COMMON FIRST REVISION TEST - 2023

Standard XII

Reg.No. | | | | | | |

MATHEMATICS

Time: 3.00 hours **Part - I** *Marks: 90*

I. Choose the correct answer: **20 x 1 = 20**

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj\,(AB)| =$

   a) –40 b) –80 c) –60 d) –20

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   a) 17 b) 14 c) 19 d) 21

3. The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is

   a) -110° b) -70° c) 70° d) 110°

4. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

   a) –2 b) –1 c) 1 d) 2

5. The minimum number of imaginary roots for the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ is

   a) 3 b) 6 c) 4 d) 9

6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

   a) [1,2] b) [–1,1] c) [0,1] d) [–1,0]

7. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

   a) 4 b) 5 c) 2 d) 3

8. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2), the coordinates of the other end are

   a) (–5,2) b) (2,–5) c) (5,–2) d) (–2,5)

9. Identify the type of the conic for the equation $x^2 - 2y = x + 3$

   a) ellipse b) circle c) parabola d) hyperbola

10. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

    a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

11. The shortest distance between the two given straight lines

    $\vec{r} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + t\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$ is

    a) $\frac{365}{3}$ b) $\frac{\sqrt{365}}{3}$ c) $\frac{365}{4}$ d) $\frac{365}{\sqrt{3}}$

12. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

    a) t = 0 b) $t = \frac{1}{3}$ c) t = 1 d) t = 3

13. The point of inflection of the curve $y = (x-1)^3$ is

    a) (0,0) b) (0,1) c) (1,0) d) (1,1)

14. The percentage error of the fifth root of 31 is approximately how many times the percentage error in 31?

a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

15. The value of $\int_0^{\infty} e^{-3x}x^2\,dx$ is

a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$

16. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

a) $\frac{x}{e^{\lambda}}$ b) $\frac{e^{\lambda}}{x}$ c) λe^x d) e^x

17. The order and degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$ are respectively

a) 2,3 b) 3,3 c) 2,6 d) 4,3

18. If in 6 trials, X is a binomial variable which follows the relation 9P(X=4) = P(X=2), then the probability of success is

a) 0.125 b) 0.25 c) 0.375 d) 0.75

19. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

20. The operation * defined by $a * b = \frac{ab}{3}$ is not a binary operation on

a) Q^+ b) Z c) R d) C

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

22. Simplify : $i\, i^2\, i^3 \ldots\ldots i^{2000}$

23. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta r}$ in terms of the coefficients.

24. Simplify : $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$

25. If $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

26. Find the value in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function

$f(x) = x + \frac{1}{x},\ x \in \left[\frac{1}{2}, 2\right]$

27. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find the approximate change in the volume.

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

29. Show that $x^2 + y^2 = r^2$ where r is a constant, is a solution of the differential equation

$\frac{dy}{dx} = \frac{-x}{y}$

30. The probability density function of X is given by $f(x) = \begin{cases} k x e^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$.

Find the value of k

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. if $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_2$.

32. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33. Solve the equation : $x^3 - 5x^2 - 4x + 20 = 0$

34. Find the value of $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

35. Find the equation of the ellipse with foci (0, ±4) and end points of major axis (0, ±5)

36. Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} - 3\hat{k}$, $4\hat{i} - 10\hat{j} + 12\hat{k}$ and $4\hat{i} + 7\hat{j}$ acting at the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$

37. Find the intervals of monotonicity and hence find the local extremum for the function $f(x) = 2x^3 + 3x^2 - 12x$

38. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$

39. Define an operation * on Q as follows $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in Q$. Examine the closure, commutative and associative properties satisfied by * on Q.

40. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2.

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Solve the following system of linear equations of Gaussian elimination method $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$ ***(OR)***

    b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42. a) Solve the equation $Z^3 + 27 = 0$ ***(OR)***

    b) Solve : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$

43. a) Assume that water issuing from the end of a horizontal pipe 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? ***(OR)***

    b) By Vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

44. a) Find the non-parametrics form of vector equation, and cartesian equations of the plane $\vec{r} = (6\hat{i} - \hat{j} + \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$ ***(OR)***

    b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

45. a) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile 10 metre high? ***(OR)***

    b) If $V(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

46. a) Prove that among all the rectangles of the given perimeter, the square has the maximum area. ***(OR)***

    b) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

47. a) The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$ (iii) $P(X \geq 2)$ ***(OR)***

    b) Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

******