COMMON SECOND REVISION TEST - 2023

Standard XI Reg.No.

MATHEMATICS

Time: 3.00 hours **Part - I** *Marks: 90*

I. Choose the correct answer: **20 x 1 = 20**

1. Let A and B be subsets of the universal set N the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is
   a) B b) A c) A^1 d) N
2. The number of relations on a set containing 3 elements
   a) 81 b) 512 c) 9 d) 1024
3. If $|x+2| \leq 9$, then x belongs to
   a) $(-\infty,-7)$ b) $[-11,-7]$ c) $(-\infty,-7) \cup (11,\infty)$ d) $(-11,7)$
4. The value of $\log_{\sqrt{2}} 512$ is
   a) 16 b) 18 c) 9 d) 12
5. If cos 28° + sin 28° = k^3, then cos 17° is equal to
   a) $\frac{-k^2}{\sqrt{3}}$ b) $\pm\frac{k^3}{\sqrt{2}}$ c) $-\frac{k^3}{\sqrt{3}}$ d) $\frac{k^3}{\sqrt{2}}$
6. $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A}$ is
   a) 1 b) cos A + cos B + cos C
   c) 0 d) sin A + sin B + sin C
7. 1 + 3 + 5 + 7 + ... + 17 is equal to
   a) 81 b) 71 c) 61 d) 101
8. The value of 2 + 4 + 6 + ... + 2n is
   a) $\frac{n(n+1)}{2}$ b) $n(n+1)$ c) $\frac{2n(2n+1)}{2}$ d) $\frac{n(n-1)}{2}$
9. The value of $1 - \frac{1}{2}\left(\frac{2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\right)^2 - \frac{1}{4}\left(\frac{2}{3}\right)^3 + \ldots$ is
   a) $\frac{3}{2}\log\left(\frac{5}{3}\right)$ b) $\frac{5}{3}\log\left(\frac{5}{3}\right)$ c) $\frac{2}{3}\log\left(\frac{2}{3}\right)$ d) $\log\left(\frac{5}{3}\right)$
10. If the point (8,–5) lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$, then the value of k is
    a) 2 b) 3 c) 1 d) 0
11. The slope of the line makes an angle 45° with the line 3x – y = –5 are
    a) $\frac{1}{2}, -2$ b) $1, \frac{1}{2}$ c) $2, -\frac{1}{2}$ d) 1,–1
12. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$ then for what value of λ, $A^2 = 0$?
    a) ±1 b) –1 c) 1 d) 0

13. If A and B are symmetric matrices of order n, where $(A \neq B)$ then
    a) A + B is skew-symmetric b) A + B is a diagonal matrix
    c) A + B is symmetric d) A – B is a zero matrix

14. The value of $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CA}$ is
    a) $\vec{CA}$ b) $\vec{O}$ c) $-\vec{AD}$ d) $\vec{AD}$

15. If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = 60^0$ then $|\vec{a} \times \vec{b}|$ is
    a) 45 b) 25 c) 35 d) 15

16. If $\lim_{x \to 0} \frac{\sin Px}{\tan 3x} = 4$, then the value of P is
    a) 9 b) 12 c) 4 d) 6

17. If $y = f(x^2 + 2)$ and $f'(3) = 5$ then $\frac{dy}{dx}$ at x = 1 is
    a) 25 b) 15 c) 5 d) 10

18. $\int \sin^3 x\, dx$ is
    a) $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$ b) $-\frac{3}{4}\cos x - \frac{\cos 3x}{12} + c$
    c) $-\frac{3}{4}\sin x - \frac{\sin 3x}{12} + c$ d) $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$

19. The number is selected from the set {1,2,3,......20}. The probability that the selected number is divisible by 3 or 4 is
    a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{8}$ d) $\frac{2}{5}$

20. A bag contains 6 green, 2 white and 7 black balls. If two balls are drawn simultaneously, then the probability that both are different colours is
    a) $\frac{68}{105}$ b) $\frac{73}{105}$ c) $\frac{71}{105}$ d) $\frac{64}{105}$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = \frac{9}{2}$, then find the value of $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$ for x > 1

22. If $A + B = 45^0$, show that $(1 + \tan A)(1 + \tan B) = 2$

23. If $(n-1)P_3 : nP_4 = 1 : 10$, find n

24. Find the expansion of $(2x + 3)^5$

25. Show that the lines $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

26. Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

27. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

28. Differentiate : $y = e^{\sin x}$

29. Integrate with respect to x : $\dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

30. If A and B are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find P(B)

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. If A and B are two sets so that $n(B-A) = 2n(A-B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(P(A))$

32. If $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$, then prove that $xyz = 1$

33. If $A + B + C = 180^\circ$, prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

34. By the principle of Mathematical Induction, prove that for $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \dots\dots\dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

35. Find the coefficient of x^{15} in $\left(x^2 + \dfrac{1}{x^3}\right)^{10}$

36. The co-ordinates of a moving point P are $\left(\dfrac{a}{2}(\operatorname{cosec}\theta + \sin\theta), \dfrac{b}{2}(\operatorname{cosec}\theta - \sin\theta)\right)$ where θ is variable parameter, show that the equation of the locus P is $b^2x^2 - a^2y^2 = a^2b^2$.

37. If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that $\cos\dfrac{\theta}{2} = \dfrac{1}{2}\left|\vec{a} + \vec{b}\right|$

38. Evalaute : $\displaystyle\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

39. Find f'(x) if $f(x) = \dfrac{1}{3\sqrt{x^2 + x + 1}}$

40. State and prove addition theorem on probability.

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Draw the functions :

i) $f(x) = x^2$ ii) $f(x) = x^2 + 1$ iii) $f(x) = (x+1)^2$ ***(OR)***

b) Resolve into partial fractions : $\dfrac{7+x}{(1+x)(1+x^2)}$

42. a) Prove that $nC_r + nC_{r-1} = n+1\,C_c$

(OR)

b) Evaluate : $\lim_{x \to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\tan x}$

43. a) State and prove Napier's formula.

(OR)

b) Evaluate : $\int \frac{3x+5}{x^2+4x+7}\,dx$

44. a) If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a G.P, show that

$(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$

(OR)

b) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively then, prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

45. a) If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\ldots\ldots\ldots$ then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots\ldots\ldots$

(OR)

b) The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$

46. a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

(OR)

b) If $y = e^{\tan^{-1}x}$, show that $(1+x^2)y'' + (2x+1)y' = 0$

47. a) Given that $P(A) = 0.52$, $P(B) = 0.43$ and $P(A \cap B) = 0.24$ find,

i) $P(A \cap \overline{B})$ ii) $P(A \cup B)$ iii) $P(\overline{A} \cap \overline{B})$ iv) $P(\overline{A} \cup \overline{B})$

(OR)

b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

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THIRUVALLUR (DIST)

SECOND REVISION TEST - 2023

SUBJECT : Mathematics

CLASS : XI

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I Choose the correct Answer

1. (D)	11. (A)
2. (B)	12. (A)
3. (B)	13. (C)
4. (B)	14. (B)
5. (D)	15. (B)
6. (C)	16. (B)
7. (A)	17. (D)
8. (B)	18. (D)
9. (A)	19. (A)
10. (B)	20. (A)

II 2 Marks

21. $(a+b)^2$ form

$\boxed{\text{EX } 2.11 \to 3 \text{ sums}}$

$$\left(x^{1/2}+x^{-1/2}\right)^2 = 9/2$$

$$x + 1/x + 2x^{1/2}\cdot\frac{1}{x^{1/2}} = 9/2$$

$$x + 1/x + 2 = 9/2$$

$$x + 1/x = 9/2 - 2 = \frac{5}{2}$$

we know

$$\left(x^{1/2}-x^{-1/2}\right) = x + 1/x - 2$$

$$= 5/2 - 2 = \frac{5-4}{2} = 1/2$$

$$\therefore \left(x^{1/2}-x^{-1/2}\right)^2 = 1/2$$

$$\left(x^{1/2}-x^{-1/2}\right) = \sqrt{1/2}$$

$$\boxed{x^{1/2}-x^{-1/2} = \pm\frac{1}{\sqrt{2}}}$$

22. $\boxed{\text{EX } 3.5 \to 6 \text{ sum}}$

$A+B = 45^\circ \quad B = 45^\circ - A$

LHS $= (1+\tan A)(1+\tan B)$

$$= (1+\tan A)(1+\tan(45^\circ - A))$$

$$= (1+\tan A)\left(1+\frac{\tan 45^\circ - \tan A}{1+\tan 45^\circ \tan A}\right)$$

$$= (1+\tan A)\left(1+\frac{1-\tan A}{1+\tan A}\right)$$

$$= \cancel{(1+\tan A)}\left(\frac{1+\cancel{\tan A}+1-\cancel{\tan A}}{\cancel{1+\tan A}}\right)$$

$$= 1+1 = 2$$

23. $\boxed{\text{EX } 4.2 \to 1}$

$$^{(n-1)}P_3 : {}^nP_4 = 1:10$$

$$\frac{^{(n-1)}P_3}{^nP_4} = \frac{1}{10}$$

$$10\,{}^{(n-1)}P_3 = {}^nP_4$$

$$10 \times \frac{(n-1)!}{(n-1-3)!} = \frac{n!}{(n-4)!}$$

$$\frac{10 \times (n-1)!}{(n-4)!} = \frac{n(n-1)!}{(n-4)!}$$

$$\boxed{10 = n}$$

24. EXP 5.1

$(2x+3)^5 \quad a = 2x \quad b = 3$

form $(ax+b)^n \quad n = 5$

$(2x+3)^n = (2x)^5 + 5(2x)^4 3 + 10(2x)^3 3^2 + 10(2x)^2 3^2 + 5(2x)3^4 + 3^5$

$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

25. EX 6.3 → 1

General form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$3x + 2y + 9 = 0$ | $12x + 8y - 15 = 0$

$a = 3 \quad b_1 = 2 \quad c_1 = 9$ | $a_2 = 12 \quad b_2 = 8 \quad c_2 = -15$

$\frac{3}{12} = \frac{2}{8}$

$1/4 = 1/4$ Line are parallel

26. Examples 8.10

$5\vec{i} + 6\vec{j} + 7\vec{k} = s(7\vec{i} - 8\vec{j} + 9\vec{k}) + t(3\vec{i} + 20\vec{j} + 5\vec{k})$

$7s + 3t = 5 \longrightarrow (1)$

$-8s + 20t = 6 \longrightarrow (2)$

$9s + 5t = 7 \longrightarrow (3)$

$s = 1/2 \qquad \therefore s = t = 1/2$

$t = 1/2$

From (3) eqn

$9(1/2) + 5(1/2) = \frac{9}{2} + \frac{5}{2}$

$= \frac{14}{2} = 7$

Vector are coplanar

27. EXPT. 22

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix}$$

$$= (y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$

$= (y-x)(z-x)(z-y)$

28. EXP 10.14

$y = e^{\sin x}$

$u = \sin x$

$y = e^u$

$\frac{du}{dx} = \cos x$

$\frac{dy}{dx} = \frac{d(e^u)}{du} \times \frac{du}{dx}$

$= e^u \cos x$

$\boxed{\frac{dy}{dx} = e^{\sin x} \cos x}$

29. EX 11.6 → 9

let $I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1}x \quad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$I = \int u \cdot du = \frac{u^2}{2} + c$

$= \frac{(\sin^{-1}x)^2}{2} + c$

(30) Exp 12.20

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

A, B are independent

$0.9 = 0.4 + P(B) - 0.4\,P(B)$

$0.9 - 0.4 = (1 - 0.4)\,P(B)$

$P(B) = \frac{5}{6}$

3 marks

(31) Exp Q1.5

$n(A\cap B) = k \quad n(A-B) = 2k$

$n(B-A) = 4k,$

$n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B) = 7k$

given $A\cup B = 14 \quad 7k = 14$

$\boxed{k=2}$

$n(A-B) = 4$

$n(B-A) = 8$

$n(A) = n(A-B) + n(A\cap B)$

$= 4 + 2$

$n(A) = 6$

$n(P(A)) = 2^6 = 64$

(32) Ex 2.12 → 10

$\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$\log x = k(y-z) = ky - zk \rightarrow (1)$

$\log y = k(z-x) = kz - kx \rightarrow (2)$

$\log z = k(x-y) = kx - ky \rightarrow (3)$

Add (1) + (2) + (3)

$\log x + \log y + \log z = \cancel{ky} - \cancel{kz} + \cancel{kz} - \cancel{kx} + \cancel{kx} - \cancel{ky}$

$\log xyz = 0$

$\boxed{xyz = 1}$

(33) Ex 3.7 → (1)

LHS

$\sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cos C$

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

$\therefore A+B+C = 180°$

$= 2\sin(180-C)\cos(A-B) + 2\sin C\cos C$

$= 2\sin C(2\sin A\sin B)$

$= 4\sin A\sin B\sin C$ (RHS)

(34) Ex 4.4 → (1)

$P(n) = 1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^3$

$n = 1$

$P(1)$ is true

$n = k \quad 1^3 + 2^3 + \dots + n^3 = \left[\frac{k(k+1)}{2}\right]^2$

$n = k+1$

$1^3 + 2^3 + \dots + n^3 = \left[\frac{(k+1)(k+2)}{2}\right]^2$

$P(k+1)$ is true.

(35) Ex 5.1 → (4)

$\left(x^2 + \frac{1}{x^3}\right)^{10} \quad n = 10 \quad x = x^2 \quad a = \frac{1}{x^3}$

$T_{r+1} = nC_r\, x^{n-r}\, a^r$

$= 10C_r\, x^{20-5r} \rightarrow (1)$

co-efficient of x^{15}

$20 - 15 = 5r$

$\boxed{r=1}$

$T_2 = 10C_1\, x^{15}$ $\boxed{x^{15} \text{ is } 10}$

(36) Ex 6.1 → (9)

$h = \frac{a}{2}(\operatorname{cosec}\theta + \sin\theta)$

$k = \frac{b}{2}(\operatorname{cosec}\theta - \sin\theta)$

$\frac{2h}{a} = \operatorname{cosec}\theta + \sin\theta$

$\frac{2k}{b} = \operatorname{cosec}\theta - \sin\theta$

$\left(\frac{2h}{a}\right)^2 - \left(\frac{2k}{b}\right)^2 = 4\operatorname{cosec}\theta\sin\theta$

$= 4$

Divide by 4

$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$

$b^2h^2 - k^2a^2 = a^2b^2$

Locus of (h, k) is

$b^2x^2 - a^2y^2 = a^2b^2$

(37) Ex 8.3 → 10(ii)

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}\cdot\vec{b})$

$= 1 + 1 + 2|\vec{a}||\vec{b}|\cos\theta$

$= 2 + 2\cos\theta$

$= 2(1+\cos\theta)$

Formulae $1+\cos\theta = \cos^2\theta/2$ (2)

$= 2\cos^2\theta/2\ (2)$

$= 4\cos^2\theta/2$

$\cos\theta/2 = \frac{1}{2}|\vec{a}+\vec{b}|$

(38) Exp 11.28

$= \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$

$= \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\left[\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\right]dx$

$= \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} = \int(\sqrt{x+1}-\sqrt{x})\,dx$

$= \int (x+1)^{1/2}\,dx - \int x^{1/2}\,dx$

$= \frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2} + c$

$= \frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + c$

(39) Exp 10.11

$f(x) = (x^2+x+1)^{-1/3}$

$f'(x) = -\frac{1}{3}(x^2+x+1)^{-4/3} \times (2x+1)$

$= -\frac{1}{3}(2x+1)(x^2+x+1)^{-4/3}$

(40) Theorem 12.6 Book page no: 249

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

Proof

$A\cup B = (A\cap\bar{B})\cup B$

$P(A\cup B) = P[(A\cap\bar{B})\cup B]$

$= P(A\cap\bar{B}) + P(B)$

$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B)$

41 (a) Illustration : 1.11 Page No : 42

$f(x^2) = x^2$

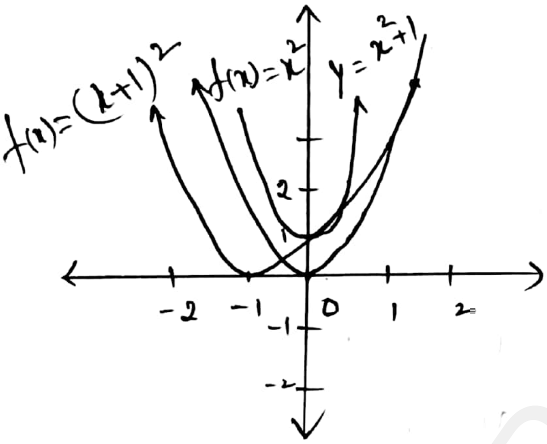
x	-2	-1	0	1	2
x^2	4	1	0	1	4

$f(x) = x^2 + 1$

x	-2	-1	0	1	2
$f(x)$	5	2	1	2	5

$f(x) = (x+1)^2$

x	-2	-1	0	1	2
$f(x)$	1	0	1	4	9



41 (b) EX 2.9 → 12

$$\frac{7+x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{x^2+1}$$

$$7+x = A(x^2+1) + Bx + C(1+x)$$

put $x=-1$ $\boxed{A=3}$

co-efficient of x^2

$\boxed{B=-3}$

put $x=0$ $\boxed{C=4}$

$$= \frac{3}{x+1} + \frac{(-3x+4)}{x^2+1}$$

42 (a) Property : 4 Page No : 180 (V-I)

$$nC_r + nC_{r-1} = n+1C_r$$

$$nC_r + nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\,n-(r-1)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{(n-r+1)}\right]$$

$$= \frac{(n+1)!}{r!(n+1-r)!}$$

$$= n+1C_r$$

42 (b) EX 9.4 → 22

$$\lim_{x\to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\tan x}$$

multiply Numerator & denominator by $\sqrt{1+\sin x} + \sqrt{1-\sin x}$

$$= \lim_{x\to 0} \frac{(1+\sin x) - (1-\sin x)}{\tan x\left[\sqrt{1+\sin x} + \sqrt{1-\sin x}\right]}$$

$$= \lim_{x\to 0} \frac{2\sin x}{\frac{\sin x}{\cos x}\left[\sqrt{1+\sin x} + \sqrt{1-\sin x}\right]}$$

$$= \lim_{x\to 0} 2\left[\frac{\cos x}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$$

$$= \frac{2(1)}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$$

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Page No : 135 Vol - I

(a)

(i) $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$

(ii) $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$

(iii) $\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$

(b)

EXP 11.40 (i)

$$I = \int \frac{3x+5}{x^2+4x+7} \, dx$$

$3x+5 = A(2x+4)+B$

Co-efficient of like terms

$A = 3/2 \qquad B = -1$

$$= \frac{3}{2} \log |x^2+4x+7| - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + c$$

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EX 5.2 → 10

(a) $a = AR^{p-1}, b = AR^{q-1}, c = AR^{r-1}$

$\log a = \log A + (p-1) \log R$

$\log b = \log A + (q-1) \log R$

$\log c = \log A + (r-1) \log R$

$= (q-r) \log a + (r-p) \log b + (p-q) \log c$

$= \log A (0) + \log R (0)$

$= 0$

(b)

EX 8.1 → 12

$\vec{OE} = \frac{\vec{a}+\vec{c}}{2}, \quad \vec{OF} = \frac{\vec{b}+\vec{d}}{2}$

$2\vec{OE} = \vec{a}+\vec{c}, \quad 2\vec{OF} = \vec{b}+\vec{d}$

let $\vec{OA} = \vec{a} \quad \vec{OB} = \vec{b} \quad \vec{OC} = \vec{c}, \vec{OD} = \vec{d}$

LHS $= AB + AD + CB + CD$

$= -2\vec{a} + 2\vec{b} - 2\vec{c} + 2\vec{d}$

$= 2[(\vec{b}+\vec{d}) - (\vec{a}+\vec{c})]$

$= 2[2\vec{OF} - 2\vec{OE}]$

$= 4[\vec{OF} - \vec{OE}]$

$= 4\vec{EF}$ (RHS)

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EX 5.4 → 7

(a) $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

multiply Both side by (-1)

$-y = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$

$-y = \log(1-x)$

Take log Both side

$e^{-y} = 1 - x$

$x = 1 - e^{-y}$

$x = 1 - \left[1 - \frac{y}{1!} + \frac{y^2}{2!} - \frac{y^3}{3!} + \cdots\right]$

$x = 1 - 1 + \frac{y}{1!} - \frac{y^2}{2!} + \frac{y^3}{3!} - \cdots$

$x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \cdots$