Class : 12

Register Number

SECOND REVISION EXAMINATION - 2023

BUSINESS MATHEMATICS AND STATISTICS

Time Allowed : 3.00 Hours] PART - I [Max. Marks : 90

I. Answer All the questions. 20×1=20

Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If $\frac{a_1}{x}+\frac{b_1}{y}=c_1, \frac{a_2}{x}+\frac{b_2}{y}=c_2, \Delta_1=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \Delta_2=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, \Delta_3=\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$ then (x, y) is ------

(a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (b) $\left(\frac{\Delta_3}{\Delta_1}, \frac{\Delta_2}{\Delta_1}\right)$ (c) $\left(\frac{\Delta_1}{\Delta_2}, \frac{\Delta_1}{\Delta_3}\right)$ (d) $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$

2. If the number of variables in a non-homogeneous system AX = B is n, then the system possesses an infinitely many solutions when,

(a) $\rho(A)=\rho(A, B)>n$ (b) $\rho(A)=\rho(A, B)=n$

(c) $\rho(A)=\rho(A, B)<n$ (d) none of these

3. $\int e^{2x}[2x^2+2x]\,dx$ = --------

(a) $e^{2x}.x^2+c$ (b) $xe^{2x}+c$ (c) $2x^2e^2+c$ (d) $\frac{x^2e^x}{2}+c$

4. The value of $\int_1^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$ is --------

(a) 1 (b) 0 (c) -1 (d) 5

5. The demand and supply functions are given by $D(x)= 16-x^2$ and $S(x) = 2x^2 + 4$ are under perfect competition, then the equilibrium price x is --------

(a) 2 (b) 3 (c) 4 (d) 5

6. The marginal cost function is $MC = 100\sqrt{x}$. find AC given that TC = 0 when the output is zero is ----

(a) $\frac{200}{3}x^{3/2}$ (b) $\frac{200}{3}x^{5/2}$ (c) $\frac{200}{3x^{3/2}}$ (d) $\frac{200}{3x^{1/2}}$

7. The differential equation $\left(\frac{dx}{dy}\right)^3+2y^{1/2}=x$ is -------

(a) of order 2 and degree 1 (b) of order 1 and degree 3

(c) of order 1 and degree 6 (d) of order 1 and degree 2

8. The general solution of differential equation $\frac{dy}{dx}=\sin x$ is -------

(a) $y= \sin x + c$ (b) $y = -\sin x + c$ (c) $y = \cos x + c$ (d) $y = -\cos x + c$

9. Lagrange's interpolation formula can be used for -------

(a) equal intervals only (b) unequal intervals only

(c) both equal and unequal intervals (d) none of these

10. $\Delta^3.\Delta^{-2}f(x)$ = ---------

(a) $\Delta^3 f(x)$ (b) $\Delta f(x)$ (c) $\Delta^{-2}f(x)$ (d) $\Delta^5 f(x)$

11. Probability which explains x is equal to or less than particular value is classified as --------

(a) discrete probability (b) cumulative probability

(c) marginal probability (d) Continuous probability

12. The heights of persons in a country is a random variable of type ----------

(a) discrete random variable (b) continuous random variable

(c) both (a) & (b) (d) neither (a) nor (b)

13. Normal distribution was invented by --------

a) Laplace b) De Moivre c) Gauss d) All tha above

14. The average percentage of failure in a certain examination is 40. The probability that out of group of 6 canditates atleast 4 passed in examination are ---------
a) 0.5443 b) 0.4543 c) 0.5543 d) 0.4573

15. If probability $P[|\hat{\theta} - \theta| < \epsilon] \to 1$ as $n \to \infty$, for any positive ϵ then $\hat{\theta}$ is said to --------- estimator of θ.
(a) efficient (b) sufficient (c) unbiased (d) consistent

16. The standard error of sample mean of 16 samples whose standard deviation 4 is ---------
(a) 1 (b) 2 (c) 4 (d) $1/4$

17. Most commonly used index number is -------
a) Volume index number b) Value index number
c) Price index number d) Simple index number

18. The LCL of R chart is given by ---------
(a) $D_2\bar{R}$ (b) $D_2\bar{\bar{R}}$ (c) $D_3\bar{\bar{R}}$ (d) $D_3\bar{R}$

19. In a degenerate solution, number of allocations is ----------
a) equal to m+n-1 b) not equal to m+n-1
c) less than m+n-1 d) greater than m+n-1

20. A type of decision making environment is ——
(a) certainity (b) uncertainity (c) risk (d) all of above

PART - II

Note : Answer any 7 questions from which Q.No.30 is compulsory. **7x2=14**

21. Akash bats according to the following traits. If he makes a hit (s) there is 25% chance that he will make a hit his next time at bat. If he fails to hit (F), there is 35% chance that he will make a hit his next time at bat. Find the transistion probability matrix.

22. The cost of overhaul of an engine is ₹10,000. The operating cost per hour is at the rate of 2x-240. where the engine has run x km. FInd out the total cost if engine run for 300 hours after overhaul.

23. Find the integrating factor of the differential equation $\cos^2 x \, dy/dx + y = \tan x$.

24. Using graphic method, find the value of y when x=48.

x	40	50	60	70
y	6.2	7.2	9.1	12

25. Suppose, the life in hours of a radis tube has following p.d.f $f(x) = \begin{cases} 100/x^2 & \text{when } x \geq 100 \\ 0 & x < 100 \end{cases}$
Find the distribution function.

26. Write down the conditions in which the normal distribution is a limiting case of binomial distribution.

27. In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian eaters. Compute the standard error assuming that both vegetarian and non-vegetarian foods are equally popular in that village?

28. Fit a trend line by method of Semi averages for given data:

Year	2000	2001	2002	2003	2004	2005	2006
Production	105	115	120	100	110	125	135

29. The research department of Hindustan Ltd. has recommended to pay marketing department to launch a shampoo of three different types. The marketing types of shampoo to be launched under following estimated pay-off for various level of sales.

Types of Shampoo	Estimated sales (in units)		
	15000	10000	5000
Egg shampoo	30	10	10
Clinic shampoo	40	15	5
Deluxe shampoo	55	20	3

What will be the marketing manager's decision if maximum principle applied?

30. Evaluate $\int_0^\infty e^{-ax} x^5 \, dx$.

PART - III

Note : Answer any 7 questions from which Q.No.40 is compulsory. 7x3=21

31. Solve by Cramer's rule : $2x + 3y = 7$; $3x + 5y = 9$.
32. Evaluate $\int \frac{x^4 - x^2 + 2}{x-1} \, dx$
33. Using integration, find the area of the region bounded between the line $x=4$ and the parabola $y^2=16x$.
34. The marginal cost function of manufacturing x gloves is $6+10x-6x^2$. The total cost of producing a pair of gloves is ₹100. Find the total cost function.
35. Evaluate $\Delta\left[\frac{1}{(x+1)(x+2)}\right]$ by taking h=1.
36. The time to failure in thousands of hours of an important piece of electronic equipment used in a manufactured DVD player has the density function.

$$f(x)=\begin{cases} 3e^{-3x} & x>0 \\ 0 & \text{otherwise} \end{cases}$$

Find the expected life of piece of equipment.

37. In a family of 3 children, what is the probability that there will be exactly 2 girls?
38. A sample of 100 items, draw from a universe with mean 4 and S.D 3, has mean value 63.5. Is the difference in mean significant at 0.05 level of significance?
39. Write the control limits for the mean chart.
40. Find an initial basic feasible solution by least cost method to transportation problem.

		Destinations				
		D_1	D_2	D_3	D_4	Supply
Origins	O_1	5	8	3	6	30
	O_2	4	5	7	4	50
	O_3	6	2	4	6	20
Demand		30	40	20	10	

PART - IV

Note : Answer All the questions. 7x5=35

41. a) An amount of ₹5000 is to be deposited in three different bonds bearing 6%, 7%, 8% per year respectively. Total annual income is ₹358. If the income from first two investments is ₹70 more than the income from the third, then find the amount of investment in each bond by rank method.

(OR)

b) If the probability that an individual suffers a bad reaction from the injection of a given serum. is 0.001, determines the probability that out of 2000 individuals, (a) exactly 3 (b) more than 2 individuals will suffer a bad reaction. ($e^{-2} = 0.1353$).

42. a) Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} \, dx$

(OR)

b) Find the optimal solution for assignment problem with following cost matrix.

		Area			
		1	2	3	4
Salesman	P	11	17	8	16
	Q	9	7	12	6
	R	13	16	15	12
	S	14	10	12	11

43. a) The demand and supply function for a commodity $P_d = 15-x$ and $P_s = 0.3x + 2$. Find consumer's surplus & producer's surplus at market equilibrium price.

(OR)

b) Find the mean, variance & Standard deviation of the probability distribution.

X	-3	-2	-1	0	1	2	3
P(x)	1/7	1/7	1/7	1/7	1/7	1/7	1/7

44. a) Evaluate $\int_0^1 (x+4)\,dx$ using limits of sums.

(OR)

b) i) A sample of 900 members has a mean 3.4 cm and S.D 2.61 cm. Is sample taken from a large population with mean 3.25 cm. and SD 2.62 cm? (95% confidence limit)

ii) If population is normal, & its mean is unknown, find 95% confidence limits of true mean.

45. a) The elasticity of demand with respect to price for a commodity is given by $\frac{4-x}{x}$, where P is price when demand is x. Find demand function when price is 4 and demand is 2. Also find the revenue function.

(OR)

b) Calculate Laspeyre's, Paasche's and Fisher Ideal Index number for data given.

Commodities	Price		Quantity	
	2000	2010	2000	2010
Rice	38	35	6	7
Wheat	12	18	7	10
Rent	10	15	10	15
Fuel	25	30	12	16
Others	30	33	8	10

46. a) Solve: $(D^2+D-6)y=e^{3x} + e^{-3x}$

(OR)

b) Fit a straight line trend by method of least squares and tabulate the trend values.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Sales	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

47. a) A discrete random variable X has following probability function.

Value of X=x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) FInd 'k' ii) Evaluate P(x<6), p(x≥6), p(0<x<5)

(OR)

b) Calculate the value of y when x=7.5 from the table given below:

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

Second Revision Exam: 2023

Business Maths.

Part I

I). Choose

1. d) $\left(-\frac{\Delta_1}{\Delta_2}, -\frac{\Delta_1}{\Delta_3}\right)$
2. (c) $P(A) = P(A,B) < n$
3. (a) $e^{2x} \cdot x^2 + c$
4. (b) 0
5. (a) 2
6. (a) $\frac{200}{3} x^{1/2}$
7. (b) of order 1 & degree 3
8. (d) ~~[illegible]~~ $y = -\cos x + c$
9. (c) both equal & unequal intervals
10. (b) $\Delta f(x)$
11. (b) Cumulative probability
12. (b) continuous random variable
13. (b) De Moivre
14. (a) 0.5443
15. (d) consistent
16. (a) 1
17. (c) price index number
18. (d) $D_3 R$
19. (c) less than, $m+n=1$
20. (d) all the above.

Part - II

II Answer the following:

21) $$T = \begin{bmatrix} 0.25 & 0.75 \\ 0.35 & 0.65 \end{bmatrix}$$

22). $C = \frac{2x^2}{2} - 240x + c$

$C = x^2 - 240x + 10000$

when $x = 300$ $C = ₹\ 28,000$

23). $\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$

$\frac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$

$I.F = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$

Sol is $y(I.F) = \int Q.(I.F)\, dx + c$

$y e^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x} dx + c$

Let $t = \tan x$ $\quad dt = \sec^2 x dx$

$= \int t e^t dt + c$

Let $u = t$ $\quad dv = \int e^t dt$

$du = dt$ $\quad v = e^t$

$= te^t - \int e^t dt + c$

$y e^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + c$

$x = 48$ $\quad y = 6.8$.

24).

25). $F(x) = \int_{-\infty}^{x} f(t)\, dt$

$F(x) = \left[1 - \frac{100}{x}\right]$. $x \geq 100$.

26). $n \to \infty$

$p \to 0$

$np = \lambda$ is finite

27). $p = \frac{230}{400} = 0.575$ $\quad q = 1 - p = 0.425$

$S.E = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.575 \times 0.425}{400}}$

$= \sqrt{0.000610937}$

$= 0.024717$.

28). 2001 ⇒ 113.33

2005 ⇒ 128.33

29). Max (10, 5, 35) = 40.

30). $\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}} = \frac{5!}{a^6}$

Part. III

31). $\Delta = 1$ $\Delta x = 8$ $\Delta y = -3$ $x = 8$ $y = -3$

32) $\int x^3 + x^2 + \frac{2}{x-1}\,dx$

$= \frac{x^4}{4} + \frac{x^3}{3} + 2\log(x-1) + C$

33. Area $= \int_a^b y\,dx = \int_0^4 4\sqrt{x}\,dx$

$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{8}{3}\left[4^{3/2}\right] = \frac{8}{3}4\sqrt{4}$

$= \frac{64}{3}$

$\therefore$ Req. area $= 2\left(\frac{64}{3}\right) = \frac{128}{3}$ sq. units

34). $C = 6x + \frac{10x^2}{2} - \frac{6x^3}{3} + K$

$C = 6x + 5x^2 - 2x^3 + K.$

If pair of gloves not taken

When $x = 2$ $C = 100$

$100 = 6(2) + 5(4) - 2(8) + K$

$100 = 12 + 20 - 16 + K$

$100 = 16 + K$

$K = 100 - 16$

$K = 84$

$C = 6x + 5x^2 - 2x^3 + 84$

If pair of gloves is taken

when $x = 1$ $C = 100$

$100 = 6 + 5 - 2 + K$

$100 = 9 + K$

$K = 100 - 9$

$K = 91$

$C = 6x + 5x^2 - 2x^3 + 91$

35) $\Delta\left[\frac{1}{x+1} - \frac{1}{x+2}\right]$

$= \frac{-2}{(x+1)(x+2)(x+3)}$

36). $E(X) = \int_0^\infty 3xe^{-3x}\,dx$

$= 3\left[\frac{1!}{3^{1+1}}\right] = 3\left[\frac{1}{3^2}\right] = \frac{1}{3}$

37). $n=3 \quad p=q=\frac{1}{2}$

$P(X=x) = {}^nC_x\, p^x q^{n-x}$

$P(X=2) = 3C_2 \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8} = 0.375$

38). Given $n=100 \quad \mu=4 \quad \sigma=3$

$\bar{x} = 3.5$

$H_0: \mu=4$

$H_1: \mu\neq 4$

$z_c = 0.05$

$|z| = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = |-1.667| = 1.667$

At 0.05 level of significance.

$z < 1.96$ Acceptance region

$z > 1.96$ Rejected region

Result: Null Hypothesis is accepted.

39). $UCL = \bar{\bar{x}} + A_2\bar{R} \quad LCL = \bar{\bar{x}} - A_2\bar{R} \quad CL = \bar{\bar{x}}$

40). $O_1 \rightarrow D_3, D_4$

$O_2 \rightarrow D_1, D_2, D_4$

$O_3 \rightarrow D_2$

Cost = ₹ 390.

Part-IV

41) (a) $x+y+z = 5000 \rightarrow (1)$ $6x+7y+8z = 35800 \rightarrow (2)$

$6x+7y-8z = 7000 \rightarrow (3)$

$x = 1000$ $y = 2200$ $z = 1800$

(b) $p = 0.0001$ $n = 2000$ $\lambda = np = 2$

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

(i) $P(X=3) = \frac{e^{-2}\,2^3}{3!} = \frac{0.1353 \times 8}{6} = 0.1804.$

(ii) $P(X>2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - [0.1353 + 0.2706 + 0.2706]$

$= 0.3235$

42). (a) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$x = 7-x$

$I + I = \int_2^5 dx$

$2I = (x)_2^5$ $\Rightarrow$ $\boxed{I = \frac{3}{2}}$

(b) ₹ 37 $P \rightarrow 3$, $Q \rightarrow 4$, $R \rightarrow 1$, $S \rightarrow 2$

43). (a) $\boxed{x_0 = 10}$ $\boxed{P_0 = 5}$

$C.S = \int_0^{x_0} f(x)\,dx - P_0 x_0$

$= \int_0^{10} 15 - x\,dx - 50$

$= \left[15x - \frac{x^2}{2}\right]_0^{10} - 50$

$= 150 - \frac{100}{2} - 50$

$= 50$ units

$P.S = P_0 x_0 - \int_0^{x_0} f(x)\,dx$

$= 50 - \int_0^{10} 0.3x + 2\,dx$

$= 50 - \left[\frac{0.3x^2}{2} + 2x\right]_0^{10}$

$= 50 - [0.3(50) + 20]$

$= 50 - [35] = 15$ units

(b) $E(X) = \sum x\,p(x)$

$= \frac{1}{7}[-3 + -2 + -1 + 0 + 1 + 2 + 3]$

$= 0$

$E(X^2) = \sum x^2 p(x)$

$= \frac{1}{7}[9 + 4 + 1 + 0 + 1 + 4 + 9]$

$= \frac{28}{7} = 4$

Mean $= 0$

$Var(X) = E(X^2) - [E(X)]^2 = 4 - 0 = 4$

$S.D = \sqrt{4} = 2$

44). (a) $a = 0, \quad b = 1 \qquad h = \frac{b-a}{n} = \frac{1}{n}$

$f(x) = x + 4$

$f(a + rh) = \frac{r}{n} + 4$

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} h\, f(a + rh)$

$= \lim_{n\to\infty} \frac{1}{n^2} \frac{n(n+1)}{2} + \frac{1}{n} 4n$

$= \lim_{n\to\infty} \frac{1}{n^2} \frac{n^2(1+\frac{1}{n})}{2} + 4$

$= \frac{1+0}{2} + 4$

$= \frac{9}{2}$

(b) (ii) $\bar{x} - Z_c \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_c \frac{\sigma}{\sqrt{n}}$

$\sigma = 2.62 \qquad \mu = 3.25 \qquad \bar{x} = 3.4 \qquad s = 2.61$

$3.229 < \mu < 3.571$

(i) Test at 0.05 level of significance.

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = 1.7254$

At 0.05 level $z < 1.96$ acceptance

$z > 1.96$ critical

$\therefore$ Null hypothesis is accepted.

45).

(a). $x - 4 = pK$

when $p = 4$ $x = 2$

$K = -\frac{1}{2}$

$p = 8 - 2x$

$R = 8x - 2x^2$

(b) $\Sigma p_0 q_0 = 952$ $\Sigma p_0 q_1 = 1236$

$\Sigma p_1 q_0 = 1110$ $\Sigma p_1 q_1 = 1460$

$P_{01}^{L} = 116.5966$

$P_{01}^{P} = 118.1229$

$P_{01}^{F} = 117.3573$

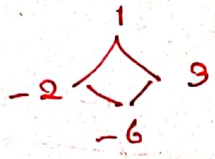
46).

(a) $y = C.F + P.I_1 + P.I_2$

$C.F = Ae^{2x} + Be^{-3x}$

$P.I_1 = \frac{e^{3x}}{6}$

$P.I_2 = \frac{xe^{-3x}}{-5}$



(b) $a = 5.975 = \frac{\Sigma y}{N}$

$b = 0.05119 = \frac{\Sigma xy}{\Sigma x^2}$

$x = \frac{x - 1998.5}{0.5}$

Trend values

5.6167
5.7190
5.8214
5.9238
6.0262
6.1286
6.2310
6.3333

$\sum P(x) = 1$

47)

(a) (i) $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k + 1$

$10k^2 + 9k - 1 = 0$

$k = -10$ $\quad$ $k = \frac{1}{10} = 0.1$

(ii) $P(X < 6) = k^2 + 8k$

$= (0.1)^2 + 8(0.1)$

$= 0.01 + 0.8 = 0.81$

$P(X \geq 6) = 2k^2 + 7k^2 + k$

$= 9k^2 + k$

$= 9(0.01) + 0.1$

$= 0.09 + 0.1 = 0.19$

$P(0 < x < 5) = 8k = 8(0.1) = 0.8$

(b) $x = -0.5$

$y = 421.88$