

HSE1\_P1\_XI\_MATHEMATICS

Sample question\_1

PART-I

20X1=20

- (i) Answer all the questions
- (ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer
- (1) If  $A = \{(x, y): y = \sin x, x \in \mathbb{R}\}$  and  $B = \{(x, y): y = \cos x, x \in \mathbb{R}\}$  then  $A \cap B$  contains  
 (a) cannot be determined (b) no element (c) infinitely many elements (d) only one element
- (2) If the function  $f: [-3, 3] \rightarrow S$  defined by  $f(x) = x^2$  is onto, then  $S$  is  
 (a)  $[0, 9]$  (b)  $[-9, 9]$  (c)  $\mathbb{R}$  (d)  $[-3, 3]$
- (3)  $\frac{1}{\cos 90^\circ} - \frac{\sqrt{3}}{\sin 90^\circ} =$  (a) 4 (b)  $\sqrt{2}$  (c)  $\sqrt{3}$  (d) 2
- (4)  $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$  is equal to  
 (a)  $2\cos x$  (b)  $\cos 2x$  (c)  $\cos x$  (d)  $\cos 3x$
- (5)  $\ln^{2n} C_2: C_3 = 11:1$  then  $n$  is  
 (a) 7 (b) 5 (c) 6 (d) 11
- (6) The value of  $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$  is  
 (a)  $\frac{e^2 - 1}{2e}$  (b)  $\frac{e^2 + 1}{2e}$  (c)  $\frac{(e+1)^2}{2e}$  (d)  $\frac{(e-1)^2}{2e}$
- (7) The slope of the line which makes an angle  $45^\circ$  with the line  $3x - y = -5$  are  
 (a)  $2, -\frac{1}{2}$  (b)  $1, -1$  (c)  $\frac{1}{2}, -2$  (d)  $1, \frac{1}{2}$
- (8) The image of the point  $(2, 3)$  in the line  $y = -x$  is  
 (a)  $(3, 2)$  (b)  $(-3, -2)$  (c)  $(-3, 2)$  (d)  $(-2, -3)$
- (9) If  $A$  is a square matrix, then which of the following is not symmetric?  
 (a)  $A - A^T$  (b)  $A + A^T$  (c)  $AA^T$  (d)  $A^T A$
- (10) If  $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ , then  $(A + I)(A - I)$  is equal to  
 (a)  $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$  (b)  $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$  (c)  $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$  (d)  $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$
- (11) If  $\vec{a}$  and  $\vec{b}$  include an angle  $120^\circ$  and their magnitudes are 2 and  $\sqrt{3}$  then  $\vec{a} \cdot \vec{b}$  is equal to  
 (a)  $\frac{-\sqrt{3}}{2}$  (b)  $\sqrt{3}$  (c)  $-\sqrt{3}$  (d) 2

- (12) If  $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$  and  $|\vec{b}| = 46$ , then  $|\vec{a}|$  is  
 (a) 32 (b) 42 (c) 12 (d) 22
- (13)  $f(x) = |x| + |x - 1|$  is  
 (1) discontinuous at  $x = 0, 1$  (2) continuous at  $x = 0$  only  
 (3) continuous at  $x = 1$  only (4) continuous at both  $x = 0$  and  $x = 1$
- (14)  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$   
 (a)  $\frac{a}{b}$  (b)  $\log ab$  (c)  $\log \left(\frac{a}{b}\right)$  (d)  $\log \left(\frac{b}{a}\right)$
- (15)  $x = at^2, y = 2at$ , then  $\frac{dy}{dx} =$  (a)  $-t$  (b)  $\frac{1}{t}$  (c)  $-\frac{1}{t}$  (d)  $t$
- (16) If  $y = \frac{1}{a-z}$ , then  $\frac{dz}{dy}$  is  
 (a)  $-(z+a)^2$  (b)  $(a-z)^2$  (c)  $-(z-a)^2$  (d)  $(z+a)^2$
- (17)  $\int \left(\frac{x-1}{x+1}\right) dx =$  (a)  $x + 2 \log(x+1) + c$  (b)  $\frac{1}{2} \left(\frac{x-1}{x+1}\right)^2 + c$   
 (c)  $x - 2 \log(x+1) + c$  (d)  $\frac{(x-1)^2}{2} \log(x+1) + c$
- (18)  $\int 2^{3x+5} dx$  is  
 (a)  $\frac{2^{3x+5}}{(a)3 \log 2} + c$  (b)  $\frac{3(2^{3x+5})}{\log 2} + c$  (c)  $\frac{2^{3x+5}}{2 \log(3x+5)} + c$  (d)  $\frac{2^{3x+5}}{2 \log 3} + c$
- (19)  $\int \frac{dx}{e^x - 1}$  is  
 (a)  $\log |e^x + 1| - \log |e^x| + c$  (b)  $\log |e^x| + \log |e^x - 1| + c$   
 (c)  $\log |e^x - 1| - \log |e^x| + c$  (d)  $\log |e^x + 1| - \log |e^x| + c$
- (20) Ten coins are tossed. The probability of getting at least 8 heads is  
 (a)  $\frac{7}{128}$  (b)  $\frac{7}{64}$  (c)  $\frac{7}{32}$  (d)  $\frac{7}{16}$

PART-II

7X2=14

Note: Answer any seven questions. Question No 30 is compulsory.

- (21) Resolve the rational expression  $\frac{1}{x^2 - a^2}$  into partial fractions.
- (22) If  $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$  then find the value of A.
- (23) Find the coefficient of  $x^5$  in the expansion of  $\left(x + \frac{1}{x^2}\right)^{17}$

(24) Find the equation of the straight line, if the perpendicular from the origin makes an angle of  $120^\circ$  with  $x$ -axis and the length of the perpendicular from the origin is 6 units?

(25) Consider the function  $f(x) = \sqrt{x}, x \geq 0$ . Does  $\lim_{x \rightarrow 0} f(x)$  exist?

(26) Calculate  $\lim_{x \rightarrow 3} \left( \frac{x^2 - 5x + 6}{x - 3} \right)$

(27) Integrate  $(1 + x^2)^{-1}$  with respect to  $x$

(28) Evaluate:  $\int \frac{1}{\sin^2 x \cos^2 x} dx$ .

(29) If two coins are tossed simultaneously, then find the probability of getting at most two tails

(30) Differentiate  $x^x$  with respect to  $x$

### PART-III

7X3=21

Note: Answer **any seven questions**. Question No **40** is compulsory.

(31) Find the range of the function  $f(x) = \frac{1}{1 - 3 \cos x}$ .

(32) In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together.

(33) If  $a_1, a_2, a_3, \dots, a_n$  is a geometric progression, every term  $a_k$  ( $k > 1$ ) is the geometric mean of its immediate predecessor  $a_{k-1}$  and immediate successor  $a_{k+1}$ .

(34) Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of  $45^\circ$  with positive the  $x$ -axis.

(35) Find the value of the product;  $\begin{vmatrix} \log_2 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ .

(36) Find the second derivative of  $\log(\log x)$  with respect to  $x$ .

(37) Integrate  $\frac{x^{15}}{1+x^{12}}$  with respect to  $x$

(38) Show that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$ .

(39) Do the limits of the functions  $\frac{\sin |x|}{x}$  exist as  $x \rightarrow 0$ ? State reasons for your answer.

(40) If  $a \sin^2 \theta + b \cos^2 \theta = c$  show that  $\tan^2 \theta = \frac{c-b}{a-b}$

### PART-IV

7x5=35

Note: Answer all the questions

(41) (a) If  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = |x| + x$  and  $g(x) = |x| - x$ , find  $g \circ f$  and  $f \circ g$ . (OR)

(b) In the set  $\mathbb{Z}$  of integers, define  $mRn$  if  $m - n$  is a multiple of 12. Prove that  $R$  is an equivalence relation.

(42) (a) Find all values of  $x$  that satisfy the inequality  $\frac{2x-3}{(x-2)(x-4)} < 0$ . (OR)

(b)  $A + B + C = \frac{\pi}{2}$ , prove that  $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$ .

(43) (a) In any triangle  $ABC$ , prove that  $\cos \left( \frac{B-C}{2} \right) = \frac{b+c}{a} \sin \frac{A}{2}$  (OR)

(b) Find the value of  $\sqrt[3]{126}$  correct to two decimal places

(44) (a) By the principle of mathematical induction, prove that, for  $n \geq 1$

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \quad (\text{OR})$$

(b) Show that the equation  $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$  represents a pair of parallel lines. Find the distance between them.

(45) (a) Show that  $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$  (OR)

(b) If  $ABCD$  is a quadrilateral and  $E$  and  $F$  are the midpoints of  $AC$  and  $BD$  respectively, then prove that  $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 4\overline{EF}$ .

(46) (a) Find the points of discontinuity of the function  $f$ , where  $f(x) = \begin{cases} 4x + 5, & \text{if } x \leq 3 \\ 4x - 5, & \text{if } x > 3 \end{cases}$  (OR)

(b) If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ , show that  $(1-x^2)y_2 - 3xy_1 - y = 0$ .

(47) (a) Integrate  $\frac{5x-2}{2+2x+x^2}$  with respect to  $x$  (OR)

(b) There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An Urn is chosen at random and a ball is drawn from it. (i) Find the probability that the ball is black (ii) If the ball is black, what is the probability that it is from the first urn?

HSE1\_P1\_XI\_MATHEMATICS\_KEY

PART\_I

QNO	ANSWER	QNO	ANSWER
1	(c) Infinitely many elements	11	C
2	(a) [0,9]	12	(c) 22
3	(a) 4	13	C
4	(a) 2cosx	14	(c) $\log\left(\frac{b}{a}\right)$
5	(c) 6	15	C
6	(d) $\frac{e^x-1}{2e}$	16	(a) $(a-z)^2$
7	(b) $\frac{1}{2}, -2$	17	C
8	(a) (-3, -2)	18	(d) $\frac{2^{3x+5}}{3 \log 2} + c$
9	(d) $A - A^T$	19	(c) $\log e^x - 1  - \log e^x  + c$
10	(a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$	20	(d) $\frac{7}{128}$

PART\_II

QNO	ANSWER	QNO	ANSWER
21	$1/2a(x-a) - 1/2a(x+a)$	26	C
22	A = 81.	27	$\tan^{-1} x + c$

23	C	28	$\tan x - \cot x + c$
24	C	29	1
25	$\lim \sqrt{x} \rightarrow$ does not exist.	30	C
<b>PART_III</b>			
	<b>QNO</b>	<b>ANSWER</b>	<b>QNO</b> <b>ANSWER</b>
31	The range of f is $(-\infty, -1/2] \cup [1/4, \infty)$ .	36	C
32	43200.	37	C
33	$a_k$ is the geometric mean of $a_{k-1}$ and $a_{k+1}$ .	38	To prove
34	$x + y = 6\sqrt{2}$	39	The limit does not exist.
35	6	40	To showed(C)
<b>PART_IV</b>			
	<b>QNO</b>	<b>ANSWER</b>	<b>ANSWER</b>
41	(a) for all x, $(f \circ g) = O, (g \circ f) = O$	OR	(b) R is an equivalence relation
42	(a) $-\infty, 3/2 \cup (2, 4)$ (3)	OR	(b) To Proved
43	(a) C (To Proved)	OR	(b) C
44	(a) To Proved	OR	(b) 8/5
45	(a) To Proved	OR	(b) To Proved
46	(a) not continuous at $x = 3$	OR	(b) To proved
47	(a) $5/2 \log x^2 + 2x + 2  - \tan^{-1}(x+1) + c$	OR	(b) 6/11

MATHS SQUARE

Part-I (One Marks)				Part-II(Two Marks)				Part-III (Three Marks)				Part-IV (Five Marks)			
Q.No	Chapter	Ex	Page	Q.No	Chapter	Ex/Eg	Page	Q.No	Chapter	Eg/Ex	Page	Q.No	Chapter	Eg/Ex	Page
1	1	Ex1.5(2)	46	21	2	Ex2.9(i)	71	31	1	Eg 1.23	29	41(A)	1	Ex1.3(10)	37
2	1	Ex1.5(20)	47	22	4	Eg4.23	165	32	4	Eg 4.32	173	(B)	1	Eg 1.13	17
3	3	Ex3.12(1)	150	23	5	C	C	33	5	Result 5.2	215	42(A)	2	Ex 2.8(2)	69
4	3	Ex3.12(16)	150	24	6	C	C	34	6	Eg 6.17	256	(B)	3	Ex3.7_4(ii)	124
5	4	Ex4.5(17)	198	25	9	Eg 9.2	93	35	7	Ex 7.4(6)	40	43(A)	3	C	C
6	5	Ex5.5(19)	233	26	9	C	C	36	10	C	C	(B)	5	C	C
7	6	Ex6.5(6)	283	27	11	Ex11.1_4(i)	188	37	10	C	C	44(A)	4	Ex4.4(2)	196
8	6	Ex6.5(16)	283	28	11	Eg 11.18	198	38	8	Ex8.4(2)	79	(B)	6	Ex6.4(14)	282
9	7	Ex7.5(8)	41	29	12	Ex12.1_2(ii)	246	39	9	Eg 9.35(i)	116	45(A)	7	Eg 7.29	35
10	7	Ex7.5(24)	43	30	10	C	C	40	3	C	C	(B)	8	Ex8.1(12)	60
11	8	C	C									46(A)	9	Ex9.5_3(i)	127
12	8	Ex8.5(15)	81									(B)	10	Ex10.4(15)	176
13	9	C	C									47(A)	11	Ex11.11_1(ii)	222
14	9	Ex9.6(7)	130									(B)	12	Ex 12.4(2)	264
15	10	C	C												
16	10	Ex10.5(5)	177												
17	11	C	C												
18	11	Ex11.13(11)	226												
19	11	Ex11.13(17)	226												
20	12	Ex12.5(23)	267												