number" on A. Write R as a subset of $A\times A$. Also, find the domain and range of R. **[S-21]**

Spolution:-

$$A = \{1,2,3,4 \ldots,45\}$$
$$A \times A = \{1,2,3 \ldots,45\} \times \{1,2,3 \ldots,45\}$$
$$= \{(1,1),(1,2) \ldots (2,1) \ldots (3,1) \ldots \ldots (45,45)\}$$

R be the relation defined as "is square of a number" on A.

$\therefore R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$

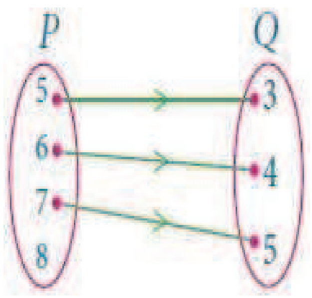
$R \subseteq A \times A$

Domain $= \{1,2,3,4,5,6\}$

Range $= \{1,4,9,16,25,36\}$

10) [M-22]

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form



Solution:-

(i) Set builder form $R = \{(x,y)/\, y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5,3), (6,4), (7,5)\}$

11) Let $X = \{3, 4, 6, 8\}$. Determine whether the relation. $R = \{(x, f(x))/\, x \in X, f(x) = x^2 + 1\}$ is a function from X to N?

Solution:-

$$f(3) = 3^2 + 1 = 9 + 1 = 10$$
$$f(4) = 4^2 + 1 = 16 + 1 = 17$$
$$f(6) = 6^2 + 1 = 36 + 1 = 37$$
$$f(8) = 8^2 + 1 = 64 + 1 = 65$$

$\therefore R = \{(3,10), (4,17), (6,37), (8,65)\}$

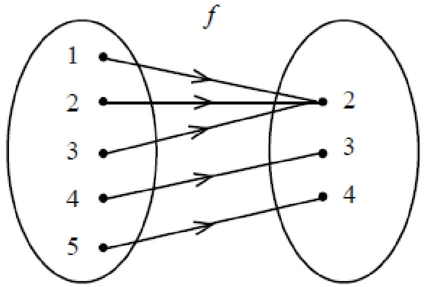
$R: X \to N$ is a function *from X to N*

12) Represent the function $f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$ through (i) an arrow diagram (ii) a table form (iii) a graph.

Solution:-

$$f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$$

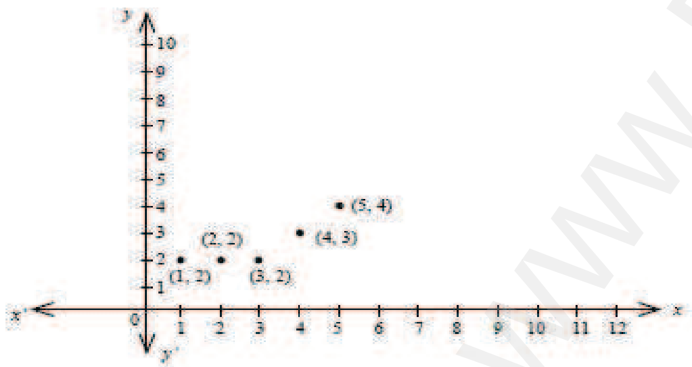
(i) **An arrow diagaram**



(ii) **A table form**

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) **A graph**



13) *If $R = \{(x,-2),(-5,y)\}$ is a identity function then find the value of x and y.* **[PTA-6]**

Solution:-

$R = \{(x,-2),(-5,y)\}$ *is a identity function*

$x = -2$ மற்றும் $y = -5$

14) Show that the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one-one function. **[S-20]**

solution:-

$$f(x) = m^2 + m + 3$$

domain , $\mathbb{N} = \{1, 2, 3, ,4, \dots\}$

Codomain , $\mathbb{N} = \{1, 2, 3, ,4, \dots\}$

$f(1) = (1)^2 + 1 + 3 = 1 + 1 + 3 = 5$

$f(2) = (2)^2 + 2 + 3 = 4 + 2 + 3 = 9$

$f(3) = (3)^2 + 3 + 3 = 9 + 3 + 3 = 15$

…

…

…

distinct elements of *DOMAIN* have distinct images in *CODOMAIN*.

∴ f is one-one function

15) Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$- as a composition of two functions.

Solution:-

$h(x) = 2x^2 - 5x + 3$ and $g(x) = \sqrt{x}$

Now $f(x) = \sqrt{2x^2 - 5x + 3}$

$= \sqrt{h(x)}$

$= g[h(x)]$

$= g \ o \ h(x)$

16) If $f(x) = 2x + 1, g(x) = x^2 - 2$ then find fog and gof.

Solution:-

$f\ o\ g = (2x + 1)\ o\ (x^2 - 2)$

$= 2(x^2 - 2) + 1$

$= 2x^2 - 4 + 1$

$= 2x^2 - 3$

$g\ o\ f = (x^2 - 2)\ o\ (2x + 1)$

$= (2x + 1)^2 - 2$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$

17) $If\ f(x) = 3 + x, g(x) = x - 4$ then verify that $fog = gof$ PTA-1]

Solution:-

$fog = (3 + x)\ o\ (x - 4)$

$= 3 + (x - 4)$

$= 3 + x - 4$

$= x - 1 \longrightarrow (1)$

$gof = (x - 4)\ o\ (3 + x)$

$= (3 + x) - 4$

$= 3 + x - 4$

$= x - 1 \longrightarrow (2)$

$\therefore from\ (1) and\ (2)\ fog = gof$

18) $If\ a^b x b^a = 800$ then find a and b.

Solution:-

a^b x $b^a = 800$

= 2 x 2 x 2 x 2 x 2 x 5 x 5

a^b x $b^a = 2^5$ x 5^2

$a = 2, b = 5$ (or) $a = 5, b = 2$

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

19) $If\ 13824 = 2^a$ x 3^b then find a and b. [M-22]

Solution:-

2^a x $3^b = 13824$

$\Longrightarrow 2^a$ x $3^b = 2^9$ x 3^3

$\therefore\ a = 9 \qquad b = 3$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

20) Find the least number that is divisible by the first ten natural numbers. [A-22]

Solution:-

lcm

$= 2^3 x 3^2 x\ 5\ x\ 7$

= 8 x 9 x 35

= 72 x 35

= 2520

2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	1, 1, 3, 2, 5, 3, 7, 4, 9, 5
2	1, 1, 3, 1, 5, 3, 7, 2, 9, 5
3	1, 1, 3, 1, 5, 3, 7, 1, 9, 5
3	1, 1, 1, 1, 5, 1, 7, 1, 3, 5
5	1, 1, 1, 1, 5, 1, 7, 1, 1, 5
7	1, 1, 1, 1, 1, 1, 7, 1, 1, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1, 1

∴ least number that is divisible by the first ten natural numbers = 2520

21) What is the time 100 hours after 7 a.m.?

Solution:-

	4
24	107
	96
	11

$$7 + 100 = 107$$

$$107\ mod\ 24 \equiv 11$$

∴ 100 hours after 7 a.m is 11 a.m.

22) What is the time 15 hours before 11p.m.??

Solution:-

11 p.m = 23 hours

$$23 - 15 = 8$$

8 mod 24 ≡ 8

∴15 hours before 11 p.m. is 8 a.m

23) Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Solution:-

If Tuesday is 2nd of the week

	6
7	47
	42
	5

$\therefore 2 + 45 = 47$

$\rightarrow 47$ mod $7 \equiv 5$

∴ 5th day of the week is Friday.

24) Find the number of terms in the A.P 3,6,9,12,. . . . ,111.

Solution:-

$a = 3$ and $d = 6 - 3 = 3$

$$n = \frac{l-a}{d} + 1$$

$$= \frac{111-3}{3} + 1$$

$$= \frac{108}{3} + 1$$

$$= 36 + 1$$

$$n = 37$$

25) Find the 19th term of an A.P $-11, -15, -19, \ldots$.

Solution:-

$a = -11$ and$d = -15 - (-11) = -4$

$$t_n = a + (n-1)d$$

$$t_{19} = -11 + (19-1)(-4)$$

$$= -11 + 18 \times (-4)$$

$$= -11 + (-72)$$
$$t_{19} = -83$$

26) Which term of an A.P $16, 11, 6, 1, \ldots$ is -54?

Solution:-

$a = 16$ and $d = 11 - 16 = -5$ then $t_n = -54$

$$t_n = a + (n-1)d$$
$$-54 = 16 + (n-1) \times -5$$
$$-54 - 16 = (n-1) \times -5$$
$$-70 = (n-1) \times -5$$
$$\frac{-70}{-5} = n - 1$$
$$14 = n - 1$$
$$14 + 1 = n$$
$$\therefore n = 15$$

27) Find x, y and z given that the numbers $x, 10, y, 24, z$ are in A.P.

Solution:-

In given sequence $10, y, 24$ are in A.P

$$\therefore 2y = 10 + 24$$
$$2y = 34$$
$$y = \frac{34}{2}$$
$$y = 17$$
$$\therefore d = 17 - 10 = 7$$
$$\therefore x = 10 - 7 = 3 \text{and} z = 24 + 7 = 31$$
$$\therefore x = 3, y = 17, z = 31$$

28) In a G.P. 729, 243, 81,… find t_7.

Solution:-

$$a = 729$$
$$r = \frac{t_2}{t_1} = \frac{243}{729} = \frac{81}{243} = \frac{27}{81} = \frac{9}{27} = \frac{1}{3}$$
$$n = 7$$
$$t_n = ar^{n-1}$$
$$\therefore t_7 = ar^6$$
$$= (729)\left(\frac{1}{3}\right)^6$$
$$= 729 \text{ x} \frac{1^6}{3^6}$$
$$= \frac{729}{3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3}$$
$$t_7 = 1$$

29) Find the sum to an infifinity of $3 + 1 + \frac{1}{3} + \cdots \infty$

Solution:-

$$a = 3, \quad r = \frac{1}{3}$$
$$S_\infty = \frac{a}{1-r}$$

$\therefore\ S_\infty = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{3-1}{3}} = \frac{3}{\frac{2}{3}} = 3 \text{ x} \frac{3}{2} = \frac{9}{2}$

30) Find the sum to an infifinity of

(i) $9 + 3 + 1 + \cdots$

Solution:-

(i) $9 + 3 + 1 + \cdots$

$$a = 9,\ r = \frac{3}{9} = \frac{1}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$\therefore\ S_\infty = \frac{9}{1-\frac{1}{3}} = \frac{9}{\frac{3-1}{3}} = \frac{9}{\frac{2}{3}} = 9 \text{ x} \frac{3}{2} = \frac{27}{2}$

31) Find the sum of $1 + 3 + 5 + \ldots + 55$.**[PTA-6]**

தீர்வு:-

WKT, $l = 55$ (odd number)

$$1 + 3 + 5 + \cdots + l = \left(\frac{l+1}{2}\right)^2$$

$$\therefore\quad 1 + 3 + 5 + \cdots + 55 = \left(\frac{55+1}{2}\right)^2$$

$$= \left(\frac{56}{2}\right)^2$$

$$= (28)^2$$

$= 784$

32) Find the sum of

(i) $1^3 + 2^3 + 3^3 + \cdots + 16^3$

(ii) $9^3 + 10^3 + 11^3 + \cdots + 21^3$

solution:-

(i) $1^3 + 2^3 + 3^3 + \cdots + 16^3$

WKT, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore\ 1^3 + 2^3 + 3^3 + \cdots + 16^3 = \left[\frac{16 \text{ x } 17}{2}\right]^2$

$= [8 \text{ x } 17]^2$

$= (136)^2$

$= 18496$

(ii) $9^3 + 10^3 + 11^3 + \cdots + 21^3$

WKT, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\therefore\ 9^3 + 10^3 + 11^3 + \cdots + 21^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 21^3) - (1^3 + 2^3 + 3^3 + \cdots + 8^3)$

$= \left(\frac{21 \text{ x } 22}{2}\right)^2 - \left(\frac{8 \text{ x } 9}{2}\right)^2$

$= (231)^2 - (36)^2$

$= 53361 - 1296$

$= 52065$

33) *If* $1 + 2 + 3 + \cdots + k = 325$ then find $1^3 + 2^3 + 3^3 + \cdots + k^3$

Solution:-

$$1 + 2 + 3 + \cdots + k = 325$$

$$\Rightarrow \quad \frac{k(k+1)}{2} = 325$$

$\therefore\ 1^3 + 2^3 + 3^3 + \cdots + k^3 = \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$

34) *If* $1^3 + 2^3 + 3^3 + \cdots + k^3 = 44100$ then find $1 + 2 + 3 + \cdots + k$

Solution:-

$1^3 + 2^3 + 3^3 + \ldots + k^3 = 44100$

$$\left[\frac{k(k+1)}{2}\right]^2 = 44100$$

$$\left[\frac{k(k+1)}{2}\right]^2 = (210)^2$$

Take square root on both sides

$$\frac{k(k+1)}{2} = 210$$

$$1+2+3+\cdots+k = 210$$

35) *Find the excluded values of* $\frac{7p+2}{8p^2+13p+5}$ **[M-22]**

solution:-

$$\frac{7p+2}{8p^2+13p+5}$$

$$= \frac{7p+2}{(p+1)(8p+5)}$$

+40	+13
$\frac{+8}{8p}$	$\frac{+5}{8p}$

$let\ (p+1)(8p+5) = 0$

$p+1 = 0$ (or) $8p+5 = 0$

$p = -10$ (or) $8p = -5$

$p = -10$ (or) $p = \frac{-5}{8}$

the excluded values -1 and $\frac{-5}{8}$

36) Find the square root of $\frac{144\, a^8 b^{12} c^{16}}{81\, f^{12} g^4 h^{14}}$ **[PTA-5]**

solution:-

$$\sqrt{\frac{144\, a^8 b^{12} c^{16}}{81\, f^{12} g^4 h^{14}}} = \left|\frac{12\, a^4 b^6 c^8}{9\, f^6 g^2 h^7}\right| = \frac{4}{3}\left|\frac{a^4 b^6 c^8}{f^6 g^2 h^7}\right|$$

37) Find the square root of $\frac{400\, x^4 y^{12} z^{16}}{100\, x^8 y^4 z^4}$ **[A-22]**

Solution:-

$$\sqrt{\frac{400\, x^4 y^{12} z^{16}}{100\, x^8 y^4 z^4}} = \sqrt{\frac{4\, y^8 z^{12}}{x^4}} = 2\left|\frac{y^4 z^6}{x^2}\right|$$

38) Determine the quadratic equations, whose sum and product of roots are -9 and 20 **[S-21]**

Solution:-

sum of the roots $= -9$

product of the roots $= 20$

$x^2 -$ (sum of the roots)x + product of the roots $= 0$

$x^2 - (-9)x + 20 = 0$

$x^2 + 9x + 20 = 0$

39) Determine the quadratic equations, whose sum and product of roots are $\frac{-3}{2}$ and -1 **[PTA-4]**

Solution:-

sum of the roots $= \frac{-3}{2}$

product of the roots $= -1$

$x^2 -$ (sum of the roots)x + product of the roots $= 0$

$= 0.$

$$x^2 - \left(\frac{-3}{2}\right)x + (-1) = 0$$

$$x^2 + \frac{3}{2}x - 1 = 0$$

$$\frac{2x^2+3x-2}{2} = 0$$

$$2x^2+3x-2 = 0$$

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40) Determine the nature of the roots for the following quadratic equation $15x^2 + 11x + 2 = 0$ **[S-21]**

Solution:-

$$15x^2 + 11x + 2 = 0$$

$$a = 15, \quad b = 11, \quad c = 2$$

$$\Delta = b^2 - 4ac$$
$$= (11)^2 - 4(15)(2)$$
$$= 121 - 120$$
$$\Delta = 1 > 0$$

Real and Unequal roots

41) If a matrix has 16 elements, what are the possible orders it can have?

Solution:-

the possible orders of a matrix with 16 elements

1×16	8×2
2×8	16×1
4×4	

42) If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:-

the possible orders of a matrix with **18 elements**	the possible orders of a matrix with **6 elements**
1×18	1×6
2×9	2×3
3×6	3×2
6×3	6×1
9×2	
18×1	

43) Construct a 3 x 3 matrix whose elements are given by

$$A = a_{ij} = i^2 j^2$$

Solution:-

$$a_{11} = 1^2 1^2 = 1 \times 1 = 1$$
$$a_{12} = 1^2 2^2 = 1 \times 4 = 4$$
$$a_{13} = 1^2 3^2 = 1 \times 9 = 9$$
$$a_{21} = 2^2 1^2 = 4 \times 1 = 4$$
$$a_{22} = 2^2 2^2 = 4 \times 4 = 16$$
$$a_{23} = 2^2 3^2 = 4 \times 9 = 36$$
$$a_{31} = 3^2 1^2 = 9 \times 1 = 9$$
$$a_{32} = 3^2 2^2 = 9 \times 4 = 36$$
$$a_{33} = 3^2 3^2 = 9 \times 9 = 81$$

$$\therefore A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$$

44) Construct a 3 x 3 matrix whose elements are given by

$$a_{ij} = |i - 2j|$$

Solution:-

The general 3 x 3 matrix is given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{ij} = |i - 2j|$$

$$a_{11} = |1 - 2(1)| = |1 - 2| = |-1| = 1$$
$$a_{12} = |1 - 2(2)| = |1 - 4| = |-3| = 3$$
$$a_{13} = |1 - 2(3)| = |1 - 6| = |-5| = 5$$
$$a_{21} = |2 - 2(1)| = |2 - 2| = |0| = 0$$
$$a_{22} = |2 - 2(2)| = |2 - 4| = |-2| = 2$$
$$a_{23} = |2 - 2(3)| = |2 - 6| = |-4| = 4$$

$$a_{31} = |3-2(1)| = |3-2| = |1| = 1$$
$$a_{32} = |3-2(2)| = |3-4| = |-1| = 1$$
$$a_{33} = |3-2(3)| = |3-6| = |-3| = 3$$

Hence the required matrix is

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$$

45) Find the values of x, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:-

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$$x = 3\ , \quad y = 12\ , \quad z = 3$$

(ii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$$x+y+z = 9 \longrightarrow (1)$$
$$x+z = 5 \longrightarrow (2)$$
$$y+z = 7 \longrightarrow (3)$$

Substitute (2) in (1)

$$5+y = 9$$
$$y = 9-5$$
$$y = 4$$

$put\ y = 4$ in(3)

$$4+z = 7$$
$$z = 7-4$$
$$z = 3$$

$put\ z = 3$ in(2)

$$x+3 = 5$$
$$x = 5-3$$
$$x = 2$$

$$\therefore\ x = 2\ ,\ y = 4\ ,\ z = 3$$

46) $If\ A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

Solution:-

$$A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

47) $If A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of -A.

[PTA-2 , S-20]

Solution:-

$$-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

48) $If\ A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

Solution:-

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$\therefore (A^T)^T = A$ is verified.

49) If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then find $2A + B$ **[PTA-3]**

Solution:-

$$2A + B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

50) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then verify $AA^T = I$

Solution:-

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$AA^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$\cos\theta \quad \sin\theta \qquad -\sin\theta \quad \cos\theta$

$$= \begin{matrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{matrix}\begin{pmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + -\sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$AA^T = I$ is verified

51) If $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$ then verify $A^2 = I$

Solution:-

$$A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$A^2 = A \times A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$5 \quad 6 \qquad -4 \quad -5$

$$= \begin{matrix} 5 & -4 \\ 6 & -5 \end{matrix}\begin{pmatrix} 25-24 & -20+20 \\ 30-30 & -24+25 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 = I$$

52) (Show that the points $(-3,-4), (7,2)$ and $(12,5)$ are collinear. **[S-21]**

Solution:-

$$(x_1, y_1) = (-3,-4)$$

$$(x_2, y_2) = (7,2)$$

$$(x_3, y_3) = (12,5)$$

$\therefore$ Area ofa a triangle

$$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -3 & 7 & 12 & -3 \\ -4 & 2 & 5 & -4 \end{Bmatrix}$$

$$= \frac{1}{2}(-6 + 35 - 48 + 28 - 24 + 15)$$

$$= \frac{1}{2}(-78 + 78)$$

$= 0$ Therefore, the given points are collinear.

53) *Show that the points* $P(-1.5, 3), Q(6, -2), R(-3, 4)$ are collinear .**[M-22]**

Solution:-

$(x_1, y_1) = P(-1.5, 3)$

$(x_2, y_2) = Q(6, -2)$

$(x_3, y_3) = R(-3, 4)$

Area ofa a ΔPQR

$$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{Bmatrix}$$

$$= \frac{1}{2}(3 + 24 - 9 - 18 - 6 + 6)$$

$$= \frac{1}{2}(27 - 27)$$

$= 0$ Therefore, the given points are collinear.

54) Find the slope of a line joining the points $(5, \sqrt{5})$ with the origin **[A-22]**

Solution:-

$(x_1, y_1) = (5, \sqrt{5})$ and $(x_2, y_2) = (0, 0)$

WKT, slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

∴ slope $m = \frac{0-\sqrt{5}}{0-5} = \frac{\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{1}{\sqrt{5}}$

55) *The line p passes through the points* $(3, -2), (12, 4)$ *and the points* $(6, -2)$ *and* $(12, 2)$ *. Is p parralel to q?* **[M-22 , A-22]**

Solution:-

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

The slope of line p is

$x_1 \to 3$ $y_1 \to -2$	$x_2 \to 12$ $y_2 \to 4$

$$m_1 = \frac{4 + 2}{12 - 3} = \frac{6}{9} = \frac{2}{3}$$

The slope of line $q$is

$x_1 \to 6$ $y_1 \to -2$	$x_2 \to 12$ $y_2 \to 2$

$$m_2 = \frac{2 + 2}{12 - 6} = \frac{4}{6} = \frac{2}{3}$$

$$m_1 = m_2$$

Therefore, line p is parallel to the line q.

56) Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$ **[S-21]**

Solution:-

$$8x - 7y + 6 = 0$$

$$-7y = -8x - 6$$

$$7y = 8x + 6$$

$$y = \frac{8}{7}x + \frac{6}{7}$$

Comparing with $y = mx + c$

Slope $m = \frac{8}{7}$ and y intercept $c = \frac{6}{7}$

57) *Find the equation of a line passing through the point* $(-1, 2)$ and having slope $\frac{-5}{4}$ **[M-22]**

Solution:-

$(x_1, y_1) = (-1,\ 2);\ = \frac{-5}{4}$.

The equation of the point-slope form of the straight line is $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{-5}{4}[x - (-1)]$$

$$4(y - 2) = -5[x + 1]$$

$$4y - 8 = -5x - 5$$

$$4y - 8 + 5x + 5 = 0$$

$$5x + 4y - 3 = 0$$

58) Show that the straight lines $2x + 3y = 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution:-

Slope $= \frac{-\text{co efficient of x}}{\text{co efficient of y}}$

Slope of the line $2x + 3y + 8 = 0$

$$= \frac{-2}{3}$$

Slope of the line $4x + 6y + 18 = 0$

$$= \frac{-4}{6} = \frac{-2}{3}$$

Here, $m_1 = m_2$ $\therefore$ Two lines are parallel.

59) Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution:-

Slope $= \frac{-\text{co efficient of x}}{\text{co efficient of y}}$

Slope of the line $x - 2y + 3 = 0$

$$= \frac{-1}{-2} = \frac{1}{2}$$

Slope of the line $6x + 3y + 8 = 0$

$$= \frac{-6}{3} = -2$$

$$\therefore m_1 \times m_2 = \frac{1}{2} \times -2 = -1$$

$\therefore$ Two lines are perpendicular

60) Find the equation of the straight line which is parallel to the line $3x - 7y = 12$ and passing through the point (6,4).

Solution:-

Equation of the straight line parallel to $3x - 7y = 12$ is $3x - 7y + k = 0$

The line passes through (6,4) $\therefore 3(6) - 7(4) + k = 0$

$$18 - 28 + k = 0 \quad \rightarrow -10 + k = 0$$

$$\therefore k = 10$$

Equation of the parallel line $3x - 7y + 10 = 0$

61) Prove that $\sqrt{\frac{1+cos\theta}{1-cos\theta}} = cosec\theta + cot\theta$ **[GMQ]**

solution:-

$$LHS = \sqrt{\frac{1+cos\theta}{1-cos\theta}}$$

$$= \sqrt{\frac{1+cos\theta}{1-cos\theta} \text{x} \frac{1+cos\theta}{1+cos\theta}}$$

$$= \sqrt{\frac{(1+cos\theta)^2}{1^2-cos^2\theta}} [\because a^2-b^2=(a+b)(a-b)]$$

$$= \sqrt{\frac{(1+cos\theta)^2}{sin^2\theta}} [\because sin^2\theta + cos^2\theta = 1]$$

$$= \frac{1+cos\theta}{sin\theta}$$

$$= \frac{1}{sin\theta} + \frac{cos\theta}{sin\theta}$$

$$= cosec\theta + cot\theta \left[\because cosec\theta = \frac{1}{sin\theta} \& \ cot\theta = \frac{cos\theta}{sin\theta}\right]$$

$$= RHS$$

62) *Prove that* $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution:-

$$LHS = \sqrt{\frac{1+sin\theta}{1-sin\theta}} + \sqrt{\frac{1-sin\theta}{1+sin\theta}}$$

$$= \frac{\sqrt{1+sin\theta}}{\sqrt{1-sin\theta}} + \frac{\sqrt{1-sin\theta}}{\sqrt{1+sin\theta}}$$

$$= \frac{\left(\sqrt{1+sin\theta}\right)^2 + \left(\sqrt{1-sin\theta}\right)^2}{\sqrt{1-sin\theta}\sqrt{1+sin\theta}}$$

$$= \frac{1+sin\theta+1-sin\theta}{\sqrt{1^2-sin^2\theta}} [\because a^2-b^2=(a+b)(a-b)]$$

$$= \frac{2}{\sqrt{cos^2\theta}} [\because sin^2\theta+cos^2\theta=1]$$

$$= \frac{2}{cos\theta}$$

$$= 2\ sec\theta$$

$$= RHS$$

63) A tower stands vertically on the ground. From a point on the ground which is 48m away from the foot of the tower, the angle of elevation of the top of the tower is 30^0. Find the height of the tower.

Solution:-

$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{PQ}{RQ}$$

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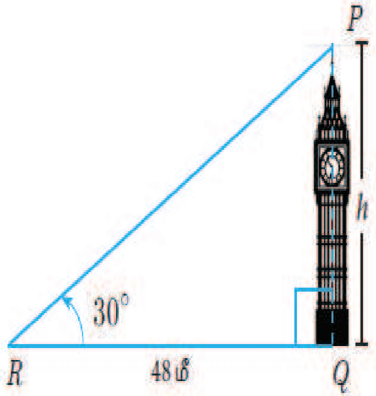
$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$\therefore h = \frac{48}{\sqrt{3}} = \frac{3 \times 16}{\sqrt{3}}$$

$$\therefore h = 16\sqrt{3}\text{ m}$$

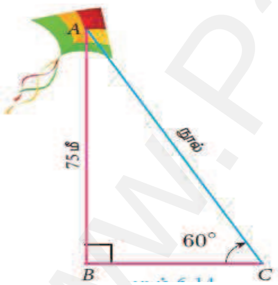


$$h = \frac{\sqrt{3}\text{x}\sqrt{3}\text{x } 16}{\sqrt{3}}$$

$$h = 16\sqrt{3}\text{மீ}$$

64) A kite is flying at a straight of 75m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^0. Find the length of the string, assuming that there is no slack in the string.

Solution:-



$$sin\theta = \frac{opp\ side}{hyp\ side}$$

$$sin60^0 = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$

$$\therefore AC = 75 \times \frac{2}{\sqrt{3}}$$

$$= \frac{3 \times 25 \times 2}{\sqrt{3}}$$

$$\therefore AC = 50\sqrt{3}\text{ m}$$

65) Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$m.

Solution:-

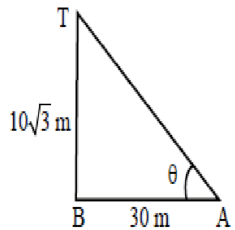
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan\theta = \frac{BT}{AB}$$

$$tan\theta = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3}$$

$$\therefore tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^0$$



66) A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30^0. Find the width of the road.

Solution:-

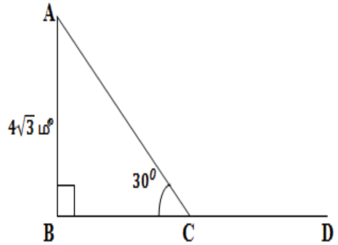
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{BC}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x}$$

$$\therefore x = 4\sqrt{3} \times \sqrt{3}$$

$$\therefore x = 4 \times 3 = 12 \text{ m}$$



67) A player siting on the top of a tower of height 20m observes the angle of depression of a ball on the ground as 60^0. Find the distance between the foot of the tower and the ball.($\sqrt{3} = 1.732$).

[PTA-3]

Solution:-

$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan60^0 = \frac{BC}{AB}$$

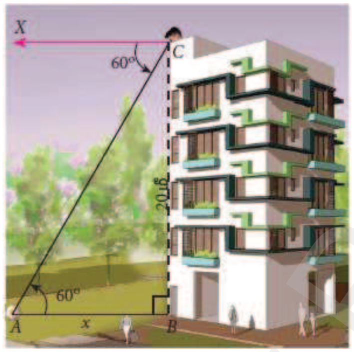
$$\sqrt{3} = \frac{20}{x}$$

$$\therefore x = \frac{20}{\sqrt{3}} = \frac{20 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\therefore x = \frac{20 \times 1.732}{3}$$

$$\therefore x = 20 \times 0.5773$$

$$x = 11.55 \text{ m}$$



68) From the top of a rock $50\sqrt{3}$m high, the angle of depression of a car on the ground is observed to be 30^0. Find the distance of the car from the rock.

Solution:-

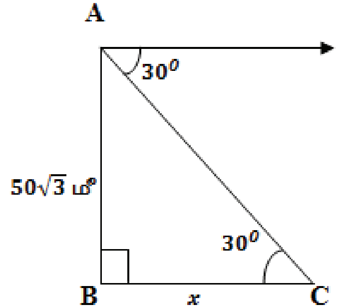
$$tan\theta = \frac{\text{opp side}}{\text{adj side}}$$

$$tan30^0 = \frac{KR}{CK}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$

$$\therefore x = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$

$$\therefore x = 150 \text{ m}$$



69) The curved surface area of a right circular cylinder of height 14cm is $88cm^2$. Find the diameter of the cylinder.

Solution:-

$h = 20$ cm

CSA of a cylinder $= 88$ sq.cm

$$2\pi rh = 88$$

$$2 \times \frac{22}{7} \times r \times 14 = 88$$

$$r = 88 \times \frac{1}{2} \times \frac{7}{22} \times \frac{1}{14}$$

$r = 1$ cm

diameter$d = 2$cm

70) The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

[A-22]

Solution:-

Let radius $r = 5x$ height $h = 7x$

Surface Area of a Cylinder$= 5500$

$$\Rightarrow 2\pi rh = 5500$$

$$2 \times \frac{22}{7} \times 5x \times 7x = 5500$$

$$x^2 = \frac{5500}{2 \times 22 \times 5}$$

$$x^2 = 25$$

$$x = 5$$

∴ radius of a cylinder $= 5x = 5 \times 5 = 25$cm.

height of a cylinder $= 7x = 7 \times 5 = 35$cm.

71) If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height. **[A-22]**

solution:-

radius, $r = 7$cm.

Total surface area of a cone $= 704$

$$\Rightarrow \pi r(l + r) = 704$$

$$\frac{22}{7} \times 7 \times (l + 7) = 704$$

$$l + 7 = \frac{704}{22}$$

$$l + 7 = 32$$

$$l = 32 - 7$$

$$l = 25\text{cm}$$

72) Find the diameter of a sphere whose surface area is 154 m^2 **[S-20]**

Solution:-

Let radius $= r$

surface area of sphere$= 154$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{4 \times 22}$$

$$r^2 = \frac{7 \times 7}{2 \times 2}$$

$$r = \frac{7}{2}$$

∴ the diameter of a sphere $= 2r = 2 \times \frac{7}{2} = 7$cm

73) If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area? **[S-20]**

Solution:-

Base area of a hemisphere=Area of a circle

$$\therefore \pi r^2 = 1386$$

TSA of a hemisphere$= 3\pi r^2$

$$= 3 \times 1386$$

$$= 4158 \text{ sq.m}$$

74) Find the volume of a cylinder whose height is 2 m and whose base area is 250 m². **[S-21]**

Solution:-

height $h = 2$மீ

base area $= 250$

$\pi r^2 = 250$ m^2

volume of a cylinder $= \pi r^2 h$

$= 250 \times 2$

$= 500$ m³

75) The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.

Solution:-

$h = 24$ cm

volume of a cone $= 11088$ cu.cm

$\frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$\therefore r^2 = 11088 \times 3 \times \frac{7}{22} \times \frac{1}{24}$

$r^2 = 63 \times 7 = 3 \times 3 \times 7 \times 7$

$\therefore r = 3 \times 7 = 21$

76) A metallic sphere of radius 16cm is melted and recast into small spheres each of radius 2cm. Howmany small spheres can be obtained?

Solution:-

Big sphere

$r = 16$cm

volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (16)^3$

$= \frac{4}{3}\pi \times 16 \times 16 \times 16$

Small sphere

$r = 2$cm

volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (2)^3$

$= \frac{4}{3}\pi \times 2 \times 2 \times 2$

No. of required sphere $= \frac{\text{volume of big sphere}}{\text{volume of small sphere}}$

$= \frac{\frac{4}{3}\pi \times 16 \times 16 \times 16}{\frac{4}{3}\pi \times 2 \times 2 \times 2}$

$= 8 \times 8 \times 8$

$= 512$

77) The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights. **[PTA-4 , M-22]**

Solution:-

Cone-1:-

Let radius $= r$

height $= h_1$

Cone-2:-

radius $= r$

height $= h_2$

volumes of two cones = 3600 : 5040

$$\frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2 = 3600 : 5040$$

$$\frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{3600}{5040}$$

$$\frac{h_1}{h_2} = \frac{360}{504}$$

$$\frac{h_1}{h_2} = \frac{30}{42}$$

$$\frac{h_1}{h_2} = \frac{5}{7}$$

$$h_1 : h_2 = 5 : 7$$

78) The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. **[M-22]**

Solution:-

$r_1 = 12$cm , $r_2 = 16$cm

ratio of C.S.A. of balloons $= \frac{4\pi {r_1}^2}{4\pi {r_1}^2}$

$$= \frac{4\pi \text{ x } 12 \text{ x } 12}{4\pi \text{ x } 16 \text{ x } 16}$$

$$= \frac{9}{16}$$

$$= 9 : 16$$

79) Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:-

Desending order 67,53,48,44,39,25,18

$$\therefore L = 67, S = 18$$

Range $= L - S = 67 - 18$

Range= 49

Co efficient of Range= $\frac{L-S}{L+s}$

$$= \frac{67 - 18}{67 + 18}$$

$$= \frac{49}{85} = 0.576$$

80) Find the range and coefficient of range of the following data.(i) 63, 89, 98, 125, 79, 108, 117, 68. (ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8.

Solution:-

(i) Decending order 125,117,108,98,89,79,68,63

$$\therefore L = 125, S = 63$$

Range= $L - S = 125 - 63$

$$= 62$$

Co efficient of Range= $\frac{L-S}{L+s}$

$$= \frac{125 - 63}{125 + 63}$$

$$= \frac{62}{188} = 0.33$$

(ii) Decending order 61.4, 43.5, 38.4, 29.8,18.9, 13.6

$\therefore L = 61.4, S = 13.6$

Range $= L - S = 61.4 - 13.6$

$= 47.8$

Co efficient of Range$= \frac{L-S}{L+s}$

$= \dfrac{61.4 - 13.6}{61.4 + 13.6}$

$= \dfrac{47.8}{75} = 0.64$

81) If the range and the smallest value of a set of data are 36.8 an 13.4 respectively, then find the largest value.

Solution:-

$\therefore$Range$= 36.8, S = 13.4$

Range$= L - S$

$36.8 = L - 13.4$

$L = 36.8 + 13.4$

$\therefore L = 50.2$

82) The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value. **[PTA-4]**

Solution:-

Range$= 13.67$, $L = 70.08$

Range$= L - S$

$13.67 = 70.08 - S$

$S = 70.08 - 13.67$

$S = 56.41$

83) Calculate the range of the following data

வயது (வருடங்களில்)	16-18	18-20	20-22	22-24	24-26	26-28
மாணவர்களின் எண்ணிக்கை	0	4	6	8	2	2

Solution:-

$L = 28, S = 18$

Range$= L - S = 28 - 18$

$\therefore$Range$= 10\ years$

84) Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution:-

$L = 450, S = 650$

Range$= L - S = 650 - 450$

$\therefore$Range$= 250$

85) If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

solution:-

standard deviation, $\sigma = 4.5$

the standard deviation will not change when we add 5 to all the values.

$\therefore$ the new standard deviation.,$\sigma = 4.5$

86) If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

solution:-

standard deviation, $\sigma = 3.6$

we divide each data by 3 the standard deviation also get divided by 3. ∴ new standard deviation

$$\sigma = \frac{3.6}{3} = 1.2$$

new variance $\sigma^2 = (1.2)^2 = 1.44$

87) If the standard deviation of 20 datas is $\sqrt{6}$. And each value of data is multiple by 3 then find the new variance and new standard deviation. **[PTA-1]**

Solution:- standard deviation $\sigma = \sqrt{6}$

each value of data is multiple by 3

∴ new standard deviation $= 3\sqrt{6}$

new variance $= (3\sqrt{6})^2 = 9 \text{ x } 6 = 54$

88) Find the standard deviation of first 21 natural numbers. **[PTA-6]**

Solution:-

standard deviation of first 'n' natural numbers $\sigma = \sqrt{\frac{n^2-1}{12}}$

$$\therefore \sigma = \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$$

$= \sqrt{36.67} = 6.06$

89) The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation. **[PTA-3]**

Solution:-

$\bar{x} = 25.6$, $C.V = 18.75$

$C.V = \frac{\sigma}{\bar{x}} \text{x } 100\%$

$18.75 = \frac{\sigma}{25.6} \text{x } 100$

$\frac{18.75 \text{ x } 25.6}{100} = \sigma$

$\sigma = \frac{480}{100}$

$\sigma = 4.8$

90) The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation. **[GMQ]**

Solution:-

$\sigma = 6.5$, $\bar{x} = 12.5$

$C.V = \frac{\sigma}{\bar{x}} \text{x } 100\%$

$= \frac{6.5}{12.5} \text{ x } 100$

$= \frac{650}{12.5}$

$= \frac{6500}{125}$

$= 52\ \%$

91) The standard deviation and coefficient of variation of a data

are 1.2 and 25.6 respectively. Find the value of mean.

Solution:-

$$\sigma = 1.2\,,\quad C.V = 25.6$$

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$25.6 = \frac{1.2}{\bar{x}} \text{ x } 100$$

$$\bar{x} = \frac{1.2 \text{ x } 100}{25.6}$$

$$\bar{x} = \frac{120}{25.6}$$

$$\bar{x} = 4.6875$$

92) If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

solution:-

$$\bar{x} = 15\quad,\quad C.V = 48$$

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$48 = \frac{\sigma}{15} \text{ x } 100$$

$$\frac{48 \text{ x } 15}{100} = \sigma$$

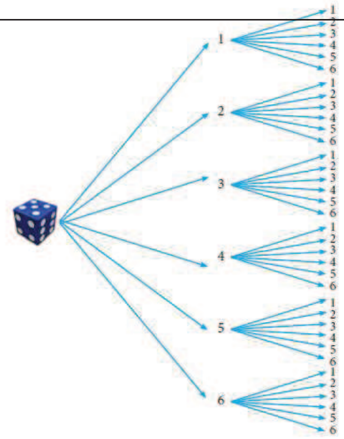
$$\sigma = \frac{720}{100}$$

$$\sigma = 7.2$$

93) Express the sample space for rolling two dice using tree diagram.

Solution:-

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$



$\therefore\ n(S) = 36$

94) Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:-

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

$A = \{\text{Getting Different Faces}\}$

$$A = \{HT, TH\}$$

$$n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

95) A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head [**S-21**]

Solution:-

$$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$$

$n(S) = 12$

$A =$ {getting the die shows odd and coin shows head}

$A = \{1H, 3H, 5H\}$

$n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

96) What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution:-

Leap year 366 = 52 weeks+ 2 days

$S =$ {(sun,mon), (mon, tue), (tue.wed), (wed,thu), (thu,fri), (fri,sat), (sat,sun)}

$n(S) = 7$

$A =$ {getting 53 Saturdays in a leap year}

= {(fri,sat),(sat,sun) }

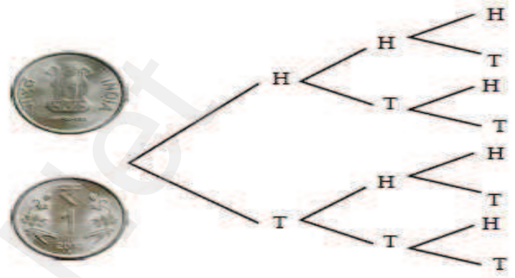
$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{2}{7}$

97) Write the sample space for tossing three coins using tree diagram.

Solution:-



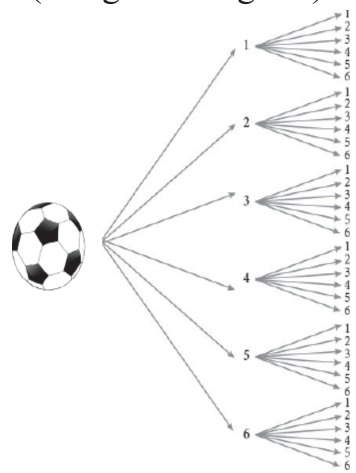
$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\therefore n(S) = 8$

98) Write a sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram). **[PTA-4]**

Solution:-

$S =$ {(1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5)}



$\therefore n(S) = 30$

99) A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black **[A-22]**

Solution:-

No. of red balls = 5

No. of white balls = 6

No. of green balls = 7

No. of black balls = 8

Total balls = 26

$\therefore\ n(S) = 26$

(i) Let A be the event of getting a white ball

. $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$$

(ii) Let B be the event of getting a black or red ball

$$n(B) = 8 + 5 = 13$$

$$P(B) = \frac{\text{n(B)}}{\text{n(S)}} = \frac{13}{26} = \frac{1}{2}$$

(iii) Let C be the event of getting a not white ball

$$n(C) = 5 + 7 + 8 = 20$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{20}{26} = \frac{10}{13}$$

(iv) Let D be the event of getting a neither white nor black ball

$$n(D) = 5 + 7 = 12$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{26} = \frac{6}{13}$$

100) two coins are tossed together. What is the probability of getting different faces on the coins? **[M-22]**

__Solution__:-

$$S = \{HH, HT, TH, TT\}$$

$$n(S) = 4$$

Let A be the event of getting different faces

$$A = \{HT, TH\}$$

$$n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

101) A coin is tossed thrice. What is the probability of getting two consecutive tails?

__Solution__:-

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$n(S) = 8$$

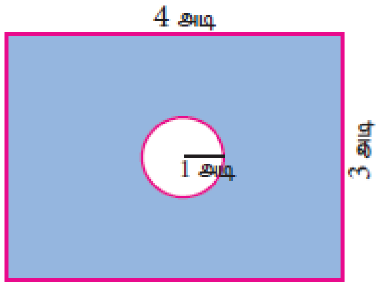
Let A be the event of getting two consecutive tails

$$A = \{HTT, TTH, TTT\}$$

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

102) Some boys are playing a game, in which the stone thrown by them landing ina circular region (given in figure) is considered as win and landind other than the circular region is considered as loss. What is the probability to win the game?



Solution:-

Area of Rectangle$= l \times b = 4 \times 3 = 12$ Sq.ft

Area of circle$= \pi r^2 = \pi \times 1^2 = \pi$

Probability of win the game$= \frac{\pi}{12} = \frac{3.14}{12}$

$= \frac{314}{1200} = \frac{157}{600}$

103) $If\ P(A) = 0.37,\ P(B) = 0.42,\ P(A \cap B) = 0.09$ then find $P(A \cup B))$

Solution:-

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.37 + 0.42 - 0.09$

$= 0.79 - 0.09$

$= 0.7$

104) From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting a king or queen card.

Solution:-

$n(S) = 52$

Let A be the event of getting a king card

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

Let B be the event of getting a queen card

$n(B) = 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$

A, B are mutually exclusive events, then

$\therefore \quad P(A \cup B) = P(A) + P(B)$

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{4+4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

105) $If\ P(A) = \frac{2}{3},\ P(B) = \frac{2}{5},\ P(A \cap B) = \frac{1}{3}$ then find $P(A \cap B))$ **[PTA-1]**

Solution:-

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10+6-5}{15}$$

$$= \frac{16-5}{15}$$

$$= \frac{11}{15}$$

106) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Solution:-

$$P(A) = 0.5, \quad P(B) = 0.3, \quad P(A\cap B) = 0$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= 0.5 + 0.3 - 0$$

$$= 0.8$$

∴ the probability that neither A nor B happen

$$= P(\overline{A\cup B})$$

$$= 1 - P(A\cup B)$$

$$= 1 - 0.8$$

$$= 0.2$$

107) Check whether AD is bisector of $\angle A$ of ΔABC in each of the following

(i) $AB = 5$ cm, $AC = 10$ cm, $BD = 1.5$ cm and $CD = 3.5$ cm. **[S-20]**

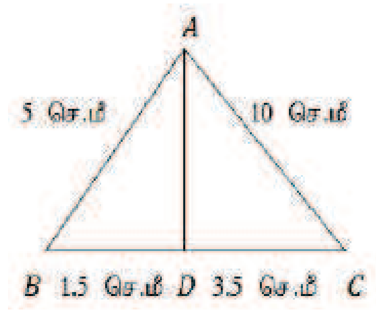
Solution:-

$AB = 5$ cm

$AC = 10$ cm

$BD = 1.5$ cm

$CD = 3.5$ cm



$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2} \dashrightarrow (1)$$

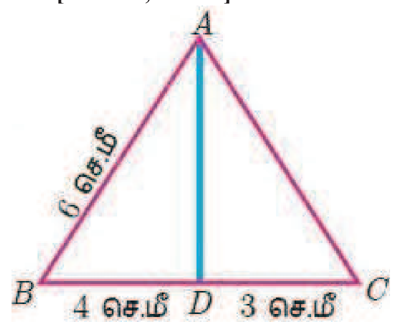
$$\frac{BD}{DC} = \frac{1.5}{3.5} = \frac{15}{35} = \frac{3}{7} \dashrightarrow (2)$$

From (1) and (2)

$$\frac{AB}{AC} \neq \frac{BD}{DC}$$

∴ AD is not a bisector of $\angle A$

108) In the Fig. AD is the bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC. **[PTA-5 , M-22]**



Solution:- $BD = 4$ cm

$DC = 3$ செ.மீ

$AB = 6$செ.மீ

$AC = ?$

By Angle Bisector Theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{6}{AC} = \frac{4}{3}$$

$$\frac{6 \text{ x } 3}{4} = AC$$

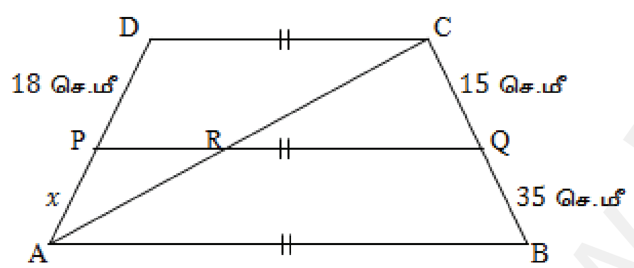
$$AC = \frac{3 \text{ x } 3}{2}$$

$$AC = \frac{9}{2}$$

$AC = 4.5$செ.மீ

109) ABCD is a trapezium in which $AB \parallel DC$ and P, Q are points on AD and BC respectively, such that $PQ \parallel DC$ if $PD = 18$ cm, $BQ = 35$ cm and $QC = 15$ cm, find AD. **[A-22]**

__Solution__:-



In ΔACD, $PR \parallel CD$	In ΔABC, $RQ \parallel AB$
By **Thales theorem**	By **Thales theorem**
$\frac{AP}{PD} = \frac{AR}{RC}$	$\frac{AR}{RC} = \frac{BQ}{QC}$
$\frac{x}{18} = \frac{AR}{RC}$	$\frac{AR}{RC} = \frac{35}{15}$
$\frac{AR}{RC} = \frac{x}{18} \longrightarrow (1)$	$\frac{AR}{RC} = \frac{7}{3} \longrightarrow (2)$

From (1) and (2)

$$\frac{x}{18} = \frac{7}{3}$$

$$x = \frac{7 \text{ x } 18}{3}$$

$$x = 7 \text{ x } 6$$

$$x = 42$$

$\therefore AP = x = 42$

$AD = AP + PD = 42 + 18 = 60$cm.

110) A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point? **[A-22]**

__Solution__:-

In right angled ΔABC –

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 24^2 + 18^2$$

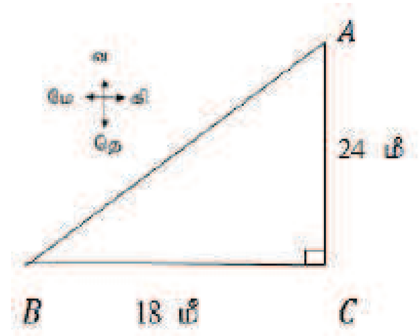
$$AB^2 = 576 + 324$$

$$AB^2 = 900$$

$$AB = \sqrt{900}$$

$$AB = \sqrt{30 \text{ x } 30}$$

$AB = 30$மீ

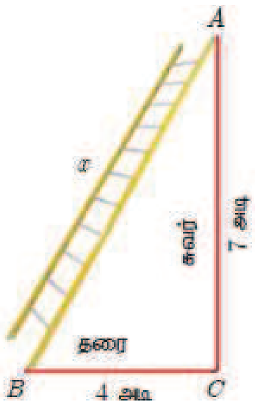


$\therefore$ தொட distance of his current position from the starting point $= 30$ m

111) What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution:-

In right angled ΔABC $AB^2 = BC^2 + AC^2$



$AB^2 = 4^2 + 7^2$

$AB^2 = 16 + 49$

$AB^2 = 65$

$AB = \sqrt{65}$

$AB = 8.062$

$AB = 8.1$அடி

Therefore, the length of the ladder is approximately 8.1 ft.