TNJ

FIRST REVISION TEST - 2023

12 - Std

MATHEMATICS

Time : 3.00 hrs. Marks : 90

I All questions are compulsory. 20 X 1 = 20

1. If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) is

a) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{pmatrix}$ d) $\begin{pmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{pmatrix}$

2. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is a) 1 b) -1 c) i d) 0

3. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) - 4

4. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

5. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [-1, 0] b) [0,1] c) [-1, 1] d) [1,2]

6. If x + y = k is a normal to the parabola $y^2 - 12x$, then the value of k is

a) 1 b) 9 c) 3 d) - 1

7. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y + c = 0$ are (11,2) the cordinates of the other end are

a) (-5, 2) b) (-3, 2) c) (5,-2) d) (-2,5)

8. Distance from the origin to the plane 3x - 6y +2z + 7 = 0 is

a) 0 b) 1 c) 2 d) 3

9. The value of $(1+i)^4 + (1-i)^4$ is

a) 8 b) 4 c) -8 d) -4

10. If A is a square matrix of order n, then |adj A| =

a) $|A|^{n-1}$ b) $|A|^{n-2}$ c) $|A|^n$ d) None

11. The point of inflection of the curve $y = (x-1)^3$ is

a) (0,0) b) (0,1) c) (1,0) d) (1,1)

12. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

13. The value of $\int_0^{\pi} \sin^4 x\,dx$ is a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

14. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

15. The solution of the differential equation $dy/dx = 2xy$ is

a) $y = Ce^{x^2}$ b) $y = 2x^2 + C$ c) $y = Ce^{-y^2} + C$ d) $y = x^2 + C$

16. The integrating factor of the differential equation $dy/dx + p(x)y = Q(x)$ is x, then p(x)

a) x b) $x^2/2$ c) $1/x$ d) $1/x^2$

17. A random variable X has binomial distribution with n= 25 and p = 0.8 then standard deviation X is

a) 6 b) 4 c) 3 d) 2

18. If p(x=0) = 1 - p(x = 1), if E(x) = 3 var (x) then p(x = 0) is

a) $2/3$ b) $2/5$ c) $1/5$ d) $1/3$

19. Subtraction is not a binary operation in

a) R b) Z c) N d) Q

20. If x + y = 8, then the maximum value of xy is ....

a) 8 b) 16 c) 20 d) 24

II Answer any seven question. Q.No. 30 Compulsory. 7 X 2 = 14

21. Prove that $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

22. Prove that $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = 2i$.

23. Form a polynomial equation with integer coefficient with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

24. Find the value of $\sin^{-1}\left(\sin\left(5\pi/4\right)\right)$.

25. Find the acute angle between the straight line $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$.

26. Prove that : $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+\cos(x)}\, dx = \frac{\pi}{4}$.

27. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

28. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

29. Prove that the identity is unique if it exists.

30. Find the equation of tangent to the curve $y = x^2 - x^4$ at (1,0).

III Answer any seven question. Q.No. 40 is compulsory. 7 X 3 = 21

31. If $A = \begin{pmatrix} 0 & -3 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -2 & -3 \\ 0 & -1 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

32. Find the square root of 6 - 8i.

33. Solve the equation $x^4 - 9x^2 + 20 = 0$.

34. With usual notation, in any triangle ABC prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35. Evaluate : $\lim_{x \to \infty} \frac{2x^2-3}{x^2-5x+3}$.

36. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{3}{2}u$.

37. Let * be defined on R by (a*b) = a + b + ab - 7. If * binary an R? If so find $3 * \left(\frac{-7}{15}\right)$.

38. Show that $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\, dx = \tan^{-1}(2)\frac{-\pi}{4}$.

39. The mean and variance of a binaryial variate x are respectively 2 and 1.5. Find p(x=0).

40. Obtain the equation of circle for which (3,4) and (2,-7) are the end of a diameter.

IV Answer all the questions. 7 x 5 = 35

41. a) Solve $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$, $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$, $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule. ***(OR)*** b) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ show that

i) $\frac{x^m}{y^n}-\frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

42. a) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2). ***(OR)***

b) By vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.

43. a) Solve : $6x^4 - 35x^3 + 6x^2 - 35x + 6 = 0$. ***(OR)***

b) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

44. a) Find vector and Cartesion equation of the plane passing through the point (0,1,5) and parallel to the straight line $\bar{r} = (i + 2j - 4k) + s(2i + 3j + 6k)$ and $\bar{r} = (i - 3j + 5k) + t(i + j - k)$ ***(OR)***

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

45. a) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)-\sec^{-1}(-2)$ ***(OR)***

b) Show that the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix and $x = \sqrt{2}$ is $y^2 = -4\sqrt{2}x$.

46. a) Show that the area of the region bounded by $3x - 2y + 6 = 0$, $x = -3$ $x = 1$ and x axis, is $\frac{15}{2}$. ***(OR)***

b) Show that the solution of the differential equation $(1+x^2)\frac{dy}{dx}=1+y^2$ is $\tan^{-1}y = \tan^{-1}x + c$ or $\tan^{-1}x = \tan^{-1}y + c$.

47. a) Prove $p \rightarrow (7qvr) = 7pr(7qvr)$ using truth table. ***(OR)***

b) The distribution function of a continuous random variable is

$$p(x)=\begin{cases} 0 & x<1 \\ \frac{x-1}{4} & 1\le x\le 5 \\ 1 & x>0 \end{cases}$$

find i) $p(x < 3)$ ii) $p(2 < x < 4)$ ii) $p(3 \le x)$.