V12M

Virudhunagar District Common Examinations
Second Revision Test - February 2023

Standard 12

MATHEMATICS

Time: 3.00 Hours | Marks: 90

PART - I

20×1=20

Answer all the questions. Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A then $\frac{|\text{adj}B|}{|C|} =$

a) 1/3 b) 1/9 c) 1/4 d) 1

2) If z is a non-zero complex number such that $2iZ^2 = \bar{z}$ then $|Z|$ is......

a) $-\frac{1}{2}$ b) 1 c) $\frac{1}{2}$ d) 3

3) The principal argument of $\frac{3}{-1+i}$ is ......

a) $\frac{-3\pi}{4}$ b) $\frac{-5\pi}{6}$ c) $\frac{-2\pi}{3}$ d) $\frac{-\pi}{2}$

4) The polynomial x^3-kx^2+9x has three real zeros if and only if, K satisfies...

a) $|K| \le 6$ b) $|k| \ge 6$ c) $|K| > 0$ d) $K = 0$

5) The sum of the squares of the root of $ax^4+bx^3+cx^2+dx+c = 0$ is

a) $\frac{2ac-b^2}{a^2}$ b) $\frac{b^2-4ac}{2a}$ c) $\frac{4ac-b^2}{a^3}$ d) $\frac{b^2-2ac}{a^2}$

6) If the function $f(x) = \sin^{-1}(x^2-3)$ then $x \in$ ....

a) $(-1, 1)$ b) $[\sqrt{2},2]$ c) $[-2i\sqrt{2},2]\cup[\sqrt{2},2]$ d) $[-2,\sqrt{2}]\cap[\sqrt{2},2]$

7) If $|x| < 1$ the value of $\sin(\tan^{-1}(x))$ is

a) $\frac{x}{\sqrt{1+x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{-x}{\sqrt{1+x^2}}$

8) If x+y = k is a normal to the parabola $y^2 = 12x$ then the value of K is....

a) 3 b) −1 c) 9 d) 1

9) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

10) If $\vec{a} = \vec{i}+\vec{j}+\vec{k}$, $\vec{b} = \vec{i}+\vec{j}$, $\vec{c} = \vec{i}$ and $(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a}+\mu\vec{b}$ then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

11) The distance between the planes $\vec{r}.(2\vec{i}-\vec{j}-2\vec{k})=6$ and $\vec{r}(6\vec{i}-3\vec{j}-6\vec{k}) = 27$ is

a) 2 b) 1 c) 3 d) 6

12) The minimum value of x log x

a) $-\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e}$ d) $\frac{4}{e^4}$

13) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \pi/12$ is

a) $-4\sqrt{3}$ b) -4 c) $\sqrt{3}/12$ d) $4\sqrt{3}$

14) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

a) $0.3\,x\,dx\ m^3$ b) $0.03\,x\ m^3$ c) $0.03\,x^3\ m^3$ d) $0.03\,x^2\ m^3$

15) If $U(x, y, z) = \dfrac{x^2+y^2}{xy} + 3z^2y$ then $\dfrac{\partial u}{\partial z}$ = ...

a) 6zy b) $\dfrac{1}{y} - \dfrac{y}{x^2}$ c) $-\dfrac{x}{y^2} + \dfrac{1}{x} + 3z^2$ d) 0

16) $\int_0^1 x(1-x)^{99}$ = ...

a) $\dfrac{1}{11000}$ b) $\dfrac{1}{10100}$ c) $\dfrac{1}{10010}$ d) $\dfrac{1}{10001}$

17) The population P in any year t is such that the rate of increase in the population is proportional to the population then, ...

a) $P = Ce^{Kt}$ b) $P = Ce^{-Kt}$ c) $P = CKt$ d) $P = C$

18) If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable then the value of a is

a) 4 b) 2 c) 3 d) 1

19) If a compound statement involves 3 simple statements, then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

20) Let * be defined on $\mathbb{R}$ by $(a*b) = a+b+ab-7$ the value of $3*\left(\frac{-7}{15}\right)$ is

a) $88/15$ b) $-88/15$ c) $8/15$ d) $-8/15$

PART - II

7×2=14

Answer any seven questions only. Q.No. 30 is compulsory.

21) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$

22) Investigate the nature roots of the polynomials $x^{2018}+1947x^{1950}+15x^8+26x^6+2019 = 0$

23) Evaluate: $\sin^{-1}(\cos\pi)$

24) Find the equation of the tangent to the parabala $y^2 = 16x$ is perpendicular to the line $2x+2y+3 = 0$

25) Find the volume of the parallelepied whose conterminous edges are represented by the vectors $2\vec{i} - 3\vec{j} + 4\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$

26) Write the maclaurin Series expansion of e^x.

27) Evaluate $\lim_{(x,y)\to(1,2)}$ if the limit exists where $g(x,y) = \dfrac{3x^2 - xy}{x^2 + y^2 + 3}$

28) Determine the orders and degree of the differential equation:
$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+\int y\,dx_2=x^3$$

29) Suppose a discrete random variable can only take the values 0, 1 and 2 the probability mass function is defined by $f(x)=\begin{cases}\frac{x^2+1}{K} & \text{for } x=0,1,2\\ 0 & \text{otherwise}\end{cases}$ for the value of K

30) Construct the truth table $\neg p \vee \neg q$

PART - III **7×3=21**

Answer any seven questions only. Q.No. 40 is compulsory.

31) If $|Z| = 1$ show that $2 \le |Z^2-3| \le 4$.

32) If $y = 4x+c$ is a tangent to the circle. $x^2+y^2 = 9$ find the value of c.

33) The concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch.

34) A particle acted on by constant forces $8\vec{i}+2\vec{j}-6\vec{k}$ and $6\vec{i}+2\vec{j}-2\vec{k}$ is displaced from the point (1, 2, 3) and (5, 4, 1). Find the total workdone by the force.

35) Show that the two curve $x^2-y^2 = r^2$ and $xy = c^2$, cuts orthogonally where c, r are constants.

36) Evaluate: $\int_{-\pi/2}^{\pi/2}\left(x^5+x\cos x+\tan^3 x+1\right)dx$

37) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}$ find the expected life of this electronic equipment.

38) Solve $\cos x \cos y\, dy - \sin x \sin y\, dx = 0$

39) If $(z, *)$ then $*$ be the $a*b = a+b+2$. Verify that closure property and commutative property?

40) Solve (by Cramer's Rule) $2x-y = 8$; $3x+2y = -2$

PART - IV **7×5=35**

Answer all the questions:

41) Solve: $x+y+z = 2$; $6x-4y+5z = 31$; $5x+2y+2z = 13$.

(OR)

Father of a family wishes to divide his square field bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife daughter and son. Is it possible to divide? If so, find the area to be divided among them.

42) If $Z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$ show that $x^2+y^2+3x-3y+2 = 0$

(OR)

Solve: $(x^3+y^3)dy = x^2y\,dx$

43) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

(OR)

Find the equation of the circle passing through the points (1, 1) (2, −1) and (3, 2)

44) Prove by vector method $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

(OR)

$W(x, y, z) = xy+yz+zx$, $x = u-v$, $y = uv$, $z = u+v$

$u, v \in R$ find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ and evaluate then at $\left[\frac{1}{2}, 1\right]$

45) Find the local maximum and minimum of the function x^2y^2 on the line $x+y=10$.

(OR)

A random variable X has the following probability mass function

X	1	2	3	4	5	6
f(x)	K	2K	6K	5K	6K	10K

46) Verify (i) closure property (ii) commutative property (iii) assiciative property (iv) existence identity (v) existence of inverse for the operation X_{11} on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(OR)

i) Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

ii) Prove that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

47) Find the vector, parametric, vector non-parametric and Cartesian equations of the plane passing through point (−1, 2, 0) (2, 2, 1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

(OR)

Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3m beyond the verified line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

II Revision Examination

XII - Mathematics

1.	b	$1/9$	11.	b	1
2.	c	$1/2$	12.	a	$-1/e$
3.	a	$-3\pi/4$	13.	c)	$\sqrt{3}/12$
4.	b	$k \geq 6$	14.	d	$0.03x^2m^3$
5.	d	$\frac{b^2-2ac}{a^2}$	15.	a	6zy
6.		M.A	16.	b	$1/10100$
7.	a)	$\frac{x}{\sqrt{1+x^2}}$	17.	a	$P = ce^{kt}$
8.	d)	1	18.	d	1
9.	a)	2ab	19.	b	8
10.	a	0	20.	b	$-88/15$

JB TUITION CENTRE

G. JAYABALAN
8056580481

21) Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Number of non zero row is 2

$\therefore \rho(A) = 2$.

22)

Let $P(x) = x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$

Number of sign change in $P(x)$ $\Big\} = 0$

$\therefore$ Number of positive real root $\Big\} = 0$

$P(-x) = x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$

Number of sign change in $P(-x)$ $\Big\} = 0$

$\therefore$ Number of negative real root $\Big\} = 0$

$\Rightarrow P(x)$ has no real root.

$\therefore$ All roots of $P(x)$ are imaginary.

23) $\sin^{-1}(\cos\pi) = \sin^{-1}(-1)$

$= -\pi/2$

24)

Equation of parabola $y^2 = 16x \to (1)$

$\therefore a = 4$

Equation of st. line

$2x + 2y + 3 = 0 \to (2)$

Slope of (2) $= \frac{-2}{2} = -1$

Let m be the slope of tangent of (1).

Gin: Tangent of (1) is $\perp^r$ to (2)

$\therefore m = \frac{-1}{-1}$

$\boxed{m = 1}$

Equation of tangent of (1) is

$y = mx + \frac{a}{m}$

$y = (1)x + \frac{4}{1}$

$y = x + 4$

$x - y + 4 = 0$

25)

Let $\vec{a}, \vec{b}$ and $\vec{c}$ are coterminus edges ~~are~~ of parallelepiped.

Gn: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Volume of parallelepiped $\} = \left|[\vec{a}, \vec{b}, \vec{c}]\right|$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(4-1) + 3(2+3) + 4(-1-6)$

$= 2(3) + 3(5) + 4(-7)$

$= 6 + 15 - 28$

$= 21 - 28$

$= -7$

$\therefore$ Volume $= |-7| = 7$ cu.units.

26) Gn: $f(x) = e^x$

$f(x) = e^x \Rightarrow f(0) = e^0 = 1$

$f'(x) = e^x \Rightarrow f'(0) = e^0 = 1$

$f''(x) = e^x \Rightarrow f''(0) = e^0 = 1$

$f'''(x) = e^x \Rightarrow f'''(0) = e^0 = 1$

Maclaurin's series is

$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots$

$= 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$

$f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

27) Gn: $g(x,y) = \frac{3x^2 - xy}{x^2+y^2+3}$

$\lim\limits_{(x,y)\to(1,2)} g(x,y) = \lim\limits_{(x,y)\to(1,2)} \frac{3x^2 - xy}{x^2+y^2+3}$

$= \frac{3(1)^2 - (1)(2)}{(1)^2 + (2)^2 + 3}$

$= \frac{3(1) - 2}{1+4+3}$

$= \frac{3-2}{8}$

$= \frac{1}{8}$

28) Given:

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int y\,dx = x^3$.

Differentiating with respect to "x",

$\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} + y = 3x^2$

Order $= 3$

degree $= 1$.

29) Given:

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & x = 0,1,2 \\ 0, & \text{otherwise}. \end{cases}$$

WKT,

$\sum f(x) = 1$

$f(0) + f(1) + f(2) = 1$

$\frac{0^2+1}{k} + \frac{1^2+1}{k} + \frac{2^2+1}{k} = 1$

$$\frac{1}{k} + \frac{2}{k} + \frac{5}{k} = 1$$

$$\frac{8}{k} = 1$$

$$\boxed{k = 8}$$

30) $\neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

31) Given $|z| = 1$

Let $z_1 = z^2,\ z_2 = 3$

WKT

$$\big||z_1| - |z_2|\big| \le |z_1 - z_2| \le |z_1| + |z_2|$$

$$\big||z^2| - |3|\big| \le |z^2 - 3| \le |z^2| + |3|$$

$$\big||z|^2 - 3\big| \le |z^2 - 3| \le |z|^2 + 3$$

$$|1^2 - 3| \le |z^2 - 3| \le 1^2 + 3$$

$$|-2| \le |z^2 - 3| \le 4$$

$$2 \le |z^2 - 3| \le 4$$

32) Given Equation of Circle is $x^2 + y^2 = 9 \rightarrow (1)$

Tangent of (1) is

$$y = 4x + c \rightarrow (2)$$

From (1) & (2),

$$a = 3,\ m = 4$$

Condition for tangent to (1) is

$$c^2 = a^2(1 + m^2)$$

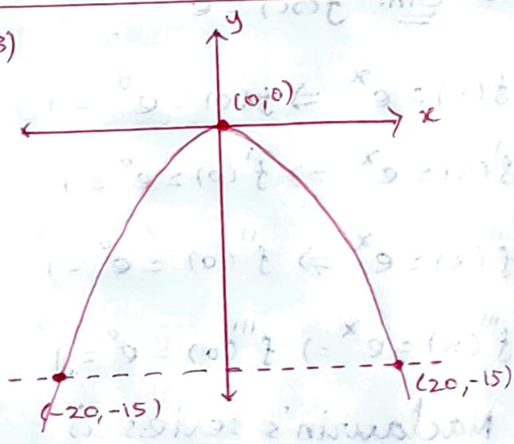
$$c^2 = 3^2(1 + 4^2)$$

$$c^2 = 9(1 + 16)$$

$$c^2 = 9(17)$$

$$\boxed{c = \pm 3\sqrt{17}}$$

33)



From the graph the vertex is at (0,0) and the parabola is open downward.

$\therefore$ Equation of parabola is

$$x^2 = -4ay \rightarrow (1)$$

From the given data,

$(-20,-15)$ and $(20,-15)$ lie on the parabola ①,

$$① \Rightarrow 20^2 = -4a(-15)$$

$$400 = 4a(15)$$

$$4a = \frac{400}{15}$$

$$4a = \frac{80}{3}$$

$$① \Rightarrow x^2 = \frac{-80}{3}y$$

$$3x^2 = -80y.$$

34) Given: $\vec{F_1} = 8\hat{i}+2\hat{j}-6\hat{k}$

$$\vec{F_2} = 6\hat{i}+2\hat{j}-2\hat{k}$$

$\therefore$ Force $\vec{F} = \vec{F_1}+\vec{F_2}$

$$= 8\hat{i}+2\hat{j}-6\hat{k}+6\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{F} = 14\hat{i}+4\hat{j}-8\hat{k}$$

Given points

$A(1,2,3)$, $B(5,4,1)$

$$\vec{OA} = \hat{i}+2\hat{j}+3\hat{k}$$

$$\vec{OB} = 5\hat{i}+4\hat{j}+\hat{k}$$

$$\vec{d} = \vec{OB} - \vec{OA}$$

$$\vec{d} = (5\hat{i}+4\hat{j}+\hat{k}) - (\hat{i}+2\hat{j}+3\hat{k})$$

$$= 5\hat{i}+4\hat{j}+\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$$

$$\vec{d} = 4\hat{i}+2\hat{j}-2\hat{k}$$

Work done by the force

$$W = \vec{F}\cdot\vec{d}$$

$$= (14\hat{i}+4\hat{j}-8\hat{k})\cdot(4\hat{i}+2\hat{j}-2\hat{k})$$

$$= 14(4)+4(2)+(-8)(-2)$$

$$= 56+8+16$$

$W = 80$ units.

35) Given two curves,

$x^2-y^2=r^2 \to ①$, $xy=c^2 \to ②$

Let m_1, m_2 be the slopes of tangents of ① & ②

Let (x_1,y_1) be the intersecting point of ① & ②.

Differentiating ① w.r.to 'x',

$$① \Rightarrow 2x - 2y\frac{dy}{dx} = 0$$

$$-2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{x_1}{y_1}$$

Differentiating ② w.r.to "x",

$$x\frac{dy}{dx} + y(1) = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

$$m_2 = \frac{-y_1}{x_1}$$

$$m_1 m_2 = \left(\frac{x_1}{y_1}\right)\left(\frac{-y_1}{x_1}\right) = -1$$

∴ ① & ② cuts orthogonally.

36)

Let $I = \int_{-\pi/2}^{\pi/2} (x^5 + x\cos x + \tan^3 x + 1)\,dx$

$$= \int_{-\pi/2}^{\pi/2} x^5\,dx + \int_{-\pi/2}^{\pi/2} x\cos x\,dx + \int_{-\pi/2}^{\pi/2} \tan^3 x\,dx + \int_{-\pi/2}^{\pi/2} 1\,dx$$

WKT, x^5, $x\cos x$, $\tan^3 x$ are odd functions

1 is an even fn;

$$\therefore I = 0+0+0+2\int_0^{\pi/2} dx$$

$$= 2\left[x\right]_0^{\pi/2}$$

$$= 2\left(\frac{\pi}{2} - 0\right)$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$I = \pi$$

37)

Given p.d.f

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Expected life time

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_0^{\infty} x f(x)\,dx$$

$$= 0 + \int_0^{\infty} x\,3e^{-3x}\,dx$$

$$= 3\int_0^{\infty} e^{-3x} x\,dx$$

$$= 3\left[\frac{1!}{3^{1+1}}\right]$$

$$= 3\left(\frac{1}{9}\right)$$

$$E(X) = \frac{1}{3}$$

38) $\cos x \cos y \, dy - \sin x \sin y \, dx = 0$

$\cos x \cos y \, dy = \sin x \sin y \, dx$

$\frac{\cos y}{\sin y} dy = \frac{\sin x}{\cos x} dx$

$\cot y \, dy = \tan x \, dx$

$\int \cot y \, dy = \int \tan x \, dx$

$\log|\sin y| = \log|\sec x| + \log|c|$

$\log|\sin y| - \log|\sec x| = \log|c|$

$\log\left|\frac{\sin y}{\sec x}\right| = \log|c|$

$\sin y \cos x = c$

39) Given set : $\mathbb{Z}$

Let $a, b \in \mathbb{Z}$

$*$ is defined by

$a * b = a + b + 2$

Since $a + b + 2 \in \mathbb{Z}$,

$a * b \in \mathbb{Z}$

$\therefore$ $*$ is closed under $\mathbb{Z}$.

Commutative:

$a * b = a + b + 2$

$= b + a + 2$

$= b * a$

$\therefore$ $*$ is commutative.

40) Given equations are

$2x - y = 8$, $3x + 2y = -2$

Matrix form of the given system of equations is $AX = B$,

where $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$,

$B = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$\Delta = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 + 3 = 7$

$\Delta_1 = \begin{vmatrix} 8 & -1 \\ -2 & 2 \end{vmatrix} = 16 - 2 = 14$

$\Delta_2 = \begin{vmatrix} 2 & 8 \\ 3 & -2 \end{vmatrix} = -4 - 24 = -28$

$x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$

$x = \frac{14}{7}$, $y = \frac{-28}{7}$

$\boxed{x = 2, \quad y = -4}$

41)

a) Solution:

Given system of equations

$x+y+z=2$

$6x-4y+5z=31$

$5x+2y+2z=13$

Matrix form of given system of equation is

$AX=B$, where

$$A=\begin{bmatrix}1 & 1 & 1\\ 6 & -4 & 5\\ 5 & 2 & 2\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

and $B=\begin{bmatrix}2\\ 31\\ 13\end{bmatrix}$.

The solution is

$$X=A^{-1}B.$$

To find A^{-1}:

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 6 & -4 & 5\\ 5 & 2 & 2\end{vmatrix}$$

$=1(-8-10)-1(12-25)+1(12+20)$

$=1(-18)-1(-13)+1(32)$

$=-18+13+32$

$|A|=27\neq 0$

$\therefore A^{-1}$ exists.

$$\text{adj } A=\begin{bmatrix}-18 & 13 & 32\\ 0 & -3 & 3\\ 9 & 1 & -10\end{bmatrix}^T$$

$$\text{adj } A=\begin{bmatrix}-18 & 0 & 9\\ 13 & -3 & 1\\ 32 & 3 & -10\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{27}\begin{bmatrix}-18 & 0 & 9\\ 13 & -3 & 1\\ 32 & 3 & -10\end{bmatrix}.$$

$$\therefore X=\frac{1}{27}\begin{bmatrix}-18 & 0 & 9\\ 13 & -3 & 1\\ 32 & 3 & -10\end{bmatrix}\begin{bmatrix}2\\ 31\\ 13\end{bmatrix}$$

$$X=\frac{1}{27}\begin{bmatrix}-36+0+117\\ 26-93+13\\ 64+93-130\end{bmatrix}.$$

$$=\frac{1}{27}\begin{bmatrix}81\\ -54\\ 27\end{bmatrix}$$

$$X=\frac{27}{27}\begin{bmatrix}3\\ -2\\ 1\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ -2\\ 1\end{bmatrix}$$

$\therefore x=3,\ y=-2,\ z=1.$

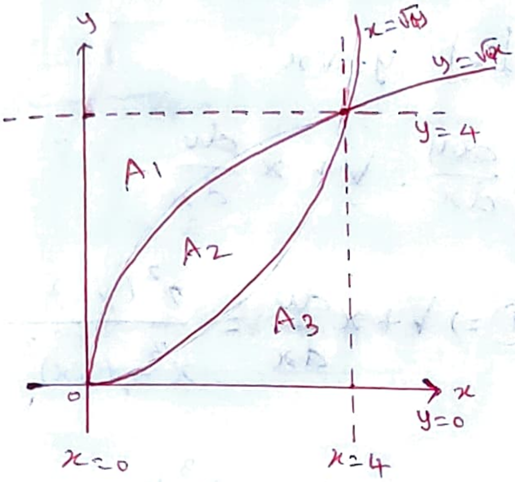
41)

i) Given curves

$y^2 = 4x, \; x^2 = 4y$

lines $x = 0, \; x = 4,$

$y = 0, \; y = 4.$



Total Area $= 4 \times 4 = 16$ sq. units

From the figure,

Area $A = A_1 + A_2 + A_3 \rightarrow ①$

$$A_1 = \int_0^4 x \, dy$$

$$= \int_0^4 \frac{y^2}{4} dy$$

$$= \frac{1}{4}\left(\frac{y^3}{3}\right)_0^4$$

$$= \frac{1}{4}\left(\frac{4^3}{3}\right)$$

$$\boxed{A_1 = \frac{16}{3} \text{ sq. units}}$$

$$A_3 = \int_0^4 y \, dx$$

$$= \int_0^4 \frac{x^2}{4} dx$$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{1}{4}\left[\frac{4^3}{3}\right]$$

$$\boxed{A_3 = \frac{16}{3} \text{ sq. units}}$$

$$① \Rightarrow 16 = \frac{16}{3} + A_2 + \frac{16}{3}$$

$$16 = \frac{32}{3} + A_2$$

$$A_2 = 16 - \frac{32}{3}$$

$$\boxed{A_2 = \frac{16}{3}} \text{ sq. units}$$

From A_1, A_2 and A_3, it is possible to divide equal parts

42)

a) Given

$$z = x+iy.$$

$$\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$$

$$\arg(z-i) - \arg(z+2) = \frac{\pi}{4}$$

$$\arg(x+iy-i) - \arg(x+iy+2) = \frac{\pi}{4}$$

$$\arg[x+i(y-1)] - \arg[(x+2)+iy] = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{\frac{y-1}{x} - \frac{y}{x+2}}{1+\left(\frac{y-1}{x}\right)\left(\frac{y}{x+2}\right)}\right] = \frac{\pi}{4}$$

$$\frac{\left(\frac{(x+2)(y-1) - yx}{x(x+2)}\right)}{\left(\frac{x(x+2)+(y-1)y}{x(x+2)}\right)} = \tan\frac{\pi}{4}$$

$$\frac{xy - x + 2y - 2 - xy}{x^2+2x+y^2-y} = 1$$

$$-x+2y-2 = x^2+2x+y^2-y$$

$$x^2+2x+y^2-y+x-2y+2=0$$

$$x^2+y^2+3x-3y+2=0$$

b) $(x^3+y^3)dy = x^2y\,dx$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \rightarrow (1)$$

It is homogeneous

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$(1) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2(vx)}{x^3+(vx)^3}$$

$$v + x\frac{dv}{dx} = \frac{x^3v}{x^3(1+v^3)}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x\frac{dv}{dx} = \frac{v - v(1+v^3)}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v - v - v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

$$\frac{(1+v^3)dv}{v^4} = -\frac{dx}{x}$$

$$\left(\frac{1}{v^4}+\frac{v^3}{v^4}\right)dv = -\frac{dx}{x}$$

$$\int\frac{dv}{v^4}+\int\frac{dv}{v} = -\int\frac{dx}{x}$$

$$\frac{-1}{3v^3}+\log|v| = -\log|x| + \log|c|$$

$$\frac{-1}{3v^3} = \log|c|-\log|x| - \log|v|$$

$$\frac{-1}{3v^3} = \log\left|\frac{c}{vx}\right|$$

$$\frac{-1}{3\left(\frac{y}{x}\right)^3} = \log\left|\frac{c}{y}\right|$$

$$\frac{x^3}{3y^3} = \log\left|\frac{y}{c}\right|$$

$$\frac{y}{c} = e^{x^3/3y^3}$$

$$y = c\,e^{x^3/3y^3}.$$

43) a) Given equation

$$6x^4-5x^3-38x^2-5x+6=0 \quad \text{(1)}$$

One solution is $\frac{1}{3}$.

Since (1) is a reciprocal equation, 3 is also one solution.

3	6	−5	−38	−5	6	
	↓	18	39	3	−6	
$\frac{1}{3}$	6	13	1	−2	$\boxed{0}$	
	↓	2	5	2		
	6	15	6	$\boxed{0}$		

∴ One factor is

$$6x^2+15x+6$$

Let $6x^2+15x+6=0$

÷ by 3 ⇒ $2x^2+5x+2=0$

$(x+2)(2x+1)=0$

4

$\frac{4}{2x}$ $\frac{1}{2x}$

$x+2=0$,

$2x+1=0$

$x=-2,\ x=-\frac{1}{2}$.

∴ Solutions of (1) are $3, \frac{1}{3}, -2, -\frac{1}{2}$.

b) Given points are $(1,1), (2,-1), (3,2)$

General equation of circle is

$$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$$

(1) passes through $(1,1)$

$$(1)^2+(1)^2+2g(1)+2f(1)+c=0$$

$$2+2g+2f+c=0$$

$$2g+2f+c=-2 \rightarrow (2)$$

(1) passes through $(2,-1)$

$$(2)^2+(-1)^2+2g(2)+2f(-1)+c=0$$

$$4+1+4g-2f+c=0$$

$$4g-2f+c=-5 \rightarrow (3)$$

(1) passes through $(3,2)$

$$3^2+2^2+2g(3)+2f(2)+c=0$$

$$9+4+6g+4f+c=0$$

$$6g+4f+c=-13 \rightarrow (4)$$

Solve (2) & (3)

$$(3) \Rightarrow 4g-2f+c=-5$$

$$(2) \Rightarrow \underset{-}{2g}\underset{-}{+2f}\underset{-}{+c}=\underset{+}{-2}$$

$$2g-4f=-3 \rightarrow (5)$$

Solve (2) & (4)

$$(4) \Rightarrow 6g+4f+c=-13$$

$$(2) \Rightarrow \underset{-}{2g}\underset{-}{+2f}\underset{-}{+c}=\underset{+}{-2}$$

$$4g+2f=-11 \rightarrow (6)$$

Solve (5) & (6)

$$(6) \Rightarrow 4g+2f=-11$$

$$(5)\times 2 \Rightarrow \underset{-}{4g}\underset{+}{-8f}=\underset{+}{-6}$$

$$10f=-5$$

$$\boxed{f=-1/2}$$

$$(6) \Rightarrow 4g+2(-1/2)=-11$$

$$4g-1=-11$$

$$4g=-10$$

$$g=\frac{-10}{4}$$

$$\boxed{g = \frac{-5}{2}}$$

$$② \Rightarrow 2\left(\frac{-5}{2}\right) + 2\left(\frac{-1}{2}\right) + c = -2$$

$$-5 - 1 + c = -2$$

$$c = -2 + 6$$

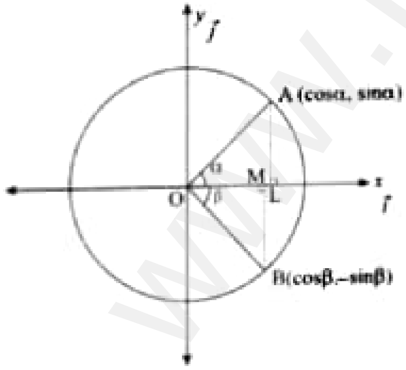
$$\boxed{c = 4}$$

∴ The required equation of circle is

$$① \Rightarrow x^2 + y^2 + 2\left(\frac{-1}{2}\right)x + 2\left(\frac{-5}{2}\right)y + 4 = 0$$

$$x^2 + y^2 - x - 5y + 4 = 0$$

44) a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.



Let $A(\cos\alpha, \sin\alpha)$ and $B(\cos\beta, -\sin\beta)$ are any two points on the unit circle.

where $\angle XOA = \alpha$, $\angle BOX = \beta$

Let $\hat{i}, \hat{j}$ are unit vector along x-axis and y-axis. $\hat{k}$ be the $\perp^r$ unit vector to $\hat{i}$ and $\hat{j}$.

$$\therefore \vec{OA} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$$

$$\vec{OB} = \cos\beta\,\hat{i} - \sin\beta\,\hat{j}$$

$$\therefore |\vec{OA}| = |\vec{OB}| = 1.$$

$$\vec{OB} \times \vec{OA} = |\vec{OB}|\,|\vec{OA}| \sin(\alpha+\beta)\hat{k}$$

$$= (1)(1)\sin(\alpha+\beta)\hat{k}$$

$$\vec{OB} \times \vec{OA} = \sin(\alpha+\beta)\hat{k} \longrightarrow ①$$

$$\vec{OB} \times \vec{OA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & -\sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$$

$$\vec{OB} \times \vec{OA} = (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\hat{k} \longrightarrow ②$$

From ① & ②,

$$\sin(\alpha+\beta)\hat{k} = (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\hat{k}$$

$$\therefore \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

44)

b) Given:

$W(x,y,z) = xy + yz + zx$

$x = u - v,\ y = uv,\ z = u + v$

$\frac{\partial w}{\partial x} = y + z$

$\frac{\partial w}{\partial y} = x + z$

$\frac{\partial w}{\partial z} = y + x$

$\frac{\partial x}{\partial u} = 1,\ \frac{\partial y}{\partial u} = v,\ \frac{\partial z}{\partial u} = 1$

$\frac{\partial x}{\partial v} = -1,\ \frac{\partial y}{\partial v} = u,\ \frac{\partial z}{\partial v} = 1$

$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$

$= (y+z)(1) + (x+z)(v) + (y+x)(1)$

$= y + z + (x+z)v + y + x$

$= (uv) + u + v + (u - v + u + v)v + uv + u - v$

$= 2uv + 2u + 2uv$

$\frac{\partial w}{\partial u} = 4uv + 2u$

$\left(\frac{\partial w}{\partial u}\right)_{(\frac{1}{2},1)} = 4(\frac{1}{2})(1) + 2(\frac{1}{2})$

$= 2 + 1$

$= 3.$

$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$

$= (y+z)(-1) + (x+z)(u) + (y+x)(1)$

$= \cancel{(y+z)}$

$\frac{\partial w}{\partial v} = (uv + u + v)(-1) + (u - v + u + v)u + (uv + u - v)(1)$

$= -uv - u - v + 2u^2 + uv + u - v$

$= 2u^2 - 2v$

$\left(\frac{\partial w}{\partial v}\right)_{(\frac{1}{2},1)} = 2(\frac{1}{2})^2 - 2(1)$

$= 2(\frac{1}{4}) - 2$

$= \frac{1}{2} - 2$

$= \frac{-3}{2}.$

Soln:

Gn: $x+y=10$

$y = x-10$.

Gn fn: $= x^2y^2$

$= x^2(x-10)^2$

Let it be f(x).

$f(x) = x^2(x-10)^2$

$\cancel{f'(x) = x^2(2(x-10)) + (x-10)^2(2x)}$

$= \cancel{2x^3 - 20x^2}$

$f(x) = x^2[x^2-20x+100]$

$f(x) = x^4-20x^3+100x^2$

$f'(x) = 4x^3-60x^2+200x$

$f''(x) = 12x^2-120x+200$

Let $f'(x) = 0$

$4x^3-60x^2+200x = 0$.

$\div$ by $4x$,

$x^2-15x+50 = 0$

$(x-10)(x-5) = 0$

$x = 10, x = 5$

$f''(5) = 12(5)^2 - 120(5) + 200$

$f''(5) = 300 - 600 + 200$

$f''(5) = -100 < 0$

$\therefore$ At $x=5$, f(x) has local maximum.

$f''(10) = 12(10)^2 - 120(10) + 200$

$= 1200 - 1200 + 200$

$f''(10) = 200 > 0$

$\therefore$ At $x=10$, f(x) has local minimum.

Local Maximum.

$f(5) = 5^2(5-10)^2$

$= 25(-5)^2$

$= 25(25)$

$= 625$

Local Minimum.

$f(10) = 10^2(10-10)^2 = 0$

45)

b) M.A

Question Incomplete.

46

a) Given set

$A = \{1, 3, 4, 5, 9\}$.

Operation: $\times_{11}$

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

(i) Closure proberty:

Since each box has an unique element of A, $\times_{11}$ is a binary operation on A.

(ii) Commutative:

The entries are symmetrical about the main diagonal. Hence $\times_{11}$ has commutative property.

(iii) Associative:

As usual, the associative property can be seen to be true.

(iv) Identity:

The entries of both the row and column headed by the element 1 are identical. Hence 1 is the identity element.

vi) Since the identity 1 exists in each row and each column, the existance of inverse property is assured for X_{11}. The inverse of 1 is 1, that of 3 is 4, that of 4 is 3, 5 is 9 and, that of 9 is 5.

b)(i)

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \frac{2\pi}{3}$

$= -\pi/3$.

(ii) $L.H.S = \tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right)$

$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\right)\left(\frac{7}{24}\right)}\right)$

$= \tan^{-1}\left(\frac{\frac{48+77}{11\times 24}}{\frac{(11\times 24)-14}{11\times 24}}\right)$

$= \tan^{-1}\left(\frac{125}{264-14}\right)$

$= \tan^{-1}\left(\frac{125}{250}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$

$= R.H.S$

44) Given points are $A(-1,2,0)$ and $B(2,2,1)$

$\therefore \vec{OA} =$

$\vec{a} = -\hat{i} + 2\hat{j}$, $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} - \vec{a} = 3\hat{i} - 0\hat{j} + \hat{k}$

Given straight line

$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

$\frac{x-1}{1} = \frac{\cancel{2}(y+1/2)}{\cancel{2}} = \frac{z+1}{-1}$

$\frac{x-1}{1} = \frac{y+1/2}{1} = \frac{z+1}{-1}$

$\therefore \vec{c} = \hat{i} + \hat{j} - \hat{k}$

(i) Parametric:

The required eqn of plane is

$\vec{r} = \vec{a} + s(\vec{b}-\vec{a}) + t\vec{c}$, $s, t \in R$

$\vec{r} = (-\hat{i}+2\hat{j}) + s(3\hat{i}+\hat{k})$

$+ t(\hat{i}+\hat{j}-\hat{k})$

(ii) Non parametric :

$\vec{r}\cdot((\vec{b}-\vec{a})\times\vec{c}) = \vec{a}\cdot[(\vec{b}-\vec{a})\times\vec{c}] \rightarrow (1)$

$(\vec{b}-\vec{a})\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= \hat{i}(0-1) - \hat{j}(-3-1) + \hat{k}(3-0)$

$= -\hat{i}+4\hat{j}+3\hat{k}$

(1) $\Rightarrow$

$\vec{r}\cdot(-\hat{i}+4\hat{j}+3\hat{k})$

$= (-\hat{i}+2\hat{j})\cdot(-\hat{i}+4\hat{j}+3\hat{k})$

$\vec{r}\cdot(-\hat{i}+4\hat{j}+3\hat{k})$

$= (-1)(-1)+2(4)+0(3)$

$\vec{r}\cdot(-\hat{i}+2\hat{j}) = 1+8+0$

$\vec{r}\cdot(-\hat{i}+2\hat{j}) = 9 \rightarrow (2)$

Cartesian form

(2) $\Rightarrow \vec{r}\cdot(-\hat{i}+2\hat{j}) = 9$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(-\hat{i}+2\hat{j}) = 9$

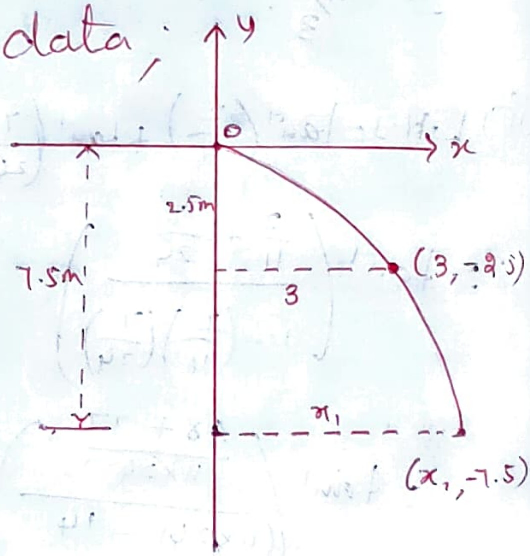
$x(-1)+y(2)+z(0) = 9$

$-x+2y+0 = 9$

$x-2y+9=0$

47)

b) From the given data,



From the figure,

Equation of the path of the water is $x^2 = -4ay \rightarrow (1)$

(1) passes through $(3, -2.5)$,

$(1) \Rightarrow 3^2 = -4a(-2.5)$

$9 = 10a$

$\boxed{a = \frac{9}{10}}$

$(1) \Rightarrow x^2 = -4\left(\frac{9}{10}\right) \cancel{a}y$

$x^2 = \frac{-18}{5} y \rightarrow (2)$

(2) passes through the point $(x_1, -7.5)$

$x_1^2 = \frac{-18}{5}(-7.5)$

$x_1^2 = 27$

$x_1 = 3\sqrt{3}$ mtr.