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PHYSICS

STUDY MATERIAL

2022 – 2023

PREPARED BY

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“thoehs; KOtJk; xt;ntthU kzpj;JspAk; NeHikaha;, cz;ikaha; cioj;jpdwtHfspd; fuq;fNs J}a;ikahd fuq;fs;.”

UNIT – I (NATURE OF PHYSICAL WORLD AND MEASUREMENT)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. Briefly explain the types of physical quantities.

Physical quantities are classified into two types. They are fundamental and derived quantities.

Fundamental or base quantities **are quantities which cannot be expressed in terms of any other physical quantities.**
These are **length, mass, time, electric current, temperature, luminous intensity and amount of substance.**

Quantities that **can be expressed in terms of fundamental quantities** are called derived quantities. For example, **area, volume, velocity, acceleration, force.**

2. Write the rules for determining significant figures.

1. All **non-zero digits** are significant. Ex.**1342** has **four** significant figures
2. All **zeros between two non-zero** digits are significant.
   Ex. **2008 has four** significant figures.
3. All **zeros to the right of a non-zero** digit but to the left of a decimal point are significant. Ex. **30700. has five** significant figures.
4. The **number without a decimal point**, the terminal or trailing zero(s) are not significant. Ex. **30700 has three** significant figures.
   All zeros are significant if they come from a measurement
   Ex. **30700 m** has **five** significant figures
5. If the number is **less than 1,** the zero (s) on the right of the decimal point but to left of the **first non-zero digit are not significant.** Ex. **0.00345** has **three** significant figures.
6. All zeros to the right of a decimal point and to the right of non-zero digit are significant. Ex. **40.00 has four** significant figures and **0.030400** has **five** significant figures.
7. The number of significant figures **does not depend on the system of units** used 1.53 cm, 0.0153 m, 0.0000153 km, all have **three** significant figures.

3. What are the limitations of dimensional analysis?

1. This method gives **no information about the dimensionless constants** in the formula like 1, 2, ...π,e, etc.
2. This method **cannot decide whether the given quantity** is a **vector or a scalar.**
3. This method is **not suitable to derive relations** involving **trigonometric, exponential and logarithmic functions.**
4. It **cannot be applied** to an equation involving **more than three physical quantities.**

5. It can only **check on whether a physical relation** is **dimensionally correct** but not the correctness of the relation. For example, using dimensional analysis,
$s = ut + \frac{1}{3}at^2$ is dimensionally correct whereas the correct relation is $s = ut + \frac{1}{2}at^2$

4. Define the terms i) Unification ii) Reductionism

1. Attempting **to explain diverse physical phenomena** with a **few concepts** and laws is **unification.**
2. An attempt **to explain a macroscopic system** in terms of its microscopic constituents is **reductionism.**

5. Define SI standard for length

One metre is the length of the **path travelled by light in vacuum** in 1/299,792,458 of a second.

6. Define SI standard for mass

One kilogram is the mass **of the prototype cylinder of platinum iridium** alloy (whose **height is equal to its diameter**), preserved at the International Bureau of Weights and Measures at Serves, near Paris, France.

7. Define SI standard for time

One second is the duration of 9,192,631,770 periods of radiation corresponding to the **transition between the two hyperfine levels of the ground state of Cesium-133 atom.**

8. Define one radian

One radian is the angle subtended at the centre of a circle by an arc **equal in length to the radius of the circle.**

9. Define steradian

One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is **equal in area, to the square of radius of the sphere.**

10. Define light year

Light year (**Distance travelled by light in vacuum in one year**)
1 Light Year = 9.467×10^{15} m

11. Define astronomical unit

Astronomical unit (the mean **distance of the Earth from the Sun**)
1 AU = 1.496×10^{11} m

12. What are systematic errors?

Systematic errors are **reproducible inaccuracies that are consistently in the same direction**. These occur often due to a problem that persists throughout the experiment.

13. How to minimize the systematic error?

Systematic errors are difficult **to detect and cannot be analyzed statistically**, because all of the data is in the same direction.

14. What is personal error?

These errors are due **to individuals performing the experiment**, may be due to **incorrect initial setting up of the experiment** or **carelessness** of the individual making the observation due to improper precautions.

15. What are least count errors? How is it minimized?

Least count is the **smallest value** that can be **measured by the measuring instrument**, and the error due to this measurement is least count error. The instrument's resolution hence is the cause of this error. Least count error can be reduced by using a **high precision instrument for the measurement**

16. What are Random errors? How is it minimized?

Random errors may arise due to **random and unpredictable variations** in experimental conditions like pressure, temperature, voltage supply
Random errors can be evaluated through **statistical analysis** and can be reduced by averaging over a **large number of observations.**

17. What are Gross errors? How is it minimized?

Reading an instrument **without setting it properly**. It can be minimized only when an **observer is careful and mentally alert.**

18. What is relative error or fractional error?

The **ratio of the mean absolute error to the mean value.** Relative error = $\frac{\Delta a_m}{a_m}$

19. What is percentage error?

The **relative error expressed as a percentage**. Percentage error = $\frac{\Delta a_m}{a_m}$ x 100%

20. Define dimensional constant and dimensionless constant

Dimensional Constant

Physical quantities which possess **dimensions and have constant** values. Examples are **Gravitational constant, Planck's constant etc.**

Dimensionless Constant

Quantities which **have constant values** and also **have no dimensions**. Examples are **π, e, numbers** etc.

21. Define dimensional variable and dimensionless variable

Dimensional variables

Physical quantities, which **possess dimensions and have variable** values. Examples are **length, velocity, and acceleration etc.**

Dimensionless variables

Physical quantities which **have no dimensions**, but **have variable** values. Examples are **specific gravity, strain, refractive index etc.**

22. What are the uses of dimensional analysis?

1. Convert a physical quantity from **one system of units to another**.
2. **Check the dimensional correctness** of a given physical equation.
3. Establish **relations among various physical quantities.**

23. Write principle of homogeneity of dimensions.

The principle of homogeneity of dimensions' states that the **dimensions of all the terms in a physical expression should be the same.** For example, in the **physical expression $v^2 = u^2 + 2as$**, the dimensions of v^2, u^2 and 2 as are the same and equal to **$[L^2T^{-2}]$.**

FIVE MARKS QUESTION WITH ANSWER QUESTIONS

24. **i) Explain the use of screw gauge and vernier caliper in measuring smaller distances.**

ii) Write a note on triangulation method and radar method to measure larger distances.

Measurement of small distances:

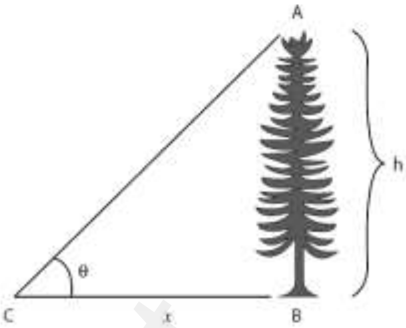
1. **Screw gauge:** The screw gauge is an instrument used for measuring accurately the **dimensions of objects up to a maximum of about 50 mm.** The principle of the instrument is the magnification of linear motion using the circular motion of a screw. The least count of the screw gauge is 0.01 mm
2. **Vernier caliper:** A vernier caliper is a **versatile instrument** for measuring the **dimensions of an object** namely diameter of a hole, or a depth of a hole.

Measurement of large distances:

3. For measuring larger distances such as the height of a tree, distance of the **Moon or a planet from the Earth,** some special methods are adopted. **Triangulation method, parallax method and radar method are used to determine very large distances.**

Triangulation method for the height of an accessible object:

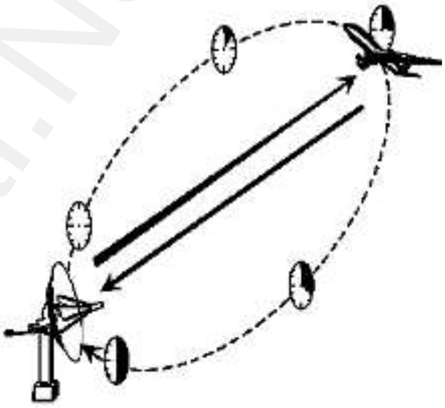
1. Let **AB = h be the height of the tree** or tower to be measured.
Let C be the point of observation at distance x from B. Place a range finder at C and measure the angle of elevation, $\angle ACB = \theta$ as shown in Figure. From right angled triangle **ABC,**



tanθ $= \frac{AB}{BC} = \frac{h}{x}$ (or) **height** $h = x \tan\theta$
Knowing the distance x, the height h can be determined.

RADAR method:

1. The word RADAR stands for **Radio Detection and Ranging.** Radar can be used **to measure accurately the distance of a nearby planet** such as Mars. In this method, **radio waves are sent from transmitters which, after reflection from the planet, are detected by the receiver.**



2. By measuring, the time interval (t) between the instants the radio waves are sent and received, the distance of the planet can be determined as $d = \frac{v \times t}{2}$.
where **v is the speed of the radio wave.**

As the time taken (t) is for the distance covered during the forward and backward path of the radio waves, it is divided by 2 to get the actual distance of the object. This method can also be **used to determine the height, at which an aero-plane flies from the ground.**

25. Explain in detail the various types of errors.

Random error, systematic error and gross error are the three possible errors

Systematic errors:

Systematic errors are reproducible inaccuracies that are consistently in the same direction.

Instrumental errors:

When an **instrument is not calibrated properly** at the time of manufacture, these errors can be corrected by **choosing the instrument carefully.**

Imperfections in experimental technique or procedure:

These errors **arise due to the limitations in the experimental** arrangement. To overcome these, necessary correction has to be applied.

Personal errors:

These errors are due to individuals performing the experiment, may be due to **incorrect initial setting up of the experiment or**

carelessness of the individual making the observation due to improper precautions

Errors due to external causes:

The **change in the external conditions** during an experiment can cause error in measurement. For example, **changes in temperature, humidity, or pressure during measurements** may affect the result of the measurement.

Least count error:

Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error.

Random errors:

1. Random errors may arise due **to random and unpredictable variations** in experimental conditions like pressure, temperature, voltage supply etc.
2. Errors may also be due to **personal errors** by the observer who performs the experiment. Random errors are sometimes called **"chance error"**
3. It can be minimized by **repeating the observations a large number of measurements** are made and then the arithmetic mean is taken.

Gross Error:

The error caused due to the shear **carelessness of an observer** is called gross error. These errors can be minimized only when an observer is **careful and mentally alert.**

26. Explain the propagation of errors in addition and multiplication.

(i) Error in the sum of two quantities:

Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively.
Then, Measured value of A = $A \pm \Delta A$; Measured value of B = $B \pm \Delta B$
Consider the sum, $Z = A + B$
The error ΔZ in Z is then given by
$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$; $= (A + B) \pm (\Delta A + \Delta B)$
$= Z \pm (\Delta A + \Delta B)$ (or) $\Delta Z = \Delta A + \Delta B$
The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

(ii) Error in the difference of two quantities:

Let ΔA and ΔB be the absolute errors in the two quantities, A and B, respectively. Then,
Measured value of A = $A \pm \Delta A$; Measured value of B = $B \pm \Delta B$
Consider the difference, $Z = A - B$
The error ΔZ in Z is then given by
$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$; $= (A - B) \pm \Delta A \pm \Delta B$
$= Z \pm \Delta A \pm \Delta B$ (or) $\Delta Z = \Delta A + \Delta B$

The maximum error in difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

(iii) Error in the product of two quantities:

Let ΔA and ΔB be the absolute errors in the two quantities A, and B, respectively. Consider the product Z = AB

The error ΔZ in Z is given by $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= (AB) \pm (A\,\Delta B) \pm (B\,\Delta A) \pm (\Delta A\,.\,\Delta B)$

Dividing L.H.S by Z and R.H.S by AB, we get, $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A}\,.\,\frac{\Delta B}{B}$

As $\Delta A / A$, $\Delta B / B$ are both small quantities,

their product term $\frac{\Delta A}{A}\,.\,\frac{\Delta B}{B}$ can be neglected.

The maximum fractional error in Z is $\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Strength does not come from winning. Your struggles develop your strengths. When you go through hardships and decide not to surrender, that is strength.

27. Write the rules for rounding off.

1. If the digit to be dropped is **smaller than 5**, then the preceding digit should be left unchanged.
   Ex. i) 7.32 is rounded off to 7.3 ii) 8.94 is rounded off to 8.9
2. If the digit to be dropped is **greater than 5**, then the preceding digit should be increased by 1
   Ex. i) 17.26 is rounded off to 17.3 ii) 11.89 is rounded off to 11.9
3. If the digit to be **dropped is 5 followed by digits other than zero**, then the preceding digit should be **raised by 1**
   Ex. i) 7.352, on being rounded off to first decimal becomes 7.4
   ii) 18.159 on being rounded off to first decimal, become 18.2
4. If the digit to be **dropped is 5 or 5 followed by zeros**, then the preceding digit is not changed if it **is even**
   Ex. i) 3.45 is rounded off to 3.4 ii) 8.250 is rounded off to 8.2
5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1 if **it is odd**
   Ex. i) 3.35 is rounded off to 3.4 ii) 8.350 is rounded off to 8.4

UNIT – II (KINEMATICS)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. Explain what is meant by Cartesian coordinate system?

At any given **instant of time**, the frame of reference with respect to which the **position of the object is described** in terms of position coordinates **(x, y, z)** (i.e., distances of the given position of an object along the x, y, and z–axes.) is called **"Cartesian coordinate system"**

2. Define a vector. Give examples

It is a quantity which is described by **both magnitude and direction**. Geometrically a vector is a directed line segment. **Examples Force, velocity, displacement, position vector, acceleration, linear momentum and angular momentum**

3. Define a scalar. Give examples

It is a property which can be described **only by magnitude**. In physics a number of quantities can be described by scalars. **Examples Distance, mass, temperature, speed and energy**

4. Write a short note on the scalar product between two vectors.

The scalar product (or dot product) of two vectors is defined **as the product of the magnitudes of both the vectors and the cosine of the angle** between them. $\vec{A}.\vec{B}$ = ABcosθ. Here, A and B are magnitudes of $\vec{A}$ and $\vec{B}$.

Properties:

The product quantity $\vec{A}$ and $\vec{B}$ is always a scalar. The **scalar product** is **commutative.**

5. Write a short note on vector product between two vectors.

The vector product or cross product of two vectors is defined as another **vector having a magnitude equal to the product of the magnitudes of two vectors** and the sine of the angle between them. $\vec{C} = \vec{A} x \vec{B}$ = (AB sin θ)n

6. Define displacement and distance.

Distance is the **actual path length** travelled by an object in the given **interval of time during the motion.** It is a positive **scalar quantity**.

Displacement the shortest **distance between these two positions** of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a **vector quantity.**

7. Define velocity and speed.

Velocity: The **rate of change of displacement** of the particle.
Velocity = Displacement / time taken. Unit: ms^{-1}. Dimensional formula: LT^{-1}
Speed: The **distance travelled in unit time**. It is a scalar quantity.

8. Define acceleration.

The acceleration of the particle at an instant is equal to **rate of change of velocity. It is a vector quantity. SI Unit: ms^{-2}. Dimensional formula: $M^0L^1T^{-2}$**

9. What is the difference between velocity and average velocity?

Velocity is **the rate at which the position changes**. But the average velocity is the **displacement or position change per time ratio.**

10. What is non uniform circular motion?

If the speed of the object in **circular motion is not constant**, then we have non-uniform circular motion. For example, **when the bob attached to a string moves in vertical circle.**

11. Write down the kinematic equations for angular motion.

1. $\omega = \omega_0 + \alpha t$ 2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ 3. $\omega^2 = \omega_0^2 + 2\alpha\theta$ 4. $\theta = \frac{(\omega + \omega_0)t}{2}$

12. What is meant by Right handed Cartesian co-ordinate system?

The x, y and z axes are drawn in **anticlockwise direction then the coordinate** system is called as "**right– handed Cartesian coordinate system".**

13. Define linear motion. Give an example.

An object is said to be in linear motion if it moves in a straight line.

Examples

i) An athlete running on a straight track

ii) A particle falling vertically downwards to the Earth.

14. Define circular motion. Give an example.

Circular motion is defined as a motion described by an **object traversing a circular path.**

Examples

1) The whirling motion of a stone attached to a string

2) The motion of a satellite around the Earth

15. Define rotational motion. Give an example.

If any **object moves in a rotational motion about an axis**, the motion is called 'rotation'.

Examples

i) Rotation of a disc about an axis through its center

ii) Spinning of the Earth about its own axis.

16. What is meant by motion in one dimension?

One dimensional motion is the motion of a **particle moving along a straight line**

Examples

i) Motion of a train along a straight railway track.

ii) An object falling freely under gravity close to Earth.

17. What is meant by motion in two dimensions?

If **a particle is moving along a curved path in a plane**, then it is said to be in two dimensional motion.

Examples

i) Motion of a coin on a carrom board.
ii) An insect crawling over the floor of a room.

18. What is meant by motion in three dimensions?

A **particle moving in usual three dimensional space** has three dimensional motion

Examples

i) A bird flying in the sky. ii) Random motion of a gas molecule.
iii) Flying of a kite on a windy day.

19. What is meant by equal vectors?

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal **magnitude and same direction** and represent the same physical quantity.

20. Define parallel and anti-parallel vectors.

Two vectors $\vec{A}$ and $\vec{B}$ act in the **same direction** along the same line or on **parallel lines**, then the angle between them is 0^0

Two vectors $\vec{A}$ and $\vec{B}$ are said to be **anti–parallel when they are in opposite directions** along the same line or on parallel lines. Then the angle between them is 180^0

21. What is retardation?

If the **velocity is decreasing with respect to time**, then the acceleration.

22. Define momentum

The linear momentum or simply momentum of a particle is defined as **product of mass with velocity**. It is denoted as '$\vec{p}$'. **Momentum is also a vector quantity.**

23. Write the kinetic equations for linear motion.

i) $v = u + at$ ii) $s = ut + ½ at^2$ iii) $v^2 = u^2 + 2as$ iv) $s = \frac{(u+v)t}{2}$

24. What is meant by projectile?

When an **object is thrown in the air with some initial velocity** and then allowed to move under the action of gravity alone, the object is known as a projectile

25. Give some examples for projectile motion.

1. An object dropped from window of a moving train
2. A bullet fired from a rifle. 3. A ball thrown in any direction.
4. A javelin or shot put thrown by an athlete.
5. A jet of water issuing from a hole near the bottom of a water tank.

26. Define Time of flight

The **time taken for the projectile to complete its trajectory** or time taken by the **projectile to hit the ground** is called time of flight.

27. What is Horizontal range?

The horizontal distance covered by the **projectile from the foot of the tower** to the point where the projectile hits the ground is called horizontal range

28. Define maximum height.

The **maximum vertical distance travelled by the projectile** during its journey is called maximum height.

29. Define horizontal range.

The **maximum horizontal distance between the point of projection** and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R).

30. Define Time of flight.

The total time taken by the **projectile from the point of projection till it hits the horizontal plane** is called time of flight.

31. Define Uniform circular motion.

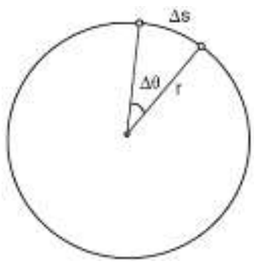
When a point **object is moving on a circular path with a constant** speed, it covers equal distances on the circumference of the circle in equal intervals of time.

32. Write the assumptions need to study about the projectile motion.

i) **Air** resistance is neglected.
ii) The effect due **to rotation of Earth and curvature of Earth** is negligible.
iii) The **acceleration due to gravity is constant** in magnitude and direction at all points of the motion of the projectile.

33. Derive the relation between linear velocity and angular velocity.

1) Consider an object moving along a circle of radius r. In a time Δ*t*, the object travels. An arc distance Δs as shown in Figure. The corresponding angle subtended is Δθ



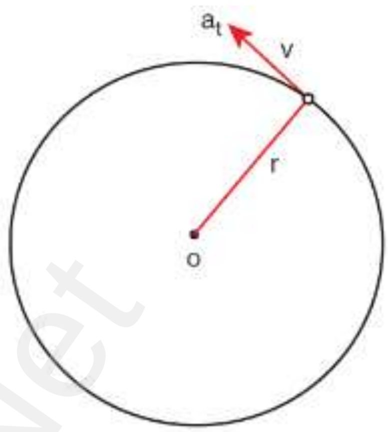
2) The Δs can be written in terms of Δθ as,

$\Delta s = r\Delta\theta$ In a time Δ*t*, we have $\frac{\Delta_s}{\Delta_t} = r\,\frac{\Delta\theta}{\Delta t}$

In the limit $\Delta t \to 0$, the above equation becomes $\frac{ds}{dt} = r\omega$ ---------(1)

3) Here $\frac{ds}{dt}$ is linear speed (v) which is tangential to the circle and ω is angular speed. So equation (1) becomes $v = r\omega$. Which gives the relation between linear speed and angular speed.

34. Find the expressions tangential acceleration.



i) In general the relation between linear and angular velocity is given by $\vec{v} = \vec{\omega} \times \vec{r}$. For circular motion equation reduces to equation v = rω. since $\vec{\omega}$ and $\vec{r}$ are perpendicular to each other. Differentiating the equation v = rω with respect to time, we get (since r is constant) $\frac{dv}{dt} = r\frac{d\omega}{dt} = r\,\alpha$

ii) Here $\frac{dv}{dt}$ is the tangential acceleration and is denoted as a_t. $\frac{d\omega}{dt}$ is the angular acceleration. Then eqn. $\vec{v} = \vec{\omega} \times \vec{r}$ becomes $a_t = r\,\alpha$

iii) The tangential acceleration a_t experienced by an object is circular motion as shown in Figure.

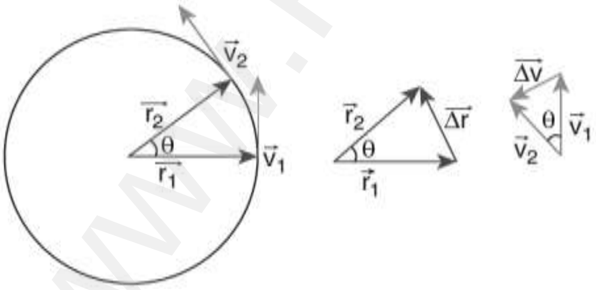
35. Derive an expression for the centripetal acceleration of a body moving in a circular path of radius 'r' with uniform speed.

i) The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors.

ii) Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt,

iii) For uniform circular motion, r = $|\vec{r_1}| = |\vec{r_2}|$ and v = $|\vec{v_1}| = |\vec{v_2}|$. If the particle moves from position vector $\vec{r_1}$ to $\vec{r_2}$, the displacement is given by $\Delta\vec{r} = \vec{r_2} - \vec{r_1}$ and the change in velocity from $\vec{v_1}$ to $\vec{v_2}$ is given by $\Delta\vec{v} = \vec{v_2} - \vec{v_1}$.

iv) The magnitudes of the displacement Δr and of Δv satisfy the following relation $\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$



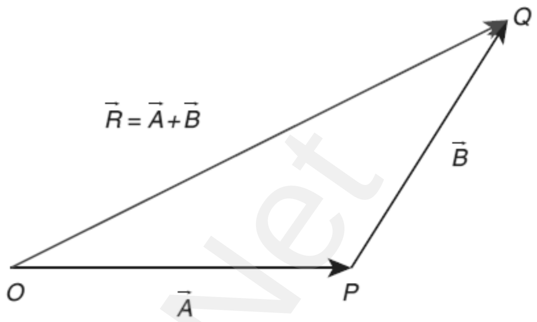
v) Here the negative sign implies that Δv points radially inward, towards the center of the circle. $\Delta v = v\left(\frac{\Delta r}{r}\right)$ then, $a = \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\frac{\Delta v}{\Delta t}\right)$; $= -\frac{v^2}{r}$

vi) For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle about the center. Then the centripetal acceleration can be written as $a = -\omega^2 r$

FIVE MARKS QUESTION WITH ANSWER QUESTIONS

36. Explain in detail the triangle law of addition.

1) Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle taken in the opposite order.

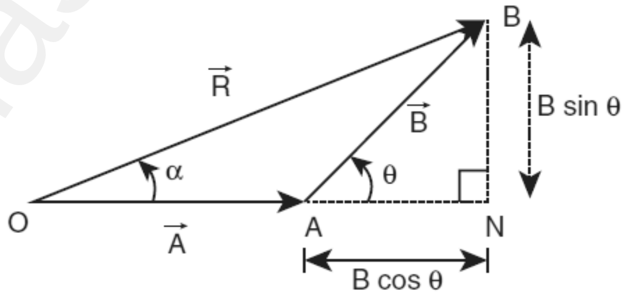


2) The **head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$ is the resultant vector connecting the tail** of the first vector $\vec{A}$ to the head of the second vector $\vec{B}$.

3) The magnitude of $\vec{R}$ (resultant) is given geometrically by the length of $\vec{R}$ (OQ) and the direction of the resultant vector is the angle between $\vec{R}$ and $\vec{A}$ Thus we write $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. $\because \overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

Magnitude of resultant vector:

4) Consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.



$\text{Cos } \theta = \frac{AN}{B} \therefore AN = B \text{ Cos } \theta$ *and*

$\text{Sin } \theta = \frac{BN}{B} \therefore BN = B \text{ Sin } \theta$

For ΔOBN,

we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + B \text{ Cos } \theta)^2 + (B \text{ Sin } \theta)^2$

$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2 (\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow \mathbf{R} = \sqrt{\mathbf{A^2 + B^2 + 2AB\ Cos\theta}}$

which is the magnitude of the resultant of *A* and *B*

Direction of resultant vectors:

5) If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \text{ Cos}\theta}$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN, $\tan \alpha = \frac{BN}{ON} = \frac{BN}{OA+AN}$

$\tan \alpha = \left(\frac{B \text{ Sin}\theta}{A+B \text{ Cos}\theta}\right)$; $\alpha = \tan^{-1}\left(\frac{B \text{ Sin}\theta}{A+B \text{ Cos}\theta}\right)$

37. Discuss the properties of scalar and vector products.

Properties of scalar products

1) The product **quantity $\vec{A}\cdot\vec{B}$ is always a scalar**. It is positive if the angle between the vectors is acute (i.e., < 90°) and negative if the angle between them is obtuse (i.e. 90°<θ< 180°).

2) The **scalar product is commutative**, i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

3) The **vectors obey distributive law** i.e. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

4) The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$

5) The **scalar product of two vectors will be maximum** when Cos θ = 1, i.e. $\theta = 0^0$, i.e., when the vectors are parallel; $(\vec{A}\cdot\vec{B})_{max} = AB$

6) The **scalar product of two vectors will be minimum**, when Cos θ = –1, i.e. $\theta = 180^0$ $(\vec{A}\cdot\vec{B})_{min} = -AB$ when the vectors are anti-parallel.

7) If **two vectors $\vec{A}$ and $\vec{B}$, are perpendicular** to each other than their scalar Product $\vec{A}\cdot\vec{B} = 0$, because Cos 90^0 =0. Then the vectors $\vec{A}$ and $\vec{B}$. are said to be mutually orthogonal.

8) The scalar product of a vector with itself is termed as self–dot product and is given by $(\vec{A})^2 = \vec{A}\cdot\vec{A} = AA\cos\theta = A^2$. Here angle θ = 0°

The **magnitude or norm of the vector $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}}$**

9) In case of a unit vector $\hat{n}$, $\hat{n}\cdot\hat{n} = 1 \times 1 \times \cos 0 = 1$.
For example, $\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$

10) In the case of **orthogonal unit vectors** $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$, $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath}$ = 1.1 Cos90^0 = 0

11) In terms of components the scalar product of $\vec{A}$ and $\vec{B}$ can be written
As $\vec{A}\cdot\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$
$= A_xB_x + A_yB_y + A_zB_z$ with all other terms zero.
The magnitude of vector $|\vec{A}|$ is given by $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Properties of vector (cross) product.

2) The **vector product of two vectors is not commutative**, i.e., $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$
But, $\vec{A}\times\vec{B} = -[\vec{B}\times\vec{A}]$. Here it is worthwhile to note that
$|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$.
i.e. in the case of the product vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$, the magnitudes are equal but directions are opposite to each other

3) The **vector product of two vectors will have maximum magnitude** when sin θ = 1, i.e., $\theta = 90^0$ i.e., when the vectors $\vec{A}$ and $\vec{B}$, are orthogonal to each other. $(\vec{A}\times\vec{B})_{max} = AB\hat{n}$

4) The **vector product of two non–zero vectors will be minimum** when sin θ = 0, i.e.,$\theta = 0^0$ *or* 180^0 $[\vec{A}\times\vec{B}]_{min} = 0$ i.e., the vector product of two non–zero vectors vanishes, if the vectors are either parallel or anti-parallel.

5) The self–cross product, i.e., **product of a vector with itself is the null vector $\vec{A} \times \vec{A}$ = AA** Sin θ $\hat{n}$ = $\vec{0}$ In physics the null vector $\vec{0}$ is simply denoted as zero.

6) The self–vector products of unit vectors are thus zero.
$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$

8) In terms of components, the vector product of two vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

$= \vec{\imath}\,(A_yB_z - A_zB_y) + \vec{\jmath}(A_zB_x - A_xB_z) + \vec{k}(A_xB_y - A_yB_x)$

38. Derive the kinematic equations of motion for constant acceleration.

Consider an object moving in a straight line with uniform or constant acceleration 'a'. Let **u be the velocity of the object** at time t = 0, and v be velocity of the body at a later time t.

Velocity - time relation:

1) The acceleration of the body at any instant is given by the **first derivative of the velocity with respect to time**, a = $\frac{dv}{dt}$ or dv = a .dt
Integrating both sides with the condition that as time changes from 0 to t, **the velocity changes from u to v.** For the constant acceleration,
$\int_u^v dv = \int_0^t a dt$
= a $\int_u^v dt \implies [v]_u^v$ = a $[t]_0^t$ --------(1)
v – u = at (or) v = u + at

Displacement – time relation:

2) The velocity of the body is given by **the first derivative of the displacement with respect to time.** v = $\frac{ds}{dt}$ or ds = v dt and since v = u + at We get ds = (u + at)dt . **Assume that initially at time** t = 0, the particle started from the origin. At a later time, t, the particle displacement is s. Further assuming that acceleration is time independent, we have $\int_0^s ds$
$= \int_0^t u dt + \int_0^t a t dt$ or s = ut + ½ at^2 -------------(2)

Velocity – displacement relation:

3) The acceleration is given by the **first derivative of velocity with respect to time.** a = $\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}$ v [since ds / dt = v where s is distance traversed] This is rewritten as a = $\frac{1}{2}\frac{dv^2}{ds}$ or ds = $\frac{1}{2a}$ d(v^2)

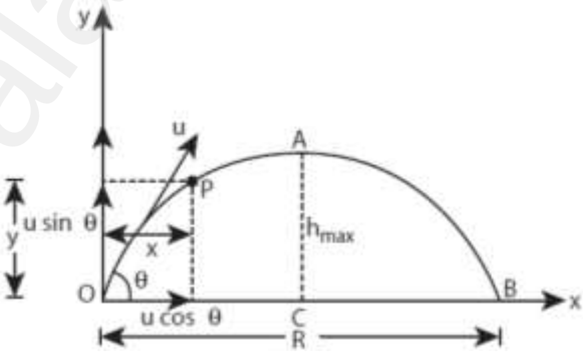
4) Integrating the above equation, using the fact when the velocity changes from u^2 to v^2, displacement changes from 0 to s,
we get $\int_0^s ds$

$= \int_u^v \frac{1}{2a} d(v^2)$; $s = \frac{1}{2a}(v^2 - u^2)$; $v^2 = u^2 + 2as$ -----------(3)

5) We can also derive the displacement s in terms of initial velocity u and final velocity v. From the equation (1) we can write, at = v – u
Substitute this in equation (2), we get s = ut + ½ (v – u)t
$s = \frac{(u+v)t}{2}$ ---------------(4)
The equations (1), (2), (3) and (4) are called kinematic equations of motion, and have a wide variety of practical applications.
Kinematic equations:
$v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$; $s = \frac{(u+v)t}{2}$

39. Derive the equation of motion, range and maximum height reached by the particle thrown at an oblique angle θ with respect to the horizontal direction.

i) Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal. then, $\vec{u} = u_x\hat{i} + u_y\hat{j}$ where $u_x = u \cos\theta$ is the horizontal component and $u_y = u \sin\theta$ the vertical component of velocity.



ii) u_x remains constant throughout the motion. u_y changes with time under the effect of acceleration due to gravity. First it decreases, becomes zero at the maximum height, after which it again increases till the prose reach be ground.

iii) Hence after the time t, the velocity along horizontal motion
$v_x = u_x + a_x t = u_x = u \cos\theta$
The horizontal distance travelled by projectile in time t is
$s_x = u_x t + \frac{1}{2} a_x t^2$. Here, $s_x = x$, $u_x = u \cos\theta$, $a_x = 0$

iv) Thus, x = u cos θ. t **or** $t = \frac{x}{u \cos\theta}$ ------------ (1)
Next, for the vertical motion $v_y = u_y + a_y t$ Here $u_y = u \sin\theta$, $a_y = -g$ (acceleration due to gravity acts opposite to the motion).
Thus, $v_y = u \sin\theta - gt$ ----------------(2)
The vertical distance travelled by the projectile in the same time *t* is
$s_y = u_y t + \frac{1}{2} a_y t^2$ Here, $s_y = y$, $u_y = u \sin\theta$, $a_x = -g$
Then, $y = u \sin\theta\, t - \frac{1}{2} g t^2$ -------------- (3)

v) Substitute the value of *t* from equation (1) in equation (3), we have
$y = u \sin\theta \frac{x}{u \cos\theta} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2\theta}$
$y = x \tan\theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2\theta}$ ----------------(4)
Thus the path followed by the projectile is an inverted parabola.

Maximum height (h_{max})

The **maximum vertical distance travelled by the projectile** during its journey is called maximum height. This is determined as follows:
For the vertical part of the motion, $v^2_y = u^2_y + 2a_y s$
Here, $u_y = u \sin\theta$, $a = -g$, $s = h_{max}$, and at the maximum height $v_y = 0$
Here, $(0)^2 = u^2 \sin^2 = gh_{max}$ (or) $h_{max} = \frac{u^2\sin^2\theta}{2g}$

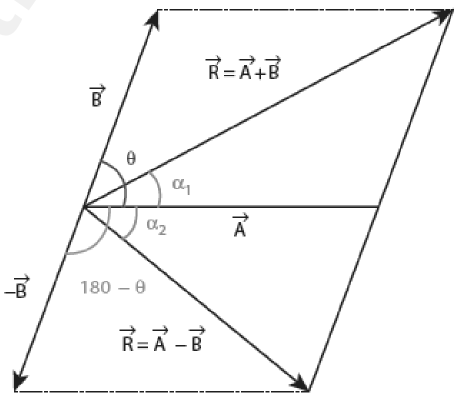
Horizontal range (R)

The **maximum horizontal distance between the point of projection** and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R). This is found easily since the horizontal component of initial velocity remains the same. We can write Range R = Horizontal component of velocity x time of flight = $u \cos\theta \times T_f$

$R = u \cos\theta \times \frac{2u \sin\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$ $R = \frac{u^2 \sin 2\theta}{g}$

40. Explain the subtraction of vectors.

i) For two non-zero vectors $\vec{A}$ and $\vec{B}$ which are inclined to each other at an angle θ, the difference $\vec{A} - \vec{B}$ is obtained as follows. First obtain $-\vec{B}$ as in Figure. The angle between $\vec{A}$ and $-\vec{B}$ is 180 – θ. The difference $\vec{A} - \vec{B}$ is the same as the resultant o $\vec{A}$ and $-\vec{B}$.
We can write $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and using the equation, we have



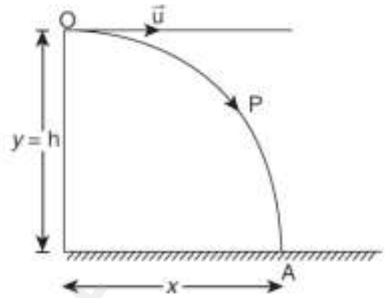
ii) $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$, we have $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$

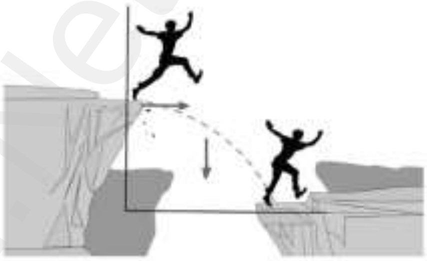
iii) since, cos (180 – θ) = −cosθ. we get,
$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$
Again from the Figure 2.19, and using an equation similar to equation
$\tan\alpha_2 = \frac{B\sin(180-\theta)}{A+B\cos(180-\theta)}$

iv) But sin(180 − θ) = sinθ, hence we get, $\tan\alpha_2 = \frac{B\sin\theta}{A-B\cos\theta}$

41. Find horizontal range and time of flight projectile in horizontal projection.

Consider a projectile, say a ball, thrown horizontally with an initial velocity $\vec{u}$ from the top of a tower of height *h* (Figure)





1. As the **ball moves, it covers a horizontal distance due to its uniform horizontal velocity *u*, and a vertical downward distance** because of constant acceleration due to gravity *g*.

2. Thus, under the combined effect **the ball moves along the path OPA. The motion is in a 2-dimensional plane. Let the ball take time *t* to reach the ground at point A,**

Then the horizontal distance travelled by the ball is *x* (t) = *x*, and the vertical distance travelled is y (t) = y

Motion along horizontal direction:

3. The particle has zero acceleration along *x* direction. So, the initial velocity u_x remains constant throughout the motion. The distance traveled by the projectile at a time *t* is given by the equation
$x = u_x t + \frac{1}{2} at^2$ Since a = 0 along *x* direction, we have
$x = u_x t$ ------------------(1)

Motion along downward direction:

4) Here $u_y = 0$ (initial velocity has no downward component), a = g (we choose the +ve y-axis in downward direction), and distance y at time t From equation, $y = u_y t + \frac{1}{2} at^2$, we get $y = \frac{1}{2} gt^2$ ---------(2)
Substituting the value of t from equation (1) in equation (2) we have

$$y = \frac{1}{2} g \frac{x^2}{u_x^2} = \left(\frac{g}{2u_x^2}\right) x^2$$

$y = Kx^2$ ------------- (3), where $K = \frac{g}{2u_x^2}$

5) Equation is the equation of a parabola. Thus, the path followed by the projectile is a parabola (curve OPA in the Figure)

Time of Flight:

h be the height of a tower. Let T be the time taken by the projectile to hit the ground, after being thrown horizontally from the tower. $\mathbf{s_y = u_y t + \frac{1}{2} at^2}$ $\mathbf{s_y = h, t = T, u_y = 0}$ (i.e no initial vertical velocity)

$$T = \sqrt{\frac{2h}{g}}$$

6) Thus, the **time of flight for projectile motion depends on the height of the tower,** but is independent of the horizontal velocity of projection.

Horizontal range:

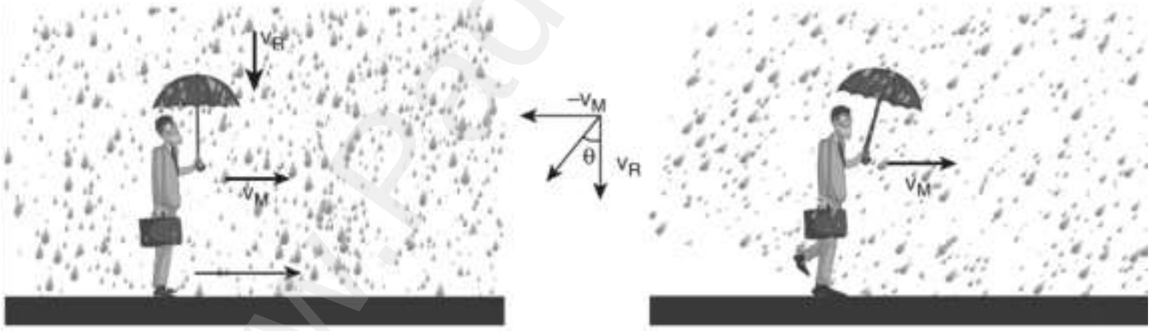
The **horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground** is called horizontal range. For horizontal motion, we have $s_x = u_x t + \frac{1}{2} at^2$. Here, s_x = R (range), $u_x = u$, a = 0 (no horizontal acceleration) T is time of flight. Then horizontal range **=** uT.

7) Since the time of flight $T = \sqrt{\frac{2h}{g}}$, we substitute this and we get the horizontal range of the particle as $R = u\sqrt{\frac{2h}{g}}$.

8) The above equation implies that the range R is directly proportional to the initial velocity u and inversely proportional to acceleration due to gravity g.

42. A man moving in rain holds an umbrella inclined to the vertical though the rain drops are falling vertically. Why?

1. Consider a person moving horizontally with velocity $\vec{V}_M$. Let rain fall vertically with velocity $\vec{V}_R$.
2. An umbrella is held to avoid the rain. Then the relative velocity of the rain with respect to the person is, $\vec{V}_{RM} = \vec{V}_R - \vec{V}_M = \vec{OB} + \vec{OC} + \vec{OD}$ which has magnitude $V_{RM} = \sqrt{v_R^2 + v_M^2}$ $\tan\theta = \frac{DB}{OB} = \frac{v_M}{v_R}$ and direction $\theta = \tan^{-1}\left(\frac{v_M}{v_R}\right)$ with the vertical as shown in Figure.
4. In order to save himself from the rain, he should hold an umbrella at an angle θ with the vertical.



UNIT – III (LAWS OF MOTION)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.

This **inability of objects to move on its own** or change its state of motion is called inertia. Inertia means resistance to change its state.

Examples:

Inertia of Rest:

i. Passengers experience a **backward push in a sudden start of bus.**

ii. Tightening of seat belts in a **car when it stops quickly**.

Inertia of Motion:

i. Passengers experience a forward push during a **sudden brake in bus.**

ii. Ripe fruits fall from the trees in the **direction of wind**.

Inertia of Direction:

i. A **stone moves tangential to Circle**.

ii. When a **car moves towards** left, we turn to the right.

2. State Newton's second law.

The force acting on an object is equal to **the rate of change of its momentum**. $\vec{F} = \frac{d\vec{p}}{dt}$

3. Define One Newton.

One Newton is defined as the **force which acts on 1 kg of mass to give an acceleration** 1 m s^{-2} in the direction of the force.

4. Show that impulse is the change of momentum.

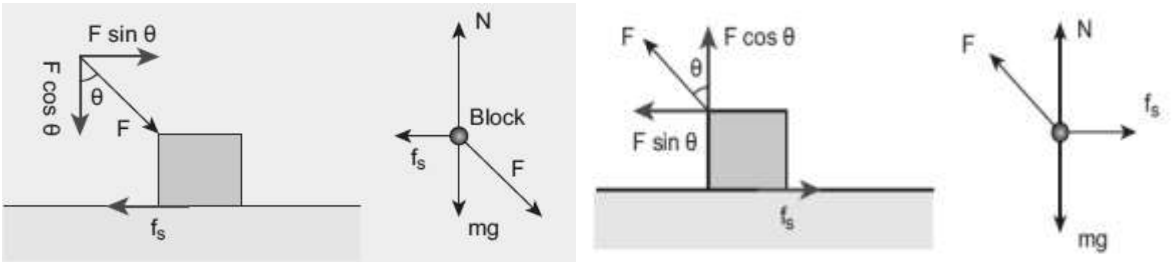
The integral $\int_{t_i}^{t_f} F\,dt = J$ is called the impulse and it is **equal to change in momentum** of the object.

Proof : If the force is constant over the time interval, then

$\int_{t_i}^{t_f} F\,dt = F\int_{t_i}^{t_f} dt = F(t_f - t_i)$

$F\Delta t = \Delta p$ is called the impulse and it is equal to change in momentum of the object.

5. Using free body diagram, show that it is easy to pull an object than to push it.



6. Explain various types of friction. Suggest a few methods to reduce friction.

Static Friction ($\vec{f_s}$):

1. Static friction is the **force which opposes the initiation of motion** of an object on the surface.
2. When the object is at **rest on the surface, only two forces act on** it. They are the **downward gravitational force and upward normal force**.
3. The resultant of these two forces on the object is zero.

Kinetic Friction ($\vec{f_k}$):

1. When an object slides, the **surface exerts a frictional force** called kinetic friction *(fk)*
2. If the external force acting on the object is **greater than maximum static friction**, the objects begin to slide.
3. The kinetic friction **does not depend on velocity**.

Rolling Friction:

The force of friction that comes into act when a wheel rolls over a surface. Methods to reduce friction:

1. By using Lubricants friction
2. By using ball bearings.

7. What is the meaning by 'pseudo force'?

Centrifugal force is called as a 'pseudo force'. A pseudo force has **no origin. A pseudo force is an apparent force** that acts on all masses whose motion is described using non inertial frame of reference such as a rotating reference frame.

8. State Newton's Third law.

For **every action** there is an **equal and opposite reaction**.

9. Under what condition will a car skid on a leveled circular road?

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid. $\mu < \frac{v^2}{rg}$ (skid)

10. Define impulse.

If a **very large force acts on an object for a very short duration**, then the force is called impulsive force or impulse.

11. State Newton's First law.

Every object continues to be in the **state of rest or of uniform motion** unless there is external force acting on it.

12. Define Inertia of rest, motion and direction.

The inability of an **object to change its state of rest** is called **inertia of rest.**
The inability of an **object to change its direction of motion** on its own is called **inertia of direction.**
The inability of an **object to change its state of uniform speed** on its own is called **inertia of motion.**

13. What is free body diagram? What are the steps to be followed for developing free body diagram?

Free body diagram is a **simple tool to analyze the motion of the object** using **Newton's laws**. The following systematic steps are followed for developing the free body diagram:

1. **Identify the forces** acting on the object.
2. **Represent the object** as a point.
3. **Draw the vectors** representing the forces acting on the object.

14. State Lami's theorem.

The magnitude of each **force of the system** is **proportional to sine of the angle between the other two forces**. The constant of proportionality is same for all three forces.

15. State the law of conservation of total linear momentum.

If there are **no external forces acting on the system**, then the **total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector.**
In other words, the total linear momentum of the system is conserved in time.

16. What is the role of air bag in a car?

cars are designed with air bags in such a way that when the car meets with an accident, the **momentum of the passengers will reduce slowly** so that **the average force acting on them will be smaller.**

17. Define frictional force.

Which always opposes the relative motion between an object and the surface where it is placed. If the **force applied is increased, the object moves** after a certain limit.

18. Define Angle of Friction.

The angle of friction is defined as the **angle between the normal force (N) and the resultant force (R)** of normal force and maximum friction force f_s^{max})

19. Define Angle of repose.

The same as angle of friction. But the difference is that the angle of repose refers to **inclined surfaces and the angle of friction is applicable** to any **type of surface.**

20. What are the applications of angle of repose?

1. The angle of inclination of **sand trap** is made to be **equal to angle of repose.**
2. Children are fond of **playing on sliding board**. Sliding will be easier when the **angle of inclination of the board is greater** than the **angle of repose.**

21. How does the rolling wheel's work in suitcase?

1. In **rolling motion** when a **wheel moves on a surface**, the point of **contact with surface is always at rest.**
2. Since the point of **contact is at rest**, there is **no relative motion** between the **wheel and surface**. Hence the **frictional force is very less.**

22. Where does the friction force act?

Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of **the frictional force between the tyre and the road**. In the braking system, kinetic friction plays a major role.

23. How did the ball bearing reduce kinetic friction?

If ball bearings are **fixed between two surfaces**, during the **relative motion only the rolling friction comes to effect and not kinetic friction.**

24. Define Centripetal force.

If a particle is in **uniform circular motion**, there must be **centripetal acceleration towards the center of the circle**. If there is acceleration, then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

25. How is the centripetal force act in whirling motion?

In the case of **whirling motion of a stone tied to a string**, the centripetal force on the particle is provided by the **tensional force on the string.** In circular motion in an **amusement park, the centripetal force is provided by the tension in the iron ropes.**

26. How did the car move on circular track?

When a car is moving on a circular track the centripetal force is given by the **frictional force between the road and the tyres**.
Frictional force = mv^2/r m-mass of the car, v-speed of the car r-radius of curvature of track, even when the **car moves on a curved track**, the car experiences the **centripetal force which is provided by frictional force between the surface and the tyre of the car**.

CONCEPTUAL QUESTIONS:

27. Why it is not possible to push a car from inside?

1. When you push on the car from inside, the reaction force of your pushing is **balanced out by our body moving backward.**
2. The seat behind you **pushes against to bring things to static equilibrium.** So, we can't push a car from inside.

28. There is a limit beyond which the polishing of a surface increase frictional resistance rather than decreasing it why?

1. Friction arises due to molecular adhesion. For more polishing the **molecules of the surface come closer.**
2. They offer greater resistance to surface.

29. Why does a parachute descend slowly?

A parachute is a device used **to slow down on object that is falling towards the ground**. When the parachute opens, the **air resistance increases**. So the person can land safely.

30. When walking on ice one should take short steps. Why?

To **avoid slipping**, take smaller steps. Because these steps causes more normal force and there by **more friction.**

31. Why is it dangerous to stand near the open door of moving bus?

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

32. When a cricket player catches the ball, he/she pulls his /her hands gradually in the direction of the ball's motion. Why?

1. If he stops his hands soon after catching the ball, the ball comes to rest very quickly.
2. It means that the momentum of the ball is brought to rest very quickly.
3. So the average force acting on the body will be very large.
4. Due to this large average force, the hands will get hurt.
5. To avoid getting hurt, the player brings the ball to rest slowly

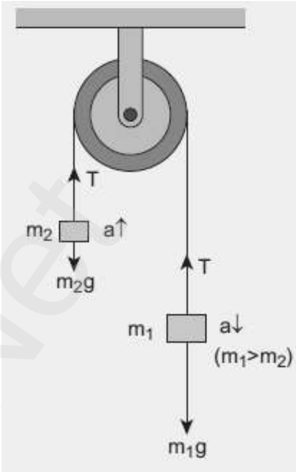
33. A man jumping on concrete floor is more dangerous than in sand floor, why?

1. Jumping on a concrete cemented floor is more dangerous than jumping on the sand.
2. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

FIVE MARKS QUESTION WITH ANSWER QUESTIONS:

34. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.

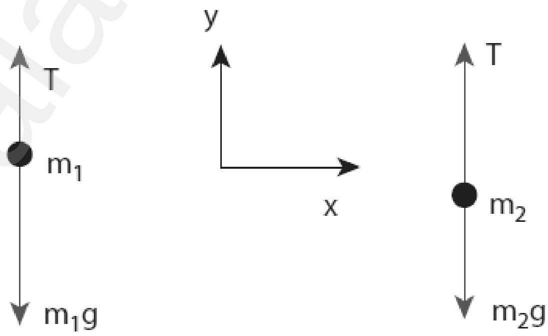
Case 1: Vertical motion:



i) Consider **two blocks of masses m_1 and m_2 ($m_1 > m_2$)** connected by a light and inextensible string that passes over a pulley.

ii) Let the tension in the string be T and acceleration *a*. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration. The gravitational force m_1g on mass m_1 is used in lifting the mass m_2.
Applying Newton's second law for mass m_2,
$T\hat{j} - m_2g\hat{j} = m_2a\hat{j}$

iii) The left-hand side of the above equation is the total force that acts on m_2 and the right-hand side is the product of mass and acceleration of m_2 in y direction.
By comparing the components on both sides, we get



$\mathbf{T - m_1g = m_2a}$ ----------- (1)
Similarly, applying Newton's second law for mass m_1
$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j}$
As mass m1 moves downward $(-\hat{j})$, its acceleration is along$(-\hat{j})$

iv) By comparing the components on both sides, we get
$\mathbf{T - m_1g = -m_1a}$; $\mathbf{m_1g - T = m_1a}$ ----------- (2)
Adding equations (1) and (2), we get
$\mathbf{m_1g - m_2g = m_1a + m_2a}$; $\mathbf{(m_1 - m_2)g = (m_1 + m_2)a}$ ------ (3)
From equation (3), the acceleration of both the masses is
$a = \left(\frac{m_1-m_2}{m_1+m_2}\right) g$ -----(4)
If both the masses are equal ($m_1=m_2$), from equation (4) a = 0

v) This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest. To find the tension acting on the string, substitute the acceleration from the equation (4) into the equation (1).

$T - m_2g = m_2\left(\frac{m_1-m_2}{m_1+m_2}\right)$; $T - m_2g + m_2\left(\frac{m_1-m_2}{m_1+m_2}\right)g$ --- (5)

By taking m2g common in the RHS of equation (5)

$T = m_2g\left(1 + \frac{m_1-m_2}{m_1+m_2}\right)$;

$T = m_2g\left(\frac{m_1+m_2+m_1-m_2}{m_1+m_2}\right)$ $\mathbf{T = \left(\frac{2m_1m_2}{m_1+m_2}\right)g}$

Case 2: Horizontal motion:

i) In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley. Assume that there is no friction on the surface.

ii) As both the blocks are connected to the un-stretchable string, if m_1 moves with an acceleration *a* downward then m_2 also moves with the same acceleration *a* horizontally.

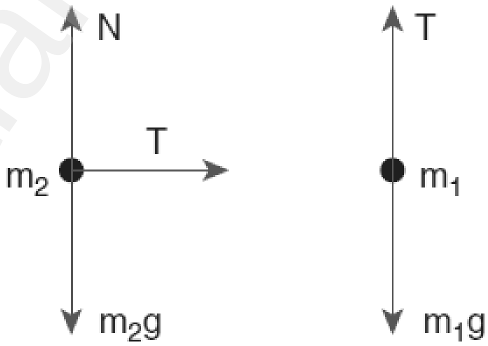
The forces acting on mass m_2 are

(i) Downward gravitational force (m_2g)
(ii) Upward normal force (N) exerted by the surface
(iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are

(i) Downward gravitational force (m_1g)
(ii) Tension (T) acting upwards

The free body diagrams for both the masses



Applying Newton's second law for m_1

$T\hat{j} - m_1g\hat{j} = m_1a\hat{j}$

By comparing the components on both sides of the above equation,

$$T - m_1g = -m_1a \text{ ------------ } (1)$$

Applying Newton's second law for m_2

$T\hat{i} - m_2a\hat{i}$, By comparing the components on both sides of above equation,

$$T = m_2a \text{ ------------ } (2)$$

There is no acceleration along y direction for m_2.

$N\hat{j} - m_2g\hat{j} = 0$, By comparing the components on both sides of the above equation

$$N - m_2g = 0\ ;\ N = m_2g \text{ ------------ } (3)$$

By substituting equation (2) in equation (1), we can find the tension T

$m_2a - m_1g = -m_1a$; $m_2a + m_1a = m_1g$; $a = \frac{m_1}{m_1+m_2}g$ ----------(4)

Tension in the string can be obtained by substituting equation (4) in equation (2)

$$T = \frac{m_1m_2}{m_1+m_2}g$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

35. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.

i) If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move.

ii) It is because of **the opposing force exerted by the surface on the object which resists its motion.**

iii) **This force is called the frictional force which always opposes the relative motion between an object and the surface** where it is placed.

iv) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ . For small angles of θ , the object may not slide down.

v) As **θ is increased, for a particular value of θ , the object begins to slide down.** This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

vi) Consider the various forces in action here. The **gravitational force mg is resolved into components parallel (mg sin θ) and perpendicular (mg cos θ) to the inclined plane.**

vii) The **component of force parallel to the inclined plane (mg sin θ) tries to move the object down**. The component of force perpendicular to the inclined plane (mg cos θ) is balanced by the Normal force (N).

$$N = mg\cos\theta \quad \text{-------------(1)}$$

When the object just begins to move, the static friction attains its maximum value,

$f_s = f_s^{max} = \mu_s$ N. This friction also satisfies the relation

$$f_s^{max} = \mu_s\, mg\sin\theta \quad \text{------------ (2)}$$

Equating the right hand side of equations (1) and (2), we get

$$(f_s^{max}) / N = \sin\theta / \cos\theta$$

From the definition of angle of friction, we also know that **$\tan\theta = \mu_s$** in which **θ** is the angle of friction.

36. State Newton's three laws and discuss their significance.

Newton's First Law:

i) Every **object continues to be in the state of rest or of uniform motion** (constant velocity) unless there is external force acting on it.

ii) This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Newton's Second Law:

i) The force acting on an object is equal to **the rate of change of its momentum** $\vec{F} = \frac{d\vec{p}}{dt}$

ii) In simple words, whenever **the momentum of the body changes, there must be a force acting on it. The momentum of the object** is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form $\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$.

$\vec{F} = m\vec{a}$

Newton's Third law:

i) Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature.

ii) Newton's third law states that **for every action there is an equal and opposite reaction.**

iii) Here, **action and reaction pair of forces do not act on the same body but on two different bodies.**

iv) Any **one of the forces** can be called as an **action force** and the **other** the **reaction force**. Newton's third law is valid in both inertial and non-inertial frames.

v) These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

37. Explain the similarities and differences of centripetal and centrifugal forces.

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is **a pseudo force or fictitious force** which cannot arise from gravitational force, tension force, normal force etc.
Acts in **both inertial and non-inertial frames**	**Acts only in rotating frames** (non-inertial frame)
It **acts** towards the **axis of rotation** or center of the circle in circular motion	It **acts outwards from the axis of rotation** or radially outwards from the center of the circular motion
$\lvert F_{cp} \rvert = m\omega^2 r = \frac{mv^2}{r}$	$\lvert F_{cf} \rvert = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects.	**Pseudo force but has real effects**
Origin of **centripetal force is interaction between two objects**	Origin of **centrifugal force** is inertia. It does not arise from interaction.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame. In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

38. Briefly explain 'Rolling Friction'.

i) One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage.

ii) When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion.

iii) **In rolling motion** when a wheel moves on a surface, **the point of contact with surface is always at rest.**

iv) **Since the point of contact is at rest**, there is no relative motion between the wheel and surface. Hence the **frictional force is very less**. At the same time if an object moves **without a wheel**, there is a relative motion between the object and the surface.

v) As a result **frictional force is larger**. This makes it **difficult to move the object**.

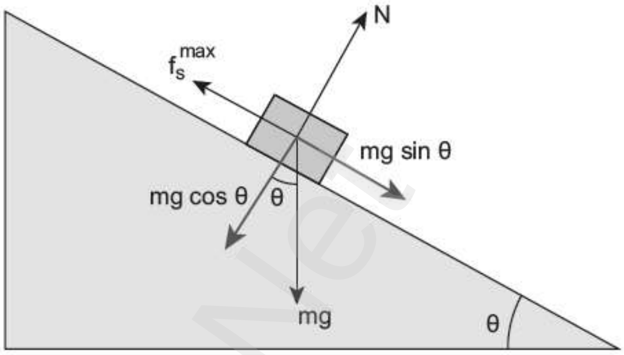
vi) Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so.

vii) Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface.

viii) **Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'**. In fact, 'rolling friction' is much smaller than kinetic friction.

39. Describe the method of measuring Angle of Repose.

i) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ . For small angles of θ,the object may not slide down.



ii) As θ is increased, for a particular value of θ , the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

iii) Consider the various forces in action here. **The gravitational force mg is resolved into components parallel (mg sin θ) and perpendicular (mg cos θ) to the inclined plane.**

iv) The component of force parallel to the inclined plane (mg sin θ) tries to move the object down. **The component of force perpendicular to the inclined plane (mg cos θ)** is balanced by the Normal force (N).

$N = mg\cos\theta$ ------------(1)

When the object just begins to move, the static friction attains its maximum value,

$f_s = f_s^{max} = \mu_s N$. This friction also satisfies the relation

$f_s^{max} = \mu_s\, mg \sin\theta$ ------------ (2)

Equating the right hand side of equations (1) and (2), we get

$(f_s^{max}) / N = \sin\theta / \cos\theta$

From the definition of angle of friction, we also know that **$\tan\theta = \mu_s$** in which **θ** is the angle of friction. **Thus the angle of repose is the same as angle of friction.**

40. Explain the need for banking of tracks.

i) In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s The coefficient of static friction depends on the nature of the surface which has a maximum limiting value.

ii) To avoid this problem, usually the **outer edge of the road is slightly raised compared to inner edge**

iii) This is **called banking of roads or tracks.** This introduces an inclination, and the angle is called banking angle.

iv) Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical.

v) When the car takes a turn, there are **two forces acting on the car**:

a) **Gravitational force mg (downwards)**

b) **Normal force N (perpendicular to surface)**

vi) We can resolve the normal force into two components. N cos θ and N sin θ

vii) The component N cos θ balances the downward gravitational force 'mg' and component N sin θ will provide the necessary centripetal acceleration. By using Newton second law

$N \cos \theta = mg$; $N \sin \theta = \frac{mv^2}{r}$

By dividing the equations we get, $\tan \theta = \frac{v^2}{rg}$

$v = \sqrt{rg \tan \theta}$

Need Banking of tracks:

1) The banking angle θ and radius of **curvature of the road or track determines the Safe speed of the car at the turning. If the speed of car exceeds this safe speed**, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding.

2) At the same time, if **the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding.**

3) However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

41. Write the salient features of Static and Kinetic friction.

Static friction	Kinetic friction
It **opposes the starting of motion**	It **opposes the relative motion** of the object with respect to the surface
Independent of surface of contact	**Independent of surface of contact**
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true <10 ms^{-1})
$f_s^{max} > f_k$	It is less than maximal value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

42. Briefly explain what are all the forces act on a moving vehicle on a leveled circular road?

i) When a **vehicle travels in a curved path**, there must be a **centripetal force acting on it**. This centripetal force is provided by the frictional force between tyre and surface of the road.

ii) Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'.

There are **three forces acting on the vehicle** when it moves

1. **Gravitational force (mg) acting downwards**
2. **Normal force (mg) acting upwards**
3. **Frictional force (F_s) acting horizontally inwards along the road**

iii) Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The **centripetal force is provided by the force of static friction F_s between the tyre and surface** of the road which acts towards the center of the circular track, $\frac{mv^2}{r} = F_s$, the static friction can increase from zero to a maximum value
$F_s \leq \mu_s mg$

iv) The static friction would be able to provide necessary centripetal force to bend the car on the road. So **the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn**. If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid. if $\frac{mv^2}{r} >$ μ_s mg , of $\mu_s < \frac{v^2}{rg}$ (skid)

UNIT – IV (WORK, ENERGY AND POWER)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. Write the differences between conservative and Non-conservative forces. Give two examples each.

Conservative forces	Non-conservative forces
Work done is **independent of the path**	Work done **depends upon the path**
Work done in a **round trip is zero**	Work done in a **round trip is not zero**
Total energy **remains constant**	Energy is **dissipated as heat energy**
Work done is **completely recoverable**	Work done is **not completely recoverable**
Force is the negative gradient of potential energy	No such relation exists.

2. Explain the characteristics of elastic and inelastic collision.

Elastic Collision	Inelastic Collision
Total **momentum** is **conserved**	Total **momentum is conserved**
Total **kinetic energy** is **conserved**	Total **kinetic energy** is **not conserved**
Forces involved are **conservative forces**	Forces involved are **non-conservative Forces**
Mechanical energy is **not dissipated**	**Mechanical energy** is **dissipated** into heat, light, sound etc.

3. Define the following
a) Coefficient of restitution b) Power c) Law of conservation of energy

a) Coefficient of restitution:

It is defined as the ratio of **velocity of separation (relative velocity)** after **collision to the velocity of approach (relative velocity)** before collision,

i.e., $e = \frac{\text{Velocity of separation (after collision)}}{\text{Velocity of approach (before collision)}}\ ;\ \frac{(v_2-v_1)}{(u_1-u_2)}$

b) Power:

The rate of work done or energy delivered.

$$\text{Power (P)} = \frac{\text{Workdone (W)}}{\text{Time taken (t)}}$$

c) Law of conservation of energy:

Energy can neither be created nor destroyed. It may be **transformed** from **one form to another but the total energy** of an isolated system remains constant.

4. Define unit of power:

One watt is defined as **the power when one joule of work is done in one second. 1W = 1Js^{-1}**

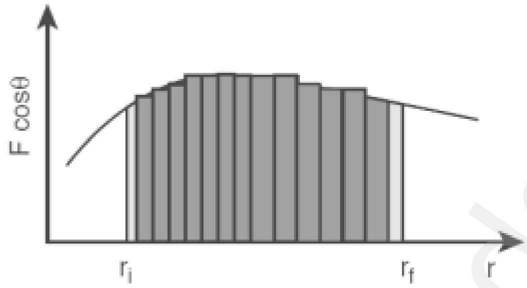
5. Explain Work done.

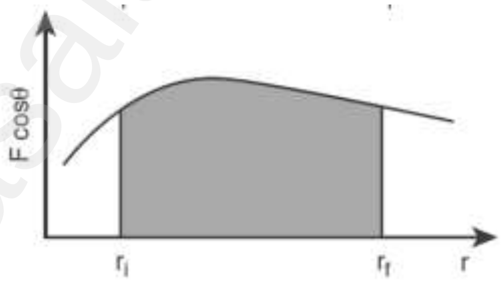
i) Work is said to be done by the **force when the force applied on a body displaces it.**
ii) **work done is a scalar quantity**. It has only magnitude and no direction.
iii) In SI system, **unit of work done is N m (or) joule (J).** Its dimensional formula is **ML^2T^{-2}**

6. Define Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, $dW= (F \cos\theta)\, dr$

7. Give the graphical representations of the work done by a variable force.





8. Define Energy, Kinetic energy and Potential Energy

Energy: The **capacity to do work**, Dimension: ML^2T^{-2} , SI Unit : Nm or joule .

Kinetic energy: The **energy possessed by a body due to its motion**.

Dimension: ML^2T^{-2} , SI Unit : Nm or joule .

Potential Energy: The **energy possessed by the body by virtue of its position**

Dimension: ML^2T^{-2} , SI Unit : Nm or joule .

9. Write the significance of kinetic energy in the work – kinetic energy theorem.

1. If the work done by **the force on the body is positive** then its **kinetic energy increases.**
2. If the work done by **the force on the body is negative** then its **kinetic energy decreases.**
3. If there is **no work done by the force** on the body then there is **no change** in its kinetic energy

10. Define Work – kinetic energy theorem.

The work done by the **force on the body changes the kinetic energy** of the body. This is called work-kinetic energy theorem.

11. Define elastic potential energy

The potential energy possessed by a spring due to a deforming force which stretches or **compresses the spring is termed** as elastic potential energy.

12. Define Conservative force

A force is said to be a conservative force if the work done by or against **the force in moving the body depends only on the initial and final positions of the body** and not on the nature of the path followed between the initial and final positions.

13. Define Non-conservative force

A force is said to be non-conservative if the work done by or against **the force in moving a body depends upon the path between the initial and final positions.** This means that the value of work done is different in different paths.

14. Define Average Power

The average power (P_{av}) is defined as **the ratio of the total work done to the total time taken.** $P_{av} = \frac{\text{Total work done}}{\text{Total time taken}}$

15. Define Instantaneous power

The instantaneous power (P_{inst}) is defined as **the power delivered at an instant** (as time interval approaches zero), $P_{inst} = \frac{dw}{dt}$

16. What is meant by collision?

Collision is a **common phenomenon that happens around us every now** and then. For example, **carom, billiards, marbles, etc.,. Collisions can happen between two bodies with or without physical contacts.**

17. What is Elastic Collision?

In a collision, **the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies** (after collision) then, it is called as elastic collision. i.e., **Total kinetic energy before collision = Total kinetic energy after collision**

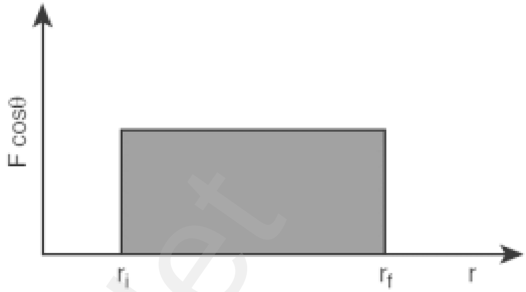
18. What is Inelastic Collision?

In a collision, the **total initial kinetic energy of the bodies** (before collision) is **not equal to the total final kinetic energy of the bodies** (after collision) then, it is called as inelastic collision. i.e., **Total kinetic energy before collision ≠ Total kinetic energy after collision**

FIVE MARKS QUESTION WITH ANSWER QUESTIONS

19. Explain with graphs the difference between work done by a constant force and by a variable force.

i) When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, dW= (F cosθ) dr



ii) The total work done in producing a displacement from initial position r_i to final position r_f is, $W = \int_{r_i}^{r_f} dW$;

$W = \int_{r_i}^{r_f} (F\cos\theta)dr = (F\cos\theta)$;

$\int_{r_i}^{r_f} dr = (F\cos\theta)(r_f - r_i)$

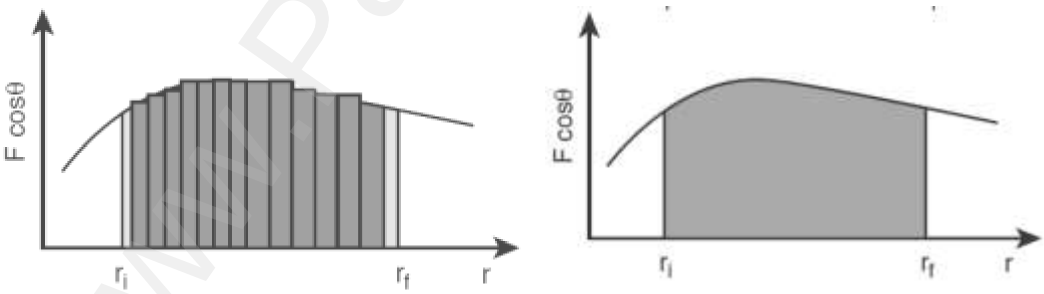
iii) The graphical representation of the work done by a constant force. The area under the graph shows the work done by the constant force.

Work done by a variable force:

i) When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation dW = (Fcos θ) dr
[F cos θ is the component of the variable force F] where, F and θ are variables.

ii) The total work done for a displacement from initial position r_i to final position r_f is given by the relation, $W = \int_{r_i}^{r_f} dW$; $= \int_{r_i}^{r_f} (F\cos\theta)dr$

iii) A graphical representation of the work done by a variable force. The area under the graph is the work done by the variable force.



20. State and explain work energy principle. Mention any three examples for it.

1) It states that work done by the force acting on a body is equal to the change produced in the kinetic energy of the body.
2) Consider a body of mass m at rest on a frictionless horizontal surface.
3) The work (W) done by the constant force (F) for a displacement (s) in the same direction is, W = Fs ------------ (1)

The constant force is given by the equation, F = ma --------- (2)

The **third equation of motion** can be written as, $v^2 = u^2 + 2as$

$a = \frac{v^2 - u^2}{2s}$ ----------------- (3)

Substituting for a in equation (2), $F = m\left(\frac{v^2-u^2}{2s}\right)$ ------------ (4)

Substituting equation (4) in (1), $W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$

$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ ------------------ (5)

The expression for kinetic energy:

i) The term $\frac{1}{2}(mv^2)$ in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v). **$KE = \frac{1}{2} mv^2$** ---------------- (6)

ii) Kinetic energy of the body is always positive.

From equations (5) and (6)

$\Delta KE = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ ---------------(7) thus, $W = \Delta KE$

iii) The expression on the right hand side (RHS) of equation (7) is the change in kinetic energy (ΔKE) of the body.

iv) This implies that **the work done by the force on the body changes the kinetic energy of the body**. This is called work-kinetic energy theorem.

21. Deduce the relation between momentum and kinetic energy.

i) Consider an object of mass m moving with a velocity $\vec{v}$. Then its linear momentum is $\vec{p} = m\vec{v}$ and its kinetic energy, $KE = \frac{1}{2} mv^2$

$KE = \frac{1}{2} mv^2$; $= \frac{1}{2} m(\vec{v}.\vec{v})$ -------- (1)

ii) Multiplying both the numerator and denominator of equation (1) by mass, m $KE = \frac{1}{2}\frac{m^2(\vec{v}.\vec{v})}{m}$;

$= \frac{1}{2}\frac{(m\vec{v}).(m\vec{v})}{m}$ $[\vec{p} = m\vec{v}]$; $= \frac{1}{2}\frac{(\vec{p}).(\vec{p})}{m}$

$= \frac{\vec{p}^2}{2m}$; $KE = \frac{p^2}{2m}$

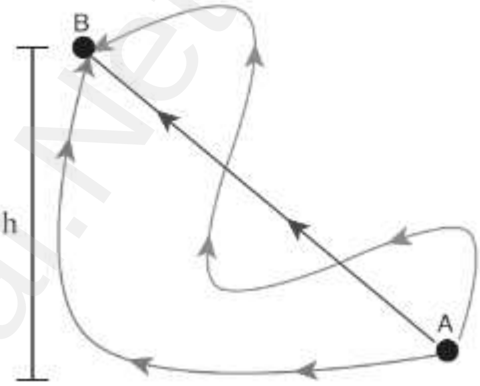
iii) Where $|\vec{p}|$ is the magnitude of the momentum. The magnitude of the linear momentum can be obtained by $|\vec{p}| = p = \sqrt{2m(KE)}$

iv) Note that if kinetic energy and mass are given, only the magnitude of the momentum can be calculated but not the direction of momentum. It is because the kinetic energy and mass are scalars.

22. What is conservative force? State how it is determined from potential energy?

i) A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

ii) Consider an object at point A on the Earth. It can be taken to another point B at a height h above the surface of the Earth by three paths.



iii) Whatever may be the path, the work done against the gravitational force is the same as long as the initial and final positions are the same.

iv) This is the reason why gravitational force is a conservative force.

v) Conservative force is equal to the negative gradient of the potential energy. In one dimensional case, $F_x = -\frac{dU}{dx}$

vi) Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

23. Derive an expression for the potential energy of a body near the surface of the Earth.

1) The gravitational potential energy (U) at some height is equal to the amount of work required to take the object from ground to that height with constant velocity.

2) Consider a body of mass being moved from ground to the height h against the gravitational force.

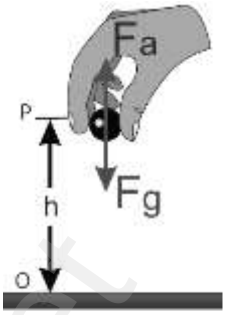
3) The gravitational force $\vec{F}_g$ acting on the body is, $\vec{F}_g = -mg\hat{j}$ (as the force is in y direction; unit vector is used). Here, negative sign implies that the force is acting vertically downwards. In order to move the body without acceleration (or with constant velocity), an external applied force $\vec{F}_a$ equal in magnitude but opposite to that of gravitational force $\vec{F}_g$ has to be applied on the body

i.e., $\vec{F}_a - \vec{F}_g$ This implies that $\vec{F}_a = -mg\hat{j}$

4) The positive sign implies that the applied force is in vertically upward direction. Hence, when the body is lifted up its velocity remains unchanged and thus its kinetic energy also remains constant.

5) The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from the ground to that height h. U = $\int \vec{F}_a . d\vec{r}$

= $\int_0^h |\vec{F}_a| \, |d\vec{r}| \cos\theta$



6) Since the displacement and the applied force are in the same upward direction, the angle between them, θ = 0 .

Hence Cos 0 = 1 and $|\vec{F}_a|$ = mg and $|d\vec{r}|$ = dr

U = mg $\int_0^h dr$; U = mg $[r]_0^h$; **U = mgh**

UNIT – V (MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

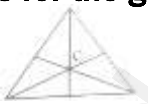
1. Define center of mass.

A point where the entire mass of the body appears to be concentrated.

2. Find out the center of mass for the given geometrical structures.

a) Equilateral triangle

Lies in center



b) Cylinder

Lies on its central axis

c) Square

Lies at their diagonals meet

3. Define torque and mention its unit.

Torque is defined as the **moment of the external applied force about a point or axis of rotation**. The expression for torque is, $\vec{\tau} = \vec{r} \times \vec{F}$. Its unit is Nm.

4. What are the conditions in which force cannot produce torque?

The torque is zero when $\vec{r}$ and $\vec{F}$ are parallel or anti-parallel. If parallel, then θ=0 and sin 0 =0. If **anti-parallel, then θ=180 and sin 180=0**. Hence, τ = 0. The torque is zero if the force acts at the reference point. i.e. as $\vec{r}$ = 0, τ = 0.

5. What is the relation between torque and angular momentum?

An external torque on a rigid body fixed to an axis produces **rate of change of angular momentum** in the body about that axis. T = $\frac{dL}{dt}$

6. What is equilibrium?

i) A rigid body is said to be in **mechanical equilibrium when both its linear momentum and angular momentum remain constant**.

ii) When all the forces act upon the object are balanced, then the object is said to be an equilibrium.

7. Give any two examples of torque in day-to-day life.

i) **Opening and closing of a door** about the hinges

ii) **Turning of a nut using a wrench**

iii) **Opening a bottle cap** (or) water top

8. How do you distinguish between stable and unstable equilibrium?

Stable equilibrium	Unstable equilibrium
Linear momentum and angular **momentum are zero.**	Linear momentum and angular **momentum are zero.**
The **body tries to come back to equilibrium** if slightly disturbed and released.	The **body cannot come back to equilibrium** if slightly disturbed and released.
The **center of mass of the body shifts slightly higher** if disturbed from equilibrium.	The **center of mass of the body shifts slightly lower** if disturbed from equilibrium.
Potential energy of the body is minimum and it **increases** if disturbed.	**Potential energy of the body is not minimum** and it **decreases** if disturbed

9. Define couple.

Pair of forces which are equal in magnitude but **opposite in direction** and separated by a **perpendicular distance** so that **their lines of action do not coincide** that causes a turning effect is called a couple

10. State principle of moments.

When an object is in equilibrium the sum of the anticlockwise moments about a turning point must be equal to the sum of the clockwise moments.

11. Define center of gravity.

The point at which the entire weight of the body acts irrespective of the position and orientation of the body.

12. Mention any two physical significance of moment of inertia.

i) For rotational motion, moment of inertia is a measure of rotational inertia.

ii) The moment of inertia of a body is not an invariable quantity. It depends not only on the mass of the body, but also on the way the mass is distributed around the axis of rotation.

13. What is radius of gyration?

The radius of gyration of an object is **the perpendicular distance from the axis of rotation to an equivalent point mass**, which would have the same mass as well as the same moment of inertia of the object.

14. State conservation of angular momentum.

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

15. What are the rotational equivalents for the physical quantities, (i) mass and (ii) force?

i) For mass : Moment of inertia , $I = mr^2$ ii) For Force : Torque $\tau = I\,\alpha$

16. What is the condition for pure rolling?

(i) The combination of translational motion and rotational motion about the center of mass. (or)

(ii) The momentary rotational motion about the point of contact.

17. What is the difference between sliding and slipping?

Sliding is the case when $v_{CM} > R\omega$ (or $v_{TRANS} > v_{ROT}$). The translation is more than the rotation.

Slipping is the case when $v_{CM} < R\omega$ (or $v_{TRANS} < v_{ROT}$). The rotation is more than the translation.

18. State the rule which is used to find the direction of torque.

The direction of torque is found using right hand rule. This rule says that if **fingers of right hand are kept along the position vector** with palm facing the **direction of the force and when the fingers are curled** the thumb points to the direction of the torque.

19. When will a body have a precession?

The torque about the axis will rotate the object about it and **the torque perpendicular to the axis will turn the axis of rotation.** When both exist simultaneously on a rigid body, the body will have a precession.

20. State Parallel axis theorem

The moment of inertia of a body about any axis is equal **to the sum of its moment of inertia** about a parallel axis through its center of mass and the product of the mass of the body and the **square of the perpendicular distance between the two axes.** $I = I_C + MR^2$

21. State Perpendicular axis theorem.

The moment of inertia of a plane laminar body about an axis perpendicular to its **plane is equal to the sum of moments of inertia** about **two perpendicular axes lying in the plane of the body** such that all the three axes are mutually perpendicular and have a common point. $I_Z = I_X + I_Y$

22. Give the scalar relation between torque and angular acceleration.

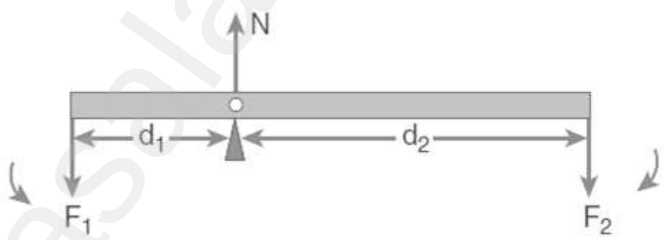
The scalar relation between the **torque and angular acceleration** is $\tau = I\alpha$ I = Moment of inertia of the rigid body. The torque in rotational motion is equivalent to the force in linear motion.

23. What are the conditions for neutral equilibrium?

1) Linear momentum and angular momentum are zero.
2) The body remains at the same equilibrium if slightly disturbed and released.
3) The center of mass of the body does not shift higher or lower if disturbed from equilibrium.
4) Potential energy remains same even if disturbed.

24. Explain the principle of moments.

1) Consider a light rod of negligible mass which is pivoted at a point along its length. Let two parallel forces F_1 and F_2 act at the two ends at distances d_1 and d_2 from the Point of pivot and the normal reaction force N at the point of pivot as shown in Figure.



2) If the rod has to remain stationary in horizontal position, it should be in translational and rotational equilibrium. Then, both the net force and net torque must be zero.

For net force to be zero, $-F_1 + N - F_2 = 0$

$\mathbf{N = F_1 + F_2}$

For net torque to be zero, $d_1 F_1 - d_2F_2 = 0$

$\mathbf{d_1 F_1 = d_2F_2}$

The above equation represents the **principle of moments.**

25. Write the principles used in beam balance and define Mechanical Advantage.

i) This forms the principle for beam balance used for weighing goods with the condition $\mathbf{d_1 = d_2; F_1 = F_2}$.

$\frac{F_1}{F_2} = \frac{d_2}{d_1}$

ii) If F_1 is the load and F_2 is our effort, we get advantage when, $d_1 < d_2$. This implies that $F_1 > F_2$. Hence, we could lift a large load with small effort. The ratio $\left(\frac{d_2}{d_1}\right)$ **is called mechanical advantage** of the simple lever. The pivoted point is called fulcrum.

Mechanical Advantage (MA) = $\frac{d_2}{d_1}$

26. Find the expression for radius of gyration.

1) A rotating rigid body with respect to any axis, is considered to be made up of point masses $m_1, m_2, m_3, \ldots m_n$ at perpendicular distances (or positions) $r_1, r_2, r_3 \ldots r_n$ respectively

2) The moment of inertia of that object can be written as,

$I = \Sigma m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$

If we take all the n number of individual masses to be equal,

$m = m_1 = m_2 = m_3 = \ldots\ldots m_n$ then

$I = mr_1^2 + mr_2^2 + mr_3^2 + \ldots mr_n^2$

$\mathbf{I = m(r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2)}$

$= nm\left(\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}\right)$

$I = MK^2$, where, nm is the total mass M of the body and K is the radius of gyration.

$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}}$

3) The expression for radius of gyration indicates that it is the root mean square (rms) distance of the particles of the body from the axis of rotation.

CONCEPTUAL QUESTIONS:

27. When a tree is cut, the cut is made on the side facing the direction in which the tree is required to fall. Why?

The weight of tree exerts a torque about the point where the cut is made. This cause rotation of the tree about the cut.

28. Why does a porter bend forward while carrying a sack of rice on his back?

Due to the added weight of rice sack, centre of gravity of the combined body weight and the carrying weight shifted to new position. Once he bends, the centre of gravity realigns as with the body's axis making his body balanced.

29. Why is it much easier to balance a meter scale on your finger tip than balancing on a Match stick?

Meter scale is longer and larger than a match stick. Meter scale's centre of gravity is higher but match stick has centre of gravity much lower as compared to scale. Higher the centre of gravity easier it is to balance.

FIVE MARKS QUESTION WITH ANSWER QUESTIONS

30. Explain the types of equilibrium with suitable examples

Translational equilibrium:

1) Linear momentum is constant 2) Net force is zero

Rotational equilibrium:

1) Angular momentum is constant 2) Net torque is zero

Static equilibrium:

1) Linear momentum and angular momentum are zero
2) Net force and net torque are zero

Dynamic equilibrium:

1) Linear momentum and angular momentum are constant
2) Net force and net torque are zero

Stable equilibrium:

1) Linear momentum and angular momentum are zero
2) The **body tries to come back to equilibrium if slightly disturbed** and released
3) The center of mass of the **body shifts slightly higher if disturbed** from equilibrium
4) Potential energy of the **body is minimum and it increases** if disturbed

Unstable equilibrium:

1) Linear momentum and angular momentum are zero
2) **The body cannot come back to equilibrium if slightly disturbed** and released
3) The **center of mass of the body shifts slightly lower if disturbed** from equilibrium
4) Potential energy of the body is **not minimum and it decreases** if disturbed

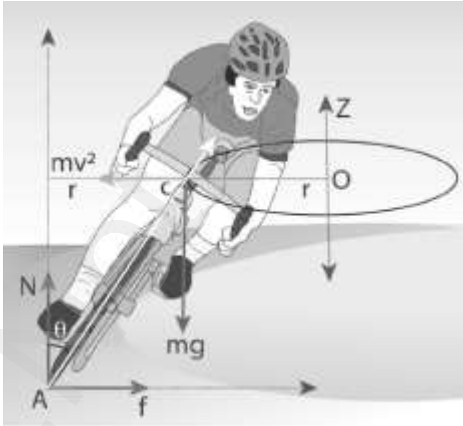
Neutral equilibrium:

1) Linear momentum and angular momentum are zero
2) The **body remains at the same equilibrium if slightly disturbed** and released
3) The **center of mass of the body does not shift higher** or lower if disturbed from equilibrium
4) **Potential energy remains same even if disturbed**

31. Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.

i) Let us consider a cyclist negotiating a circular level road (not banked) of radius r with a speed v.

ii) The cycle and the cyclist are considered as one system with mass m**. The center gravity of the system is C and it goes in a circle of radius r with center at O.**

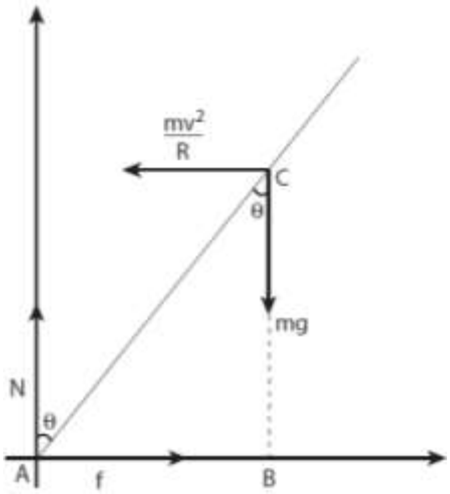


iii) Let us choose the line OC as X-axis and the vertical line through O as Z-axis as shown in Figure

iv) The system as a frame is rotating about Z-axis. **The system is at rest in this rotating frame. To solve problems in rotating frame of reference,** we have to apply a centrifugal force (pseudo force) on the system which will be $\frac{mv^2}{r}$.This force will act through the center of gravity.

v) The **forces acting on the system are, (i) gravitational force (mg), (ii) normal force (N), (iii) frictional force (f) and (iv) centrifugal force $\left(\frac{mv^2}{r}\right)$**

vi) As the system is in equilibrium in the rotational frame of reference, the net external force and net external torque must be zero. Let us consider all torques about the point A in Figure



vii) For **rotational equilibrium, $\vec{\tau}$net = 0**. The torque due to the gravitational force about point A is (mgAB) which causes a **clockwise turn that is taken as negative.** The torque due to the centripetal force is $\left(\frac{mv^2}{r}BC\right)$ which causes an anticlockwise turn that is taken as positive.

$- mgAB + \frac{mv^2}{r}BC = 0$; $mg\,AB = \frac{mv^2}{r}BC$

From ΔABC,

AB = ACsinθ and BC = ACcosθ ;

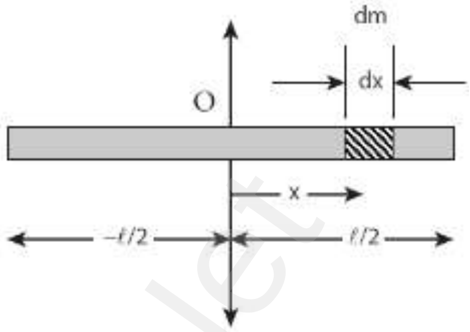
$mg\,AC\sin\theta = \frac{mv^2}{r}AC\cos\theta$; $\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

viii) While negotiating a circular level road of radius r at velocity v, a cyclist has to bend by an angle θ from vertical given by the above expression to stay in equilibrium (i.e. to avoid a fall).

32. Derive the expression for moment of inertia of a rod about its center and perpendicular to the rod.

1) Let us consider a uniform rod of mass (M) and length (*l*) as shown in Figure. Let us find an expression for moment of inertia of this rod about an **axis that passes through the center of mass and perpendicular to the rod.**



2) **First an origin is to be fixed for the coordinate system** so that it coincides with the center of mass, which is also the geometric center of the rod. The rod is now along the x axis.

3) We take an infinitesimally small mass (dm) at a distance (x) from the origin. The moment of inertia (dI) of this mass (dm) about the axis is,

dI = (dm)x²

As the mass is uniformly distributed, the mass per unit length (λ) of the rod is, $\boldsymbol{\lambda = \frac{M}{l}}$

The (dm) mass of the infinitesimally small length as, dm = λ, dx $= \frac{M}{l}$dx.

The moment of inertia (I) of the entire rod can be found by integrating dI, I = $\int dI = \int (dm)x^2$;

$\int \left(\frac{M}{l}dx\right)x^2$;

I = $\frac{M}{l}\int x^2 dx$

4) As the **mass is distributed on either side of the origin, the limits for integration are taken from** $-\frac{l}{2}$ **to** $\frac{l}{2}$

I = $\frac{M}{l}\int_{-\frac{l}{2}}^{\frac{l}{2}} x^2 dx \quad = \frac{M}{l}\left[\frac{x^3}{3}\right]_{\frac{-l}{2}}^{\frac{l}{2}}$

I = $\frac{M}{l}\left[\frac{l^3}{24} - \left(-\frac{l^3}{24}\right)\right] \quad = \frac{M}{l}\left[\frac{l^3}{24} + \frac{l^3}{24}\right]$

I = $\frac{M}{l}\left[2\left(\frac{l^3}{24}\right)\right]$;

I = $\frac{1}{12}$m*l*²

33. Derive the expression for moment of inertia of a uniform ring about an axis passing through the center and perpendicular to the plane.

1) Consider a **uniform ring of mass M and radius R.** To find the moment of inertia of the **ring about an axis passing through its center** and perpendicular to the plane, let us take an infinitesimally small mass (dm) of length (dx) of the ring.

2) This (dm) is located at a distance R, which is the radius of the ring from the axis as shown in Figure. The moment of inertia (dI) of this small mass (dm) is, **dI = (dm)R²**

The **length of the ring is its circumference (2πR)**. As the mass is uniformly distributed, the mass per unit length (λ - is, $\lambda = \frac{M}{2\pi R}$

The (dm) mass of the infinitesimally small length as,

dm = λ, dx = $\frac{M}{2\pi R}$dx.

Now, the moment of inertia (I) of the entire ring is,

I = $\int dI = \int (dm)R^2$;

$\int \left(\frac{M}{2\pi R} dx\right) R^2$

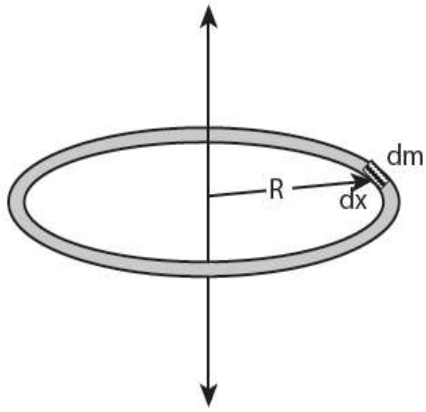
I = $\frac{MR}{2\pi} \int dx$

To cover the entire length of the ring, the limits of integration are taken from 0 to 2πR

I = $\frac{MR}{2\pi} \int_0^{2\pi R} dx$; = $\frac{MR}{2\pi} [x]_0^{2\pi R}$;

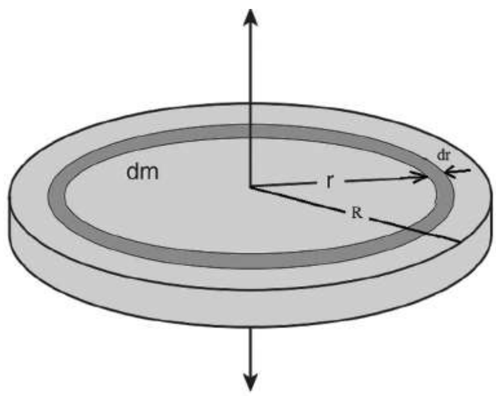
= $\frac{MR}{2\pi} [2\pi R - 0]$

I = MR²



34. Derive the expression for moment of inertia of a uniform disc about an axis passing through the center and perpendicular to the plane.

i) Consider a **disc of mass M and radius R.** This disc is **made up of many infinitesimally small rings** as shown in Figure. Consider one such ring of mass (dm) and thickness (dr) and radius (r). The moment of inertia (dI) of this small ring is, **dI = (dm)r²**



ii) As the mass is uniformly distributed, the mass per unit area (σ) is, σ = $\frac{M}{\pi R^2}$

The mass of the infinitesimally small ring is, dm = σ 2 πrdr = $\frac{M}{\pi R^2}$ 2 πrdr

where, the term (2πr dr) is the area of this elemental ring (2πr is the length and dr is the thickness) dm = $\frac{2M}{R^2}$rdr. ; dI = $\frac{2M}{R^2}$ r³dr

The moment of inertia (I) of the entire disc is, I = $\int dI$

I = $\int_0^R \frac{2M}{R^2}$ r³dr; = $\frac{2M}{R^2} \int_0^R$ r³dr ; I = $\frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R$; = $\frac{2M}{R^2}\left[\frac{R^4}{4} - 0\right]$ **I = ½ MR²**

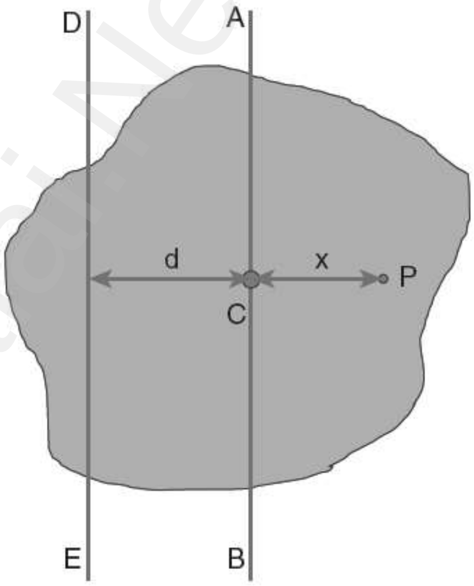
35. State and prove parallel axis theorem.

i) Parallel axis theorem states that **the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.**

ii) If IC is the moment of inertia of the body of mass M about an axis passing through the center of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation,

$$I = I_C + Md^2$$

iii) let us consider a rigid body as shown in Figure. Its **moment of inertia about an axis AB passing through the center of mass is I_C.** DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of I_C. For this, let us consider a point mass m on the body at position x from its center of mass.



iv) The moment of inertia of the point mass about the axis DE is, $m(x + d)^2$. The moment of inertia I of the whole body about DE is the summation of the above expression.

$\mathbf{I = \Sigma m(x + d)^2}$ This equation could further be written as,

$I = \Sigma m(x^2 + d^2 + 2xd)$

$I = \Sigma(mx^2 + md^2 + 2dmx)$

$\mathbf{I = \Sigma mx^2 + \Sigma md^2 + 2d\Sigma mx}$

v) Here, Σmx^2 is the moment of inertia of the body about the center of mass. Hence, $I_C = \Sigma mx^2$

The term, $\Sigma mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation (Σmx) will be zero

Thus, $\mathbf{I = I_C + \Sigma md^2 \,;\, I_C + (\Sigma m)d^2}$

vi) Here, Σm is the entire mass M of the object ($\Sigma m = M$)

$\mathbf{I = I_C + Md^2}$

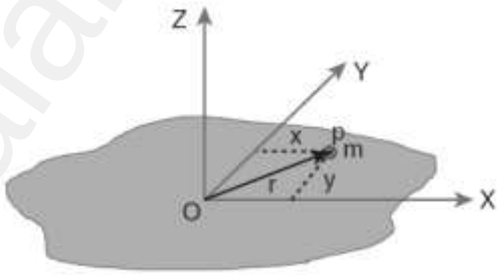
Hence, the parallel axis theorem is proved.

36. State and prove perpendicular axis theorem.

i) The theorem states that **the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.**

ii) Let the **X and Y-axes lie in the plane and Z-axis perpendicular to the plane of the laminar object.** If the moments of inertia of the body about X and Y-axes are I_X and I_Y respectively and I_Z is the moment of inertia about Z-axis, then **the perpendicular axis theorem could be expressed as, $I_Z = I_X + I_Y$**

iii) To prove this theorem, let us consider a plane laminar object of negligible thickness on which lies the origin (O). **The X and Y-axes lie on the plane and Z-axis is perpendicular** to it as shown in Figure. The lamina is considered to be made up of a large number of particles of mass m. Let us choose one such particle at a point P which has coordinates (x, y) at a distance r from O.



iv) The moment of inertia of the particle about Z-axis is, mr^2 The summation of the above expression gives the moment of inertia of the entire lamina about Z-axis as, $I_Z = \Sigma\, mr^2$

Here, $r^2 = x^2 + y^2$; Then, $I_Z = \Sigma m\,(x^2 + y^2)$

$\mathbf{I_Z = \Sigma\, mx^2 + \Sigma my^2}$

In the above expression, the term Σmx^2 is the moment of inertia of the body about the Y-axis and similarly, the term $\Sigma\, my^2$ is the moment of inertia about X-axis. Thus, $I_X = \Sigma\, my^2$ and $I_Y = \Sigma\, mx^2$

Substituting in the equation for I_z gives, $\mathbf{I_Z = I_X + I_Y}$

Thus, the perpendicular axis theorem is proved.

37. Discuss rolling on inclined plane and arrive at the expression for the acceleration.

1) Let us **assume a round object of mass m and radius R is rolling down an inclined Plane without slipping** as shown in Figure. There are two forces acting on the object along the inclined plane.

2) One is the component **of gravitational force (mg sinθ)** and the other is **the static frictional force (f).** The other component of gravitation force (mg cosθ) is **cancelled by the normal force (N) exerted by the plane**. As the motion is happening along the incline, we shall write the equation for motion from the free body diagram (*FBD*) of the object.

3) For translational motion, mg sinθ is the supporting force and f is the opposing force, $\mathbf{mg\sin\theta - f = ma}$ —(1)

For rotational motion, let us take the torque with respect to the center of the object.
Then mg sinθ cannot cause torque as it passes through it but the frictional force f can set torque of Rf. **Rf = Iα**

4) By using the relation, a = r α, and moment of inertia I = mK^2, we get,

$Rf = mk^2 \frac{a}{R}$; $\mathbf{f = ma \left(\frac{K^2}{R^2}\right)}$

Now equation (1) becomes

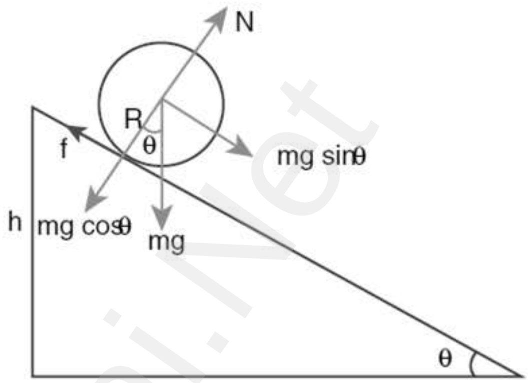
$mg\sin\theta - ma\left(\frac{K^2}{R^2}\right) = ma$

$\mathbf{mg\sin\theta = ma + ma\left(\frac{K^2}{R^2}\right)}$

$a\left(1 + \frac{K^2}{R^2}\right) = g\sin\theta$

After rewriting it for acceleration,

we get, $\mathbf{a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}}$



5) We can also find the expression for final velocity of the rolling object by using third equation of motion for the inclined plane. $v^2 = u^2 + 2as$. If the body starts rolling from rest, u = 0. When h is the vertical height of the incline, the length of the incline s is, $\mathbf{S = \frac{h}{Sin\theta}}$ **;**

$v^2 = 2\frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}\left(\frac{h}{Sin\theta}\right) = \frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}$

By taking square root, $\mathbf{v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}}$

6) The time taken for rolling down the incline could also be written from first equation of motion as, v = u + at. For the object which starts rolling from rest, u = 0. Then,

$t = \frac{v}{a}$; $t = \left(\sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}\right)\left(\frac{\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}\right)$

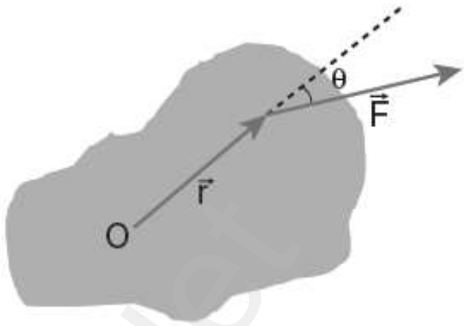
$\mathbf{t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{gSin^2\theta}}}$

7) The equation suggests that for a given incline, the object with the least value of radius of gyration K will reach the bottom of the incline first.

38. Define Torque and derive its expression.

1) Torque is defined as the **moment of the external applied force** about a point or axis of rotation. The expression for torque is, $\vec{\tau} = \vec{r} \times \vec{F}$

2) where, $\vec{r}$ is the position vector of the point where the force $\vec{F}$ is acting on the body as shown in Figure.



3) Here, **the product of $\vec{r}$ and $\vec{F}$ is called the vector product or cross product.** The vector product of two vectors results in another vector that is perpendicular to both the vectors. Hence, torque ($\vec{\tau}$) is a vector quantity.

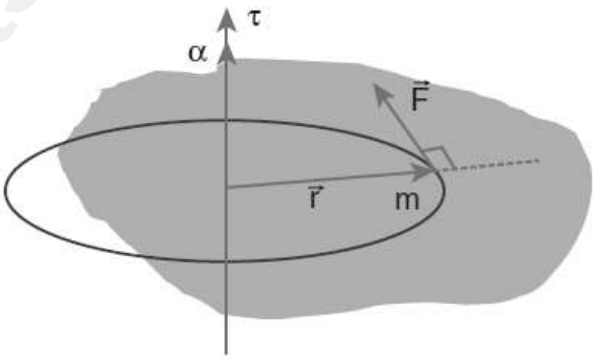
4) **Torque has a magnitude (rFsinθ) and direction perpendicular to, $\vec{r}$ and $\vec{F}$. Its unit is N m. $\vec{\tau} = (rF\sin\theta)\hat{n}$**

5) Here, θ is the angle between $\vec{r}$ and $\vec{F}$ and $\hat{n}$ is the unit vector in the direction of $\vec{\tau}$.

39. Obtain the relation between torque and angular acceleration.

i) Let us consider a **rigid body rotating about a fixed axis.** A point mass m in the body will execute a circular motion about a fixed axis

ii) A tangential force $\vec{F}$ acting on the point mass produces the necessary torque for this rotation. This **force $\vec{F}$ is perpendicular to the position vector $\vec{r}$ of the point mass.**



iii) The **torque produced by the force on the point mass m about the axis** can be written as, $\tau = r F \sin 90$ $= r F$; $(\sin 90 = 1)$

$\tau = rma$ $(F = ma)$

$\tau = rmr\alpha$; $= mr^2\alpha$ $(a = r\alpha)$

$\boldsymbol{\tau = mr^2\alpha}$ ————(1)

iv) Hence, the torque of **the force acting on the point mass produces an angular acceleration (α) in the point mass about the axis of rotation**. In vector notation,

$\vec{\tau} = (mr^2)\vec{\alpha}$ ------------(2)

v) The directions of τ and α are along the axis of rotation. If the direction of τ is in the direction of α, it **produces angular acceleration. On the other hand if, τ is opposite to α, angular deceleration or retardation is produced on the point mass.**

vi) The term mr^2 in equations 1 and 2 is called moment of inertia (I) of the point mass. A rigid body is made up of many such point masses. Hence, the moment of inertia of a rigid body is the sum of moments of inertia of all such individual **point masses that constitute the body ($I = \Sigma m_i r_i^2$) .Hence, torque for the rigid body can be written as,** $\vec{\tau} = (\Sigma m_i r_i^2)\,\vec{\alpha}$; $\vec{\tau} = I\,\vec{\alpha}$

40. Write an expression for the kinetic energy of a body in pure rolling.

1) The total kinetic energy (KE) as the sum of kinetic energy due to translational motion (KE_{TRANS}) and kinetic energy due to rotational motion (KE_{ROT}). **$KE = KE_{TRANS} + KE_{ROT}$**

2) If the mass of the rolling object is M, the velocity of center of mass is v_{CM}, its moment of inertia about center of mass is I_{CM} and angular velocity is ω, then **$KE = \frac{1}{2} Mv^2_{CM} + \frac{1}{2} I_{CM}\,\omega^2$**

With center of mass as reference:

3) The moment of inertia (I_{CM}) of a rolling object about the center of mass is, $I_{CM} = MK^2$ and $v_{CM} = R\omega$. Here, K is radius of gyration.

$$KE = \frac{1}{2} Mv^2_{CM} + \frac{1}{2} (MK^2)\frac{v^2_{CM}}{R^2}$$

$$KE = \frac{1}{2} Mv^2_{CM} + \frac{1}{2} Mv^2_{CM}\left(\frac{K^2}{R^2}\right)$$

$$\mathbf{KE = \frac{1}{2} Mv^2_{CM}\left(1 + \frac{K^2}{R^2}\right)}$$

With point of contact as reference:

4) We can also arrive at the same expression by taking the momentary rotation happening with respect to the point of contact (another approach to rolling). If we take the point of contact as O, then,

$KE = \frac{1}{2} I_o\,\omega^2$

5) Here, I_o is the moment of inertia of the object about the point of contact. By parallel axis theorem, $I_o = I_{CM} + MR^2$. Further we can write, **$I_o = MK^2 + MR^2$.**

With $v_{CM} = R\omega$ or $\omega = \frac{v_{CM}}{R}$

$$KE = \frac{1}{2} (MK^2 + MR^2)\frac{v^2_{CM}}{R^2}$$

$$\mathbf{KE = \frac{1}{2} Mv^2_{CM}\left(1 + \frac{K^2}{R^2}\right)}$$

6) **KE in pure rolling can be determined by any one of the following two cases.**

(i) **The combination of translational motion and rotational motion about the center of mass.** (or)

(ii) The **momentary rotational motion about the point of contact.**

UNIT – VI (GRAVITATION)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. State Kepler's three laws.

1. **Law of orbits:** Each planet moves around the Sun in an **elliptical orbit with the Sun at one of the foci.**
2. **Law of area:**
   The radial vector (line joining the Sun to a planet) **sweeps equal areas in equal intervals of time.**
3. **Law of period:**
   The **square of the time period of revolution of a planet** around the Sun in its Elliptical orbit is **directly proportional to the cube of the semi-major axis of the ellipse.**

2. State Newton's Universal law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of **this force of attraction** was found to be **directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.**

3. Will the angular momentum of a planet be conserved? Justify your answer.

Yes, Because $\vec{\tau} = \vec{r} \times \vec{F}$; $\vec{r} \times \left(\frac{GM_S M_E}{r^2}\hat{r}\right) = 0$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$ So, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

It implies that angular momentum is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion.

4. Define the gravitational field. Give its unit.

The gravitational force experienced by **unit mass placed at that point.**

Unit $\vec{E}_1 = \frac{\vec{F}_{21}}{m_2}$ in equation we get, $\vec{E} = -\frac{Gm_1}{r^2}\hat{r}$. its unit is N kg^{-1} (or) m s^{-2} .

5. What is meant by superposition of gravitational field?

Consider 'n' particles of masses, m_1 , m_2 ...m_n distributed in space at positions $\hat{r}_1, \hat{r}_2, \hat{r}_3$ ....etc, with respect to point P. **The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses.** This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + ...\vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\vec{r}_1 - \frac{Gm_2}{r_2^2}\vec{r}_2 - .... \frac{Gm_n}{r_n^2}\vec{r}_n \; ; = -\sum_{i=1}^{n}\frac{Gm_i}{r_i^2}\vec{r}_i$$

6. Define gravitational potential energy.

Gravitational potential energy associated with this conservative force field. The gravitational potential energy is defined **as the work done to bring the mass m_2 from infinity to a distance 'r' in the gravitational field of mass m_1. Its unit is joule.**

7. Define gravitational potential.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r.

8. What is meant by escape speed in the case of the Earth?

The minimum speed required by an object to escape from Earth's gravitational field. ie. **Ve = $\sqrt{2gR_E}$; Ve = 11.2 kms^{-1}**

9. Why is the energy of a satellite (or any other planet) negative?

The **negative sign in the total energy implies that the satellite** is bound to the Earth and it cannot escape from the Earth.

As h approaches, ∞ the **total energy tends to zero**. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

10. Define weight

The weight of an object is defined as the **downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth**. The magnitude of weight of an object is denoted as, **W=N=mg.**

11. Why is there no lunar eclipse and solar eclipse every month?

Moon's orbit is tilted 5^0 with respect to Earth's orbit, only during certain periods of the year; the **Sun, Earth and Moon align in straight line leading to either lunar eclipse** or solar eclipse depending on the alignment.

12. How will you prove that Earth itself is spinning?

The Earth's spinning motion can be proved by **observing star's position over a night. Due to Earth's spinning motion**, the stars in sky appear to move in circular motion about the pole star.

13. What is meant by state of weightlessness?

When downward acceleration of the **object is equal to the acceleration due to the gravity of the Earth**, the object appears to be weightless

14. Why do we have seasons on Earth?

The seasons in the Earth arise due to the rotation of Earth around the **Sun with 23.5° tilt. Due to this 23.5° tilt, when the northern part of Earth is**

farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

15. Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of **higher gravitational potential than the surface of the Earth.** i.e. **$V_{hill} > V_{ground}$.**

16. Why does a tide arise in the ocean?

Tides arise in the ocean due to the **force of attraction between the moon and sea water.**

17. When a man is standing in the elevator, what are forces acting on him.

1. **Gravitational force which acts downward**. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. **The normal force exerted by floor** on the man which acts vertically upward, $\vec{N} = N\hat{j}$

18. Find the expression of the orbital speed of satellite revolving around the earth.

Satellite of **mass M to move in a circular orbit, centripetal force must be acting on the satellite.** This centripetal force is provided by the Earth's gravitational force.

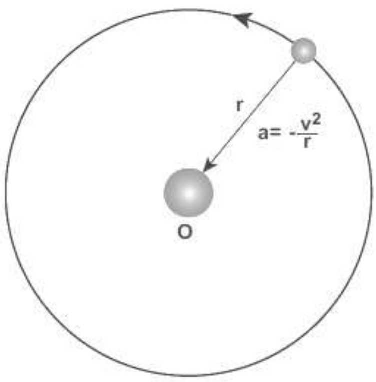
$$\frac{MV^2}{(R_E+h)} = \frac{GMM_E}{(R_E+h)^2} \; ; \quad V^2 = \frac{GM_E}{(R_E+h)} \; ; \; V = \sqrt{\frac{GM_E}{(R_E+h)}}$$

As h increases, the speed of the satellite decreases.

FIVE MARKS QUESTION WITH ANSWER:

19. Explain how Newton arrived at his law of gravitation from Kepler's third law.

Newton's inverse square Law:



Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is

$$a = \frac{v^2}{r} \text{ ------------------- } 1$$

Here v is the velocity and r, the distance of the planet from the center of the orbit. The velocity in terms of known quantities r and T, is

$$V = \frac{2\pi r}{T} \text{ ------------------- } 2$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (1) we get,

$$a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \text{ ------- } 3$$

Substituting the value of 'a' from (3) in Newton's second law, F = ma, where 'm' is the mass of the planet.

$$F = \frac{4\pi mr}{T^2} \text{ ------------------ } 4$$

From Kepler's third law, $\frac{r^3}{T^2}$ = k (Constant) --------- 5

$$\frac{r}{T^2} = \frac{k}{r^2} \text{ --------------- } 6$$

By substituting equation 6 in the force expression, we can arrive at the law of gravitation. $F = \frac{4\pi^2 mk}{r^2}$ --------7

Here **negative sign implies that the force is attractive and it acts towards the center**. In equation (7), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, **if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force.** So he felt that the Sun's mass (M) should also occur explicitly in the expression for force. From this insight, he equated the constant $4\pi^2 k$ to GM which turned out to be the law of gravitation.

$$F = \frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

20. Explain how Newton verified his law of gravitation.

1) Newton verified **his law of universal gravitation by comparing the acceleration of a terrestrial object to the acceleration of the moon.**
2) He knew that the **distance from the center of earth to the center of two spheres of known mass at either end of a light** rod suspended by a then fiber from the center of the rod.
3) He had earlier found **the small force that was needed to twist the fiber.**
4) By bringing a third sphere close to one of the suspended spheres.
5) He was **able to measure the force of gravity between the spheres** and hence gravitation.

21. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is U = mgh.

1) Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$$U = -\frac{GM_e m}{r} \text{ --------------- } 1$$

2) Here r = Re+h, where Re is the radius of the Earth. h is the height above the Earth's surface, $U = -G\frac{M_e m}{(R_e+h)}$ ----------- 2

If h << Re, equation (2) can be modified as

$$U = -G\frac{M_e m}{R_e\left(1+\frac{h}{R_e}\right)} \;;\quad U = -G\frac{M_e m}{R_e}\left(1+\frac{h}{R_e}\right)^{-1} \text{ -------- } 3$$

3) By using Binomial expansion and neglecting the higher order terms, we get **$U = -G\frac{M_e m}{R_e}\left(1 - \frac{h}{R_e}\right)$** --------- 4

We know that, for a mass m on the Earth's surface,

$G\frac{M_e m}{R_e}$ = mgR_e -------------- 5

Substituting equation (5) in (4) we get, $U = -mgR_e + mgh$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from h and it can be omitted. **U = mgh**

22. Explain in detail the idea of weightlessness using lift as an example.

i) **When the lift falls (when the lift wire cuts)** with downward acceleration **a = g, the person inside the elevator is in the state of weightlessness** or free fall.

ii) As they fall freely, they are **not in contact with any surface** (by neglecting air friction). The **normal force acting on the object is zero.** The **downward acceleration is equal to the acceleration due to the gravity of the Earth**. i.e (a = g). From equation N = m (g − a) we get,
a = g ∴ N = m (g – g) = 0. This is called the state of weightlessness.

23. Derive an expression for escape speed.

1) Consider an object of mass M on the surface of the Earth. **When it is thrown up with an initial speed v_i , the initial total energy of the object is $E_i = \frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E}$** --------------- 1

Where M_E, is the mass of the Earth and R_E - the radius of the Earth.

The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

2) When the object reaches a height far away from Earth and hence treated as approaching infinity, **the gravitational potential energy becomes zero [$U(\infty) = 0$] and the kinetic energy becomes zero as well**. Therefore, **the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape**. Otherwise Kinetic energy can be non-zero.

$E_f = 0$, According to the law of energy conservation, **$E_i = E_f$** ------- 2

Substituting (1) in (2) we get,

$\frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} = 0$ ‘ **$\frac{1}{2} MV_i^2 = \frac{GMM_E}{R_E}$** --------------- 3

3) The escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace, V_i with V_e. i.e,

$\frac{1}{2} MV_e^2 = \frac{GMM_E}{R_E}$

$V_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M}$; $V_e^2 = \frac{2GM_E}{R_E}$ ------------ 4

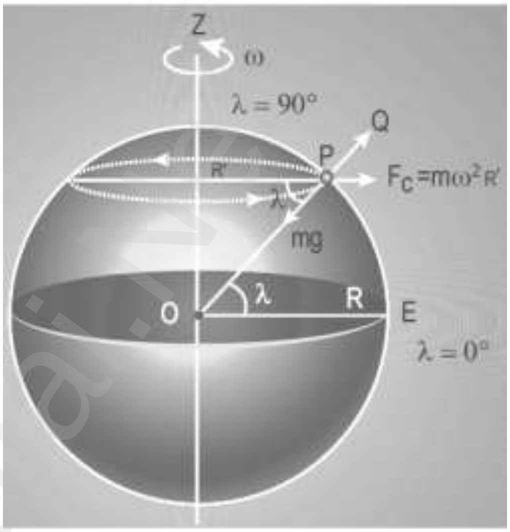
Using $g = \frac{GM_E}{R_e}$ − − − − − − − − − − − − − − − − 5

$V_e^2 = 2gR_E$; $V_e = \sqrt{2gRE}$ ------------ 6

From equation (6) the escape speed depends on two factors: **acceleration due to gravity and radius of the Earth**. It is completely independent of the mass of the object.

24. Explain the variation of g with latitude.

Variation of g with latitude:



Whenever we analyze the motion of objects in rotating frames, we must take into account the centrifugal force. **Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis**. So when an object is on the surface of the Earth, it experiences a **centrifugal force that depends on the latitude of the object on Earth**. If the Earth were not spinning, the force on the object would have been mg. However, **the object experiences an additional centrifugal force due to spinning of the Earth.**

This centrifugal force is given by $m\omega^2 R'$

$\mathbf{R'} = \mathrm{R} \cos \lambda$ ---------- 1

Where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is $\mathbf{a_c = \omega^2 R' \cos \lambda}$

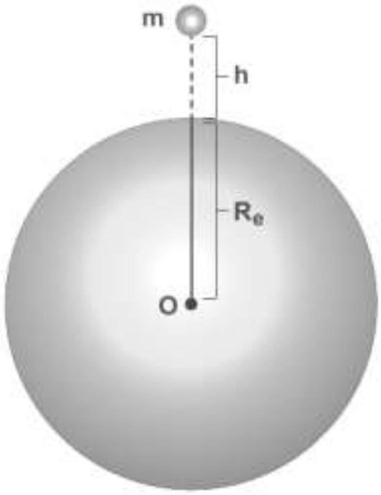
$= \boldsymbol{\omega^2 R \cos^2 \lambda}$ since $R' = R \cos \lambda$ Therefore,

$\mathbf{g' = g - \omega^2 R \cos^2 \lambda}$ ---------- 2

From the expression (2), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles $\lambda = 90$; $g' = g$, it is maximum. At the equator, g' is minimum.

25. Explain the variation of g with altitude.

Variation of g with altitude:



Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is $\mathbf{g' = \frac{GM}{(R_e+h)^2}}$

$g' = \frac{GM}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$; $g' = \frac{GM}{R_e^2}\left(1+\frac{h}{R_e}\right)^{-2}$

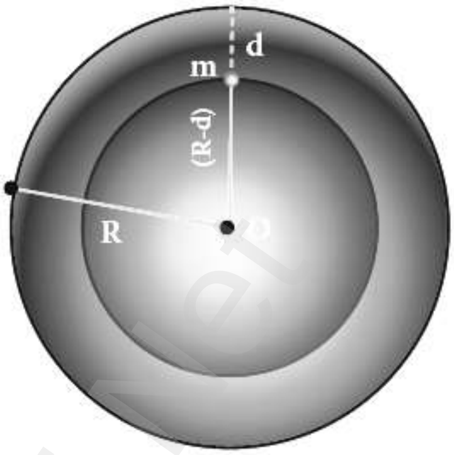
If h << R_e. We can use Binomial expansion. Taking the terms upto first order

$g' = \frac{GM}{R_e^2}\left(1 - 2\frac{h}{R_e}\right)$; $\mathbf{g' = g\left(1 - 2\frac{h}{R_e}\right)}$

We find that $\mathbf{g' < g}$. This means that **as altitude h increases the acceleration due to gravity g decreases.**

26. Explain the variation of g with depth from the Earth's surface.

Variation of g with depth:



Consider a **particle of mass m which is in a deep mine on the earth. Ex. Coal mines – in Neyveli)**. Assume the **depth of the mine as d**. To Calculate g at a depth d, consider the following points. The **part of the Earth which is above the radius (R_e – d) do not contribute to the acceleration.** The result is proved earlier and is given as $g' = \frac{GM'}{(R_e - d)^2}$ Here M is the mass of the Earth of radius (R_e – d). Assuming the density of earth ρ to be constant,

$\rho = \frac{M'}{V'}$; $\frac{M'}{V'} = \frac{M}{V}$ and $M' = \frac{M}{V}V'$

$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$;

$M' = \frac{M}{R_e^3}(R_e - d)^3$

$\mathbf{g' = G\frac{M}{R_e^3}(R_e - d)^3 \cdot \frac{1}{(R_e - d)^2}}$;

$g' = GM\,\frac{R_e\left(1 - \frac{d}{R_e}\right)}{R_e^3}$

$g' = GM\,\frac{\left(1 - \frac{d}{R_e}\right)}{R_e^2}$ thus $\mathbf{g' = g\left(1 - \frac{d}{R_e}\right)}$. Here also **g′<g .**

As depth increases, g′ decreases.

27. Derive the time period of satellite orbiting the Earth.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to 2π (R_E +h) and time taken for it is the time period, T. Then

$$\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi\,(R_E + h)}{T}$$

From equation, $\sqrt{\frac{GM_E}{(R_E + h)}} = \frac{2\pi\,(R_E + h)}{T}$ --------------- 1

$T = \frac{2\pi}{\sqrt{GM_E}}(R_E + h)^{\frac{3}{2}}$ ------------------ 2

Squaring both sides of the equation (2), we get $T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$

$\frac{4\pi^2}{GM_E}$ = Constant say c, $T^2 = c\,(R_E + h)^3$ -------------- 3

Equation (3) implies that a satellite orbiting the **Earth has the same relation between time and distance as that of Kepler's law of planetary motion.** For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E . Then, $T^2 = \frac{4\pi^2}{GM_E}R_E^3$; $T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}}$

$T^2 = \frac{4\pi^2}{g} R_E$ Since $\frac{GM_E}{R_E^2} = g$; $T = 2\pi\sqrt{\frac{R_E}{g}}$

28. Explain in detail the geostationary and polar satellites.

Geo-stationary and polar satellite

1) The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital **radius of a satellite if its time period is 24 hours is calculated below.** Kepler's third law is used to find the radius of the orbit.

$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$; $(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$

$(R_E + h) = \left(\frac{GM_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$

2) Substituting for the **time period (24 hrs = 86400 seconds)**, mass, and **radius of the Earth, h turns out to be 36,000 km**. Such satellites are called **"geo-stationary satellites"**, since they appear to be stationary when seen from Earth.

3) Geo-stationary satellites for the purpose of telecommunication. Another type of **satellite which is placed at a distance of 500 to 800 km** from the surface of the Earth orbits the Earth from north to south direction.

4) This type of satellite that **orbits Earth from North Pole to South Pole** is called a polar satellite. The **time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day.**

5) **A Polar satellite covers a small strip of area from pole to pole during one revolution**. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

UNIT – VII (PROPERTIES OF MATTER)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. Define stress and strain.

The **force per unit area is called as stress**. Stress, $\sigma = \frac{Force}{Area} = \frac{F}{A}$

The SI unit of **stress is N m^{-2} or Pascal (Pa)** and its dimension is **$[ML^{-1}T^{-2}]$.**

The fractional change in the size of the object, in other words, strain measures the degree of deformation. Strain, **e** = $\frac{\textbf{Change in Size}}{\textbf{Original size}} = \frac{\Delta l}{l}$

2. State Hooke's law of elasticity.

Hooke's law is for a small deformation, when **the stress and strain are proportional to each other.**

3. Define Poisson's ratio.

The ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

Poisson s ratio, μ = **Lateral strain / Longitudinal strain**

4. Which one of these is more elastic, steel or rubber? Why?

Steel is **more elastic than rubber because the steel has higher young's modulus than rubber**. That's why**, if equal stress is applied on both steel and rubber, the steel produces less strain**.

5. State Pascal's law in fluids.

If the **pressure in a liquid is changed at a particular point**, the change is transmitted to the entire liquid without being diminished in magnitude.

6. State Archimedes principle.

It states that when a **body is partially or wholly immersed in a fluid**, it experiences an **upward thrust equal to the weight of the fluid displaced** by it and its up-thrust acts through the centre of gravity of the liquid displaced.

7. What do you mean by up-thrust or buoyancy?

The **upward force exerted by a fluid that opposes the weight of an immersed object in a fluid** is called up-thrust or buoyant force and the phenomenon is called buoyancy.

8. State the law of floatation.

The law of floatation states **that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body.**

9. Define coefficient of viscosity of a liquid.

The coefficient of viscosity is defined as **the force of viscosity acting between two layers per unit area** and unit velocity gradient of the liquid. Its unit is **Nsm^{-2} and dimension is $[ML^{-1}T^{-1}]$.**

10. Distinguish between streamlined flow and turbulent flow.

Streamlined flow: When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a streamlined flow.

The velocity of the particle at any point is constant. It is also referred to as steady or laminar flow.

The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point.

Turbulent flow: When the speed of the moving fluid exceeds the critical speed, v_c the motion becomes turbulent.

The velocity changes both in magnitude and direction from particle to particle.

The path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies.

11. What is Reynold's number? Give its significance.

Reynold's number(R_C) is **a dimensionless number, which is used to find out the nature of flow of the liquid. $R_C = \frac{\rho v D}{\eta}$**

Where, ρ- density of the liquid, v –The velocity of flow of liquid.
D- Diameter of the pipe, η - The coefficient of viscosity of the fluid.

12. Define terminal velocity.

The **maximum constant velocity acquired by a body while falling freely through a viscous medium** is called the terminal velocity.

13. Write down the expression for the Stoke's force and explain the symbols involved in it.

Viscous force F acting on a spherical body of **radius r depends directly on**
i) radius (r) of the sphere
ii) velocity (v) of the sphere and
iii) coefficient of viscosity η of the liquid $F = 6\pi\eta r v$

14. State Bernoulli's theorem.

According to Bernoulli's theorem, **the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant**.

15. Two streamlines cannot cross each other. Why?

No two streamlines can cross each other. If they do so, the **particles of the liquid at the point of intersection will have two different directions for their flow**, which will destroy the steady nature of the liquid flow.

16. Define surface tension of a liquid. Mention its S.I unit and dimension.

The surface tension of a liquid is defined as the energy per unit area of the surface of a liquid. (or) **The surface tension of a liquid is defined as the force of tension acting perpendicularly on both sides of an imaginary line of unit length drawn on the free surface of the liquid.**

Its unit is N m^{-1} and dimension is $[MT^{-2}]$.

17. Define angle of contact for a given pair of solid and liquid.

The **angle between the tangent to the liquid surface at the point of contact and the solid surface** is known as the angle of contact.

18. Distinguish between cohesive and adhesive forces.

The force between the like molecules which holds the liquid together is called 'cohesive force'. **When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force** which is called **'adhesive force'.**

19. What are the factors affecting the surface tension of a liquid?

(1) **The presence of any contamination or impurities** considerably affects the **force of surface tension depending upon the degree of contamination.**

(2) **The presence of dissolved substances** can also **affect the value of surface tension**. For example, **a highly soluble substance like sodium chloride (NaCl) when dissolved in water (H_2O) increases the surface tension of water.** But the sparingly soluble substance **like phenol** or **soap solution when mixed in water decreases the surface tension of water.**

(3) **Electrification** affects the surface tension. **When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases** which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) **Temperature** plays a **very crucial role in altering the surface tension of a liquid.** Obviously, the surface tension decreases linearly with the rise of temperature.

20. What do you mean by capillarity or capillary action?

The **rise or fall of a liquid in a narrow tube** is called capillarity or capillary action.

21. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?

A **drop of oil placed on the surface of water spreads** because the force of **adhesion between water and oil molecules dominates the cohesive force** of oil molecules.

On the other hand, **cohesive force of water molecules dominates the adhesive force between water and oil molecules**. So **drop of water on oil contracts to a spherical shape.**

22. State the principle and usage of Venturimeter.

Bernoulli's theorem is the **principle of Venturimeter**.

Venturimeter is **used to measure the rate of flow or flow speed of the incompressible fluid flowing through a pipe**.

23. What are the applications of surface tension?

1) **Oil pouring on the water** reduces surface tension. So that the **floating mosquitos' eggs drown and killed.**
2) Finely adjusted surface tension of the liquid makes droplets of desired size, which **helps in desktop printing, automobile painting and decorative items.**
3) **Specks of dirt are removed** from the cloth when it is washed in detergents added hot water, which has low surface tension.
4) **A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric.** This increases the angle of contact due to surface tension.

24. Give some examples for surface tension.

Clinging of painting brush hairs, when taken out of water.
Needle float on the water, Camphor boat.

25. How do water bugs and water striders walk on the surface of water?

When the water bugs or water striders are on the surface of the water, its weight is balanced by the surface tension of the water. Hence, they can easily walk on it.

26. What are the applications of viscosity?

1) Viscosity of liquids helps in choosing **the lubricants for various machinery parts. Low viscous lubricants** are used in light machinery parts and high viscous lubricants are used in heavy machinery parts.
2) As **high viscous liquids damp the motion**; they are used **in hydraulic brakes as brake oil.**
3) **Blood circulation** through arteries and veins depends upon the viscosity of fluids.
4) Viscosity is used in **Millikan's oil-drop method** to find the **charge of an electron.**

27. Explain the Stoke's law application in raindrop falling.

According to Stoke's law**, terminal velocity is directly proportional to square of radius of the spherical body.** So that smaller raindrops having less terminal velocity float as cloud in air. When they gather as bigger drops get higher terminal velocity and start falling.

28. Define Young's modulus. Give its unit.

Young's modulus is defined as **the ratio of tensile or compressive stress to the tensile or compressive strain. Its unit is N m^{-2} or pascal.**

29. Define Pressure. Give its unit and dimension.

The pressure is defined as **the force acting per unit area. Its unit is N m^{-2} or pascal and dimension is $[ML^{-1}T^{-2}]$.**

30. What is elasticity? Give examples.

Elasticity is the **property of a body in which it regains its original shape and size after the removal of deforming force**.

Ex: Rubber, metals, steel ropes.

31. What is plasticity? Give an example.

Plasticity is the **property of a body in which it does not regains its original shape and size after the removal of deforming force. Ex: Glass.**

CONCEPTUAL QUESTIONS

32. Why two holes are made to empty an oil tin?

When oil comes out from a hole of an oil tin, **pressure inside it decreased than the atmosphere.** Therefore, the surrounding air rush up into the same hole prevents the oil to come out. Hence **two holes are made to empty the oil tin.**

33. We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?

Since the stress produced on **the vegetables by the sharp knife is higher than the blunt knife, vegetables can be cut easily with the sharp knife.**

34. Why the passengers are advised to remove the ink from their pens while going up in an aero-plane?

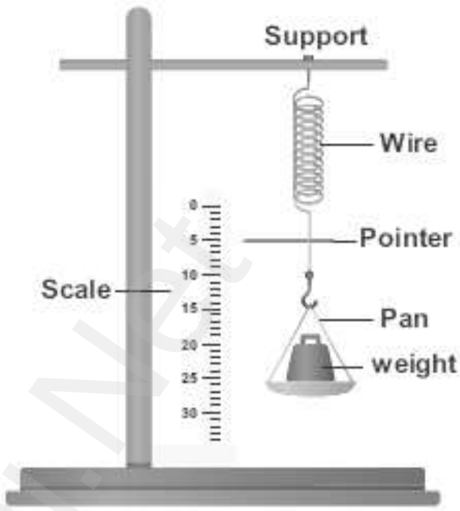
When an aero-plane ascends, **the atmospheric pressure is decreased.** Hence, the ink from the pen will leak out. So that, the passengers are advised **to remove the ink from their pens while going up in the aero-plane.**

35. We use straw to suck soft drinks, why?

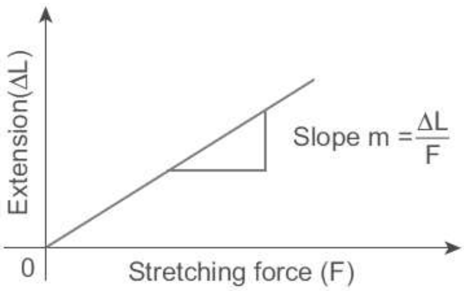
When we suck the soft drinks through the straw, **the pressure inside the straw becomes less than the atmospheric pressure.** Due to the difference in pressure, the soft drink rises in the straw and we are able to enjoy it conveniently.

FIVE MARKS QUESTION WITH ANSWER

36. State Hooke's law and verify it with the help of an experiment.



1) Hooke's law is for a small deformation, when the stress and strain are proportional to each other.
2) It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed-point O.
3) A pan and a pointer are attached at the free end of the wire as shown in Figure (a).
4) The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL.
5) It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis.
6) This graph is a straight line passing through the origin as shown in Figure (b).



Therefore, ΔL = (slope)F
Multiplying and dividing by volume,
V = A L,

F (slope) = $\frac{AL}{AL}\Delta L$

Rearranging, we get, $\frac{F}{A} = \left[\frac{L}{A(slope)}\right]\frac{\Delta L}{L}$

Therefore, , $\frac{F}{A}\ \alpha \left[\frac{\Delta L}{L}\right]$

Comparing with stress equation and strain equation, we get $\sigma\ \alpha\ \varepsilon$
i.e., the stress is proportional to the strain in the elastic limit.

37. Explain the different types of modulus of elasticity.

There are three types of elastic modulus.

(a) Young's modulus, (b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, **then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.**

$= \frac{\text{Tensile stress or compressive stress}}{\text{Tensile starin or compressive strain}}$ $Y = \frac{\sigma_t}{\varepsilon_t}$ or $Y = \frac{\sigma_c}{\varepsilon_c}$

The unit for Young modulus has the **same unit of stress because, strain has no unit.** So, S.I. unit of **Young modulus is N m^{-2} or pascal.**

Bulk modulus:

Bulk modulus is defined as **the ratio of volume stress to the volume strain.**

Bulk modulus, K = $\frac{\text{Normal (Perpendicular)stress or pressure}}{\text{Volume strain}}$

The normal stress or pressure is $\sigma_n = \frac{F_n}{\Delta A} = \Delta p$

The volume strain is $\varepsilon_v = \frac{\Delta V}{V}$

Therefore, Bulk modulus is $K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta p}{\frac{\Delta V}{V}}$

The **negative sign in the equation means** that **when pressure is applied on the body, its volume decreases**. Further, the equation implies that a material can be easily compressed if it has a small value of bulk modulus.

The rigidity modulus or shear modulus:

The rigidity modulus is defined **as Rigidity modulus or Shear modulus,**

$$\eta_R = \frac{\text{Shearing stress}}{\text{Angle of shear or shearing strain}}$$

The shearing stress is $\sigma_s = \frac{\text{Trangential force}}{\text{Area over which it is applied}} = \frac{F_t}{\Delta A}$

The angle of shear or shearing strain $\varepsilon_s = \frac{x}{h} = \theta$

Therefore, Rigidity modulus is $\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta}$

Further, the equation implies, that **a material can be easily twisted if it has small value of rigidity modulus**. For example, **consider a wire, when it is twisted through an angle θ, a restoring torque is developed**, that is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, **rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small.**

38. State and prove Pascal's law in fluids.

Hydraulic lift which is **used to lift a heavy load with a small force**. It is a **force multiplier.** It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure). They are fitted with **frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston**, the pressure of the liquid under this piston increases to $P\left(\text{where}, P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1$$

Therefore by **changing the force on the smaller piston A, the force on the piston B has been increased by the factor** $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

39. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.

Expression for terminal velocity:

Consider a sphere of radius *r* which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

Gravitational force acting on the sphere, $F_G = mg = \frac{4}{3}\pi r^3 \rho g$

(Downward force)

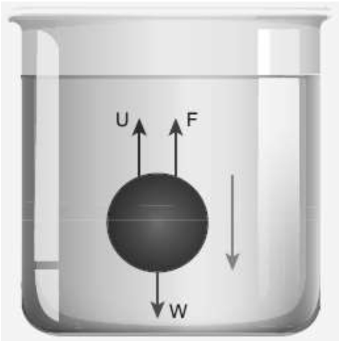
Up thrust, $U = \frac{4}{3}\pi r^3 \sigma g$ (upward force)

Viscous force $F = 6\pi\eta r v_t$

At terminal velocity v_t, downward force = upward force

$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - = \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$

$$V_t = \frac{2}{9} \times \frac{r^2(\rho-\sigma)}{\eta} g \Rightarrow V_t \propto r^2$$



Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity.

40. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.

Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a **capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient** $\left(\frac{P}{l}\right)$. Then, $v \; \alpha \; \eta^a r^b \left(\frac{P}{l}\right)^c$;

$v = k\, \eta^a r^b \left(\frac{P}{l}\right)^c$ where, k is a dimensionless constant. Therefore,

$[v] = \frac{\text{Volume}}{\text{time}} = [L^3T^{-1}]$, $\left[\frac{dP}{dx}\right] = \frac{\text{Pressure}}{\text{distance}} = [ML^{-2}T^{-2}]$,

$[\eta] = [ML^{-1}T^{-1}]$ and $[r] = [L]$

Substituting in equation, So, equating the powers of M, L, and T on both sides, we get **$a + c = 0$, $-a + b - 2c = 3$, and $-a - 2c = -1$**

We have three unknowns *a*, *b*, and *c*. We have three equations, on solving, we get $a = -1$, $b = 4$, and $c = 1$

Therefore, equation becomes, $v = k\eta^{-1} r^4 \left(\frac{P}{l}\right)^1$

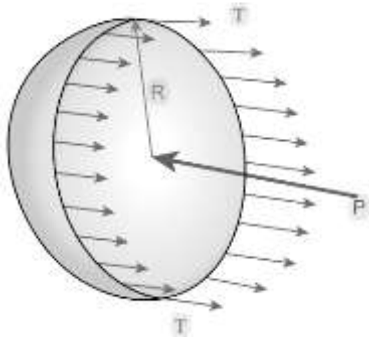
Experimentally, the value of *k* is shown to be $\frac{\pi}{8}$, we have $v = \frac{\pi r^4 P}{8\eta l}$

41. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.

i) Excess of pressure inside air bubble in a liquid.

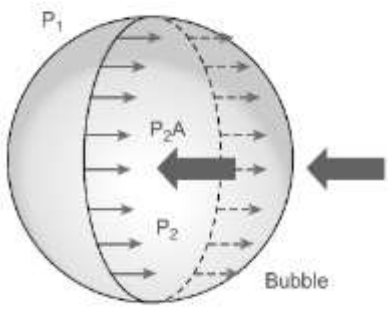
Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is

$\Delta P = P_1 - P_2$. To find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble.



ii) Excess pressure inside a soap bubble.

Consider a soap bubble of radius *R* and the surface tension of the soap bubble be T as shown in Figure. **A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other** outside the bubble. Therefore, the force on the soap bubble due to surface tension is **$2 \times 2\pi RT$**. The various forces acting on the soap bubble are,



i) **Force due to surface tension $F_T = 4\pi RT$ towards right**

ii) Force due to **outside pressure $F_{P1} = P_1\pi R^2$ towards right**

iii) Force due to **inside pressure $F_{P2} = P_2\pi R^2$ towards left**

As the bubble is in equilibrium, $F_{P2} = F_T + F_{P1}$

$P_2\pi R^2 = 4\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

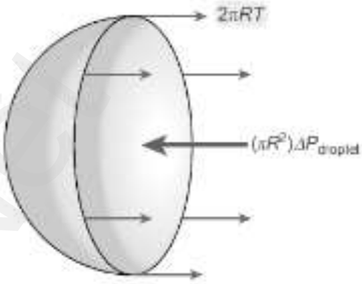
iii) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure. The various forces acting on the liquid drop are,

i) Force due to **surface tension $F_T = 2\pi RT$ towards right**

ii) Force due to **outside pressure $F_{P1} = P_1\pi R^2$ towards right**

iii) Force due to inside pressure $F_{P2} = P_2\pi R^2$ towards left



As the liquid drop is in equilibrium, **$F_{P2} = F_T + F_{P1}$**

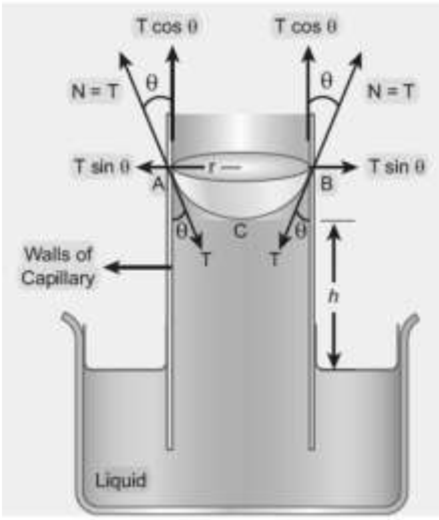
$P_2\pi R^2 = 2\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{2T}{R}$

42. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.

Consider a capillary tube which is held vertically in a beaker containing water; **the water rises in the capillary tube to a height h due to surface tension.**

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into **two components i) Horizontal component Tsinθ and ii) Vertical component Tcosθ acting upwards,** all along the whole circumference of the meniscus.



Total upward force = (Tcosθ) (2πr) = 2πrTcosθ

Where θ is the angle of contact, *r* is the radius of the tube. Let ρ be the density of water and *h* be the height to which the liquid rises inside the tube.

$$\text{Then,}\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{Volume of the liquid}\\ \text{column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{Volume of liquid of}\\ \text{radius r and height}\\ \text{r} - \text{Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$$

$$V = \pi r^2 h + \left(\pi r^2 x\, r - \frac{2}{3}\pi r^3\right) \Rightarrow \pi r^2 h + \frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

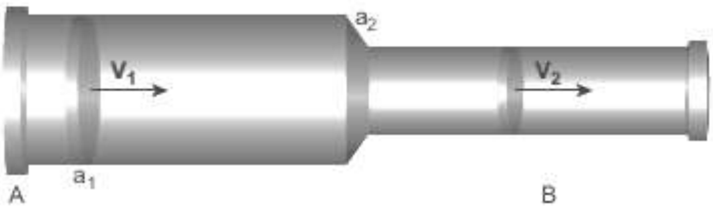
$$2\pi rT\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow T = \frac{r\left(h+\frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$$T = \frac{r\rho g h}{2\cos\theta}$$

43. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure.



Let m_1 be the mass of fluid flowing through section *A* in time Δt,

$\mathbf{m_1 = (a_1 v_1 \Delta t)\, \rho}$

Let m_2 be the mass of fluid flowing through section *B* in time Δt,

$\mathbf{m_2 = (a_2 v_2 \Delta t)\, \rho}$

For an **incompressible liquid, mass is conserved $m_1 = m_2$**

$$a_1 v_1 \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$$

$$\mathbf{a_1 v_1 = a_2 v_2 \Rightarrow a\, v = constant}$$

which is called **the equation of continuity and it is a statement of conservation of mass in the flow of fluids.**

In general, a v = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

44. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

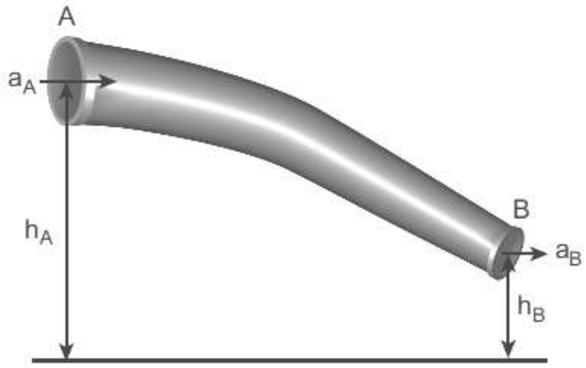
Bernoulli's theorem:

According to Bernoulli's theorem, **the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.**

$\frac{P}{\rho} + \frac{1}{2}v^2 + gh =$ **Constant**, this is known as Bernoulli's equation.

Proof:

Let us consider a flow of liquid through a pipe AB as shown in Figure. Let V be the volume of the liquid when it enters A in a time *t* which is equal to the volume of the liquid leaving *B* in the same time. Let a_A, v_A and PA be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at *A* respectively.



Let the **force exerted by the liquid at A is $F_A = P_A a_A$**

Distance travelled by the liquid in time t is $d = v_A t$

Therefore, the work done is $W = F_A d = P_A a_A v_A t$

But $\mathbf{a_A v_A t = a_A d = V}$, volume of the liquid entering at A.

Thus, the **work done is the pressure energy (at A), $W = F_A d = P_A V$**

Pressure energy per unit volume at A = $\frac{\text{Pressure energy}}{\text{Volume}} = \frac{P_A V}{V} = P_A$

Pressure energy per unit mass at A = $\frac{\text{Pressure energy}}{\text{Mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is $E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$

Potential energy of the liquid at A, $P_{EA} = mg\,h_A$,

Due to the flow of liquid, the **kinetic energy of the liquid at A, $KE_A = \frac{1}{2} mV_A^2$**

Therefore, the total energy due to the flow of liquid at A,

$$E_A = EP_A + KE_A + PE_A$$

$$\mathbf{E_A = m\frac{P_A}{\rho} + \frac{1}{2} mV_A^2 + mgh_A}$$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get $\mathbf{E_B = m\frac{P_B}{\rho} + \frac{1}{2} mV_B^2 + mgh_B}$

From the **law of conservation of energy, $E_A = E_B$**

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2} mV_A^2 + mgh_A = E_B = m\frac{P_B}{\rho} + \frac{1}{2} mV_B^2 + mgh_B$$

$$\mathbf{\frac{P_A}{\rho} + \frac{1}{2} V_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2} V_B^2 + gh_B = constant}$$

Thus, the above equation can be written as $\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \textbf{constant}$

45. Write any two applications of Bernoulli's theorem.

(a) Blowing off roofs during wind storm:

1) In olden days, **the roofs of the huts or houses were designed with a slope**. One important scientific reason is that as per the Bernoulli's principle**, it will be safeguarded except roof during storm or cyclone.**
2) **During cyclonic condition, the roof is blown off without damaging the other parts of the house.**
3) In accordance with the **Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1.**
4) The pressure under the roof P_2 is greater. Therefore, this **pressure difference ($P_2 - P_1$) creates an up thrust and the roof is blown off.**

(b) Aerofoil lift:

1) The wings of an airplane (aerofoil) are so designed that its **upper surface is more curved than the lower surface and the front edge** is **broader than the real edge.**
2) As the aircraft moves, the **air moves faster above the aerofoil than at the bottom.**
3) According to Bernoulli's Principle, **the pressure of air below is greater than above, which creates an up-thrust called the dynamic lift to the aircraft.**

46. Write the applications of elasticity.

1) The elastic behavior is one such property which especially decides the **structural design of the columns and beams of a building.**
2) As far as the **structural engineering is concerned**, the amount of stress that the **design could withstand is a primary safety factor**.
3) A bridge has to be designed in such a way that it should have the **capacity to withstand the load of the flowing traffic, the force of winds**, and even its own weight.
4) The elastic behavior or in other **words the bending of beams is a major** concern over **the stability of the buildings or bridges.**
5) To reduce the bending of a beam for a given load, one should **use the material with a higher Young's modulus of elasticity (Y).**
6) The Young's modulus of steel is **greater than aluminium or copper. Iron comes next to steel.**
7) This is the reason why steel is mostly preferred in **the design of heavy duty machines and iron rods in the construction of buildings**.

UNIT – VIII (HEAT AND THERMODYNAMICS)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. 'An object contains more heat'- is it a right statement? If not why?

Heat is not a quantity. **Heat is energy in transit which flows from higher temperature object to lower temperature object.** Once the heating process is stopped we cannot use the word heat. When we use the word 'heat', it is the **energy in transit but not energy stored in the body.** An Object has more heat is wrong, instead object is hot will be appropriate.

2. Obtain an ideal gas law from Boyle's and Charles' law.

1) Acceleration to Boyle's law P $\alpha \frac{1}{V}$
2) Acceleration to Charle's law V α T. By combining these two equations We have PV = CT. Here C is a positive constant.
3) So we can write the constant C as k times the number of particles N. Here k is the **Boltzmann constant (1.381×10^{-23} JK^{-1})** and it is found to be a **universal constant**. So the **ideal gas law** can be stated as follows **PV = NkT**

3. Define one mole.

One mole of any substance is **the amount of that substance which contains Avogadro number (N_A) of particles (such as atoms or molecules)**.

4. Define specific heat capacity and give its unit.

Specific heat capacity of a substance is defined as **the amount of heat energy required to raise the temperature of 1kg of a substance by 1 Kelvin or 1°C**

ΔQ = ms ΔT

Therefore, s = $\frac{1}{m}\frac{\Delta Q}{\Delta T}$

Where, s – Specific heat capacity of a substance and its value depends only on the nature of the substance not amount of substance.

ΔQ - Amount of heat energy ; ΔT - Change in temperature ;

m – Mass of the substance; **The SI unit for specific heat capacity is $Jkg^{-1}K^{-1}$**

5. Define molar specific heat capacity.

Molar specific heat capacity is defined as **heat energy required to increase the temperature of one mole of substance by 1K or 1°C.** C = $\frac{1}{\mu}\frac{\Delta Q}{\Delta T}$

Here C is known as molar specific heat capacity of a substance and μ is number of moles in the substance.

The **SI unit for molar specific heat capacity is $J\ mol^{-1}\ K^{-1}$.**

6. What is a thermal expansion?

Thermal expansion is **the tendency of matter to change in shape, area, and volume due to a change in temperature.**

All three states of matter (solid, liquid and gas) expand when heated. When **a solid is heated, its atoms vibrate with higher amplitude** about their fixed points. The relative change in the size of solids is small.

7. Define latent heat capacity. Give its unit.

Latent heat capacity of a substance is defined **as the amount of heat energy required to change the state of a unit mass of the material.**

Q = m x L ; L = $\frac{Q}{m}$

Where L = Latent heat capacity of the substance; Q = Amount of heat m = mass of the substance. The SI unit for Latent heat capacity is $J\ kg^{-1}$.

8. State Stefan-Boltzmann law.

Stefan Boltzmann law states that, **the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.**

E α T^4 or E = σT^4 ; Where, **σ is known as Stefan's constant.** Its value is $5.67\times10^{-8}\ Wm^{-2}\ k^{-4}$

9. What is Wien's law?

Wien's law states that, **the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body**. $\lambda_m \alpha \frac{1}{T}$ or $\lambda_m = \frac{b}{T}$. Where, **b is known as Wien's constant. Its value is 2.898× 10^{-3} m K**

10. Define thermal conductivity. Give its unit.

The **quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions** is known as thermal conductivity of a material.
$\frac{Q}{L} = \frac{KA\Delta T}{L}$; Where, K is known as the coefficient of thermal conductivity.
The SI unit **of thermal conductivity is J s^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1}.**

11. What is a black body?

A black body is an object **that absorbs all electromagnetic radiations. It is a perfect absorber and radiator of energy with no reflecting power.**

12. What are the different types of thermodynamic systems?

Open system can **exchange both matter and energy** with the environment.
Closed system exchange energy, but not matter with the environment.
Isolated system can **exchange neither energy nor matter** with the environment.

13. What is meant by 'thermal equilibrium'?

Two systems are said to be in **thermal equilibrium with each other if they are at the same temperature, which will not change with time**.

14. What is mean by state variable? Give example.

In thermodynamics, the state of a thermodynamic system is **represented by a set of variables called thermodynamic variables.**
Examples: Pressure, temperature, volume and internal energy etc.

The values of these **variables completely describe the equilibrium state of a thermodynamic system**.

15. What are intensive and extensive variables? Give examples.

Extensive variable depends on the size or mass of the system.
Example: Volume, total mass, entropy, internal energy, heat capacity etc.
Intensive variables do not depend on the size or mass of the system.
Example: Temperature, pressure, specific heat capacity, density etc.

16. What is an equation of state? Give an example.

Equation of state:

The equation which connects the state variables in a specific manner is called equation of state. **A thermodynamic equilibrium is completely specified by these state variables by the equation of state**. If the system is not in thermodynamic equilibrium, then **these equations cannot specify the state of the system.**

Example of equation of state called vander Waals equation. Real gases obey this equation at thermodynamic equilibrium. The air molecules in the room truly obey vander Waals equation of state. But at room temperature with low density we can approximate it into an ideal gas.

17. State Zeroth law of thermodynamics.

The zeroth law of thermodynamics states that **if two systems, A and B, are in thermal equilibrium with a third system, C, then A and B are in thermal equilibrium with each other.**

18. Define the internal energy of the system.

The internal energy of a thermodynamic system is **the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.**

The energy due to molecular motion including translational, rotational and vibrational motion is called internal kinetic energy (E_K) The energy due to molecular interaction is called internal potential energy (E_P).

Example: Bond energy. $U = E_K + E_P$

19. Define one calorie.

The amount of heat required at a pressure of standard atmosphere to **raise the temperature of 1g of water 1^0C.**

20. State the first law of thermodynamics.

Change in internal energy (ΔU) of the system is **equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings**.

21. Define the quasi-static process.

A quasi-static process is an **infinitely slow process in which the system changes its variables (P,V,T)** so slowly such that it remains in **thermal, mechanical and chemical equilibrium with its surroundings throughout**. By this infinite slow variation, the system is always almost close to equilibrium state.

22. What is PV diagram?

PV diagram is a **graph between pressure P and volume V of the system**. The P-V diagram is **used to calculate the amount of work done by the** gas **during expansion** or on the gas during compression.

23. Give the equation of state for an adiabatic process.

The equation of state for an adiabatic process is given by PV^{γ} **= Constant.** Here γ is called adiabatic exponent $\left(\gamma = \frac{C_p}{C_v}\right)$ which depends on the nature of the gas. **The equation implies that if the gas goes from an equilibrium state (P_i,V_i) to another equilibrium state (P_f ,V_f) adiabatically then it satisfies the relation.**

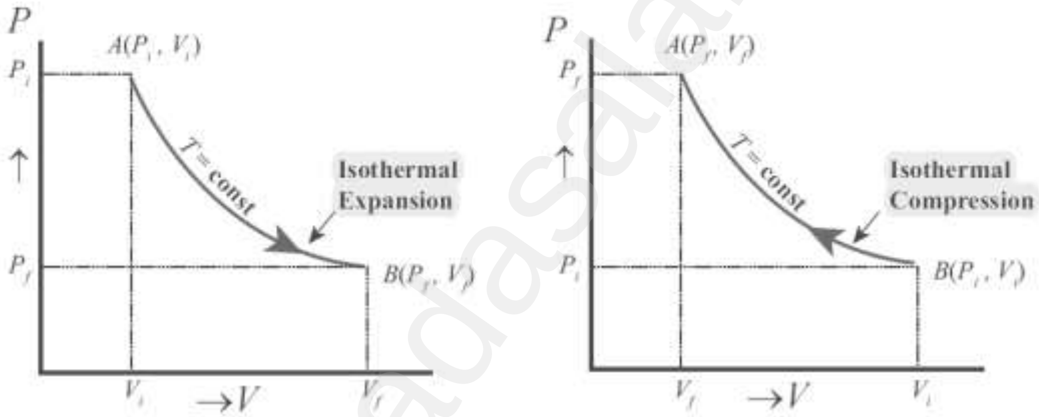
24. Give an equation state for an isochoric process.

The equation of state for an **isochoric process** is given by P= $\left(\frac{\mu R}{V}\right)$T,

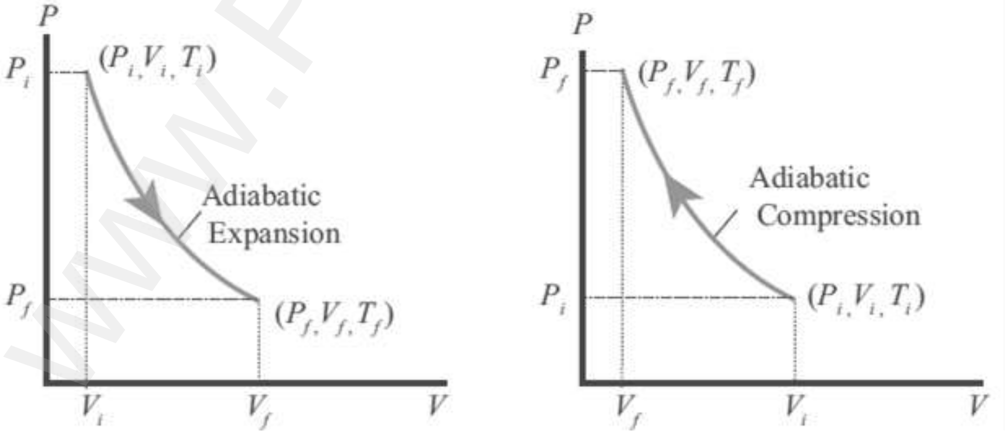
Where, $\left(\frac{\mu R}{V}\right)$ = Constant

25. Draw the PV diagram for ;

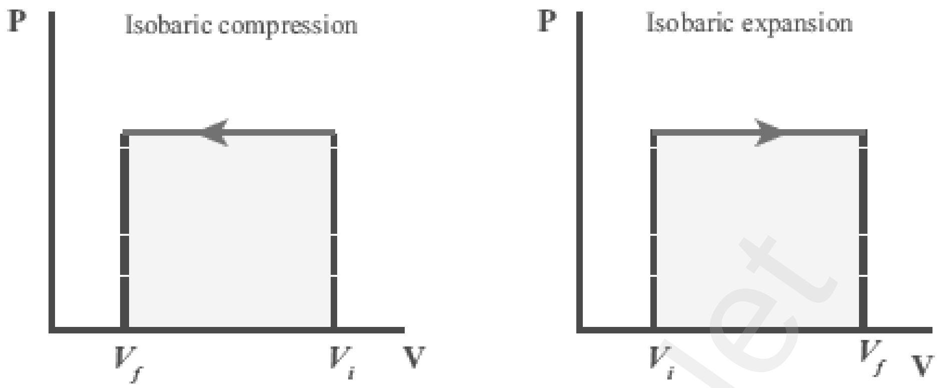
a. Isothermal process



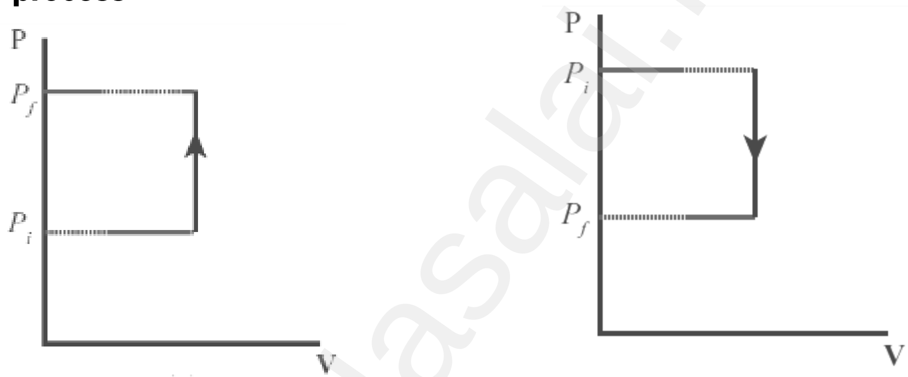
b. Adiabatic process



c. isobaric process



d. Isochoric process



26. **What is a cyclic process?**

This is a thermodynamic process in which the thermodynamic system returns **to its initial state after undergoing a series of changes**. Since **the system comes back to the initial state, the change in the internal energy is zero.** In cyclic process, **heat can flow in to system and heat flow out of the system.**

27. **What is meant by reversible and irreversible processes?**

Reversible process: A thermodynamic process can be considered **reversible only if it possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states** as in the initial, direct process. Example: **A quasi–static isothermal expansion of gas, slow compression and expansion of a spring.**

Irreversible process: All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

28. **State Clausius form of the second law of thermodynamics**

"Heat always flows from hotter object to colder object spontaneously". This is known as the **Clausius form of second law of thermodynamics.**

29. State Kelvin-Planck statement of second law of thermodynamics.

Kelvin-Planck statement: It is impossible to construct **a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work.** This implies that **no heat engine in the universe can have 100% efficiency.**

30. Define heat engine.

Heat engine is **a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.**

31. State the second law of thermodynamics in terms of entropy.

"For all the processes that occur in nature (irreversible process), **the entropy always increases**. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.

32. Define the coefficient of performance.

COP is a **measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor W. COP =** $\beta = \frac{Q_L}{W}$

33. Can water be boiled without heating?

Yes, at low pressure, **the water boils fast at low temperature below the room temperature, when the pressure is made low**, the water starts boiling without supplying any heat.

34. As air is a bad conductor of heat, why do we not feel warm without clothes?

This is conductor when we are without clothes air carries **away heat from our body due to convection and hence we feel cold.**

35. Why is it hotter at the same distance over the top of a fire than in front of it?

At a point in front of fire, heat is received due to the process of radiation only, while at a point above the fire, **heat reaches both due to radiation and convection.**

36. Define Triple point.

Triple point the triple point of a substance is **the temperature and pressure at which the three phases (gas, liquid and solid)** of that substance coexist in thermodynamic equilibrium. The **triple point of water is at 273.1 K**

37. Write the applications of thermal conversion.

1) **Boiling water in a cooking pot is an example of convection**. Water at the bottom of the pot receives more heat. Due to heating, **the water expands and the density of water decreases at the bottom.**
2) Due to this decrease in density, molecules rise to the top. At the same time the molecules at the **top receive less heat and become denser and come to the bottom of the pot.**
3) This process goes on continuously**. The back and forth movement of molecules is called convection current.**
4) To keep the room warm, we use room heater. **The air molecules near the heater will heat up and expand.**
5) As they expand, the density of **air molecules will decrease and rise up while the higher density cold air will come down**. This circulation of air molecules are called convection current.

38. Write the main features of Prevost theory?

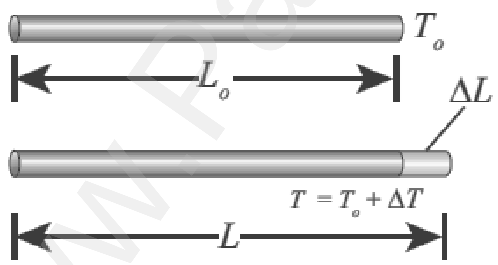
1) Every object emits heat radiations at all finite temperatures (except 0 K) as well as it absorbs radiations from the surroundings. For example, **if you touch someone, they might feel your skin as either hot or cold.**
2) A body at high temperature radiates more heat to the surroundings than it receives from it. Similarly, **a body at a lower temperature receives more heat from the surroundings than it loses to it.**
3) Prevost applied the idea of **'thermal equilibrium'** to radiation. He suggested that **all bodies radiate energy but hot bodies radiate more heat than the cooler bodies**. At one point of time the rate of exchange of heat from both the bodies will become the same. Now the bodies are said to be in 'thermal equilibrium'. Only at absolute zero temperature a body will stop emitting.

FIVE MARKS QUESTION WITH ANSWER:

39. Explain in detail the thermal expansion.

1) Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.
2) All three states of matter (solid, liquid and gas) expand when heated. **When a solid is heated, its atoms vibrate with higher amplitude about their fixed points**. The relative change in the size of solids is small. **Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle**. Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes.
3) **Liquids**, have less intermolecular forces than solids and hence they expand more than solids. This is the **principle behind the mercury thermometers.**
4) In the case of **gas molecules, the intermolecular forces are almost negligible and hence they expand much more than solids**. For example, in **hot air balloons when gas particles get heated, they expand and take up more space.**
5) The **increase in dimension of a body due to the increase in its temperature is called thermal expansion.**
6) The expansion in length is called **linear expansion**. Similarly, the expansion in area is termed as **area expansion** and the expansion in volume is termed as **volume expansion**.

Linear Expansion:



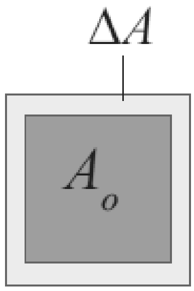
In solids, for a small change in temperature ΔT, the fractional change in length $\left(\frac{\Delta L}{L}\right)$ is directly proportional to ΔT. $\frac{\Delta L}{L} = \alpha_L \Delta T$

Therefore, $\alpha_L = \frac{\Delta L}{L\Delta T}$; Where, α_L = coefficient of linear expansion.

ΔL = Change in length; L = Original length ;

ΔT = Change in temperature.

Area Expansion:



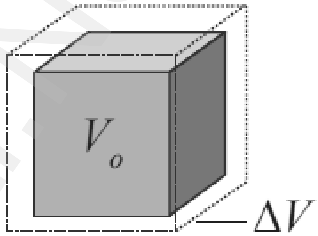
For a small change in temperature ΔT the fractional change in area $\left(\frac{\Delta A}{A}\right)$ of a substance is directly proportional to ΔT and it can be written as $\frac{\Delta A}{A} = \alpha_A \Delta T$ Therefore,

$\alpha_A = \frac{\Delta A}{A \Delta T}$; Where, αA = coefficient of area expansion.

ΔA = Change in area; A = Original area;

ΔT = Change in temperature

Volume Expansion:



For a small change in temperature ΔT the fractional change in volume $\left(\frac{\Delta V}{V}\right)$ of a substance is directly proportional to ΔT.

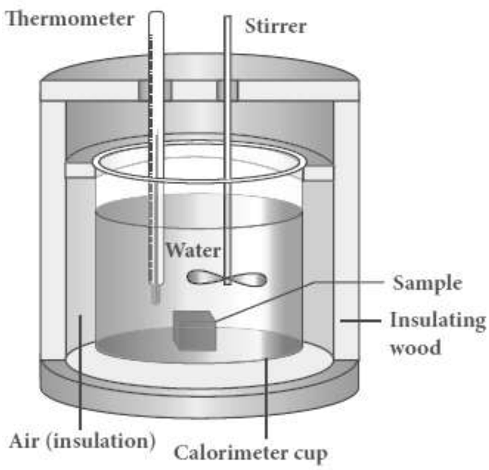
$\frac{\Delta V}{V} = \alpha_V \Delta T$, Therefore, $\alpha_V = \frac{\Delta V}{V \Delta T}$

Where, αV = coefficient of volume expansion;
ΔV = Change in volume; V = Original volume;
ΔT = Change in temperature. Unit of coefficient of linear, area and volumetric expansion of solids is $^\circ C^{-1}$ or K^{-1}

40. Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.

Calorimetry:

1) Calorimetry means **the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process.** When a body at higher temperature is brought in contact with **another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body.** No heat is allowed to escape to the surroundings. It can be mathematically expressed as **$Q_{gain} = -Q_{lost}$; $Q_{gain} + Q_{lost} = 0$**



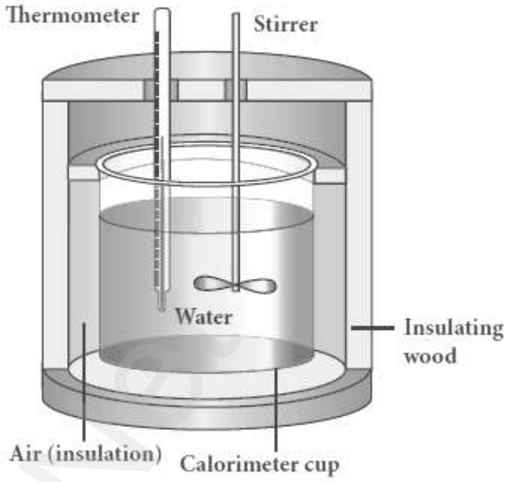
2) Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure.

3) A sample **is heated at high temperature (T_1) and immersed into water at room temperature (T_2) in the calorimeter.** After some time both sample and water reach a final equilibrium temperature T_f . Since **the calorimeter is insulated, heat given by the hot sample is equal to heat gained by the water.** It is shown in the Figure.

$\mathbf{Q_{gain} = -\,Q_{lost}}$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity



$$Q_{gain} = m_2 s_2 (T_f - T_2) \; ;$$

$$Q_{lost} = m_1 s_1 (T_f - T_1)$$

Here s_1 and s_2 specific heat capacity of hot sample and water respectively. So we can write

$$m_2 s_2 (T_f - T_2) = - m_1 s_1 (T_f - T_1)$$

$$\mathbf{m_2 s_2 T_f - m_2 s_2 T_2 = - m_1 s_1 T_f + m_1 s_1 T_1}$$

$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

The final temperature $T_f = \dfrac{m_1 S_1 T_1 + m_2 S_2 T_2}{m_1 S_1 + m_2 S_2}$

41. Discuss various modes of heat transfer.

Conduction:

Conduction is the process of **direct transfer of heat through matter due to temperature difference**. When two objects are in **direct contact with one another, heat will be transferred from the hotter object to the colder one. Thermal conductivity depends on the nature of the material.**

Convection:

Convection is the process in which **heat transfer is by actual movement of molecules in fluids such as liquids and gases**. In convection, **molecules move freely from one place to another.**

Radiation:

Radiation is a form of **energy transfer from one body to another by electromagnetic waves.** Radiation which requires **no medium to transfer energy from one object to another.**

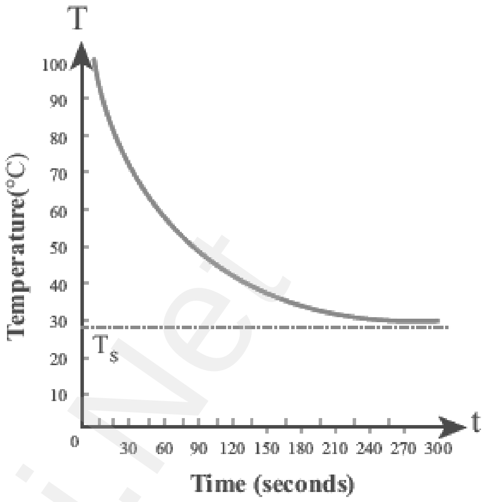
Example: 1. Solar energy from the Sun. 2. Radiation from room heater.

42. Explain in detail Newton's law of cooling.

Newton's law of cooling:

1) Newton's law of cooling states that the **rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings .**

$\frac{dQ}{dt} \alpha$ – (T – T) ------------ 1



2) The **negative sign indicates that the quantity of heat lost by liquid goes on decreasing with time.** Where, T = Temperature of the object

T_S = Temperature of the surrounding.

From the **graph** in Figure, it is clear that **the rate of cooling is high initially and decreases with falling temperature.**

3) Let us consider an object of mass m and specific heat capacity s at temperature T. Let **Ts be the temperature of the surroundings.** If the temperature falls by a small amount dT in time dt, then the amount of heat lost is, **dQ = msdT** --------------- 2

4) Dividing both sides of equation (2) by $\frac{dQ}{dt} = \frac{\text{msdT}}{dt}$ ------------3

From Newton's law of cooling $\frac{dQ}{dt} \alpha$ – (T – T_S)

$\frac{dQ}{dt}$ = – a (T – T_S) ---------- 4

Where a is some positive constant. From equation (2) and (4)

– a (T – T_S) = ms $\frac{dT}{dt}$

$\frac{dT}{(T - TS)} = -\frac{a}{ms}dt$ -------------------- 5

Integrating equation (5) on both sides,

$\int_0^{\infty} \frac{dT}{(T - TS)} = -\int_0^t \frac{a}{ms}\,dt$

In (T – T_S) = $\frac{a}{ms}$t + b_1

Where b_1 is the constant of integration. taking exponential both sides, we get,

T = T_S + b_2 e $\frac{a}{ms}$t . Here b_2 = eb_1 = Constant

43. Discuss the

a. Thermal equilibrium **b. Mechanical equilibrium**

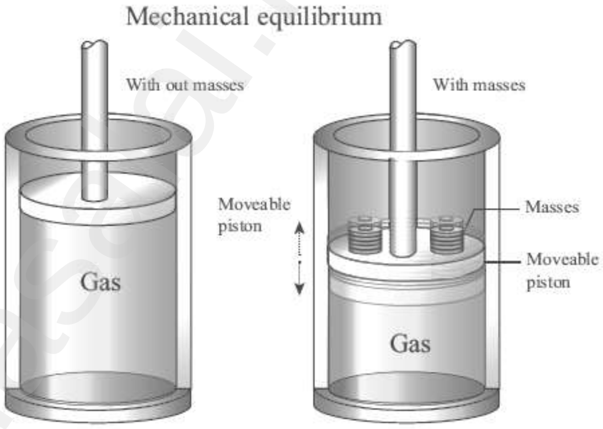
c. Chemical equilibrium **d. Thermodynamic equilibrium.**

a. Thermal equilibrium:

Two systems are said to be in **thermal equilibrium with each other if they are at the same temperature, which will not change with time.**

b. Mechanical equilibrium:

Consider a gas container with piston as shown in Figure. When some mass is placed on the piston, it will **move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position**. When the downward gravitational force given by the **piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium**. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermodynamic system or on the surrounding by thermodynamic system.



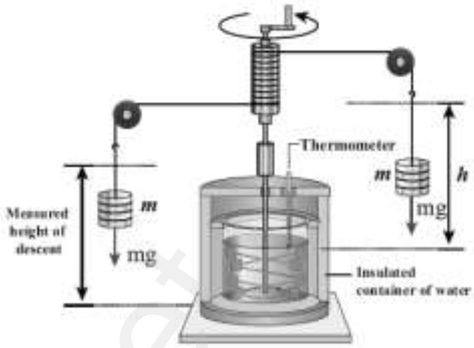
c. Chemical equilibrium:

If there is **no net chemical reaction between two thermodynamic systems in contact** with each other then it is said to be in chemical equilibrium.

d. Thermodynamic equilibrium:

If two systems are set to be in thermodynamic equilibrium, then the **systems are at thermal, mechanical and chemical equilibrium with each other**. In a state of thermodynamic equilibrium, the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

44. Explain Joule's Experiment of the mechanical equivalent of heat.



1) Joule showed that **mechanical energy can be converted into internal energy** and vice versa. In his experiment, two masses were attached with a rope and a paddle wheel.
2) When these masses fall through a distance h due to gravity, **both the masses lose potential energy equal to 2mgh.**
3) When the masses fall, **the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel.**
4) This causes a rise in temperature of the water. This implies that **gravitational potential energy is converted to internal energy of water.**
5) The temperature of water increases due to the work done by the masses. In fact, **Joule was able to show that the mechanical work has the same effect as giving heat.**
6) He found that to raise 1 g of an object by 1°C, 4.186 J of energy is required. In earlier days the heat was measured in calorie. **1 cal = 4.186 J**.This is called Joule's mechanical equivalent of heat.

45. Derive Mayer's relation for an ideal gas.

Meyer's relation

1) Consider μ mole of an ideal gas in a container with volume V, pressure P and temperature T.
2) When **the gas is heated at constant volume the temperature increases by dT. As no work is done by the gas, the heat that flows into the system will increase only the internal energy**. Let the change in internal energy be dU.
If Cv is the molar specific heat capacity at constant volume,
$$dU = \mu C_v dT \text{ ------------- } 1$$
3) Suppose the gas is heated at constant pressure so that the temperature increases by dT. If 'Q' is the heat supplied in this process and 'dV' the change in volume of the gas. $Q = \mu C_p dT$ ----------- 2
4) If W is the work done by the gas in this process, then
$$W = PdV \text{ ------------3}$$
But from the first law of thermodynamics, $Q = dU + W$ --------4
Substituting equations (1), (2) and (3) in (4), we get,
$$\mu C_p dT = \mu C_v dT + PdV \text{ -----------5}$$
5) For mole of ideal gas, the equation of state is given by
$$PV = \mu RT \Rightarrow PdV + VdP = \mu RdT \text{ --------- } 6$$
Since the pressure is constant, dP=0
$$\therefore C_p dT = C_v dT + RdT$$
$$\therefore C_P = C_v + R \text{ (or) } C_p - C_v = R \text{ ---------- } 7$$
This relation is called Meyer's relation

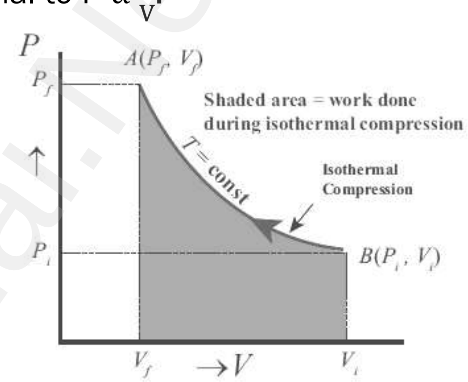
46. Explain in detail the isothermal process.

Isothermal process

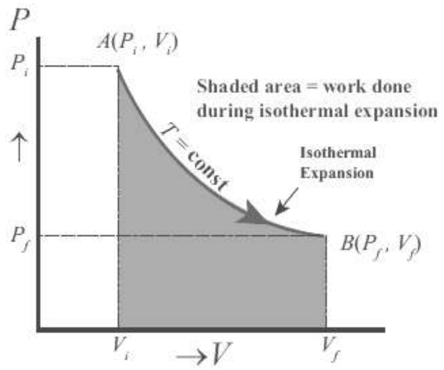
1) It is a process in which **the temperature remains constant but the pressure and volume of a thermodynamic system will change**. The ideal gas equation is PV= μRT , Here, T is constant for this process
So the equation of state for isothermal process is given by
PV = constant ----- 1

2) This implies that if the gas goes from one equilibrium state (P_1,V_1) to another equilibrium state (P_2,V_2) the following relation holds for this process **$P_1V_1 = P_2V_2$** -------2

3) Since PV = constant, P is inversely proportional to P $\alpha \frac{1}{V}$.
This implies that **PV graph is a hyperbola. The pressure-volume graph for constant temperature is also called isotherm. PV diagram for quasi-static isothermal expansion and quasi-static isothermal compression.**



4) We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the **internal energy is also constant.**
This implies that **dU or ΔU = 0**. For an isothermal process, **the first law of thermodynamics** can be written as follows, **Q = W** ----------- 3



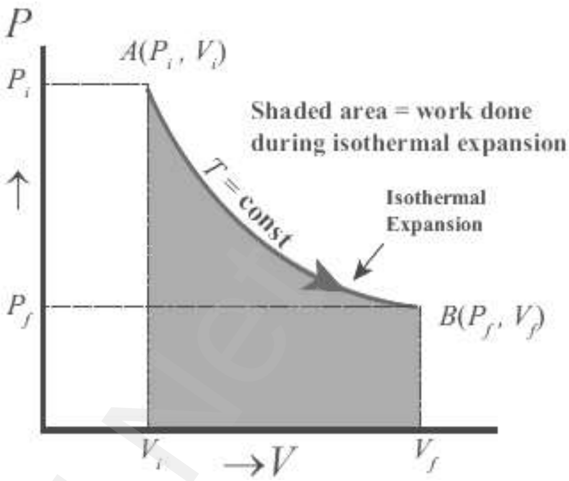
5) From equation (3), we infer that the heat **supplied to a gas is used to do only external work.**

6) The isothermal compression takes place when the **piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact.**

47. Derive the work done in an isothermal process

Work done in an isothermal process:

1) Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state (P_i,V_i) to the final state (P_f , V_f). We can calculate the work done by the gas during this process. From equation the work done by the gas,



W = $\int_{V_i}^{V_f} PdV$ ----------- 1

2) As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

P = $\frac{\mu RT}{V}$ ----------------2

Substituting equation (2) in (1) we get,

W = $\int_{V_i}^{V_f} \frac{\mu RT}{V} dV$;   W = $\mu RT \int_{V_i}^{V_f} \frac{dV}{V}$ ----------- 3

In equation (3), we take μRT out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (3),

we get W = $\mu RT \ln\left(\frac{V_f}{V_i}\right)$ ------------ 4

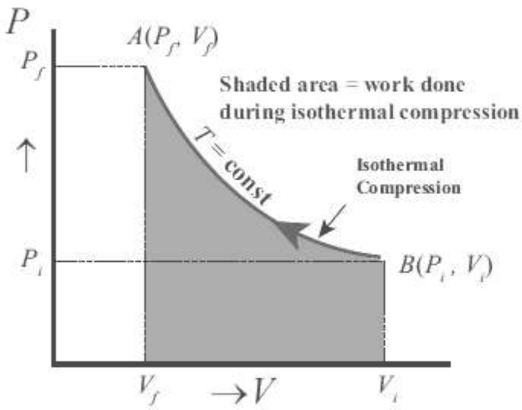
3) Since we have an isothermal expansion, $\frac{V_f}{V_i} > 1$, So $\ln\left(\frac{V_f}{V_i}\right) > 0$.

As a result, the work done by the gas during an isothermal expansion is positive.

The above result in equation (4) is true for isothermal compression also.

But in an isothermal compression $\frac{V_f}{V_i} < 1$, So $\ln\left(\frac{V_f}{V_i}\right) < 0$. As a result, the work done on the gas in an isothermal compression is negative.

4) In the **PV diagram the work done during the isothermal expansion is equal to the area under the graph**. Similarly, for an isothermal compression, **the area under the PV graph is equal to the work done on the gas which turns out to be the area with a negative sign.**



48. Explain in detail an adiabatic process.

Adiabatic process:

1) This is a process in which **no heat flows into or out of the system (Q=0). But the gas can expand by spending its internal energy or gas can be compressed through some external work.** So the pressure, volume and temperature of the system may change in an adiabatic process.

2) The equation of state for an adiabatic process is given by

PV^{γ} **= Constant**------------1

Here γ is called adiabatic exponent $\left(\gamma = \frac{C_p}{C_v}\right)$ which depends on the nature of the gas.

3) The equation (1) implies that if the gas goes from an equilibrium state (P_i,V_i) to another equilibrium state (P_f ,V_f) adiabatically then it satisfies the relation

$P_iV_i^{\gamma} = P_fV_f^{\gamma}$ ------------ 2

4) The **PV diagram of an adiabatic expansion and adiabatic compression process.** The PV diagram for an adiabatic process is also called **adiabat.**

5) Note that the PV diagram for isothermal and adiabatic processes look similar. But **actually the adiabatic curve is steeper than isothermal curve.**

6) To rewrite the equation (1) in terms of T and V. From ideal gas equation, the pressure P = $\frac{\mu RT}{V}$. Substituting this equation in the equation (1), we have $\frac{\mu RT}{V}V^{\gamma}$ = Constant or $\frac{T}{V}V^{\gamma} = \frac{Constant}{\mu R}$

7) Note here that is another constant. So it can be written as

$TV^{\gamma-1}$ **= Constant** -----------3

The equation (3) implies that if the gas goes from an initial equilibrium state (T_i, V_i) to final equilibrium state (T_f, V_f) adiabatically then it satisfies the **relation** $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$ -------------- 4

The equation of state for adiabatic process can also be written in terms of T and P as $T^{\gamma}P^{1-\gamma}$ = constant.

49. Derive the work done in an adiabatic process

Work done in an adiabatic process:

1) Consider μ moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. **A frictionless and insulating piston of cross sectional area A is fitted in the cylinder.** Let W be the work done when the system goes from the initial state (P_i,V_i,T_i) to the final state (P_f,V_f,T_f) adiabatically. W = $\int_{V_i}^{V_f} PdV$ -------------- 1

2) By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is PV^{γ} = constant (or) P = $\frac{Constant}{V^{\gamma}}$ can be substituted in the equation (1), we get $W_{adia} = \int_{V_i}^{V_f} \frac{Constant}{V^{\gamma}} dV$

$= \text{Constant} \int_{V_i}^{V_f} V^{\gamma} dV$

$$= \text{Constant}\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{\text{Constant}}{1-\gamma}\left[\frac{\text{Constant}}{V_f^{\gamma-1}} - \frac{\text{Constant}}{V_i^{\gamma-1}}\right]$$

But, $P_iV_i^\gamma = P_fV_f^\gamma$ = constant.

$$W_{adia} = \frac{1}{1-\gamma}\left[\frac{P_fV_f^\gamma}{V_f^{\gamma-1}} - \frac{P_iV_i^\gamma}{V_i^{\gamma-1}}\right]$$

$$W_{adia} = \frac{1}{1-\gamma}[P_fV_f - P_iV_i] \text{ ------- } 2$$

From ideal gas law, $P_fV_f = \mu RT_f$ and $P_iV_i = \mu RT_i$

Substituting in equation (2), we get, $W_{adia} = \frac{\mu R}{\gamma-1}[T_i - T_f]$

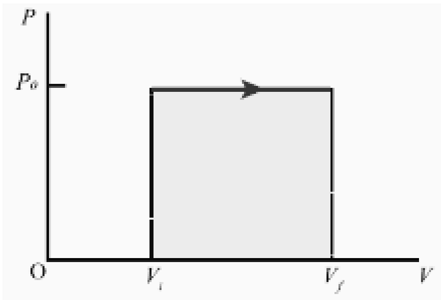
3) In adiabatic expansion, work is done by the gas. i.e., W_{adia} is positive. As $T_i > T_f$, **the gas cools during adiabatic expansion. In adiabatic compression, work is done on the gas. i.e., W_{adia} is negative.** As $T_i < T_f$, the temperature of the gas increases during adiabatic compression.

50. Explain the isobaric process and derive the work done in this process

Isobaric process:

1) This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$V = \left(\frac{\mu R}{P}\right)T$ ---------1 Here $\frac{\mu R}{P}$ = Constant



2) In an isobaric process the temperature is directly proportional to volume. $V \propto T$ (Isobaric process) ---- (2)
This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

3) If a gas goes from a state (V_i, T_i) to (V_f, T_f) at constant pressure, then the system satisfies the following equation $\frac{T_f}{V_f} = \frac{T_i}{V_i}$

The work done in an isobaric process:

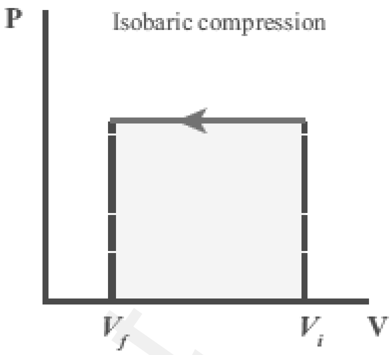
Work done by the gas W = $\int_{V_i}^{V_f} PdV$

In an isobaric process, the pressure is constant, so P comes out of the integral,

$W = P\int_{V_i}^{V_f} dV$ W = P [$V_f - V_i$] = PΔV ----------3

4) Where ΔV denotes change in the volume. If ΔV is negative, W is also negative. This implies that the work is done on the gas. If ΔV is positive, W is also positive, implying that work is done by the gas.

5) The equation (3) can also be rewritten using the **ideal gas equation.**



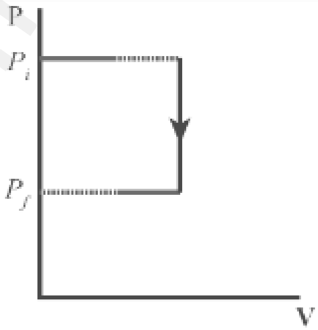
From ideal gas equation **PV = μRT and** $V = \frac{\mu RT}{P}$ Substituting this in equation (3) we get, $W = \mu RT_f\left(1 - \frac{T_i}{T_f}\right)$

6) In the PV diagram, **area under the isobaric curve is equal to the work done in isobaric process**. The shaded area in the following Figure is equal to the work done by the gas.

7) The first law of thermodynamics for isobaric process is given by $\Delta U = Q - P\Delta V$

51. Explain in detail the isochoric process.

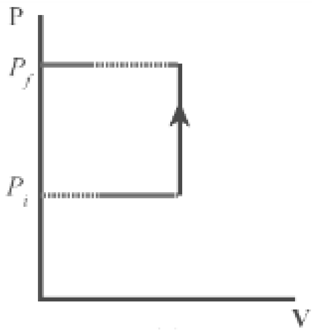
Isochoric process:



1) This is a thermodynamic process in which the volume of the system is kept constant. But pressure, **temperature and internal energy continue to be variables**. The pressure - volume graph for an isochoric process is a vertical line parallel to pressure axis.

2) The equation of state for an isochoric process is given by $P = \left(\frac{\mu R}{V}\right)$; Where, $\left(\frac{\mu R}{V}\right)$ = Constant

It that the pressure is **directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin.** If a gas goes from state (P_i, T_i) to (P_f, T_f) at constant volume, then the system satisfies the following equation $\frac{P_i}{T_i} = \frac{P_f}{T_f}$



For an isochoric process, $\Delta V = 0$ and $W = 0$. Then the first law becomes $\Delta U = Q$

3) Implying that **the heat supplied is used to increase only the internal energy.** As a result, the temperature increases and pressure also increases.

4) Suppose a **system loses heat to the surroundings through conducting walls by keeping the volume constant**, then its **internal energy decreases**. As a result, the **temperature decreases; the pressure also decreases.**

52. What are the limitations of the first law of thermodynamics?

Limitations of first law of thermodynamics

The first law of **thermodynamics explains well the inter convertibility of heat and work**. But it does **not indicate the direction of change.**

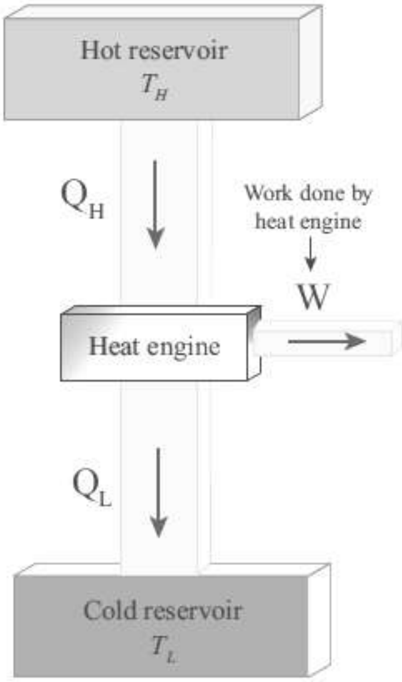
For example,

a. When a hot object is in contact with a cold object, **heat always flows from the hot object to cold object but not in the reverse direction.** According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature **the direction of heat flow is always from higher temperature to lower temperature.**

b. When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this **heat is not reconverted to the kinetic energy of the car. So the first law is not sufficient to explain many of natural phenomena.**

53. Explain the heat engine and obtain its efficiency.

Heat engine is a device which takes **heat as input and converts this heat in to work by undergoing a cyclic process.**



A heat engine has three parts:

(a) Hot reservoir (b) Working substance

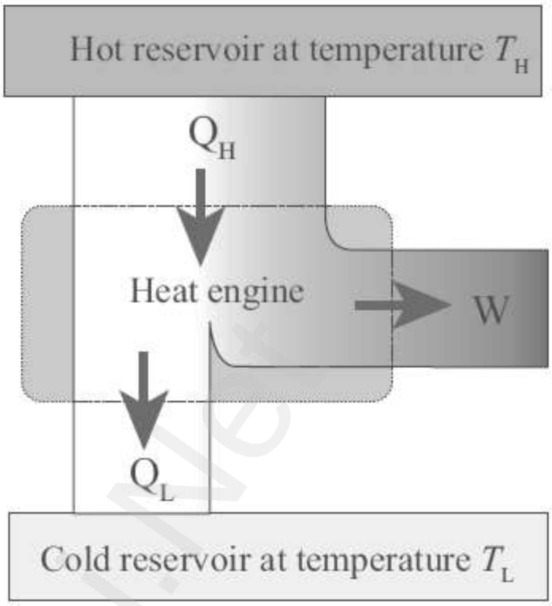
(c) Cold reservoir

A Schematic diagram for heat engine is given below in the figure

1) **Hot reservoir (or) Source:**
It supplies heat to the engine. It is always maintained at a high temperature T_H

2) **Working substance:** It is a substance like gas or water, which converts the heat supplied into work.

i) A simple example of a heat engine is a steam engine. **In olden days' steam engines were used to drive trains**. The working substance in these is water which absorbs heat from the burning of coal.



ii) The heat converts the water into steam. This steam is does work by rotating the wheels of the train, thus making the train move.

3) **Cold reservoir (or) Sink:** The heat engine ejects some amount of heat (Q_L) **in to cold reservoir after it doing work.** It is always maintained at a low temperature T_L.
For example, in the automobile engine, the cold reservoir is the surroundings at room temperature. The automobile ejects heat to these surroundings through a silencer.

4) The **heat engine works in a cyclic process.** After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, **the change in the internal energy of the heat engine is zero.**

5) The efficiency of the heat engine is defined as the ratio of the work done (output) to the heat absorbed (input) in one cyclic process. Let the working substance absorb heat Q_H units from the source and reject Q_L units to the sink after doing work W units
We can write **Input heat = Work done + ejected heat**

$$Q_H = W + Q_L$$
$$W = Q_H - Q_L$$

Then the efficiency of heat engine $\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

6) Note here that Q_H, Q_L and W all are taken as positive, a sign convention followed in this expression.
Since $Q_L < Q_H$, **the efficiency (η) always less than 1. This implies that heat absorbed is not completely converted into work.** The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

54. Explain in detail the working of a refrigerator.

Refrigerator:

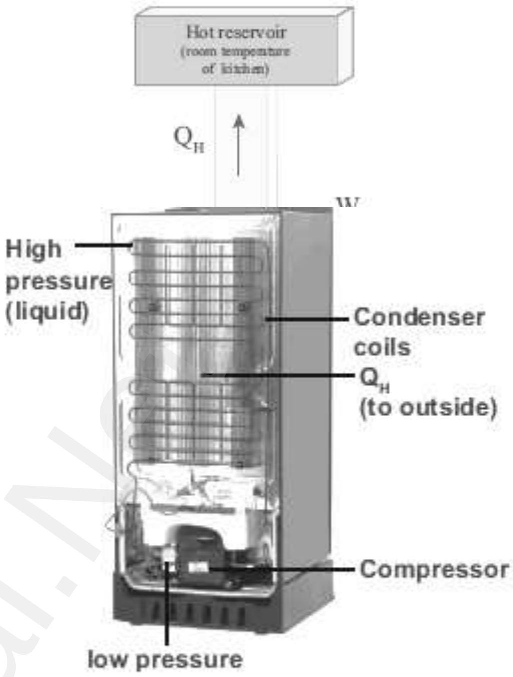
A refrigerator is a **Carnot's engine working in the reverse order.**

Working Principle:

The working substance (gas) absorbs a quantity of heat Q_L from the cold body (sink) at a lower temperature T_L. **A certain amount of work W is done on the working substance by the compressor and a quantity of heat Q_H is rejected to the hot body (source) ie, the atmosphere at T_H.** When you stand beneath of refrigerator, you can feel warmth air. rom the first law of thermodynamics,

we have $Q_L + W = Q_H$

As a result, the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.



UNIT – IX (KINETIC THEORY OF GASES)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. What is the microscopic origin of pressure?

With the help **of kinetic theory of gases, the pressure is linked to the velocity of molecules.** $P = \frac{1}{3}\frac{N}{V} mV^{-2}$ m – mass of a molecule;

N - Avogadro Number; V – Volume; V^{-2} – Avogadro velocity molecules.

2. What is the microscopic origin of temperature?

Average Kinetic Energy / Molecule : KE = ε = $\frac{3}{2}$ NkT

3. Why moon has no atmosphere?

The escape speed of gases on **the surface of Moon is much less than the root mean square speeds of gases due to low gravity. Due to this all the gases escape from the surface of the Moon.**

4. Write the expression for rms speed, average speed and most probable speed of a gas molecule.

$V_{rms} = \sqrt{\frac{3RT}{M}}$; V_{ave} $\overline{V} = \sqrt{\frac{8RT}{\pi M}}$; $V_{mp} = \sqrt{\frac{2RT}{M}}$

5. Define the term degrees of freedom.

The minimum number of independent coordinates needed to specify **the position and configuration of a thermo-dynamical system in space** is called the degree of freedom of the system.

6. State the law of equipartition of energy.

According to kinetic theory, the average kinetic energy of system of **molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom** (x or y or freedom will get ½ kT of energy. This is called law of equipartition of energy.

7. Define mean free path and write down its expression.

Average distance travelled **by the molecule between collisions is called mean free path (λ).** We can calculate the mean free path based on kinetic theory.

8. Deduce Charles' law based on kinetic theory.

Charles' law: From the equation P = $\frac{2}{3}\frac{U}{V} = \frac{2}{3}$u we get PV = $\frac{2}{3}$ U

For a fixed pressure, the volume of **the gas is proportional to internal energy of the gas or average kinetic energy of the gas and the average kinetic energy is directly proportional to absolute temperature.**

It implies that V α T or $\frac{V}{T}$ = Constant.

9. Deduce Boyle's law based on kinetic theory.

Boyle's law: From the equation P = $\frac{2}{3}\frac{U}{V} = \frac{2}{3}$u we get PV = $\frac{2}{3}$ U

But the internal energy of an ideal gas is equal to N times the average kinetic energy (∈) of each molecule. U = N∈

For a fixed temperature, the average translational kinetic energy ∈ will remain constant. It implies that **PV = $\frac{2}{3}$N∈ Thus PV = constant**

Therefore, pressure of a given **gas is inversely proportional to its volume provided the temperature remains constant. This is Boyle's law.**

10. List the factors affecting the mean free path.

1) Mean free path **increases with increasing temperature.** As the **temperature increases, the average speed of each molecule will increase.** It is the reason why the smell of hot sizzling food reaches several meter away than smell of cold food.

2) Mean free **path increases with decreasing pressure of the gas and diameter of the gas molecules.**

11. What is the reason for Brownian motion?

According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from **all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random** and zig–zag manner.

12. What are the factors which affect Brownian motion?

1) Brownian motion **increases with increasing temperature.**
2) Brownian **motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.**

13. Why No hydrogen in Earth's atmosphere?

As the **root mean square speed of hydrogen is much less than that of nitrogen, it easily escapes from the earth's atmosphere**. In fact, the presence of nonreactive nitrogen instead of highly combustible hydrogen deters many disastrous consequences.

14. What is an ideal gas? (or) What is perfect gas?

An **ideal gas is that gas which obeys the gas laws.** i.e. Charle's law, Boyle's law etc, at all **values of temperature and pressure.** Molecules of such a gas should be free from intermolecular attraction.

FIVE MARKS QUESTION WITH ANSWER

15. Write down the postulates of kinetic theory of gases.

1) All the molecules of a gas **are identical, elastic spheres**.
2) The molecules of **different gases are different.**
3) The number of molecules in a gas is **very large and the average** separation **between them is larger than size of the gas molecules.**
4) The molecules of a gas are in **a state of continuous random motion.**
5) The molecules collide with one another and also with the walls of the container.
6) These **collisions are perfectly elastic so that there is no loss of kinetic energy during collisions.**
7) Between two successive collisions, **a molecule moves with uniform velocity.**
8) The molecules do not exert any force of attraction or repulsion on each other except during collision. **The molecules do not possess any potential energy and the energy is wholly kinetic.**
9) The collisions are instantaneous. The time spent by a molecule in each collision is very small compared to the time elapsed between two consecutive collisions.
10) These **molecules obey Newton's laws of motion even though they move randomly.**

16. Explain in detail the kinetic interpretation of temperature.

1) To understand the microscopic origin of temperature in the same way,

Rewrite the equation P = nm$\overline{v_x^2}$

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2} \;;\; PV = \frac{1}{3}m\overline{v^2} \text{ -------------1}$$

Comparing the equation (1) with ideal gas equation PV=NkT,

$$NkT = \frac{1}{3}Nm\overline{v^2} \;;$$

$$KT = \frac{1}{3}m\overline{v^2} \text{ ---------------2}$$

Multiply the above equation by 3/2 on both sides,

$$\frac{3}{2}KT = \frac{1}{2}m\overline{v^2} \text{ ------------------- 3}$$

R.H.S of the equation (3) is called average kinetic energy of a single molecule ($\overline{KE}$). Th e average kinetic energy per molecule

$$\overline{KE} = \epsilon = \frac{3}{2}KT \text{ ----------- 4}$$

2) Equation (3) implies that **the temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas. Equation 4 is a very important result from kinetic theory of gas.** We can infer the following from this equation.

3) The average kinetic energy of the molecule is **directly proportional to absolute temperature of the gas. The equation (3) gives the connection between the macroscopic world** (temperature) to microscopic world (motion of molecules).

4) The **average kinetic energy of each molecule depends only on temperature of the gas not on mass of the molecule**. In other words, if the temperature of an ideal gas is measured using thermometer, the **average kinetic energy of each molecule can be calculated without seeing the molecule through naked eye.**

5) By multiplying **the total number of gas molecules with average kinetic energy of each molecule, the internal energy of the gas is obtained.**

Internal energy of ideal gas U = N$\left(\frac{1}{2}m\overline{v^2}\right)$

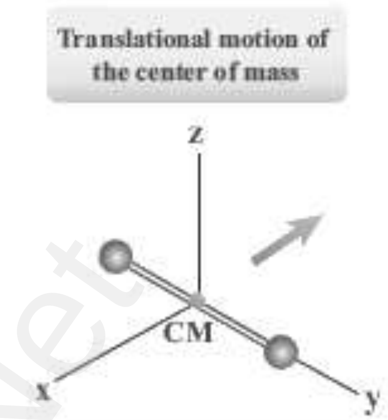
By using equation (3) U = $\frac{3}{2}$NKT ---------- 5

From equation (5), we understand that the internal energy of an ideal gas depends only on absolute temperature and is independent of pressure and volume.

17. Describe the total degrees of freedom for mono-atomic molecule, diatomic molecule and tri-atomic molecule.

Mono-atomic molecule: A mono-atomic molecule by virtue of its nature has only three translational degrees of freedom. Therefore, f = 3

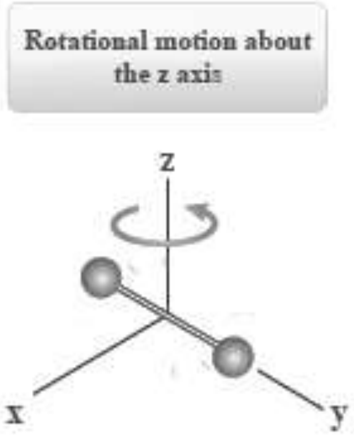
Example: Helium, Neon, Argon



Diatomic molecule: There are two cases.

1) **At Normal Temperature A molecule of a diatomic gas consists of two atoms bound to each other by a force of attraction. Physically the molecule can be regarded** as a system of two point masses fixed at the ends of a mass less elastic spring. The center of mass lies in the center of the diatomic molecule. So, **the motion of the center of mass requires three translational degrees of freedom** (figure a). In addition, the diatomic molecule can rotate about three mutually perpendicular axes (figure b). But the moment of inertia about its own axis of rotation is negligible (about y axis in the figure). Therefore, **it has only two rotational degrees of freedom (one rotation is about Z axis and another rotation** is about Y axis). Therefore, totally there are five degrees of freedom.

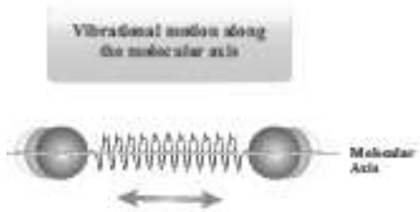
f = 5



2) **At High Temperature At a very high temperature such as 5000 K, the diatomic molecules possess additional two degrees of freedom due to vibrational** motion [one due to kinetic energy of vibration and the other is due to potential energy] (Figure c). So totally there are seven degrees of freedom.

f = 7.

Examples: Hydrogen, Nitrogen, Oxygen.



3) **Tri-atomic molecules** There are two cases.

Linear tri-atomic molecule In this type, two atoms lie on either side of the central atom as shown in the Figure. **Linear tri-atomic molecule has three translational degrees of freedom. It has two rotational degrees of freedom because it is similar to diatomic molecule except there is an additional atom at the center.** At normal temperature, linear tri-atomic molecule will have five degrees of freedom. **At high temperature it has two additional vibrational degrees of freedom. So a linear tri-atomic molecule has seven degrees of freedom. Example: Carbon dioxide**

Non-linear tri-atomic molecule In this case, the three atoms lie at the vertices of a triangle as shown in the Figure. **It has three translational degrees of freedom and three rotational degrees of**

freedom about three mutually orthogonal axes. The total degrees of freedom, f = 6

Example: Water, Sulphurdioxide.

18. Derive the ratio of two specific heat capacities of mono-atomic, diatomic and Tri-atomic molecules.

Application of law of equipartition energy in specific heat of a gas:

Meyer's relation $C_P - C_V = R$ connects the two specific heats for one mole of an ideal gas. Equipartition law of energy is used to calculate the value of $C_P - C_V$ and the ratio between them $\gamma = \frac{C_P}{C_V}$. Here γ is called adiabatic exponent.

i) **Monatomic molecule:**

Average kinetic energy of a molecule = $\left[\frac{3}{2}\mathbf{kT}\right]$

Total energy of a mole of gas $\frac{3}{2}$ kT x N_A ; = $\frac{3}{2}$ RT

For one mole, the molar specific heat at constant volume

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{3}{2}RT\right]$$

$$C_V = \left[\frac{3}{2}R\right] ; C_P = C_V + R$$

$$= \frac{3}{2}R + R = \frac{5}{2}R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} \quad \gamma = 1.67$$

ii) **Diatomic molecule:**

Average kinetic energy of a diatomic molecule at low temperature = $\frac{5}{2}$ kT

Total energy of one mole of gas = $\frac{5}{2}$ kT x N_A ; = $\frac{5}{2}$ RT

(Here, the total energy is purely kinetic) For one mole Specific heat at constant volume. $C_V = \frac{dU}{dT}$; $=\left[\frac{5}{2}RT\right]$; $C_V = \frac{5}{2}R$

But, $C_P = C_V + R$

$$= \frac{5}{2}R + R = \frac{7}{2}R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$; $= \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$ $\gamma = 1.40$

Energy of a diatomic molecule at high temperature is equal to $\frac{7}{2}$ RT

$$C_V = \frac{dU}{dT} ; = \left[\frac{7}{2}RT\right] ; C_V = \frac{7}{2}R$$

But, $C_P = C_V + R$

$$= \frac{7}{2}R + R = \frac{9}{2}R$$

Note that the C_V and C_P are **higher for diatomic molecules than the mono atomic molecules.** It implies that to increase the temperature of diatomic gas molecules by 1°C it require more heat energy than mono-atomic molecules.

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7} \quad \gamma = 1.28$$

iii) Tri-atomic molecule:

a) Linear molecule:

Energy of one mole = $\frac{7}{2}$ kT x N_A ; $= \frac{7}{2}$ RT

$C_V = \frac{dU}{dT}$; $= \frac{d}{dT}\left[\frac{7}{2}RT\right]$; $C_V = \frac{7}{2}R$

But, $C_P = C_V + R$

$$= \frac{7}{2}R + R = \frac{9}{2}R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$; $= \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7}$ $\gamma = 1.28$

b) Non-linear molecule:

Energy of a mole = $\frac{6}{2}$ kT x N_A; $= \frac{6}{2}$ RT = 3RT

$C_V = \frac{dU}{dT}$; = 3R ;

But, $C_P = C_V + R$;

= 3R + R = 4R

The ratio of specific heats, $\gamma = \frac{C_P}{C_V} = \frac{4R}{3R} = \frac{4}{3}$ $\gamma = 1.33$

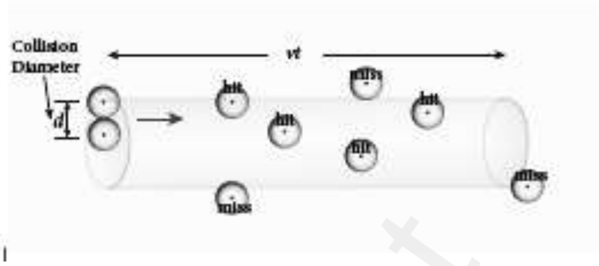
Note that according to kinetic theory model of gases the **specific heat capacity at constant volume and constant pressure are independent of temperature.** But in reality it is not sure. The specific heat capacity varies with the temperature.

19. Derive the expression for mean free path of the gas.

Expression for mean free path

1) We know from postulates of **kinetic theory that the molecules of a gas are in random motion and they collide with each other**. Between two successive collisions, a molecule moves along a straight path with uniform velocity.
2) This path is called mean free path. Consider a system of molecules each with diameter d. Let n be the number of molecules per unit volume. Assume that **only one molecule is in motion and all others** are at rest as shown in the Figure.

3) If a molecule moves with average speed v in a time t, the distance travelled is vt. In this **time t, consider the molecule to move in an imaginary cylinder of volume πd^2vt.**



4) It collides with **any molecule whose center is within this cylinder. Therefore, the number of collisions is equal to the number of molecules in the volume of the imaginary cylinder**. It is equal to πd^2vtn. The total path length divided by the number of collisions in time t is the mean free path.

Mean free path = $\frac{\text{Distance travelled}}{\text{Number of collisions}}$; $\lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2}$ ------------- 1

5) Though we have assumed that **only one molecule is moving at a time and other molecules are at rest, in actual practice all the molecules are in random motion.** So the average relative speed of one molecule with respect to other molecules has to be taken into account. After some detailed calculations (you will learn in higher classes) the correct expression for mean free path. $\lambda = \frac{1}{\sqrt{2}n\pi d^2}$ ----------------- 2

6) The equation (1) implies that **the mean free path is inversely proportional to number density. When the number density increases the molecular collisions increases** so it decreases the distance travelled by the molecule before collisions.

Case1: Rearranging the equation (2) using 'm' (mass of the molecule)

$$\lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

But mn=mass per unit volume = ρ (density of the gas)

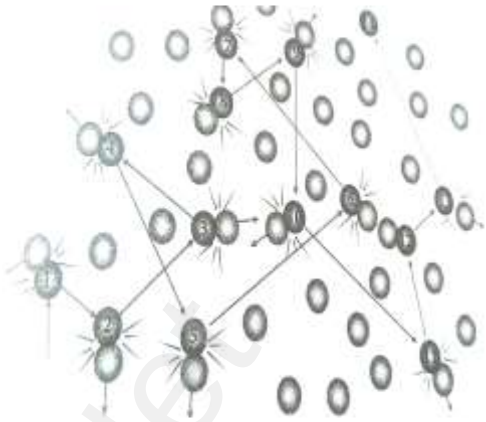
$\lambda = \frac{m}{\sqrt{2}\pi d^2 \rho}$ Also we know that PV = NkT

P = $\frac{N}{V}$ kT = nkT ; n = $\frac{P}{kT}$

Substituting n = $\frac{P}{kT}$ in equation (2), we get

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

20. Describe the Brownian motion.



1) Brownian motion is due **to the bombardment of suspended particles by molecules of the surrounding fluid.**
2) According to kinetic theory, **any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible.** This leads to the motion of the particles in a random and zig–zag manner

Factors affecting Brownian motion:

1) Brownian motion **increases with increasing temperature**.
2) Brownian motion **decreases with bigger particle size, high viscosity and density of the liquid (or) gas.**

UNIT – X (OSCILLATIONS)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. What is meant by periodic and non-periodic motion? Give any two examples, for each motion.

1) **Periodic motion** Any motion which **repeats itself in a fixed time interval** is known as periodic motion.
Examples: Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.
2) **Non-Periodic Motion** Any motion which **does not repeat itself after a regular interval of time** is known as non-periodic motion.
Example: Occurrence of Earth quake, eruption of volcano, etc.

2. What is meant by force constant of a spring?

The displacement of the particle is measured **in terms of linear displacement $\vec{r}$. The restoring force is $\vec{F} = -k\vec{r}$**, where k is a spring constant or force constant.

1) Oscillations of a loaded spring 2) Vibrations of a turning force

3. Define time period of simple harmonic motion.

The time period is defined **as the time taken by a particle to complete one oscillation. It is usually denoted by T.** For one complete revolution, the time taken is t = T, therefore, **$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$**

4. Define frequency of simple harmonic motion.

The **number of oscillations produced by the particle per second** is called frequency. It is denoted by f. **SI unit for frequency is s^{-1} or hertz** (In symbol, Hz).

Angular frequency is related to time period by $f = \frac{1}{T}$

The number of cycles (or revolutions) per second is called angular frequency.

It is usually denoted by the Greek small letter 'omega', ω.

Angular frequency and frequency are related by $\omega = 2\pi f$

SI unit for angular frequency is rad s^{-1}.

5. What is an epoch?

The displacement time t = 0 s (initial time), the phase $\varphi = \varphi_0$ is called **epoch.** (initial phase) where φ_0 is called the angle of epoch.

6. Write down the time period of simple pendulum.

The angular frequency of this oscillator (natural frequency of this system) is **$\omega^2 = \frac{g}{l} \Rightarrow \omega = \sqrt{\frac{g}{l}}$ in rads^{-1}**

The frequency of oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ in Hz, and time period of oscillations is $T = 2\pi\sqrt{\frac{l}{g}}$

7. State the laws of simple pendulum?

Law of length: For a given value of acceleration due to gravity, the time period of a simple pendulum is **directly proportional to the square root of length of the pendulum. $T \propto \sqrt{l}$**

Law of acceleration: For a fixed length, the time period of a simple pendulum is **inversely proportional to square root of acceleration due to gravity. $T \propto \frac{1}{\sqrt{g}}$**

8. Write down the equation of time period for linear harmonic oscillator.

From Newton's second law, we can write the equation for the particle executing simple harmonic motion $m\frac{d^2x}{dt^2} = kx$;

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Comparing the equation with simple harmonic motion equation, we get, $\omega = \sqrt{\frac{k}{m}}$ rad s^{-1}

Natural **frequency of the oscillator** is f **=** $\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ **Hertz.**

and the **time period of the oscillation** is T = $\frac{1}{f}$ = $2\pi\sqrt{\frac{m}{k}}$ **second.**

9. What is meant by free oscillation?

When the oscillator is **allowed to oscillate by displacing its position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator.** Such an oscillation or vibration is known as free oscillation or free vibration.

10. Explain damped oscillation. Give an example.

1) Due to the presence of friction and air drag, **the amplitude of oscillation decreases as time progresses.** It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy.

2) **The energy lost is absorbed by the surrounding medium. This type of oscillatory motion** is known as damped oscillation.
Examples (i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container. (ii) Electromagnetic oscillations in a tank circuit. (iii) Oscillations in a dead beat and ballistic galvanometers.

11. Define forced oscillation. Give an example.

In this type of vibration, **the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force**. Such vibrations are known as forced vibrations. **Example:** Sound boards of stringed instruments.

12. What is meant by maintained oscillation? Give an example.

While playing in swing, **the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source**, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example: The vibration of a tuning fork getting energy from a battery or from external power supply.

13. Explain resonance. Give an example.

The frequency of external periodic force (or driving force) matches with the **natural frequency of the vibrating body (driven). As a result, the oscillating body begins to vibrate such that its amplitude increases** at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

Example: The breaking of glass due to sound.

14. State five characteristics of SHM.

Displacement: The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement.
Velocity: The rate of change of displacement of the particle is velocity.
Acceleration: The rate of change of velocity of the particle is acceleration.
Amplitude: The maximum displacement on either side of mean position.
Time Period: The time taken by the particle executing SHM to complete one vibration.

15. Will a pendulum clock loss or gain time when taken to the top of a mountain?

On the top of the mountain, **the value of g is less than that on the surface of the earth the decreases in the value of g increases** the time period of the pendulum on the top of the mountain. So the pendulum clock loses time.

16. Why are army troops not allowed to march in steps while crossing the bridge?

Army troops are not allowed to march in **steps because it is quite likely that the frequency of the footsteps may match with the natural frequency of the bridge and due to resonance the bridge** may pick up large amplitude and break.

17. How can earthquakes cause disaster sometimes?

The resonance may cause **disaster during the earthquake, if the frequency of oscillation present within the earth per chance coincides** with natural frequency of some building, which may start vibrating with large amplitude due to resonance and may get damaged.

18. Glass windows may be broken by a far-away explosion. Explain why?

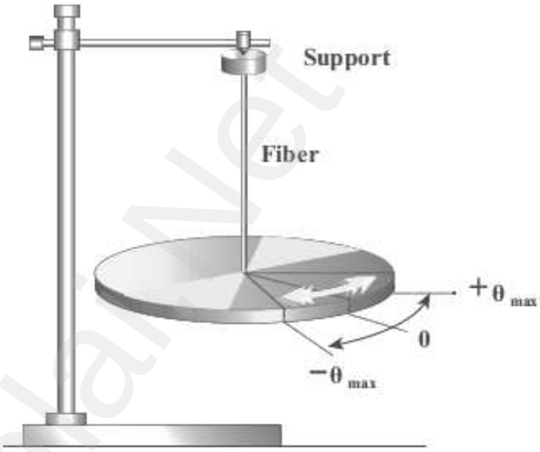
A large amplitude in all directions. As these sound waves strike the glass windows, they set them into forced oscillations.

Since glass is brittle, so the glass windows break as soon as they start oscillating due to forced oscillations.

FIVE MARKS QUESTION WITH ANSWER

19. What is meant by angular harmonic oscillation? Compute the time period of angular harmonic oscillation.

1) **When a body is allowed to rotate freely about a given axis then the oscillation** is known as the angular oscillation. The point at which the resultant **torque acting on the body is taken to be zero is called mean position.**



2) If the body is displaced from the mean position, then the resultant torque acts such that it is **proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position.** Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is $\vec{\tau}\ \alpha\ \vec{\theta}$ ------------ 1

$\vec{\tau} = -k\vec{\theta}$ ------------ 2

k is the restoring torsion constant, which is torque per unit angular displacement. If I is the moment of inertia of the body and $\vec{\alpha}$ is the angular acceleration then $\vec{\tau} = I\vec{\alpha} = -k\vec{\theta}$.

But $\vec{\alpha} = \frac{d^2\vec{\theta}}{dt^2}$ and therefore, $\vec{\alpha} = \frac{d^2\vec{\theta}}{dt^2} = \frac{k}{I}\vec{\theta}$ ------------ 3

3) This differential equation resembles simple harmonic differential equation. So, comparing equation with simple harmonic motion given in equation, we have $\omega = \sqrt{\frac{k}{I}}$ rad s^{-1} ---------- 4

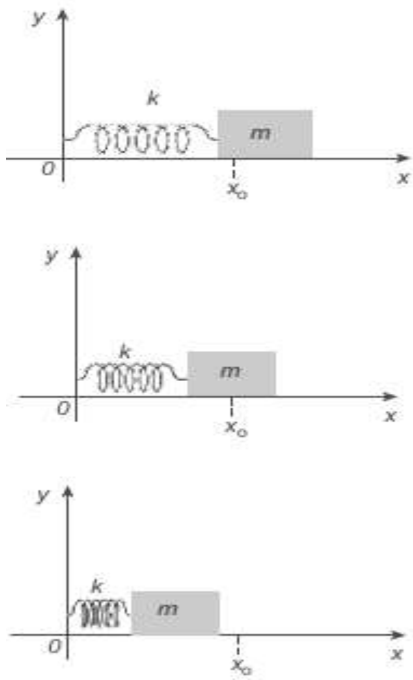
The frequency of the angular harmonic motion is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$ Hz...5

and the time period of the oscillation is $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{k}}$ **second.**

20. Write down the difference between simple harmonic motion and angular simple harmonic motion.

S. No.	Simple Harmonic Motion	Angular Harmonic Motion
1	The displacement of the particle is measured in terms of linear displacement $\vec{r}$	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$
2	Acceleration of the particle is $\vec{\boldsymbol{a}} = -\boldsymbol{\omega}^2\vec{\boldsymbol{r}}$	Angular Acceleration of the particle is $\vec{\boldsymbol{\alpha}} = -\boldsymbol{\omega}^2\vec{\boldsymbol{\theta}}$
3	Force , $\vec{F} = m\vec{a}$ where m is called mass of the particle.	Torque, $\vec{\tau} = I\vec{\alpha}$ where I is called moment of inertia of a body.
4	The restoring force $\vec{F} = -k\,\vec{r}$ where k is restoring force constant	The restoring torque $\vec{\tau} = -\kappa\,\vec{\theta}$ where k is restoring torsion constant. Note: k pronounced "kappa"
5	Angular frequency $\omega = \sqrt{\frac{k}{m}}$ rad^{-1}	Angular frequency $\omega = \sqrt{\frac{k}{I}}$ rad^{-1}

21. Explain the horizontal oscillations of a spring.



1) Consider a system containing a block of mass m attached to a mass less spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in Figure.
2) **Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed.** Suppose the mass is displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position x_0.
3) **Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block.** For one dimensional motion, mathematically, we have $F \alpha\, x$; $F = -k\,x$
4) **Where negative sign implies that the restoring force will always act opposite to the direction of the displacement**. Notice that, the restoring force is linear with the displacement.
5) This is not always true; in case if we apply **a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher** powers of x) and therefore, the oscillation of the system is not linear and hence, it is called non-linear oscillation.

6) We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).

From Newton's second law, we can write the equation for the particle executing simple harmonic motion m $\frac{d^2x}{dt^2}$ =– k x ; $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ -----------1

Comparing the equation (1) with simple harmonic motion equation, we get $\omega^2 = \frac{k}{m}$

Which means the angular frequency or natural frequency of the oscillator is $\omega = \sqrt{\frac{k}{m}}$ rsd s^{-1} ------------- 2

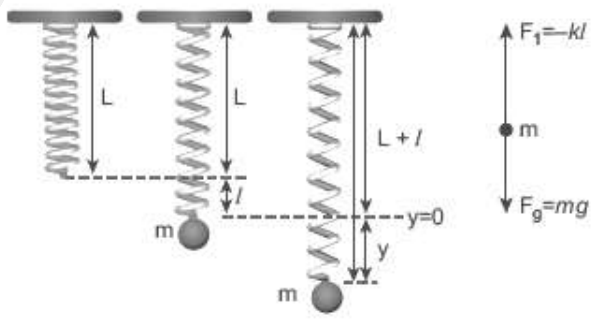
Natural frequency of the oscillator is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hertz -------- 3

and the time period of the oscillation is $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ second -------- 4

Notice that **in simple harmonic motion, the time period of oscillation is independent of amplitude**. This is valid only if the amplitude of oscillation is small.

22. Describe the vertical oscillations of a spring.

1) **Consider a mass less spring with stiffness constant or force constant k attached to a ceiling** as shown in Figure. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l.



2) **Let F_1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward.** We can draw free-body diagram for this system as shown in Figure. When the system is under equilibrium,

$F_1 + mg = 0$-------------- 1

3) But the spring elongates by small displacement l,

therefore, $F_1 \propto \boldsymbol{l} \Rightarrow F_1 = -k\,\boldsymbol{l}$ ----------- 2

Substituting equation (2) in equation (1), we get $-k\,l + mg = 0$

$mg = kl$ or $\frac{m}{k} = \frac{l}{g}$ ---------- 3

4) Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is $y + l$) is

$F_2 \propto (y + \boldsymbol{l})$ $F_2 = -k(y + \boldsymbol{l}) = -ky - k\boldsymbol{l}$ -----4

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get $-ky - kl + mg = m\frac{d^2y}{dt^2}$ —— 5

The net force acting on the mass due to this stretching is $F = F_2 + mg$

$F = -ky - kl + mg$ ———— 6

5) The gravitational force opposes the restoring force. Substituting equation (3) in equation (6), we get $F = -ky - kl + kl = -ky$

Applying Newton's law, we get $m\frac{d^2y}{dt^2} = -ky$; $m\frac{d^2y}{dt^2} = -\frac{k}{m}y$ ———— 7

6) The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ second

The time period can be rewritten using equation (3)

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$ second

The acceleration due to gravity g can be computed from the formula

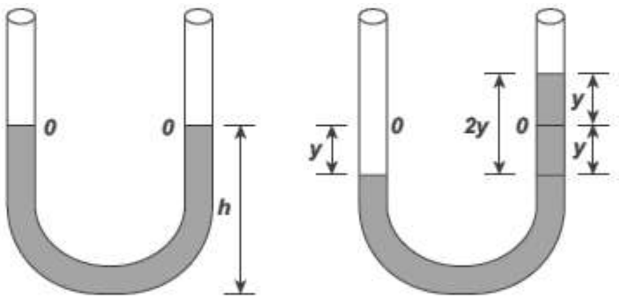
$g = 4\pi^2\left(\frac{l}{T^2}\right)ms^{-1}$

23. Write short notes on the oscillations of liquid column in U-tube.

1) **Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area A. Let us pour a non-viscous uniform incompressible liquid of density** ρ in the U-shaped tube to a height h as shown in the Figure.

2) If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position O. **It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm** (edge of the tube on either side), which balances with the atmospheric pressure.

3) **Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm**, and the liquid gets disturbed from equilibrium position O, which means, the pressure at blown arm is higher than the other arm.



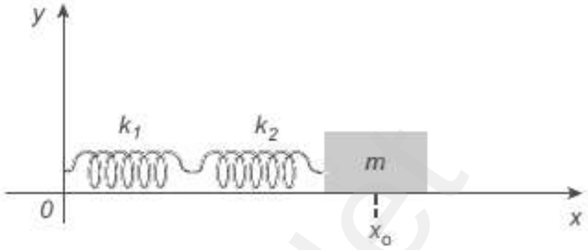
4) This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest.

Time period of the oscillation is $T = 2\pi\sqrt{\frac{l}{g}}$ second

24. Explain the effective spring constant in series connection and parallel connection

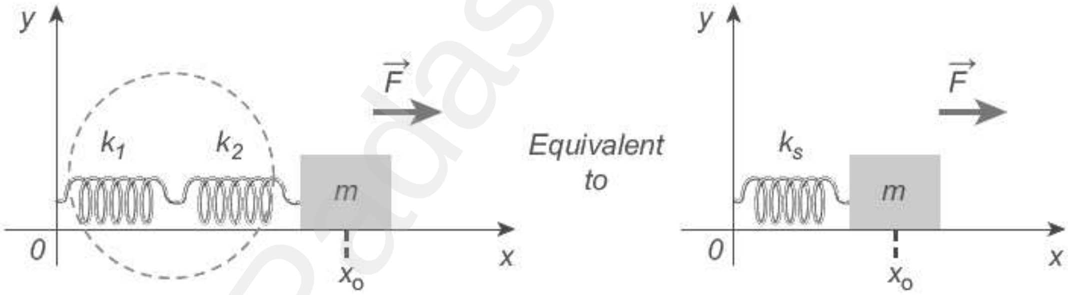
a) Springs connected in series

1) **When two or more springs are connected in series, all the springs in series with an equivalent spring (effective spring)** whose net effect is the same as if all the springs are in series connection.



2) Given the **value of individual spring constants k_1, k_2, k_3,... (known quantity), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_s (unknown quantity).** For simplicity, let us consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m as shown in Figure.

3) The results thus obtained can be generalized for any number of springs in series. Let F be the applied force towards right as shown in Figure.

4) 

Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths.

5) Let x_1 and x_2 be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force F. Then, the net displacement of the mass point is $x = x_1 + x_2$ -----------1

From Hooke's law, the net force

$F = -k_s(x_1+x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s}$ -----------2

For springs in series connection

$-k_1x_1 = -k_2x_2 = F$

$x_1 = -\frac{F}{k_1}$ and $x_2 = -\frac{F}{k_2}$ ----------3

Therefore, substituting equation (3) in equation (2), the effective spring constant can be calculated as $-\frac{F}{k_1} - \frac{F}{k_2} = \frac{F}{k_s}$

$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$ or $k_s = \frac{k_1 k_2}{k_1+k_2}$ Nm^{-1} ------------ 4

Suppose we have n springs connected in series, the effective spring constant in series is $\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ... + \frac{1}{k_n} = \sum_{i=1}^{n} \frac{1}{k_i}$ ---------5

If all spring constants are identical i.e., $k_1 = k_2 = ... = k_n = k$

then $\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n}$ ——— 6

6) This means that the effective spring constant. reduces by the factor n. Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

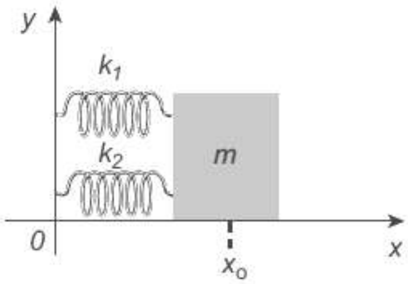
7) From equation (3), we have, $k_1x_1 = k_2x_2$ Then the ratio of compressed distance or elongated distance x_1 and x_2 is $\frac{x_2}{x_1} = \frac{k_1}{k_2}$ -----------7

The elastic potential energy stored in first and second springs are $v_1 = \frac{1}{2}k_1x_1^2$ and $v_2 = \frac{1}{2}k_2x_2^2$ respectively.

Then, their ratio is $\frac{v_1}{v_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2}$ ----------- 8

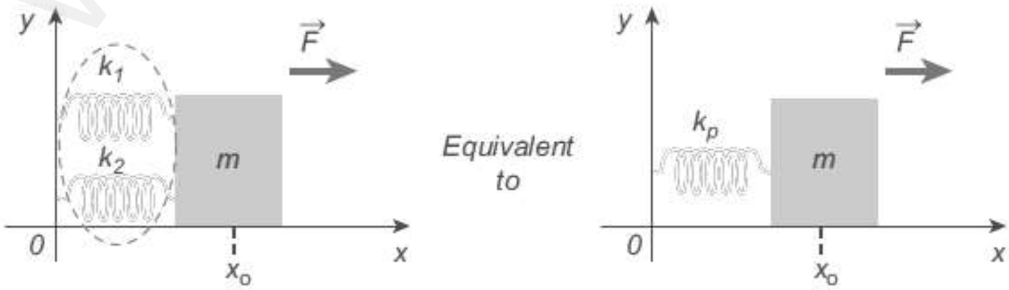
b) Springs connected in parallel

1) **When two or more springs are connected in parallel, we can replace, all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection.**



2) Given the values of individual spring constants to be k_1, k_2, k_3, ... (known quantities), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_p (unknown quantity).

3) For simplicity, let us consider only two springs of spring constants k_1 and k_2 attached to a mass m as shown in Figure. The results can be generalized to any number of springs in parallel.

4) 

Let the force F be applied towards right as shown in Figure. In this case, both the springs elongate or compress by the same amount of displacement. Therefore, net force for the displacement of mass m is

$F = -k_p x$ ------------ 1

where k_p is called effective spring constant.

5) Let the first spring be elongated by a displacement x due to force F_1 and second spring be elongated by the same displacement x due to force F_2, then the net force $F = -k_1 x - k_2 x$ ----------- 2

Equating equations (2) and (1), we get

$$k_p = k_1 + k_2 \quad \text{------------------ 3}$$

Generalizing, for n springs connected in parallel, $k_p = \sum_{i=1}^{n} k_i$ --------- 4

If all spring constants are identical i.e., $k_1 = k_2 = \dots = k_n = k$

then $k_p = nk$ ------------ 5.

6) This implies **that the effective spring constant increases by a factor n.** Hence, for the springs in parallel connection, the effective spring constant is greater than individual spring constant.

UNIT – XI (WAVES)

TWO MARKS AND THREE MARKS QUESTION WITH ANSWER:

1. What is meant by waves?

The **disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium** is known as a wave.

2. What are transverse waves? Give one example.

In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves.

Example: light (electromagnetic waves)

3. What are longitudinal waves? Give one example.

In **longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel** to the direction of propagation (direction of energy transfer) of waves.

Example: Sound waves travelling in air.

4. Define wavelength.

For **transverse waves,** the **distance between two neighbouring crests or troughs** is known as the wavelength.

For **longitudinal waves, the distance between two neighbouring compressions or rarefactions** is known as the wavelength.

The SI unit of wavelength is meter.

5. What is meant by interference of waves?

Interference is a phenomenon in which **two waves superimpose to form a resultant wave of greater, lower or the same amplitude**.

6. Explain the beat phenomenon.

When two or more waves superimpose each other with slightly different frequencies, **then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats.** The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second n = | f_1 - f_2| per second.

7. Define intensity of sound and loudness of sound.

The **intensity of sound** is defined as "**the sound power transmitted per unit area taken normal to the propagation of the sound wave**".

The **loudness of sound** is defined as "**the degree of sensation of sound produced in the ear or the perception of sound by the listener**".

8. Explain Doppler Effect.

When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.

9. Explain red shirt and blue shift in Doppler Effect.

The **spectral lines of the star are found to shift towards red end of the** spectrum **(called as red shift)** then the **star is receding away from the Earth.** Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum **(called as blue shift)** then the star is approaching Earth.

10. What is meant by end correction in resonance air column apparatus?

The antinodes are not exactly formed at the open end, we have to include a correction, called end correction. $L_1 + e = \frac{\lambda}{4}$ and $L_2 + e = \frac{3\lambda}{4}$

11. What is meant by an echo? Explain.

1) An echo is a repetition of sound produced by the reflection of sound waves from a wall, **mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m s^{-1}.** If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall.
2) After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{\text{th}}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

   Velocity = $\frac{\text{Distance travelled}}{\text{Time taken}}$; = $\frac{2d}{t}$; **2d = 344 x 0.1=34.1m; d = 17.2 m**

   The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

12. What is reverberation?

In a **closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function.** The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation.

13. Write characteristics of wave motion.

1) For the propagation of the waves, **the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.**
2) In a given medium, **the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum** at their mean position and zero at extreme positions.
3) Waves **undergo reflections, refraction, interference, diffraction and Polarization.**

FIVE MARKS QUESTION WITH ANSWER

14. Discuss how ripples are formed in still water.

1) **A stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface**. We find that this **disturbance spreads out (diverges out)** in the form of concentric **circles of ever increasing radii (ripples)** and strike the boundary of the trough.
2) This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance.
3) This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the

water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

15. Briefly explain the difference between travelling waves and standing waves.

S. No.	Progressive waves	Stationary waves
1	**Crests and troughs are formed in transverse progressive waves**, and compression and rarefaction are formed in **longitudinal progressive** waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	**Crests and troughs are formed in transverse stationary waves**, and compression and rarefaction are formed in **longitudinal stationary waves. These waves neither move** forward nor backward in a medium i.e., they will not advance in a medium.
2	All the particles in the medium vibrate such that the **amplitude of the vibration for all particles is same.**	Except at nodes, all other particles of the medium vibrate such that amplitude of **vibration is different for different particles**. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3	These wave **carry energy while propagating.**	These waves **do not transport energy.**

16. Show that the velocity of a travelling wave produced in a string is v = $\sqrt{\frac{T}{\mu}}$

1) Consider an elemental segment in the string as shown in the Figure. **Let A and B be two points on the string at an instant of time. Let d*l* and dm be the length and mass of the elemental string**, respectively. By definition, linear mass density, μ is $\boldsymbol{\mu} = \frac{dm}{dl}$ -------- 1

   dm = μdl ----------- 2

2) The elemental string **AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ** at the origin O as shown in Figure. The angle θ can be written in terms of arc length and radius as θ=$\frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is $a_{cp} = \frac{v^2}{R}$ -------------- 3

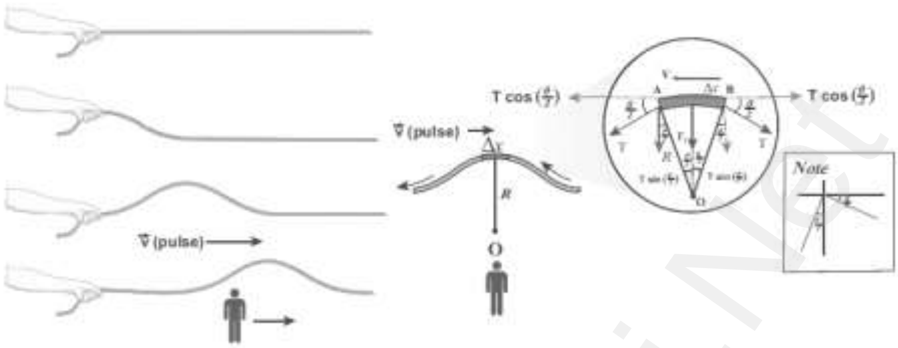
3) Then, **centripetal force can be obtained when mass of the string (dm) is included in equation** (3)

   $F_{cp} = \frac{(dm)v^2}{R}$ -------------- 4

4) The centripetal force experienced by elemental string can be calculated by substituting equation (2) in equation (4) we get,

   $\frac{(dm)v^2}{R} = \frac{\mu v^2 dl}{R}$ ----------- 5

5) The **tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored.** We can resolve T into horizontal component T $\cos\left(\frac{\theta}{2}\right)$ and vertical component T $\sin\left(\frac{\theta}{2}\right)$.



6) The **horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small,** the vertical components at A and B appears to acts vertical towards the centre of the arc and hence, they add up. The net radial force F_r is

$F_r = 2T \sin\left(\frac{\theta}{2}\right)$ ---------6

7) Since the amplitude of **the wave is very small when it is compared with the length of the string, the sine of small angle is approximated as $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$.** Hence, equation (6) can be written as

$F_r = 2T \times \frac{\theta}{2} = T\,\theta$ --------7

8) But $\theta = \frac{dl}{R}$, therefore substituting in equation (7),

we get $F_r = T\frac{dl}{R}$ -----------8

Applying Newton's second law **to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force.** Hence equating equation (5) and equation (8), we have

$T\frac{dl}{R} = \mu v^2 \frac{dl}{R}$ v = $\sqrt{\frac{T}{\mu}}$ measured in ms^{-1} --------9

17. Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.

1) Newton assumed that when sound propagates in air, the formation of **compression and rarefaction takes place in a very slow manner** so that the process is isothermal in nature.

2) That is, the **heat produced during compression (pressure increases, volume decreases)**, and **heat lost during rarefaction (pressure decreases, volume increases)** occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law,

PV = Constant ----------- 1

3) Differentiating equation (1), we get **PdV + VdP = 0 or**

$P = -V\frac{dP}{dV} = B_T$ ------- 2

where, BT is an isothermal bulk modulus of air. Substituting equation (2) in equation $V = \sqrt{\frac{B}{\rho}}$ the speed of sound in air is

$V_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}}$ -------------3

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$\mathbf{P = (0.76 \times 13.6 \times 10^3 \times 9.8)\ N\ m^{-2}}$

ρ = 1.293 kg m-3. here ρ is density of air

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is $V_T = \sqrt{\frac{0.76 \text{ x} 13.6 \text{ x } 10^3 \text{x } 9.8}{1.293}}$ **=279.80 ms^{-1} ≈ 280 ms-1**

(theoretical value)

But the speed of sound in air at 0°C is experimentally observed as **332ms^{-1} which is close upto 16% more than theoretical value**

(Percentage error is $\frac{(332-280)}{332}$ x 100% = 15.6%) This error is not small.

Laplace's correction:

1) Laplace assumed that when the sound propagates through a medium, the particles oscillate very rapidly such that the **compression and rarefaction occur very fast.** Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat.

2) Since, temperature is no longer considered as a constant here, **sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law** (not Boyle's law as Newton assumed), which is

Pv^{γ} = Constant ----------4

Where, $\gamma = \frac{C_P}{C_V}$, which is the ratio between specific heat at constant pressure and specific heat at constant volume. Differentiating equation (4) on both the sides, we get

v^{γ} dP + P$(\gamma V \gamma^{-1} dV)$ = 0 or γ^{P}= $-V\frac{dP}{dV}$ B_A ------------5

where, BA is the adiabatic bulk modulus of air. Now, substituting equation (5) in equation V = $\sqrt{\frac{B}{\rho}}$ the speed of sound in air is

$V_A = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} V_T$ ------------6

V_A = 331ms^{-1}

18. Discuss the law of transverse vibrations in stretched strings.

i) The law of length:

For a given wire with **tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length.**

Therefore, $f \alpha \frac{1}{l} \Rightarrow f = \frac{c}{l} \Rightarrow l \times f$ = C, where C is a constant.

ii) The law of tension:

For a given **vibrating length *l* (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T**, $f \alpha \sqrt{T}$

$\Rightarrow f = A\sqrt{T}$ where A is a constant

iii) The law of mass:

For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ, f $\alpha \frac{1}{\sqrt{\mu}} \Rightarrow f = \frac{B}{\sqrt{\mu}}$, where B is a constant.

19. Explain the concepts of fundamental frequency, harmonics and overtones in detail.

1) Keep the **rigid boundaries at x = 0 and x = L and produce a standing wave by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced**. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary y(x = 0, t) = 0 and y(x = L, t) = 0 ------------1

Since the nodes formed are at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left[\frac{\lambda_n}{2}\right]$=L

2) where n is an integer, L is the **length between the two boundaries and λ_n is the specific wavelength that satisfy the specified boundary** conditions. Hence, $\lambda_n = \left(\frac{2L}{n}\right)$ ---------------2

3) Therefore, **not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions**, i.e., for n = 1, the first mode of vibration has specific wavelength λ_l = 2L. Similarly for n = 2, the second mode of vibration has specific wavelength

$\lambda_2 = \left(\frac{2L}{2}\right)$ = L

For n = 3, the third mode of vibration has specific wavelength $\lambda_3 = \left(\frac{2L}{3}\right)$

and so on. The frequency of each mode of vibration (called natural frequency) can be calculated. $f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right)$ ------------3

4) The lowest natural frequency is called the fundamental frequency.
$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right)$ ------------4
The second natural frequency is called the first over tone.
$f_2 = 2\left(\frac{v}{2L}\right) \;\; = \frac{1}{L}\sqrt{\frac{T}{\mu}}$
The third natural frequency is called the second over tone.
$f_3 = 3\left(\frac{v}{2L}\right) \;\; = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$ and so on. Therefore, the nth natural frequency can be computed **as integral (or integer) multiple of fundamental frequency, i.e., $f_n = nf_1$, where n is an integer** ------------ 5

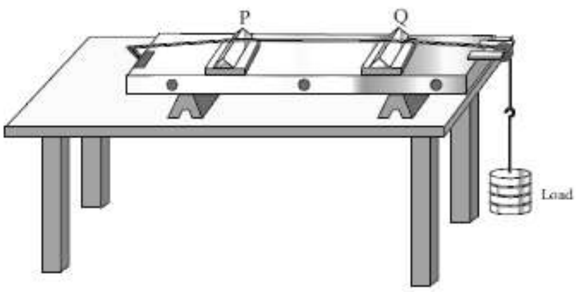
5) If natural frequencies are written as **integral multiple of fundamental frequencies, then the frequencies are called harmonics**. Thus, the first harmonic is **$f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$ etc.**

20. What is a sonometer? Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

1) Sono means *sound* related, and **sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced** in the transverse standing wave in a string, and the tension, length and mass per unit length of the string.

2) Therefore, using this device, we can determine the following quantities:
a) the frequency of the tuning fork or frequency of alternating current
b) the tension in the string
c) the unknown hanging mass

Construction:



3) The sonometer is made up of a **hollow box which is one-meter-long with a uniform metallic thin string attached** to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in Figure.

4) Since only one string is used, it is also known as monochord. The weights are **added to the free end of the wire to increase the tension of the wire.**

5) Two adjustable **wooden knives are put over the board, and their positions are adjusted to change the vibrating length** of the stretched wire.

Working:

6) A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is l then $l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$

7) Let f be the frequency of the vibrating element, T the tension of in the string and μ the mass per unit length of the string. Then using equation

$v = \sqrt{\frac{T}{\mu}}$, we get $f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ in Hz -----------1

8) Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ,

$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$; $f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi\rho d^2}{4}}}$ $f = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}$

21. Write short notes on intensity and loudness.

Intensity of sound:

1) **When a sound wave is emitted by a source, the energy is carried to all possible surrounding points.** The average sound energy emitted or transmitted per unit time or per second is called sound power.

2) Therefore, **the intensity of sound is defined as "the sound power transmitted per unit area taken normal to the propagation of the sound wave".**

3) For a particular source (fixed source), **the sound intensity is inversely proportional to the square of the distance from the source.**

$I = \frac{\text{power of the source}}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

This is known as inverse square law of sound intensity.

Loudness of sound:

1) Two sounds with same intensities need not have the same loudness. For example, the **sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market.**

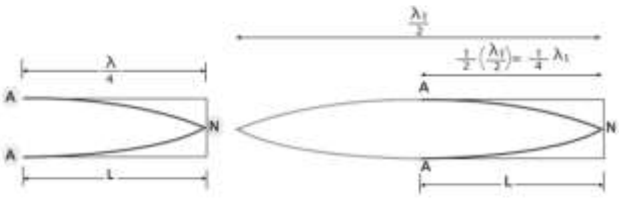
2) Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased, then loudness also increases. But additionally, **not only does intensity matter, the internal and subjective experience of "how loud a sound** is" i.e., the sensitivity of the listener also matters here.

3) This is often called loudness. That is, **loudness depends on both intensity of sound waves and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person)** whereas the intensity of sound does not depend on the observer.

4) The loudness of sound is defined as "**the degree of sensation of sound produced in the ear or the perception of sound by the listener".**

22. **Explain how overtones are produced in a**
(a) Closed organ pipe (b) Open organ pipe

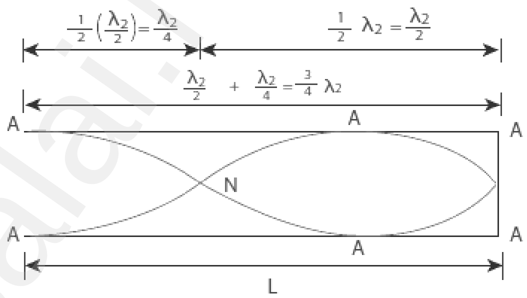
a) Closed organ pipes:



1) It is a pipe with one end closed and the other end open. If one end of a pipe is closed, the wave reflected at this closed end is 180° out of phase with the incoming wave.

2) Thus there is **no displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.**

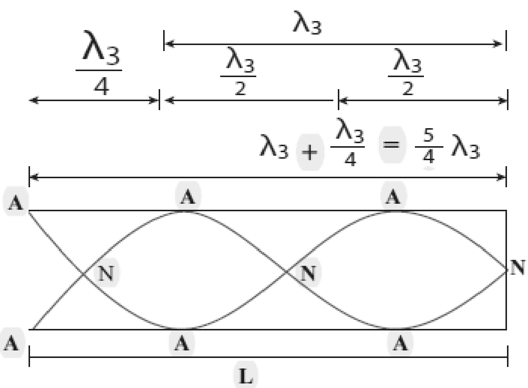
3) Consider the simplest mode of vibration of the air column called the fundamental mode. **Anti-node is formed at the open end and node at closed end**. From the Figure, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,



$L = \frac{\lambda_1}{4}$ or $\lambda_1 = 4L$; The frequency of the note emitted is

$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$ **which is called the fundamental note.**

4) The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.



The Figure 2 shows the second mode of vibration having two nodes and two anti-nodes. $4L = 3\lambda_2$ $L = \frac{3\lambda_2}{4}$ or $\lambda_2 = \frac{4L}{3}$

The frequency of this $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$ is called first over tone, since here, the frequency is three times the fundamental frequency it is called third harmonic.

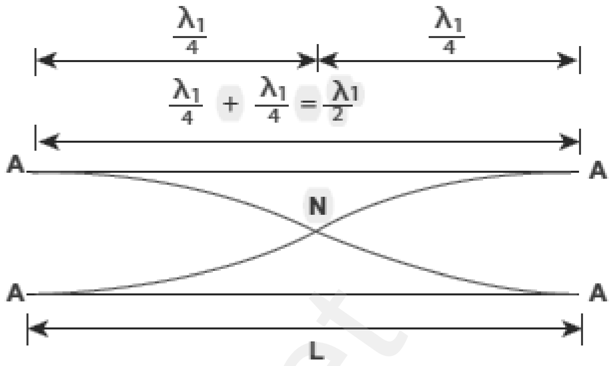
5) The Figure 3 shows third mode of vibration having three nodes and three anti-nodes. $4L = 5\lambda_3$ $L = \frac{5\lambda_3}{4}$ or $\lambda_3 = \frac{4L}{5}$

The frequency of this $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$ is called second over tone, and since n = 5 here, this is called fifth harmonic.

6) Hence, the closed organ pipe has only odd harmonics and frequency of the nth harmonic is **$f_n = (2n+1)f_1$. Therefore, the frequencies of harmonics are in the ratio $f_1 : f_2 : f_3 : f_4 :... = 1 : 3 : 5 : 7 : ...$**

b) Open organ pipe:

1) **It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end,** a node is formed at the mid-point of the pipe.



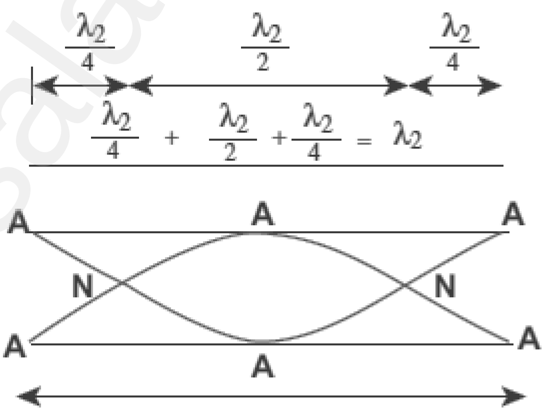
2) From Figure, if L be the length of the tube, the wavelength of the wave produced is given by

$L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$

The frequency of the note emitted is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

which is called the fundamental note.

3) The **frequencies higher than fundamental frequency can be produced by blowing** air strongly at one of the open ends. Such frequencies are called overtones.



4) The Figure shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore, **L =λ_2 or λ_2 = L .**

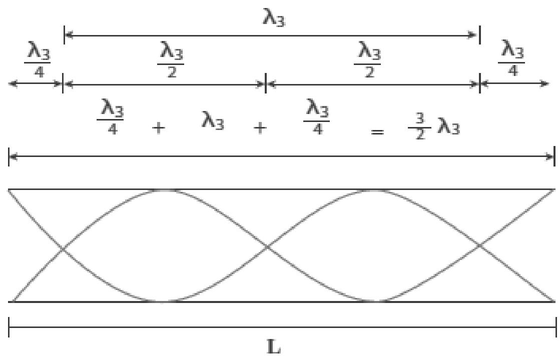
The frequency $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

$= 2 \times \frac{v}{2L} = 2f_1$ is called **first over tone**. Since n = 2 here, it is called the **second harmonic.**

5) The Figure shows the third mode of vibration having three nodes and four anti-nodes $L = \frac{3}{2}\lambda_3$ or $\lambda_3 = \frac{2L}{3}$;

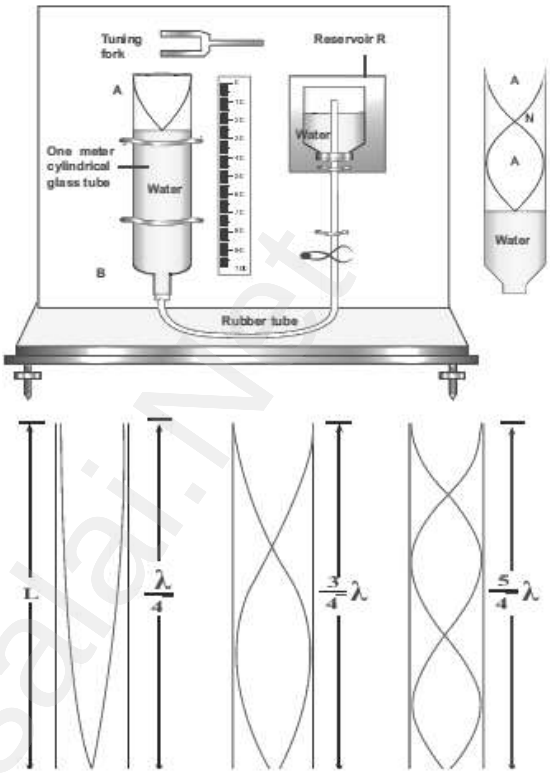
$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 \times \frac{v}{2L} = 3f_1$

is called second over tone. Since n = 3 here, it is called the third harmonic.



6) Hence, the open organ pipe has all the harmonics and frequency of n^{th} harmonic is fn = nf1. Therefore, **the frequencies of harmonics are in the ratio $f_1 : f_2 : f_3 : f_4 : ... = 1 : 2 : 3 : 4 :$ ....**

23. How will you determine the velocity of sound using resonance air column apparatus?



1) The resonance air column apparatus is **one of the simplest techniques to measure the speed of sound in air at room temperature.**
2) It consists of a cylindrical glass tube of **one-meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube** as shown in Figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it.
3) The **tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R.** The surface of the water will act as a closed end and other as the open end.
4) Therefore, it **behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end.**
5) When a **vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column**. These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves.
6) The **length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork).**
7) At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced. Let the **first resonance occur at length L_1, then $\frac{1}{4}\lambda = L_1$**
8) But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some **small distance above the open end. Including this end correction, the first resonance is $\frac{1}{4}\lambda = L_1 + e$**
9) Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which **the second resonance occurs. Again taking end correction into account, $\frac{3}{4}\lambda = L_2 + e$**

In order to avoid end correction,

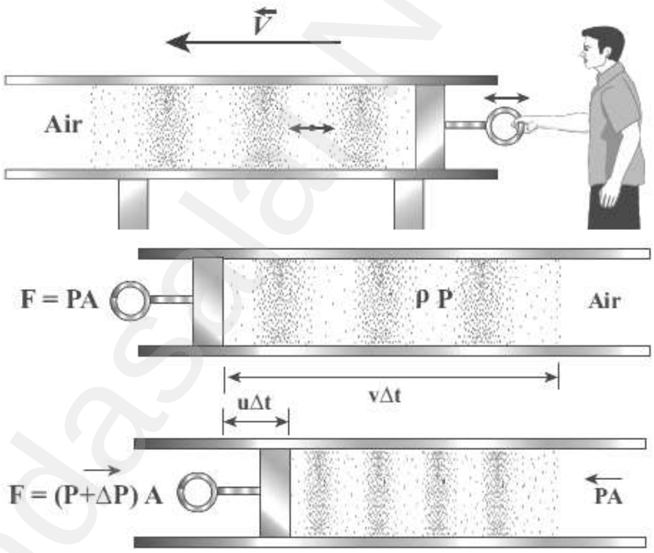
let us take the difference of equation $\frac{1}{4}\lambda$ = L,
and equation $f_1 : f_2 : f_3 : f_4 :... = 1 : 2 : 3 : 4 : ...$

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e - L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L \Rightarrow \lambda = 2\,\Delta L$$

24. Write the expression for the velocity of longitudinal waves in an elastic medium.

1) **Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is A and maintained under a pressure P.** One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube.



2) Let us **assume that the direction of propagation of waves coincides with the axis of the cylinder. Let ρ be the density of the fluid which is initially at rest. At t = 0,** the piston at left end of the tube is set in motion toward the right with a speed u.

3) Let u be the velocity of the piston and v be the velocity of the elastic wave. **In time interval Δt, the distance moved by the piston Δd = u Δt. Now, the distance moved by the elastic disturbance is Δ*x* = vΔt**. Let Δm be the mass of the air that has attained a velocity v in a time Δt .
Therefore, **$\Delta m = \rho A \Delta x = \rho A (v \Delta t)$**

4) Then, the momentum imparted due to motion of piston with velocity u is **$\Delta p = [\rho A (v \Delta t)]u$**
But the change in momentum is impulse.
The net impulse is **$I = (\Delta P A)\Delta t$ Or $(\Delta P A)\Delta t = [\rho A (v \Delta t)]u$**

$$\Delta P = \rho v u \text{ ———— } 1$$

5) When the sound waves passes through air, the small volume element (ΔV) of the air undergoes regular compressions and rarefactions. So, the change in pressure can also be written as $\Delta P = B\frac{\Delta V}{V}$

where, V is original volume and B is known as bulk modulus of the elastic medium.

But $V = A\,\Delta x = A\,v\,\Delta t$ and $\Delta V = A\,\Delta d = A\,u\,\Delta t$

Therefore, $\Delta P = B\frac{Au\,\Delta t}{Av\,\Delta t} = B\frac{u}{v}$ ———— 2

Comparing equation (1) and equation (2),

we get $\rho\, v\, u = B\frac{u}{v}$ or $v^2 = \frac{B}{\rho} \Rightarrow v = \sqrt{\frac{B}{\rho}}$ ——3

In general, the velocity of a longitudinal wave in elastic medium is $v = \sqrt{\frac{E}{\rho}}$, where E is the modulus of elasticity of the medium.

Cases: For a solid:

i) **One-dimension rod (1D)**

$v = \sqrt{\frac{Y}{\rho}}$, ----------4 where Y is the Young's modulus of the material of the rod and ρ is the density of the rod. The 1D rod will have only Young's modulus.

ii) **Three-dimension rod (3D)**

The speed of longitudinal wave in a solid is $v = \sqrt{\frac{4+\frac{3}{4}\eta}{\rho}}$ --------5

where η is the modulus of rigidity, K is the bulk modulus and ρ is the density of the rod.

Cases: For liquids: $v = \sqrt{\frac{K}{\rho}}$, ------- 6 where, K is the bulk modulus and ρ is the density of the rod.

25. Write the applications of reflection of sound waves:

a) Stethoscope: It works on the principle of multiple reflections.

It consists of three main parts: i) Chest piece (ii) Ear piece (iii) Rubber tube

i) Chest piece:

It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

ii) Ear piece:

It is made up of metal tubes which are used to hear sounds detected by the chest piece.

iii) Rubber tube:

This tube connects both chest piece and ear piece. It is used to **transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected**, and it reaches the ear piece through this tube by multiple reflections.

b) Echo:

1) An echo is **a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m s^{-1}**. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall.

2) After reflection**, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds**. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{th}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

Velocity = $\frac{\text{Distance travelled}}{\text{Time taken}}$; = $\frac{2d}{t}$; 2d = 344 x 0.1=34.1m; d = 17.2 m

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

c) SONAR:

SOund NAvigation and Ranging. Sonar systems make **use of reflections of sound waves in water to locate the position or motion of an object**. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

d) Reverberation:

In a closed room the sound is repeatedly **reflected from the walls and it is even heard long after the sound source ceases to function.** The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation.

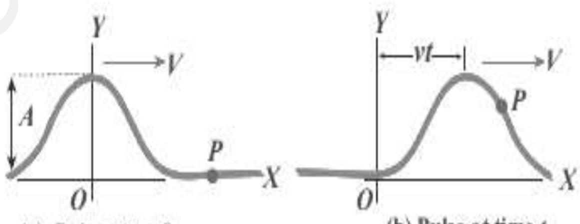
The **duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall.** Therefore, halls are constructed with some optimum reverberation time.

26. Write characteristics of progressive waves:

1) Particles in the medium **vibrate about their mean positions with** the same amplitude.
2) The phase **of every particle ranges from 0 to 2π.**
3) No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.
4) Transverse progressive waves are characterized by **crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.**
5) When the **particles pass through the mean position they always move with the same maximum velocity.**
6) The displacement, velocity and acceleration of particles separated from each other by $n\lambda$ are the same. where n is an integer, and λ is the wavelength.

27. Derive the equation of a plane progressive wave.

1) A jerk on a stretched string at time t = 0 s. Let us assume that the wave pulse created during this disturbance moves along positive x direction with constant speed v as shown in Figure.
2) We can represent the shape of the wave pulse, mathematically as $y = y(x, 0) = f(x)$ at time $t = 0$ s. Assume that the shape of the wave pulse remains the same during the propagation. After some time t, the pulse moving towards the right and any point on it can be represented by x' (read it as x prime) as shown in Figure. Then, $y(x, t) = f(x') = f(x - vt)$



(a) Pulse at t = 0 (b) Pulse at time t

3) Similarly, **if the wave pulse moves towards left with constant speed v, then $y = f(x + vt)$. Both waves $y = f(x + vt)$ and $y = f(x - vt)$** will satisfy the following one **dimensional differential equation known as the wave equation** $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.
4) where the symbol ∂ represent partial derivative. Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of x and t.
5) But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the

partial derivative with respect to x is the same as total derivative in coordinate x, we write $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

28. Derive the relation between intensity and loudness.

1) According to Weber-Fechner's law, **"loudness (L) is proportional to the logarithm of the actual intensity (I) measured with an accurate non-human instrument". This means that $L \propto \ln I$, $L = k\ln I$** where k is a constant, which depends on the unit of measurement.
2) The **difference between two loudness, L_1 and L_0 measures the relative loudness between two precisely measured intensities and is called as sound intensity level.**
3) Sound intensity level is $\Delta L = L_1 - L_0 = k\ln I_1 - k\ln I_0 = k\ln\left[\frac{I_1}{I_0}\right]$ if k = 1, then **sound intensity level is measured in bel,** in honour of Alexander Graham Bell. Therefore, $\Delta L = \ln\left[\frac{I_1}{I_0}\right]$ bel
4) However, this is practically a bigger unit, so we use a convenient smaller unit, called decibel. Thus, **decibel = $\frac{1}{10}$ bel ,**
5) Therefore, by multiplying and dividing by 10,
we get $\Delta L = 10\left(\ln\left[\frac{I_1}{I_0}\right]\right)\frac{1}{10}$ bel ; $\Delta L = 10\ln\left[\frac{I_1}{I_0}\right]$ decibel with k = 10
For practical purposes, we use logarithm to base 10 instead of natural logarithm, $\Delta L = 10\log_{10}\left[\frac{I_1}{I_0}\right]$ decibel.

UNIT – 1 (NATURE OF PHYSICAL WORLD AND MEASUREMENT)

1. From a point on the ground, the top of a tree is seen to have an angle of elevation 60^0. The distance between the tree and a point is 50 m. Calculate the height of the tree?

Solution

$\theta = 60^0$, The distance between the tree and a point x = 50 m,
Height of the tree (h)=?

For triangulation method $\tan\theta = \frac{h}{x}$

h = $\boldsymbol{x}$ **tan θ** ; $= 50 \times \tan 60^0$; $= 50 \times 1.732$

h = 86.6 m; **The height of the tree is 86.6 m.**

2. A RADAR signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the Earth is 6.3×10^{10} m. Calculate the speed of the signal?

Solution

The distance of the planet from the Earth
$d = 6.3 \times 10^{10}$ m
Time t = 7 minutes = 7 × 60 s.
the speed of signal V =?

The speed of signal $V = \frac{2d}{t} = \frac{2x6.3x10^{10}}{7x60}$;

$V = 3x10^8\ ms^{-1}$

No.	Log
12.6	1.1004
42.0	2.6232
(−)	$\bar{2}.4772$
Antilog	3.000×10^8

3. Two resistances $R_1 = (100 \pm 3)\ \Omega$, $R_2 = (150 \pm 2)\ \Omega$, are connected in series. What is their equivalent resistance?

Solution

$R_1 = 100 \pm 3\ \Omega$, $R_2 = 150 \pm 2\Omega$
Equivalent resistance R =?
Equivalent resistance $R = R_1 + R_2$
$= (100 \pm 3) + (150 \pm 2)$; $= (100 + 150) \pm (3 + 2)$
$R = (250 \pm 5)\ \Omega$

4. The temperatures of two bodies measured by a thermometer are $t_1 = (20 + 0.5)°C$, $t_2 = (50 \pm 0.5)°C$. Calculate the temperature difference and the error therein.

Solution

$t_1 = (20 \pm 0.5)°C$ $t_2 = (50 \pm 0.5)°C$ ‘ temperature difference t=?
$t = t_2 - t_1$; $= (50 \pm 0.5) - (20 \pm 0.5)°C$
$= (50 - 20) \pm (0.5+0.5)$; **$t = (30 \pm 1)°C$**

5. State the number of significant figures in the following

i) 600800 - Four ii) 400 - One iii) 0.007 - One
iv) 5213.0 - Five v) 2.65×10^{24}m - Three vi) 0.0006032 – Four

6. Round off the following numbers as indicated

i) 18.35 up to 3 digits	:	1.84
ii) 19.45 up to 3 digits	:	19.4
iii) 101.55 × 10^6 up to 4 digits	:	101.6 x 10^6
iv) 248337 up to digits 3 digits	:	248000
v) 12.653 up to 3 digits	:	12.7

7. Convert 76 cm of mercury pressure into Nm^{-2} using the method of dimensions.

Solution

In cgs system 76 cm of mercury pressure = 76 × 13.6 × 980 dyne cm^{-2}

The dimensional formula of pressure P is $[ML^{-1}T^{-2}]$

$$P_1[M_1^a\, L_1^b\, T_1^c] = P_2[M_2^a\, L_2^b\, T_2^c]\ ;\ P_2 = P_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

M_1 = 1g, M_2 = 1kg; L_1 = 1 cm, L_2 = 1m; T_1 = 1 s, T_2 = 1s

As a =1, b= -1, and c = -2

$$\text{Then } P_2 = 76 \times 13.6 \times 980 \left[\frac{1kg}{1kg}\right]^1 \left[\frac{1\text{cm}}{1\text{m}}\right]^{-1} \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$$= 76 \times 13.6 \times 980 \left[\frac{10^{-3}kg}{1\text{kg}}\right]^1 \left[\frac{10^{-2}\text{m}}{1\text{m}}\right]^{-1} \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$= 76 \times 13.6 \times 980 \times [10^{-3}] \times 10^2$

$P_2 = 1.01 \times 10^5\ Nm^{-2}$

8. If the value of universal gravitational constant in SI is $6.6\text{x}10^{-11}\ Nm^2\ kg^{-2}$, then find its value in CGS System?

Solution

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the cgs system. Then $G_{SI} = 6.6 \times 10^{-11} Nm^2\ kg^{-2}$

G_{cgs} =?

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c\ ;\ G_{cgs} = G_{SI}\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

M_1 = 1 kg, L_1 = 1m, T_1 = 1s ; M_2 = 1 g, L_2 = 1 cm, T_2 = 1s

The dimensional formula for G is $M^{-1}L^3T^{-2}$; a = -1, b = 3 and c = -2

$$G_{cgs} = 6.6 \times 10^{-11}\left[\frac{1kg}{1\text{g}}\right]^{-1} \left[\frac{1\text{m}}{1\text{cm}}\right]^3 \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$$= 6.6 \times 10^{-11}\left[\frac{1kg}{10^{-3}kg}\right]^{-1} \left[\frac{1\text{m}}{10^{-2}\text{m}}\right]^3 \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6 \times 1$; **$G_{cgs} = 6.6 \times 10^{-8}$ dyne cm^2g^{-2}**

9. Check the correctness of the equation ½ mv^2 = mgh using dimensional analysis method.

Solution

Dimensional formula for ½ mv^2 = $[M][LT^{-1}]^2 = [ML^2T^{-2}]$

Dimensional formula for mgh = $[M][LT^{-2}][L]=[ML^2T^{-2}]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

Both sides are dimensionally the same, hence the equations ½ mv^2 = mgh is dimensionally correct.

10. Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended. (Constant k = 2π)

Solution

T∝ $m^a l^b g^c$; T = k $m^a l^b g^c$

Here k is the dimensionless constant. Rewriting the above equation with dimensions

$[T^1] = [M^a] [L^b] [LT^{-2}]^c$ $[M^0L^0T^1] = [M^a L^{b+c} T^{-2c}]$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a,b and c a = 0, b = 1/2, and c = −1/2

From the above equation T = k. m^0 $l^{1/2}$ $g^{1/2}$

T = k $\left(\frac{l}{g}\right)^{1/2}$; k $\sqrt{l/g}$; Experimentally k = 2π, hence T = 2π $\sqrt{l/g}$

EXERCISE PROBLEM

11. In a submarine equipped with sonar, the time delay between the generation of a pulse and its echo after reflection from an enemy submarine is observed to be 80 s. If the speed of sound in water is 1460 ms^{-1}. What is the distance of enemy submarine?

Solution

v = 80s , v = 1460 ms^{-1}, D = ?

D = $\frac{v\,t}{2} = \frac{1460 \times 80}{2}$; = 1460 x 40 ; 58400m

D = 58.4km

12. Jupiter is at a distance of 824.7 million km from the Earth. Its angular diameter is measured to be 35.72˝. Calculate the diameter of Jupiter

Solution

X = 824.7 million km = 824.7 x 10^6 x 10^3 m

θ = 35.72" = 35.72 x 4.85 x 10^{-6} rad; b = ?

$x = \frac{b}{\theta}$; $b = x\theta$;

= 824.7 x 10^9 x 35.72 x 4.85 x 10^{-6}

= 1.428 x 10^5 x 10^3 m ; **b = 1.428 x 10^5km**

UNIT – 2 (KINEMATICS)

13. Two vectors $\vec{A}$ and $\vec{B}$ of magnitude 5 units and 7 units respectively make an angle 60⁰ with each other.Find the magnitude of the resultant vector and its direction with respect to the vector $\vec{A}$

Solution

The magnitude of the resultant vector $\vec{R}$ is given by

$R = |\vec{R}| = \sqrt{5^2 + 7^2 + 2x5x7\cos 60^0}$; = $\sqrt{25 + 49 + \frac{70x1}{2}}$;

$\mathbf{R} = \sqrt{\mathbf{109}}$ units

The angle α between $\vec{R}$ and $\vec{A}$ is given by $\tan\alpha = \frac{\mathbf{B \, Sin\,\theta}}{\mathbf{A+B\, Cos\theta}}$

$= \frac{7 \text{ x } \sin 60^0}{5+7\cos 60^0}$; $= \frac{7 \text{ x } \sqrt{3}}{10+7}$ $= \frac{7 \text{ x } \sqrt{3}}{17}$; ≅ 0.713 ; $\alpha \cong 35^0$

14. Two vectors $\vec{A}$ and $\vec{B}$ are given in the component form as $\vec{A} = 5\vec{i} + 7\vec{j} - 4\vec{k}$ and $\vec{B} = 6\vec{i} + 3\vec{j} + 2\vec{k}$. Find $\vec{A} + \vec{B}$, $\vec{B} + \vec{A}$, $\vec{A} - \vec{B}$, $\vec{B} - \vec{A}$.

Solution

$\vec{A} + \vec{B} = (5\vec{i} + 7\vec{j} - 4\vec{k}) + (6\vec{i} + 3\vec{j} + 2\vec{k})$; $= \mathbf{11\vec{i} + 10\vec{j} - 2\vec{k}}$.

$\vec{B} + \vec{A} = (6\vec{i} + 3\vec{j} + 2\vec{k}) + (5\vec{i} + 7\vec{j} - 4\vec{k})$; $= \mathbf{11\vec{i} + 10\vec{j} - 2\vec{k}}$.

$\vec{A} - \vec{B} = (5\vec{i} + 7\vec{j} - 4\vec{k}) - (6\vec{i} + 3\vec{j} + 2\vec{k})$; $= \mathbf{-\vec{i} + 4\vec{j} - 6\vec{k}}$.

$\vec{B} - \vec{A} = (6\vec{i} + 3\vec{j} + 2\vec{k}) - (5\vec{i} + 7\vec{j} - 4\vec{k})$; $= \mathbf{\vec{i} - 4\vec{j} + 6\vec{k}}$.

Note that the vectors $\vec{A} + \vec{B}$ and $\vec{B} + \vec{A}$ **are same** and the vectors $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ **are opposite to each other.**

15. Given the vector $\vec{A} = 2\vec{i} + 3\vec{j}$, what is $3\vec{A}$?

Solution

$3\vec{A} = 3(2\vec{i} + 3\vec{j}) = 6\vec{i} + 9\vec{j}$. The vector $3\vec{A}$ is given as in the same direction as vector $\vec{A}$

16. Check whether the following vectors are orthogonal.

i) $\vec{A} = 2\vec{i} + 3\vec{j}$ and $\vec{B} = 4\vec{i} - 5\vec{j}$ ii) $\vec{C} = 5\vec{i} + 2\vec{j}$ and $\vec{D} = 2\vec{i} - 5\vec{j}$

Solution

$\vec{A}.\vec{B}$ = 8 -15 = -7≠0 . Here $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

$\vec{C}.\vec{D}$ = 10 -10 = 0 . Here $\vec{C}$ and $\vec{D}$ are orthogonal to each other.

17. Two vectors are given as $\vec{r} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ and $\vec{F} = 3\vec{i} - 2\vec{j} + 4\vec{k}$. Find the resultant vector $\vec{\tau} = \vec{r} \times \vec{F}$

Solution

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

$= (12-(-10))\,\hat{\imath} + (15-8)\,\hat{\jmath} + (-4-9)\,\hat{k}$; $\boldsymbol{\vec{\tau} = 22\hat{\imath} + 7\hat{\jmath} - 13\hat{k}}$

18. The position vector of a particle is given $\vec{r} = 2t\vec{i} + 3t^2\vec{j} - 5\vec{k}$.
a) Calculate the velocity and speed of the particle at any instant *t*
b) Calculate the velocity and speed of the particle at time *t* = 2 s

Solution

The velocity $\bar{\bar{v}} = \frac{d\bar{r}}{dt} = 2\vec{i} + 6t\vec{j}$;

The speed $v(t) = \sqrt{2^2 + (6t)^2}$ ms^{-1}

The velocity of the particle at *t* = 2 s ;

$\vec{v}(2sec) = 2\vec{i} + 12\vec{j}$

The speed of the particle at *t* = 2 s ;

$v(2s) = \sqrt{2^2 + 12^2} = \sqrt{4 + 144}$

$= \sqrt{148}$; ≈ 12.16 ms^{-1}

No.	Log
½ x 148	½ x 2.1703 1.0851
Antilog	1.216×10^1

19. The velocity of three particles A, B, C are given below. Which particle travels at the greatest speed?

$$\bar{v}_A = 3\vec{i} - 5\vec{j} + 2\vec{k}\ ;\ \bar{v}_B = \vec{i} + 2\vec{j} + 3\vec{k}\ ;\ \bar{v}_C = 5\vec{i} + 3\vec{j} + 4\vec{k}$$

Solution

Speed of A = $|\bar{v}_A| = \sqrt{(3)^2 + (-5)^2 + (2)^2}$; $= \sqrt{9 + 25 + 4}$; ; $= \sqrt{38}$ ms^{-1}

Speed of B = $|\bar{v}_B| = \sqrt{(1)^2 + (2)^2 + (3)^2}$; $= \sqrt{1 + 4 + 9}$; ; $= \sqrt{14}$ ms^{-1}

Speed of C = $|\bar{v}_C| = \sqrt{(5)^2 + (3)^2 + (4)^2}$; $= \sqrt{25 + 9 + 16}$; ; $= \sqrt{50}$ ms^{-1}

The particle C has the greatest speed $\sqrt{50} > \sqrt{38} > \sqrt{14}$

20. A train was moving at the rate of 54 km h^{-1} when brakes were applied. It came to rest within a distance of 225 m. Calculate the retardation produced in the train.

Solution

The final velocity of the particle v = 0

The initial velocity of the particle u = 54 x $\frac{5}{18}$ ms^{-1} = 15 ms^{-1} ; s = 225 m

Retardation is always against the velocity of the particle.

$v^2 = u^2 - 2aS$; $0 = (15)^2 - 2a(225)$; 450 a = 225

a = $\frac{225}{450}$ ms^{-2} ; =0.5 ms^{-2} ; Retardation = =0.5 ms^{-2}

21. In the cricket game, a batsman strikes the ball such that it moves with the speed 30 m s^{-1} at an angle 30^0 with the horizontal. The boundary line of the cricket ground is located at a distance of 75 m from the batsman? Will the ball go for a six? (Neglect the air resistance and take acceleration due to gravity g = 10 m s^{-2}).

Solution

The motion of the cricket ball in air is essentially a projectile motion. As we have already seen, the range (horizontal distance) of the projectile motion is given by

No.	Log
45	1.6532
1.732	0.2385
(−)	1.8917
Antilog	7.793 x 10^1

R = $\frac{u^2 sin2\theta}{g}$; The initial speed u= 30m s^{-1}

The projection angle θ = 30^0

The horizontal distance travelled by the cricket ball

R = $\frac{(30)^2 \text{x} \, sin60^0}{10}$ = $\frac{900 \text{ x} \frac{\sqrt{3}}{2}}{10}$; = 77.94 m

This distance is greater than the distance of the boundary line. Hence the ball will cross this line and go for a six.

EXERCISE PROBLEM

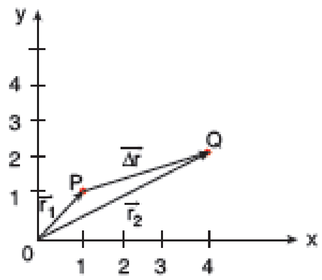
22. The position vectors particle has length 1m and makes 30^0 with the *x*-axis. What are the lengths of the x and y components of the position vector?

Solution

l = 1m , θ= 30^0 ; Length of x – component $l_x = lcos\theta = 1 \text{ x } cos\, 30^0 = \frac{\sqrt{3}}{2}$

Length of y – component $l_y = lsin\theta = 1 \text{ x } sin\, 30^0 = \frac{1}{2} = 0.5$

23. A particle has its position moved from $\vec{r_1} = 3\hat{\imath} + 4\hat{\jmath}$ to $\vec{r_2} = \hat{\imath} + 2\hat{\jmath}$ Calculate the displacement vector $(\Delta\vec{r})$ and draw the $\vec{r_1}$, $\vec{r_2}$ and $\Delta\vec{r}$ vector in a two dimensional Cartesian coordinate system.



$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$; ; = $(\hat{\imath} + 2\hat{\jmath}) - (3\hat{\imath} + 4\hat{\jmath})$; = $\hat{\imath} + 2\hat{\jmath} - 3\hat{\imath} - 4\hat{\jmath}$

$\Delta\vec{r} = -2\hat{\imath} - 2\hat{\jmath}$

24. Calculate the average velocity of the particle whose position vector changes from $\vec{r_1} = 3\hat{\imath} + 6\hat{\jmath}$ to $\vec{r_2} = \widehat{2\imath} + 3\hat{\jmath}$ in a time 5 second.

Solution

The average velocity $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t}$; = $\frac{(\widehat{2\imath}+3\hat{\jmath})-(3\hat{\imath}+6\hat{\jmath})}{5}$

= $\frac{2\hat{\imath}+3\hat{\jmath}-5\hat{\imath}-6\hat{\jmath}}{5}$; = $\frac{-3\hat{\jmath}-3\hat{\jmath}}{5}$; $\vec{v}_{avg} = -\frac{3}{5}(\hat{\imath}+\hat{\jmath})$

25. Convert the vector $\hat{r} = 3\hat{\imath} + 2\hat{\jmath}$ into a unit vector.

Solution

The magnitude of the vector $\hat{r} = 3\hat{\imath} + 2\hat{\jmath}$

$|\hat{r}| = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$; $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{\imath}+2\hat{\jmath}}{\sqrt{13}}$

26. What are the resultants of the vector product of two given vectors given by $\vec{A} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{B} = 5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

Solution

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 1 \\ 5 & 3 & -4 \end{vmatrix}$$

$= (8 - 3)\,\hat{\imath} + (5+16)\,\hat{\jmath} + (12 + 10)\,\hat{k}$;

$\vec{A} \times \vec{B} = 5\hat{\imath} + 21\hat{\jmath} + 22\,\hat{k}$

27. An object at an angle such that the horizontal range is 4 times of the maximum height. What is the angle of projection of the object?

Solution

Horizontal Range $R = \frac{u^2\sin2\theta}{g}$; maximum height $h_{max} = \frac{u^2\sin^2\theta}{2g}$;

$R = 4h_{max}$; $\frac{u^2\sin2\theta}{g} = 4 \times \frac{u^2\sin^2\theta}{2g}$

$\frac{u^2 2\sin\theta\cos\theta}{g} = 4 \times \frac{u^2\sin^2\theta}{2g}$; $\cos\theta = \sin\theta$; $\frac{\sin\theta}{\cos\theta} = 1$; $\tan\theta = 1$

$$\boldsymbol{\theta = tan^{-1}(1) = 45^0}$$

28. If Earth completes one revolution in 24 hours, what is the angular displacement made by Earth in one hour. Express your answer in both radian and degree.

Solution

Angular displacement for one complete revolution (i.e.) for 24 hours = 360^0

Hence, angular displacement for one hour, $\theta = \frac{360^0}{24}$; $= 15^0$ or

$\theta = \frac{\pi}{180^0} \times 15^0$; $= \frac{\pi}{12}$ rad [$180^0 = \pi$ rad]

29. A object is thrown with initial speed 5 ms^{-1} with an angle of projection 30^0. What is the height and range reached by the particle?

Solution

No.	Log
25	1.3979
78.4	1.8943
(−)	$\bar{1}$.5036
Antilog	3.188×10^{-1}

i) maximum height of the projectile, $h_{max} = \frac{u^2\sin^2\theta}{2g}$

$h_{max} = \frac{5^2 \sin30^0 \sin 30^0}{2 \times 9.8}$; $= \frac{25 \times \left[\frac{1}{2}\right] \times \left[\frac{1}{2}\right]}{2 \times 9.8}$; $= \frac{25}{8 \times 9.8}$; $= \frac{25}{78.4}$; $h_{max} = 0.3188$m

No.	Log
43.3	1.6365
19.6	1.2922
(−)	0.3443
Antilog	2.209×10^0

ii) Horizontal Range $R = \frac{u^2\sin2\theta}{g}$; $= \frac{u^2 2\sin\theta\cos\theta}{g}$; $= \frac{5^2 \times 2\sin30^0\cos30^0}{9.8}$

$= \frac{25 \times 2 \left[\frac{1}{2}\right] \times \left[\frac{\sqrt{3}}{2}\right]}{9.8}$; $= \frac{25 \times 1.732}{2 \times 9.8} = \frac{43.300}{19.6}$; $R = 2.21$m

30. A foot-ball player hits the ball with speed 20 ms^{-1} with angle 30^0 with respect to horizontal direction. The goal post is at distance of 40 m from him. Find out whether ball reaches the goal post?

Solution

Here the reaches ball is considering as a projectile. Its range

Horizontal Range R = $\frac{u^2 \sin 2\theta}{g}$; $= \frac{u^2 2 \sin\theta \cos\theta}{g}$; $= \frac{20^2 \times 2 \sin 30^0 \cos 30^0}{9.8}$

$= \frac{400 \times 2 \left[\frac{1}{2}\right] \times \left[\frac{\sqrt{3}}{2}\right]}{9.8}$; $= \frac{400 \times 1.732}{2 \times 9.8} = \frac{692.800}{19.6}$; R = 35.35m

Thus the range of the reaches ball is 35.35 m. But the goal post is at a distance of 40 m from him. So ball will not reach the goal post.

31. If an object is thrown horizontally with an initial speed 10 m s^{-1} from the top of a building of height 100 m. what is the horizontal distance covered by the particle?

Solution

Horizontal range of the object projected horizontally,

$R = u\sqrt{\frac{2h}{g}}$; $= 10\sqrt{\frac{2 \times 100}{9.8}} = 10\sqrt{\frac{200}{9.8}}$; $= \sqrt{\frac{200 \times 100}{9.8}}$; $= \sqrt{\frac{20000}{9.8}}$;

= 45.18m ≅ 45m

No.	Log
20000	4.3010
9.8	0.9912
(−)	3.3098
½ x 3.3098 = 1.6549	
Antilog	4.518×10^1

32. An object is executing uniform circular motion with an angular speed of $\frac{\pi}{12}$ radian per second. At t = 0 the object starts at an angle θ = 0. What is the angular displacement of the particle after 4 s ?

Solution

$\omega = \frac{\theta}{t}$ or $\theta = \omega t$; $\theta = \frac{\pi}{12} \times 4$; $= \frac{\pi}{3}$ rad ; $= 60^0$

33. The Moon is orbiting the Earth approximately once in 27 days, what is the angle transverse by the Moon per day?

Solution

Angle traversed by the Moon for one complete rotation

(i.e.) for 27days = 360^0 = $2\pi rad$

Angle traversed by the Moon for one day,

$\theta = \frac{2\pi}{27}$; $= \frac{2 \times 3.14}{27}$; $= \frac{6.28}{27}$

θ =0.2362 rad = 13.33^0 [1 rad = 57.27^0]

No.	Log
6.28	0.7980
27	1.4314
(−)	$\bar{1}.3666$
Antilog	2.326×10^{-1}

34. An object of mass m has angular acceleration α = 0.2 $rads^{-1}$. What is the angular displacement covered by the object after 3 second? (Assume that the object started with angle zero with zero angular velocity).

Solution

From equation for uniform circular motion, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ [$\omega_0 = 0$]

$\theta = 0 + \frac{1}{2} \times 0.2 \times 3^2$; $\theta = \frac{1}{2} \times 0.2 \times 9$; = 0.1 x 9

$\boldsymbol{\theta = 0.9}$ **rad ≅ 51^0** [1 rad = 57.27^0]

UNIT – III (LAWS OF MOTION)

35. A book of mass m is at rest on the table. (1) What are the forces acting on the book? (2) What are the forces exerted by the book? (3) Draw the free body diagram for the book.

Solution

(1) There are two forces acting on the book.

(i) Gravitational force (mg) acting downwards on the book

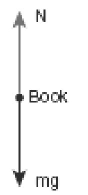
(ii) Normal contact force (N) exerted by the surface of the table on the book. It acts upwards as shown in the figure.

(2) According to Newton's third law, there are two reaction forces exerted by the book.

(i) The book exerts an equal and opposite force (mg) on the Earth which acts upwards.

(ii) The book exerts a force which is equal and opposite to normal force on the surface of the table (N) acting downwards.

Free body diagram



36. If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?

Solution

For the object of mass 2.5 kg, the acceleration is a = $\frac{F}{m} = \frac{5}{2.5}$; = 2ms^{-2}

For the object of mass 100 kg, the acceleration is a = $\frac{F}{m} = \frac{5}{100}$; = 0.05ms^{-2}

37. The position vector of a particle is given by $\vec{r} = 3t\vec{i} + 5t^2\vec{j} + 7\vec{k}$. Find the direction in which the particle experiences net force?

Solution

Velocity of the particle, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\vec{i} + \frac{d}{dt}(5t^2)\vec{j} + \frac{d}{dt}(7)\vec{k}$

$\frac{d\vec{r}}{dt} = 3\vec{i} + 10t\vec{j}$; Acceleration of the particle $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\vec{j}$

Here, the particle has acceleration only along positive y direction. According to Newton's second law, net force must also act along positive y direction. In addition, the particle has constant velocity in positive x direction and no velocity in z direction. Hence, there are no net force along x or z direction.

38. A particle of mass 2 kg experiences two forces, $\vec{F}_1 = 5\vec{i} + 8\vec{j} + 7\vec{k}$ and , $\vec{F}_2 = 3\vec{i} - 4\vec{j} + 3\vec{k}$. What is the acceleration of the particle?

Solution

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. Acceleration is $\vec{a} = \frac{\vec{F}_{net}}{m}$

$\vec{F}_{net} = (5+3)\vec{i} + (8-4)\vec{j} + (7+3)\vec{k}$; $\vec{F}_{net} = 8\vec{i} + 4\vec{j} + 10\vec{k}$

$\vec{a} = \left(\frac{8}{2}\right)\vec{i} + \left(\frac{4}{2}\right)\vec{j} + \left(\frac{10}{2}\right)\vec{k}$; $\mathbf{\vec{a} = 4\vec{i} + 2\vec{j} + 5\vec{k}}$

39. An object of mass 10 kg moving with a speed of 15 m s^{-1} hits the wall and comes to rest within a) 0.03 second b) 10 second .Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object P_I = 10x15 = 150 $kgms^{-1}$

final momentum of the object P_f = 0 ; Δp = 150 – 0 = 150 $kgms^{-1}$

(a) Impulse J = Δp = 150 Ns ; (b) Impulse J = Δp = 150 Ns

(a) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{0.03}$; = 5000N ;

(b) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{10}$; = 15N

impulse is the same in both cases, but the average force is different.

40. If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 ms^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?

Solution : $F_{cp} = \frac{mv^2}{r}$; $\frac{\frac{1}{4} \text{x} (2)^2}{3}$ = 0.333N

41. Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1}, 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})

Solution

From the safe turn condition, the speed of the vehicle (v) must be less than or equal $\sqrt{\mu_s rg}$; $v \leq \sqrt{\mu_s rg}$; $\sqrt{\mu_s rg} = \sqrt{0.81 \text{ x } 10 \text{ x } 10}$ = 9 ms^{-1}

For car C, $\sqrt{\mu_s rg}$ is less than v

The speed of car A, B and C are 7 ms^{-1}, 8 ms^{-1} and 10 ms^{-1} respectively. The **cars A and B will have safe turns.** But the **car C has speed 10 ms^{-1}** while it turns which exceeds the safe turning speed. Hence, **the car C will skid.**

42. Consider a circular road of radius 20 meter banked at an angle of 15 degrees. With what speed a car has to move on the turn so that it will have safe turn?

Solution

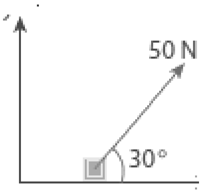
$v = \sqrt{(rg \tan\theta)} = \sqrt{20 \text{ x } 9.8 \text{ x } \tan 15^0}$

$= \sqrt{20 \text{ x } 9.8 \text{ x } 0.26}$ = 7.1 ms^{-1}

The safe speed for the car on this road is 7.1 ms^{-1}

EXERCISE PROBLEM

43. A force of 50N act on the object of mass 20 kg. shown in the figure. Calculate the acceleration of the object in x and y directions.



Solution

From Newton's second law; $F=m\,a$

Hence the acceleration ; a = $\frac{F}{m} = \frac{50}{20}$; = 2.5 ms^{-2}

The acceleration in x-axis; a_x = acos θ; = 2.5 x cos30^0 ; = 2.5 x $\frac{\sqrt{3}}{2}$

= 1.25 x 1.732; **a_x = 1.165 ms^{-2}**

The acceleration in y-axis; a_y = asin θ; = 2.5 x sin30^0 ; = 2.5 x $\frac{1}{2}$;

a_y= 1.25 ms^{-2}

44. A spider of mass 50 g is hanging on a string of a cob web. What is the tension in the string?

Solution

Here two forces acting on the spider.

(1) Downward gravitational force (mg) (2) Upward tension (T)

Hence, $\boldsymbol{T}=\boldsymbol{mg}$; = 50 x 10^{-3} x 9.8 =490 X 10^{-3} ; **T = 0.49 N**

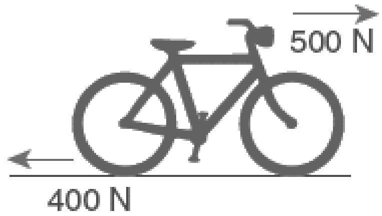
45. Calculate the acceleration of the bicycle of mass 25 kg as shown in Figures 1 and 2.

Solution

Apply Newton's second law in figure (1)

500 − 400 = $m\,a$

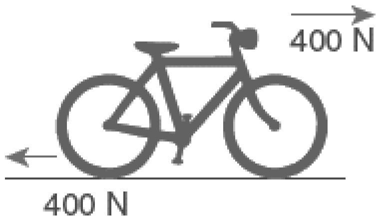
$a = \frac{F}{m}$; $= \frac{500-400}{25} = \frac{100}{25}$; a = 4ms^{-2}



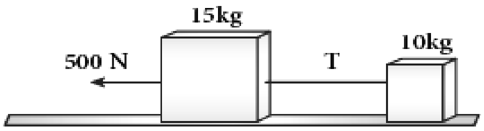
Apply Newton's second law in figure (2)

400 − 400 = $m\,a$

$a = \frac{F}{m}$; $= \frac{400-400}{25} = \frac{0}{25}$; a = 0ms^{-2}



46. Two bodies of masses 15 kg and 10 kg are connected with light string kept on a smooth surface. A horizontal force F=500 N is applied to a 15 kg as shown in the figure. Calculate the tension acting in the string.



Solution

Here motion is along horizontal direction only.
Consider the motion of mass m1 ; $F - T = m_1 a$ (or) $500 - T = 15\,a$
(or) **T = 500 – 15 a** – – – – (1)
Consider the motion of mass m_2 ; **$T = m_2 a = 10\,a$** – – – – (2)
From equation (1) and (2)
$500 - 15\,a = 10\,a$;
$25\,a = 500$; $\boldsymbol{a} = \frac{500}{25}$; $= \mathbf{20\ ms^{-2}}$
Put this in equation (2), we get **T = 10 a = 10 x 20 = 200N**

47. A car takes a turn with velocity 50 ms-1 on the circular road of radius of curvature 10 m. calculate the centrifugal force experienced by a person of mass 60kg inside the car?

Solution

Centrifugal force is given by, $F_{cf} = \frac{mv^2}{r}$; $= \frac{60 \times 50 \times 50}{10}$; = 6 x 2500
F_{cf} =15000 N

UNIT – IV (WORK, ENERGY AND POWER)

48. A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30⁰, find the work done by the force.

Solution

Force, F = 25 N; Displacement, dr = 15 m;
Angle between F and dr, $\theta = 30^0$
Work done, $W = Fdr \cos\theta$; $W = 25 \times 15 \times \cos 30^0$
$= 25 \times 15 \times \frac{\sqrt{3}}{2}$; **W = 324.76 J**

No.	Log
12.5	1.0969
15	1.1761
1.732	0.2385
(+)	2.5115
Antilog	3.247×10^2

49. A variable force $F = k\,x^2$ acts on a particle which is initially at rest. Calculate the work done by the force during the displacement of the particle from x = 0 m to x = 4 m. (Assume the constant k =1 N m⁻²)

Solution

Work done, $W = \int_{x_i}^{x_f} F(x)dx = k\int_0^4 x^2 dx$; $\frac{64}{3}$ Nm

50. Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg m s^{-1}.
(a) Will they have same kinetic energy?
(b) Will they have same speed?
Solution

(a) The kinetic energy of the mass is given by KE $=\frac{P^2}{2m}$

For the object of mass 2kg, kinetic energy is KE$_1$ = $\frac{(20)^2}{2x2} = \frac{400}{4}$ = 100J

For the object of mass 4kg, kinetic energy is KE$_2$ = $\frac{(20)^2}{2x4} = \frac{400}{8}$ = 50J

the kinetic energy of **both masses is not the same.** The kinetic energy of the **heavier object has lesser kinetic energy than smaller mass.**

(b) As the momentum, p = mv, the two objects **will not have same speed.**

51. Water in a bucket tied with rope is whirled around in a vertical circle of radius 0.5 m. Calculate the minimum velocity at the lowest point so that the water does not spill from it in the course of motion. (g = 10 ms^{-2})
Solution

Radius of circle r = 0.5m

The required speed at the highest point $v_2 = \sqrt{gr} = \sqrt{10 \text{ x } 0.5} = \sqrt{5}$ ms^{-1}

The speed at the lowest point $v_1 = \sqrt{5gr} = \sqrt{5} \text{ x } \sqrt{gr} = \sqrt{5} \text{ x } \sqrt{5}$ = 5ms^{-1}

52. Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).
Solution

Power, P = 75 W
Time of usage, t = 8 hour × 30 days = 240 hours
Electrical energy consumed is the product of power and time of usage.
Electrical energy = power × time of usage = P × t
=75 watt x 240 hour
=18000 watt hour
=18 kilowatt hour = 18kWh
1 electrical unit = 1kWh ; Electrical energy =18 unit

53. Show that the ratio of velocities of equal masses in an inelastic collision when one of the masses is stationary is $\frac{v_1}{v_2} = \frac{1-e}{1+e}$
Solution

e= $\frac{\text{Velocity of seperation (after collision)}}{\text{Velocity of approach (before collision)}}$; $= \frac{(V_2-V_1)}{(u_1-u_2)} = \frac{(V_2-V_1)}{(u_1-0)} = \frac{(V_2-V_1)}{u_1}$; $v_2 - v_1 = eu_1$ -------1

From the law of conservation of linear momentum, $mu_1 = mv_1 + mv_2$; $u_1 = v_1 + v_2$ ---------2

using the equation 2 for u_1 in 1, we get $v_2 - v_1 = e(v_1+v_2)$

on simplification, we get $\frac{v_1}{v_2} = \frac{1-e}{1+e}$

UNIT – V (MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES)

54. The position vectors of two point masses 10 kg and 5 kg are $(-3\vec{i} + 2\vec{j} + 4\vec{k})$m and $(3\vec{i} + 6\vec{j} + 5\vec{k})$m respectively. Locate the position of centre of mass.

Solution

m_1= 10kg, m_2 = 5kg ; $\vec{r}_1 = (-3\vec{i} + 2\vec{j} + 4\vec{k})$m ; $\vec{r}_2 = (3\vec{i} + 6\vec{j} + 5\vec{k})$m

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \;;\; \vec{r} = \frac{10(-3\vec{i}+2\vec{j}+4\vec{k})+5(3\vec{i}+6\vec{j}+5\vec{k})}{10+5} \;;$$

$$= \frac{30\vec{i}+2\vec{j}+40\vec{k}+15\vec{i}+30\vec{j}+25\vec{k}}{10+5} \;;= \frac{-15\vec{i}+50\vec{j}+65\vec{k}}{15} \;;= \left(-\vec{i} + \frac{10}{3}\vec{j} + \frac{13}{3}\vec{k}\right)\text{m}$$

The centre of mass is located at position $\vec{r}$

55. A force of $(4\vec{i} - 3\vec{j} + 5\vec{k})$N is applied at a point whose position vector is $(7\vec{i} + 4\vec{j} - 2\vec{k})$m. find the torque of force about the origin.

Solution

$$\vec{\tau} = \vec{r} \text{x} \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & 4 & -2 \\ 4 & -3 & 5 \end{vmatrix}$$

= (20 −6) $\hat{\imath}$ − (35 +8) $\hat{\jmath}$ + (−21−16) $\hat{k}$;

= (14$\hat{\imath}$ − 43$\hat{\jmath}$ − 37 $\hat{k}$) Nm

56. A cyclist while negotiating a circular path with speed 20 m s^{-1} is found to bend an angle by 30o with vertical. What is the radius of the circular path? (given, g = 10 ms^{-2})

Solution

Speed of the cyclist, v = 20 ms^{-1} ;
Angle of bending with vertical, θ = 30^0

Equation for angle of bending, $\tan\theta = \frac{v^2}{rg}$

Rewriting the above equation for radius $r = \frac{v^2}{\tan\theta g}$

$$r = \frac{(20)^2}{\tan 30^0 \text{ x } 10} \;;= \frac{20 \text{ x } 20}{(\tan 30^0) \text{ x } 10} \;;= \frac{400}{\left(\frac{1}{\sqrt{3}}\right)\text{x } 10} \;;= (\sqrt{3}) \text{ x } 40 \;;$$

= 1.732 x 40 ; **r = 69.28m**

57. Find the rotational kinetic energy of a ring of mass 9 kg and radius 3 m rotating with 240 rpm about an axis passing through its centre and perpendicular to its plane. (rpm is a unit of speed of rotation which means revolutions per minute)

Solution

The rotational kinetic energy is , KE = $\frac{1}{2}I\omega^2$.
The moment of Inertia of the ring is, I = MR^2
I = 9 x 3^2 ; = 9 x 9; = 81 kgm^2
The angular speed of the ring is, ω = 240 rpm ; = $\frac{240 \text{ x } 2\pi}{60}$ $rads^{-1}$
KE = $\frac{1}{2}$ x 81 x $\left(\frac{240 \text{ x } 2\pi}{60}\right)^2$. ; = $\frac{1}{2}$ x 81 x $(8\pi)^2$; KE = $\frac{1}{2}$ x 81 x 64 $(\pi)^2$;
= 2592 x $(\pi)^2$; KE ≈ 25920J **KE = 25.920 kJ [$(\pi)^2$ ≈ 10]**

58. A rolling wheel has velocity of its centre of mass as 5 ms^{-1}. If its radius is 1.5 m and angular velocity is 3 $rads^{-1}$, then check whether it is in pure rolling or not.

Solution

Translational velocity (v_{TRANS}) or velocity of centre of mass,
v_{CM} = 5 m s^{-1}
The radius is, R = 1.5 m and the angular velocity is, ω = 3 $rads^{-1}$
Rotational velocity, v_{ROT} = Rω
v_{ROT} = 1.5×3; v_{ROT} = 4.5 ms^{-1}
As v_{CM} > Rω (or) v_{TRANS} > Rω, It is not in pure rolling, but sliding.

59. Four round objects namely a ring, a disc, a hollow sphere and a solid sphere with same radius R start to roll down an incline at the same time. Find out which object will reach the bottom first.

Solution

For all the four objects namely the ring, disc, hollow sphere and solid sphere, the radii of gyration K are R, $\sqrt{\frac{1}{2}}$ R, $\sqrt{\frac{2}{3}}$ R, $\sqrt{\frac{2}{5}}$ R With numerical values the radius of gyration K are 1R, 0.707R, 0.816R, 0.632R respectively. The expression for time taken for rolling has the radius of gyration K.

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}.$$

The one with least value of radius of gyration K will take the shortest time to reach the bottom of the inclined plane. The order of objects reaching the bottom is first, solid sphere; second, disc; third, hollow sphere and last, ring.

EXERCISE PROBLEM

60. Find the moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis. Given: mass of hydrogen atom 1.7×10^{-27} kg and inter atomic distance is equal to 4×10^{-10}m

Solution

The moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis,

$I_{CM} = 2m_H d^2 + m_H d^2 = 2m_H d^2$

$I_{CM} = 2 \times 1.7 \times 10^{-27} \times 2 \times 10^{10} \times 2 \times 10^{10}$

$I_{CM} = 13.6 \times 10^{-47}$

$I_{CM} = 1.36 \times 10^{-46}$ **kgm**

UNIT – VI (GRAVITATION)

EXERCISE PROBLEM

61. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T_1, what is the time period of this unknown planet?

Solution

$a_2 = 2a_1$. From Kepler's third law $T_1^2 \propto a_1^3$; $T_2^2 \propto a_2^3$

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} ; \frac{T_1^2}{T_2^2} = \frac{a_1^3}{(2a_1)^3} ; = \frac{a_1^3}{(8a_1)^3} ; = \frac{1}{8} ; T_2^2 = 8T_1^2$$

$$T_2 = \sqrt{8}T_1 ; = 2\sqrt{2}\, T_1$$

62. Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Planet (imaginary)	Time period(T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

Solution

1) For Kurunji ; T = 2 years , a = 8AU = 2 x 4 = 2 $(2)^2$ = 2T^2
2) For Mullai ; = 3 years, a = 18AU = 2 x 9 = 2 $(3)^2$ = 2 T^2
3) For Marutham ; 4 years, a = 32AU = 2 x 16 = 2 $(4)^2$ = 2 T^2
4) For Neithal ; 5 years, a = 50AU = 2 x 25 = 2 $(5)^2$ = 2 T^2
5) For Paalai ; 6 years, a = 72AU = 2 x 36 = 2 $(6)^2$ = 2 T^2

Hence the relation connecting semi major axis and time period; a= $2T^2$

63. If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

Solution

By Newton's law of gravitation, $F = \frac{Gm_1m_2}{r^2}$

If $m_1 \to 2m_1$, $m_2 \to 2m_2$ and $r = 2r$

$= \frac{G2m_12m_2}{(2r)^2}$; $= \frac{4Gm_1m_2}{4r^2}$; $= \frac{Gm_1m_2}{r^2}$; $= F$; There is no change in the force.

64. If the angular momentum of a planet is given by $\vec{L}=5\,t^2\hat{i}-6\,t\,\hat{j}+3\,\hat{k}$. What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Solution

Torque is given by, $\vec{\tau}= \frac{d\vec{L}}{dt}= \frac{d}{dt}(5\,t^2\hat{i}-6t\hat{j}+3\hat{k})$; $\vec{\tau}=$ $\mathbf{10t\,\hat{i}-6\hat{j}}$

Here the torque produced will be in the direction of angular momentum.

UNIT – VII (PROPERTIES OF MATTER)

65. A wire 10 m long has a cross-sectional area 1.25×10^{-4} m². It is subjected to a load of 5 kg. If Young's modulus of the material is 4×10^{10} Nm⁻², calculate the elongation produced in the wire. Take g = 10 ms⁻².

Solution

$\frac{F}{A} = Y \times \frac{\Delta L}{L}$; $\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$; $\left(\frac{150}{1.25 \times 10^{-4}}\right)\left(\frac{10}{4 \times 10^{10}}\right)$; $=10^{-4}$ m

66. A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m³, calculate the bulk modulus of the material.

Solution

$K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\frac{V}{\Delta V}$; $K = \frac{10^6 \times 1}{1.5 \times 10^{-5}}$; $= 6.67 \times 10^{10}$ Nm^{-2}

No.	Log
10^{11}	11.0000
1.5	0.1761
(–)	10.8239
Antilog	6.666×10^{10}

67. A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20m, F=4000N, x=0.50cm ; =0.005m and Area A = L² = 0.04 m²

Therefore, $\eta_R = \left(\frac{F}{A}\right) \times \left(\frac{L}{x}\right)$; $= \left(\frac{4000}{0.04}\right) \times \left(\frac{0.20}{0.005}\right)$; $= 4 \times 10^6$ Nm^{-2}

68. A wire of length 2 m with the area of cross-section 10^{-6} m² is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: Y=12 × 10^{10}N m^{-2}

Solution

i) Stress = $\frac{F}{A} = \frac{980}{10^{-6}}$; = 98 x 10^7 Nm^{-2}

ii) Strain = $\frac{\text{Stress}}{Y}$; $\frac{98 \text{ x } 10^7}{12 \text{ x } 10^{10}}$; =8.17 x 10^{-3}

iii) Volume = 2 x 10^{-6} m^3

Energy = ½ (Stress x Strain) x Volume

= ½ (98x10^7) x8.17x10^{-3}) x2x 10^{-6} = 8J

No.	Log
980	2.9912
81.66	1.9120
(+)	4.9032
Antilog	8.002 x 10^4

69. A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

Solution

Radius of the sphere R = 1.5 cm; mass m = 0.038 kg

Volume of the sphere V= $\frac{4}{3}\pi R^3$; = $\frac{4}{3}(3.14) \text{ x } (1.5 \text{ x } 10^{-2})^3$; 1.413 x $10^{-5}m^3$

Density $\rho = \frac{m}{V} = \frac{0.038 \text{ kg}}{1.413 \text{ x } 10^{-5} m^3}$; = 2690 kgm^{-3}

Hence, the specific gravity of the sphere = $\frac{2690}{1000}$ = 2.69

70. A metal plate of area 2.5×10^{-4} m² is placed on a 0.25×10^{-3} m thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity 3×10–2 m s^{-1}, calculate the coefficient of viscosity of castor oil.
Given: A=2.5×10^{-4} m², dx = 0.25×10^{-3}m, F=2.5 N and dv = 3×10^{-2} ms^{-1}

Solution

$F = -\eta A \frac{dv}{dx}$; $\eta = \frac{F}{A}\frac{dx}{dv}$; = $\frac{(2.5 \text{ N})\ (0.25 \text{ x } 10^{-3} m)}{(2.5 \text{ x } 10^{-4} m^2)(3 \text{ x } 10^{-2} \text{ ms}^{-1})}$;

= 0.083 x 10^3 N m^{-2}s

71. Let 2.4×10^{-4} J of work is done to increase the area of a fi lm of soap bubble from 50 cm2 to 100 cm². Calculate the value of surface tension of soap solution.

Solution

A soap bubble has two free surfaces, therefore increase in surface area $\Delta A = A_2 - A_1$ = 2(100–50) × $10^{-4}m^2$ = 100 × $10^{-4}m^2$.

Since, work done W = T x ΔA ; T = $\frac{W}{\Delta A}$; = $\frac{2.4 \text{ x } 10^{-4} J}{100 \text{ x } 10^{-4} m^2}$;

= 2.4 x 10^{-2} Nm^{-1}

72. If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where R = 2.0 cm, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is $\Delta p = P_2 - P_1 = \frac{4T}{R}$

$\Delta p = P_2 - P_1 = \rho g h \;\; [\rho g h = \frac{4T}{R}]$

Surface tension, T = $\frac{\rho g h R}{4}$; = $\frac{(800)(9.8)(4 \times 10^{-3})(2 \times 10^{-2})}{4}$;T=15.68×10^{-2} Nm^{-1}

73. Mercury has an angle of contact equal to 140^0 with soda lime glass. A narrow tube of radius 2 mm, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury T=0.456 Nm^{-1}; Density of mercury $\rho = 13.6\times10^3$ kgm^{-3}

Solution

Capillary descent, cos140 = cos(90+50) –sin50 = –0.7660

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times (0.456\ Nm^{-1})\ (\cos 140^0)}{(2 \times 10^{-3}\ m)\ (13.6 \times 10^3)\ (9.8\ ms^{-2})} ; = \frac{2 \times 0.456 \times (-0.7660)}{2 \times 13.6 \times 9.8}$$

$= \frac{-0.6986}{266.56}$; $= -2.62 \times 10^{-3}$ m

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

74. In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

No.	Log
1.32	0.1206
30	1.4771
(–)	$\bar{2}$.6435
Antilog	4.400 x 10^{-2}

Solution

$a_1 v_1 = 30 a_2 v_2$; $\pi r_1^2 v_1 = 30\ \pi r_2^2 v_2$; $V_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$

$V_2 = \frac{1}{30}\left(\frac{0.8 \times 10^{-2} m}{0.4 \times 10^{-2}\ m}\right)^2 \times (0.33\ ms^{-1})$; $V_2 = 0.044\ ms^{-1}$

EXERCISE PROBLEM

75. A capillary of diameter d mm is dipped in water such that the water rises to a height of 30*mm*. If the radius of the capillary is made 2/3 of its previous value, then compute the height up to which water will rise in the new capillary?

Solution

Surface tension by capillary rise method is, $T = \frac{\rho r h g}{2\cos\theta}$; $h = \frac{2T\cos\theta}{\rho r g}$

$h \propto \frac{1}{r}$ (or) $hr = \text{constant}$

$h_1 r_1 = h_2 r_2$; $h_2 = \frac{h_1 r_1}{r_2}$; $= \frac{30 \times 10^{-3} \times r}{\left(\frac{2r}{3}\right)}$; $= \frac{3 \times 30 \times 10^{-3} \times r}{2r}$

$h_2 = 45 \times 10^{-3}$ m = 45mm

UNIT – VIII (HEAT AND THERMODYNAMICS)

76. Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2° C and in hot summer its average temperature 25° C. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron α = 10 ×10⁻⁶ per °C

Solution

$\frac{\Delta L}{L_0} = a_L \Delta T$; $\Delta L = a_L L_0 \Delta T$;

$\Delta L = 10 \times 10^{-6} \times 300 \times 23 = 0.069$ m=69 mm

77. A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?

Solution

Work done on the system (by the person while stirring), W = −30 kJ = −30,000J

Heat flowing out of the system, Q = −5 kcal = −5×4184 J =−20920 J

Using First law of thermodynamics, ΔU = Q−W

ΔU = −20,920 J− (−30,000) J

ΔU = −20,920 J+30,000 J = 9080 J

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

78. Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?

Solution

Work done by the system (body), W = +500 kJ

Heat released from the system (body), Q = –230 kJ

The change in internal energy of a body = ΔU = –230 kJ – 500 kJ = –730 kJ

79. 500 g of water is heated from 30^0 C to 60^{0}C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)

Solution

When the water is heated from 30^{0}C to 60^{0}C, there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy. $\Delta U = Q = m s_v \Delta T$

The mass of water = 500 g = 0.5 kg;

The change in temperature = 30K

The heat Q = 0.5×4184×30 = 62.76 kJ

80. During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?

Solution

The efficiency of heat engine is given by $\eta = 1 - \frac{Q_L}{Q_H}$; $\eta = 1 - \frac{300}{500}$;

$= 1 - \frac{3}{5}$; $\eta = 1 - 0.6$; 0.4

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

81. There are two Carnot engines A and B operating in two different temperature regions. For Engine A, the temperatures of the two reservoirs are 150^{0}C and 100^{0}C. For engine B the temperatures of the reservoirs are 350^{0}C and 300^{0}C. Which engine has lesser efficiency?

Solution

The efficiency for engine A $= 1 - \frac{373}{623}$; = 0.11. Engine A has 11% efficiency

No.	Log
373	2.5717
423	2.6263
(−)	$\bar{1}$.9454
Antilog	8.819 x 10^{-1}

The efficiency for engine B $= 1 - \frac{573}{623}$; = 0.08.

Engine B has 8% efficiency

No.	Log
573	2.7582
623	2.7945
(−)	$\bar{1}$.9637
Antilog	9.198 x 10^{-1}

82. A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interion?

Solution COP = $\beta = \frac{Q_L}{W}$; W = $\frac{Q_L}{COP}$; = $\frac{200}{3}$; = 66.67J

EXERCISE PROBLEM

83. Calculate the number of moles of air is in the inflated balloon at room temperature The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa.

Solution

From ideal gas equation PV = μRT or $\mu = \frac{PV}{RT}$ --------1

Volume of spherical shaped balloon, V = $\frac{4}{3}\pi r^3$

= $\frac{4}{3}$ x 3.14 x $(10 \times 10^{-2})^3$; 4 x 1.046 x 10^{-3} ;

V = 4.184 x 10^{-3}

From equation 1,= $\frac{180 \times 10^3 \times 4.184 \times 10^{-3}}{8.31 \times (27+273)}$;

$= \frac{180 \times 4.184}{8.31 \times 300}$;

$= \frac{180 \times 4.184}{2493}$; = 0.3021 ; $\mu \cong$ 0.3 moles

No.	Log
180	2.2553
4.184	0.6216
(+) 2493	2.8769 3.3967
(−)	$\bar{1}$.4802
Antilog	3.021 x 10^{-1}

84. In the planet Mars, the average temperature is around -53°C and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

Solution

Number of molecules per unit volume in Mars planet is $\mu_{mars} = \frac{PV}{RT}$

$= \frac{0.9 \times 1000 \times 1}{8.31 \times (-53+273)}$; $= \frac{900}{8.31 \times 220}$; $= \frac{90}{8.31 \times 22}$; $= \frac{90}{182.82}$;

μ_{mars} = 0.4922 moles

Number of molecules per unit volume in Earth planet is $\mu_{earth} = \frac{PV}{RT}$

$= \frac{101.3 \times 1000 \times 1}{8.31 \times (27+273)}$; $= \frac{101300}{8.31 \times 300}$; $= \frac{1013}{8.31 \times 3}$; $= \frac{1013}{24.93}$;

μ_{earth} = 40.46 moles

85. A man starts bicycling in the morning at a temperature around 25^0 C, he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

Solution

Let this is considered as isochoric process, then P = $\left[\frac{\mu R}{V}\right]$T

In morning , $P_1 = \left[\frac{\mu R}{V}\right]T_1$ -------------1 In afternoon , $P_2 = \left[\frac{\mu R}{V}\right]T_2$ -------------2

Divide equation (1) by (2)

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$; $T_2 = \frac{P_1}{P_2} T_1$; $T_2 = \frac{520 \times 1000}{500 \times 1000}(25+273)$; $= \frac{52}{50} \times 298$
$= 1.04 \times 298$; $= 309.92$ k, **T_2 = 309.92 k or 36.92 ^{0}c**

86. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of 327°C. To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

Solution

Efficiency when , $T_H = (327+ 273) = 600$ K & $\eta = 45\% = 0.45$

$\eta = 1 - \frac{T_L}{T_H}$; $0.45 = 1 - \frac{T_L}{600}$; $\frac{T_L}{600} = 1 - 0.45$; $= 0.55$

$T_L = 0.55 \times 600 = 330$ K ; **T_L = 330 K or 57^{0}C**

Efficiency when , $T_H = (57+ 273) = 330$ K & $\eta = 60\% = 0.60$

$\eta = 1 - \frac{T_L}{T_H}$; $0.60 = 1 - \frac{330}{T_H}$; $\frac{330}{T_H} = 1 - 0.60$; $= 0.40$

$T_H = \frac{330}{0.4} = \frac{3300}{4}$; $= 825$ K ; **T_H = 825 K or 552^{0}C**

87. An ideal refrigerator keeps its content at 0°C while the room temperature is 27°C. Calculate its coefficient of performance.

Solution

Coefficient of performance (COP) of refrigerator, when

$T_H = (27 + 273) = 300$ K & $T_L = (0 +273) = 273$ K

COP = $\beta = \frac{T_L}{T_H - T_L}$; $\beta = \frac{273}{300-273}$; $= \frac{273}{27}$;

$\beta = 10.11$

No.	Log
273	2.4362
27	1.4314
(−)	1.0048
Antilog	1.011×10^1

UNIT – IX (KINETIC THEORY OF GASES)

88. Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 m s^{-1}. Calculate rms speed, average speed and most probable speed.

Solution

The average speed $\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10}$; = 5.2 ms^{-1}

To find the rms speed, first calculate the mean square speed

$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10}$; = 30.6 m^2s^{-2}

$V_{max} = \sqrt{\overline{v^2}} = \sqrt{30.6}$; = 5.53 ms^{-1}

The most probable speed is 5 ms^{-1} because three of the particles have that speed.

89. Find the adiabatic exponent γ for mixture of μ_1 moles of monoatomic gas and μ_2 moles of a diatomic gas at normal temperature (27^0C) .

Solution

The specific heat of one mole of a monoatomic gas $C_V = \frac{3}{2}R$

For μ_1 mole, $C_V = \frac{3}{2}\mu_1 R$, $C_P = \frac{5}{2}\mu_1 R$

The specific heat of one mole of a diatomic gas , $C_V = \frac{5}{2}R$

For μ_2 mole, $C_V = \frac{5}{2}\mu_2 R$, $C_P = \frac{7}{2}\mu_2 R$

specific heat of the mixture at constant volume $C_V = \frac{3}{2}\mu_1 R + \frac{5}{2}\mu_2 R$

The specific heat of the mixture at constant pressure

$C_P = \frac{5}{2}\mu_1 R + \frac{7}{2}\mu_2 R$

The adiabatic exponent $\gamma = \frac{C_P}{C_V} = \frac{5\mu_1 + 7\mu_2}{3\mu_1 + 5\mu_2}$

90. An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is 1.2×10^{-10}m. Calculate the mean free path of oxygen molecule.

Solution

$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$.

We have to find the number density n By using ideal gas law

$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$; = 2.499 x 10^{25} molecules / m^3

$$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2}$$

$= \frac{1}{15.65 \times 10^5}$; $\lambda = 0.63 \times 10^{-6}$ m

EXERCISE PROBLEM

91. A fresh air is composed of nitrogen N_2(78%) and oxygen O_2(21%). Find the rms speed of N_2 and O_2 at 20°C.

Solution

N_A = Avogadro Number and R = 8.314 J mol^{-1} K^{-1}

1) **Nitrogen molecule (N_2)**

Atomic mass of Nitrogen = 14,

Then One Nitrogen molecule = 2×14=28

Thus 28 *g* nitrogen gas contains N_A number of nitrogen molecules Hence, Molecular mass of one mole of nitrogen molecule, M = 28 g/mol = 0.028 kg/mol

RMS speed of nitrogen molecule,

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} ; = \sqrt{\frac{3 \times 8.314 \times (20+273)}{0.028}}$$

$$= \sqrt{\frac{3 \times 8.314 \times 293}{0.028}} ; = 510.9 \text{ ms}^{-1} ; (v_{rms}) = 511 \text{ ms}^{-1}$$

No.	Log
3	0.4771
8.314	0.9198
293	2.4669
(+)	3.8638
0.028	$\bar{2}$.4472
(−)	5.4166 x ½ = 2.7083
Antilog	5.109 x 10^2

2) **Oxygen molecule (O_2)**

Atomic mass of Oxygen = 16. Then One Oxygen molecule =2×16=32

Thus 32 *g* Oxygen contains NA number of oxygen molecules

Hence, Molecular mass of one mole of oxygen molecule M = 32 g/mol = 0.032kg/mol

RMS speed of oxygen molecule, (v_{rms})

$$= \sqrt{\frac{3RT}{M}} ; = \sqrt{\frac{3 \times 8.314 \times (20+273)}{0.032}}$$

$$= \sqrt{\frac{3 \times 8.314 \times 293}{0.032}} ; = 477.9 \text{ ms}^{-1} ; (v_{rms}) = 478 \text{ ms}^{-1}$$

No.	Log
3	0.4771
8.314	0.9198
293	2.4669
(+)	3.8638
0.032	$\bar{2}$.5051
(−)	5.3587 x ½ = 2.6793
Antilog	4.779 x 10^2

92. If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 ms^{-1}, show that the surface temperature of Jupiter is sub-zero.

Solution

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} \text{ or } v_{rms}^2 = \frac{3RT}{M} \text{ or } T = \frac{v_{rms}^2 M}{3R}$$

$$T = \frac{(471.8)^2 x\ 0.016}{3\ x\ 8.314} ; \frac{471.8\ x\ 471.8\ x\ 0.016}{24.942} ;$$

T = 142.8 K ≈143 K

or T = 143 – 273 ; = – 130^{0}C

Thus surface temperature of Jupiter planet is less than 0°C

No.	Log
471.0	2.6738
471.0	2.6738
0.016	$\bar{2}$.2041
(+)	3.5517
24.942	1.3969
(−)	2.1548
Antilog	1.428 x 10^2

93. Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P. (standard temperature T_1 = 273 K)

Solution

At standard temperature and pressure (STP) ; $v_{rms\,1} = v$ & T_1 = 273 K

At new temperature and pressure ; $v_{rms\,2} = 3v$ & $T_2 = ?$

By definition, $v_{rms\,1} = \sqrt{\frac{3RT_1}{M}}$ -----------1 ; $v_{rms\,2} = \sqrt{\frac{3RT_2}{M}}$ -----------2

Divide equation 2 by 1

$\frac{v_{rms\,2}}{v_{rms\,1}} = \frac{\sqrt{\frac{3RT_2}{M}}}{\sqrt{\frac{3RT_1}{M}}} = \sqrt{\frac{T_2}{T_1}}$; $\left[\frac{v_{rms\,2}}{v_{rms\,1}}\right]^2 = \frac{T_2}{T_1}$; $T_2 = T_1 \left[\frac{v_{rms\,2}}{v_{rms\,1}}\right]^2$; 273 x $\left[\frac{3v}{v}\right]^2$

= 273 x 9 ; **T_2 = 2557 K**

94. A gas is at temperature 80°C and pressure 5×10^{-10}N m^{-2}. What is the number of molecules per m3 if Boltzmann's constant is 1.38×10^{-23} J K^{-1}

Solution

Ideal gas constant PV = nkT;

$n = \frac{PV}{kT} = \frac{5 \text{ x } 10^{-10} \text{x } 1}{1.38 \text{ x } 10^{-23} \text{x}(80+273)}$; $= \frac{5 \text{ x } 10^{13}}{1.38 \text{ x } 353}$

n = 1.026 x 10^{-2} x 10^{13}; **n = 1.026 x 10^{11}**

95. A gas made of a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Calculate the energy of the gas in terms of RT. Neglect the vibrational modes.

Solution

Oxygen (O_2) is a di atomic molecule. Its number of degrees of freedom

f = 5

Argon (Ar) is a mono atomic molecule. Its number of degrees of freedom

f =3

For mono atomic molecule, total energy of μ mole of gas,

$U_1 = \mu_1 \text{ x } \frac{3}{2} N_A kT; = \mu_1 \text{ x } \frac{3}{2} RT$

For di atomic molecule, total energy of μ mole of gas,

$U_2 = \mu_2 \text{ x } \frac{5}{2} N_A kT; = \mu_2 \text{ x } \frac{5}{2} RT$

Total energy of gas mixture $U = U_1 + U_2$

$U = \mu_1 \text{ x } \frac{3}{2} RT + \mu_2 \text{ x } \frac{5}{2} RT$; $U = 4 \text{ x } \frac{3}{2} RT + 2 \text{ x } \frac{5}{2} RT$

U = 6 RT + 5 RT; **U = 11RT**

96. Estimate the total number of air molecules in a room of capacity 25m³ at a temperature of 27°C.

Solution

Ideal gas constant PV = NkT or $N = \frac{PV}{kT}$;

$$= \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times (27+273)}$$

$$= \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300}; = \frac{1.013 \times 25 \times 10^{28}}{414}$$

$N = 6.116 \times 10^{-3} \times 10^{28}$;

$N = 6.116 \times 10^{25}$ molecules

No.	Log
1.013 25	0.0056 1.3979
(+) 414	1.4035 2.6170
(−)	$\bar{2}$.7865
Antilog	6.116×10^{-2}

UNIT – X (OSCILLATIONS)

97. If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.

Solution

$T \propto \sqrt{l}$; T = constant $\sqrt{l}$

$\frac{T_f}{T_i} = \sqrt{\frac{1+\frac{44}{100}l}{l}}$; $\sqrt{1.44}$ = 1.2 ;

Therefore, $T_f = 1.2\,T_i = T_i + 20\%\,T_i$

EXERCISE PROBLEM

98. Consider a simple pendulum of length l = 0.9 m which is properly placed on a trolley rolling down on a inclined plane which is at θ = 45° with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Solution

The effective value of acceleration due to gravity will be equal to the component of g normal to the inclined plane which is $g' = g\cos\theta$

Then the time period is given by $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

$T = 2\pi\sqrt{\frac{0.9}{9.8 \times \cos 45^0}}; = 2\pi\sqrt{\frac{0.9}{9.8 \times \frac{1}{\sqrt{2}}}}; ; = 2 \times 3.14\sqrt{\frac{0.9}{9.8 \times 0.707}}$

$T = 6.28\sqrt{\frac{0.9}{6.9286}}; = 6.28\sqrt{0.1290}$; = 6.28 x 0.3604 ; **T= 2.263s**

99. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released, then executes simple harmonic motion. Show that its time period of oscillation is T = $2\pi\sqrt{\frac{m}{A\,g\rho}}$

Solution

Let the wood piece of mass 'm' and area 'A' floating in liquid of density 'ρ' is pressed down by a distance 'x' and released, so that it executes SHM.

The restoring force is given by, $F = k\,x$ (or) mg = kx (or) k = $\frac{mg}{x}$;

$$= \frac{(\rho v)g}{x}\ ; = \frac{(\rho A x)g}{x} ; = \ \rho A g$$

The time period of vertical oscillation is, T = $2\pi\sqrt{\frac{m}{k}}$; = $2\pi\sqrt{\frac{m}{A\,g\rho}}$

UNIT – XI (WAVES)

100. The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 ms^{-1}.

Solution

$\lambda_1 = \frac{v}{f_1} = \frac{340}{20}$; = 17 m ; $\lambda_2 = \frac{v}{f_2} = \frac{340}{20 \times 10^3}$; = 0.017m

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 ms^{-1}.

101. Calculate the speed of sound in a steel rod whose Young's modulus Y = 2 × 10^{11} Nm^{-2} and ρ = 7800 kg m^{-3}.

Solution

$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}}$; $\sqrt{0.2564 \times 10^8}$;

= 0.506 x 10^4 ms^{-1} ; **= 5 x 10^3 ms^{-1}**

No.	Log
10^{11} 3900	11.0000 3.5911
(−)	7.4089 x ½ 3.7044
Antilog	5.062 x 10^3

Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safeguard his cattle.

102. An increase in pressure of 100 kPa causes a certain volume of water to decrease by 0.005% of its original volume. (a) Calculate the bulk modulus of water? (b) Compute the speed of sound (compressional waves) in water?

Solution

(a) Bulk modulus B = V $\left|\frac{\Delta P}{\Delta V}\right| = \frac{100 \times 10^3}{0.005 \times 10^{-2}}$; = $\frac{100 \times 10^3}{5 \times 10^{-5}}$; = 2000Mpa, where Mpa is mega pascal

(b) Speed of sound in water is $v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2000 \times 10^6}{1000}}$ = 1414 ms^{-1}

103. Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 ms^{-1}.

Solution

The time taken by the sound to come back as echo is $2t = 4 \Rightarrow t = 2$ s

∴The distance is $d = vt = (343 \text{ m s}^{-1})(2 \text{ s}) = 686$ m.

104. A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.

Solution

Frequency, f = 900 MHz; = 900×10^6 Hz

The speed of wave is $c = 3 \times 10^8 ms^{-1}$

$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{900 \times 10^6}$; = 0.33m

105. Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$. Compute the number of beats per second.

Solution : Given $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$

Comparing with $y = A \sin(2\pi f_1 t)$, we get

$2\pi f_1 = 240\pi \Rightarrow f_1 = 120$Hz ; $2\pi f_2 = 244\pi \Rightarrow f_2 = 122$Hz

The number of beats produced is $| f_1 - f_2| = |120 - 122| = |-2|$

=2 beats per sec

106. The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. The intensity of the sound from each instrument is 10^{-12} W m^{-2}

Solution

$\Delta L = 10\log_{10}\left[\frac{I_1}{I_0}\right] = 50$ dB ; $\log_{10}\left[\frac{I_1}{I_0}\right] = 5$ dB

$\frac{I_1}{I_0} = 10^5$; $I_1 = 10^5 I_0$; $= 10^5 \times 10^{-12}$ Wm^{-2}; $I_1 = 10^{-7}$ Wm^{-2}

Since three musical instruments are played, therefore $I_{total} = 3I_1 = 3\times10^{-7}$ Wm^{-2}

107. If a flute sounds a note with 450Hz, what are the frequencies of the second, third, and fourth harmonics of this pitch? If the clarinet sounds with a same note as 450Hz, then what are the frequencies of the lowest three harmonics produced.

Solution

For a flute which is an open pipe, we have

Second harmonics $f_2 = 2 f_1 = 900$ Hz

Third harmonics $f_3 = 3 f_1 = 1350$ Hz

Fourth harmonics $f_4 = 4 f_1 = 1800$ Hz

For a clarinet which is a closed pipe, we have

Second harmonics $f_2 = 3 f_1 = 1350$ Hz

Third harmonics $f_3 = 5 f_1 = 2250$ Hz

Fourth harmonics $f_4 = 7 f_1 = 3150$ Hz

108. If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

Solution

Let l_2 be the length of the open organ pipe, with l_1=30 cm the length of the closed organ pipe.

It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.

The third harmonic of a closed organ pipe $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1}$; = 3$f_1$

The fundamental frequency of open organ pipe is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$;

Therefore $\frac{v}{2l_2} = \frac{3v}{4l_1}$; $l_2 = \frac{2l_1}{3}$ = 20 cm

109. A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water? (speed of sound in air is 343 m s^{-1})

Solution

The wavelength , $\lambda = \frac{C}{f}$; $\lambda = \frac{343\ ms^{-1}}{343\ Hz}$; = 1.0m

Let the length of the resonant columns be L_1, L_2 and L_3.

The first resonance occurs at length L_1. $L_1 = \frac{\lambda}{4} = \frac{1}{4}$ = 0.25 m

The second resonance occurs at length L_2. $L_2 = \frac{3\lambda}{4} = \frac{3}{4}$ = 0.75 m

The third resonance occurs at length L_3. $L_3 = \frac{5\lambda}{4} = \frac{5}{4}$ = 1.25 m and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water H_{min} for resonance is, H_{min} = 1.0 m – 0.75 m = 0.25 m

110. A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

Solution

End correction e= $\frac{L_2 - 3L_1}{2}$; = $\frac{0.7-3\ (0.2)}{2}$; = 0.05 m

111. A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 ms^{-1}.

(a) Calculate the frequency of the sound which is coming directly from the source.

(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 ms^{-1}.

Solution

(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$f' = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+6}\right]$ x 1500 = 1473 Hz

(b) Sound is reflected from the cliff and reaches observer, therefore,

$f' = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-6}\right]$ x 1500 = 1528 Hz

112. An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 m s−1. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

Solution

Observer is stationary

(i) Source (train) is moving towards an observer:
The observed frequency due to train arriving station is

$f_{in} = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-8}\right]$ x 240 = 246 Hz

(ii) Source (train) is moving away from an observer:
The observed frequency due to train leaving station is

$f_{out} = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+8}\right]$ x 240 = 234 Hz

So the number of beats = $|\mathbf{f_{in}} - \mathbf{f_{out}}|$ = (246 – 234) = 12

EXERCISE PROBLEM

113. The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?

Solution

Since 3000 waves passes over in 2 minutes (120 s), the number of waves passes per second is, $f = \frac{3000}{120}$; = 25 per second.

The wavelength $\lambda = \frac{v}{f} = \frac{900}{25}$; λ =36m

114. A ship in a sea sends SONAR waves straight down into the seawater from the bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases.

Solution

Depth at first place, $d_1 = \frac{v \times t_1}{2}$; $= \frac{1533 \times 3.5}{2}$; $= \frac{5365.5}{2}$; $d_1 = 2682.75\text{m}$

Depth at second place, $d_2 = \frac{v \times t_2}{2}$; $= \frac{1533 \times 2}{2}$; $= \frac{5365.5}{2}$; $d_1 = 1533\text{m}$

The difference in height between two cases $\Delta d = d_1 - d_2$

$= 2682.75 - 1533$; $\Delta d = 1149.75\text{m}$

115. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

Solution

The third harmonic of a closed organ pipe $f_c = \frac{v}{4L}$ -----------1

The fundamental frequency of open organ pipe is $f_0 = \frac{v}{2L}$ ---------2

Divide equation 2 by 1 $\frac{f_0}{f_c} = \frac{\left[\frac{v}{2L}\right]}{\left[\frac{v}{4L}\right]} = 2$ $f_0 = 2f_c = 2 \times 250 = 500\text{Hz}$

$2.1798 v_t = -80.652$; $v_0 = \frac{80.652}{2.1798}$; $= 36.99$ $v_0 = 37\text{m/s}$

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