Sun Tuition Center - 9629216361
Important - 2 & 3 Mark Question

Std-XII Volume - 1 & 2 (2022-2023)

CHAPTER-1 APPLICATIONS OF MATRICES AND DETERMINANTS

1 EXERCISE 1.1

8. If $\text{adj}(A)=\begin{bmatrix}2 & -4 & 2\\ -3 & 12 & -7\\ -2 & 0 & 2\end{bmatrix}$, find A.

Example 1.5

Find a matrix A if $\text{adj}(A)=\begin{bmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{bmatrix}$.

2 EXERCISE 1.1

9. If $\text{adj}(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

Example 1.6

If $\text{adj } A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}.

3 EXERCISE 1.1

7. If $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

Example 1.9

Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{bmatrix}0 & -3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & -3\\ 0 & -1\end{bmatrix}$.

4 EXERCISE 1.2

2. Find the rank of the following matrices by row reduction method:

(i) $\begin{bmatrix}1 & 1 & 1 & 3\\ 2 & -1 & 3 & 4\\ 5 & -1 & 7 & 11\end{bmatrix}$ (iii) $\begin{bmatrix}3 & -8 & 5 & 2\\ 2 & -5 & 1 & 4\\ -1 & 2 & 3 & -2\end{bmatrix}$

Example 1.18

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

5

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(i) $2x+5y=-2,\ x+2y=-3$

(ii) $2x-y=8,\ 3x+2y=-2$

Example 1.22

Solve the following system of linear equations, using matrix inversion method:

$5x+2y=3,\ 3x+2y=5$.

6

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule:

(i) $5x-2y+16=0, x+3y-7=0$

(ii) $\frac{3}{x}+2y=12, \frac{2}{x}+3y=13$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

CHEAPER - 2 COMPLEX NUMBERS

1 **Example 2.8**

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real

EXERCISE 2.4

7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

2 **Example 2.11**

Which one of the points $i, -2+i$, and 3 is farthest from the origin?

EXERCISE 2.5

3. Which one of the points $10-8i$, $11+6i$ is closest to $1+i$.

3 EXERCISE 2.5

4. If $|z|=3$, show that $7 \le |z+6-8i| \le 13$.

5. If $|z|=1$, show that $2 \le |z^2-3| \le 4$.

6. If $|z|=2$, show that $8 \le |z+6+8i| \le 12$.

Example 2.13

If $|z|=2$ show that $3 \le |z+3+4i| \le 7$

4 EXERCISE 2.5

10. Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$.

Example 2.17

Find the square root of $6-8i$.

5 EXERCISE 2.5

9. Show that the equation $z^3+2\overline{z}=0$ has five solutions.

Example 2.16

Show that the equation $z^2=\overline{z}$ has four solutions.

6

EXERCISE 2.6

1. If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$ show that the locus of z is real axis.

5. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases:

(i) $|z-4|=16$ (ii) $|z-4|^2-|z-1|^2=16$.

Example 2.21

Obtain the Cartesian form of the locus of z in each of the following cases.

(i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$

7

EXERCISE 2.7

1. Write in polar form of the following complex numbers

(i) $2+i2\sqrt{3}$ (ii) $3-i\sqrt{3}$ (iii) $-2-i2$

Example 2.23

Represent the complex number (i) $-1-i$ (ii) $1+i\sqrt{3}$ in polar form.

8 **Example 2.32**

Find the cube roots of unity.

Example 2.33

Find the fourth roots of unity.

Example 2.29

Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

Example 2.31

Simplify (i) $(1+i)^{18}$ (ii) $(-\sqrt{3}+3i)^{31}$.

9

EXERCISE 2.8

1. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

2. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$.

8. If $\omega \neq 1$ is a cube root of unity, show that

   (i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$.

   (ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots(1+\omega^{2^{11}})=1$.

CHAPTER -3 THOERY OF EQUATION

1 EXERCISE 3.1

5. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$.

7. If α,β, and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the value of $\sum\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

Example 3.3

If α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

Example 3.4

Find the sum of the squares of the roots of $ax^4+bx^3+cx^2+dx+e=0$.

2 EXERCISE 3.1

9. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$.

10. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

3 **Example 3.7**

If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.

EXERCISE 3.2

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$, in terms of k.

4 EXERCISE 3.2

4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

Example 3.10

Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

5 EXERCISE 3.2

5. Prove that a straight line and parabola cannot intersect at more than two points.

Example 3.14

Prove that a line cannot intersect a circle at more than two points.

6. EXERCISE 3.3

7. Solve the equation : $x^4-14x^2+45=0$

Example 3.16

Solve the equation $x^4-9x^2+20=0$.

7. EXERCISE 3.3

2. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

Example 3.19

Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

8

EXERCISE 3.5

1. Solve the following equations

(i) $\sin^2 x-5\sin x+4=0$ (ii) $12x^3+8x=29x^2-4$

Example 3.29

Find solution, if any, of the equation

$$2\cos^2 x-9\cos x+4=0$$

9. EXERCISE 3.6

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

Example 3.30

Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

CHAPTER-4 INVERSE TRIGONOMETRICAL FUNCTIONS

1 EXERCISE 4.1

4. Find the value of (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

Example 4.3

Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

2 EXERCISE 4.2

5. Find the value of

(i) $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

(iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

Example 4.6

Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

3 EXERCISE 4.2

7. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

8. Find the value of

(i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$ (ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

4 EXERCISE 4.3

4. Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$.

Example 4.10

Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

5 EXERCISE 4.4

2. Find the value of

(i) $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$ (ii) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$

(iii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$

Example 4.14

If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

6 EXERCISE 4.5

3. Find the value of

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$ (iii) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

8. Simplify: $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$.

Example 4.19

Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

Example 4.26

Show that $\cot(\sin^{-1}x)=\frac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\neq 0$

Example 4.27

Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$.

CHAPTER-5 2D ANALYTICAL GEOMETRY

EXERCISE 5.1

4. Find the equation of the circle with centre (2,3) and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$.

12. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

2 EXERCISE 5.2

6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

Example 5.15

Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

3 EXERCISE 5.4

5. Find the equation of the tangent at $t=2$ to the parabola $y^2=8x$. (Hint: use parametric form)

8. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

4. **Example 5.32**

The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

Example 5.34

The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

Example 5.36

A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is $40\,cm$ wide from rim to rim and $30\,cm$ deep. The bulb is located at the focus .

(1) What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

CHAPTER -6 APPLICATION OF VECTOR ALGEBRA

1 **Example 6.1 (Cosine formulae)**

With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a^2=b^2+c^2-2bc\cos A$ (ii) $b^2=c^2+a^2-2ca\cos B$

(iii) $c^2=a^2+b^2-2ab\cos C$

2 **Example 6.2**

With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a=b\cos C+c\cos B$ (ii) $b=c\cos A+a\cos C$

(iii) $c=a\cos B+b\cos A$

3 **Example 6.4**

With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

4 EXERCISE 6.1

3. Prove by vector method that an angle in a semi-circle is a right angle.

5 6. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$.

6 EXERCISE 6.3

3. Prove that $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}]=0$.

Example 6.18

If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}]=-[\vec{a},\vec{b},\vec{c}]$.

7 **Example 6.19**

Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

8. EXERCISE 6.4

5. Find the angle between the following lines.

(i) $\vec{r}=(4\hat{i}-\hat{j})+t(\hat{i}+2\hat{j}-2\hat{k}),\ \vec{r}=(\hat{i}-2\hat{j}+4\hat{k})+s(-\hat{i}-2\hat{j}+2\hat{k})$

(ii) $\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5},\ \vec{r}=4\hat{k}+t(2\hat{i}+\hat{j}+\hat{k})$.

(iii) $2x=3y=-z$ and $6x=-y=-4z$.

Example 6.29

Find the angle between the lines $\vec{r}=(\hat{i}+2\hat{j}+4\hat{k})+t(2\hat{i}+2\hat{j}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.

9. EXERCISE 6.5

2. Show that the lines $\vec{r}=(6\hat{i}+\hat{j}+2\hat{k})+s(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(3\hat{i}+2\hat{j}-2\hat{k})+t(2\hat{i}+4\hat{j}-5\hat{k})$ are skew lines and hence find the shortest distance between them.

5. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

Example 6.36

Find the shortest distance between the two given straight lines $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$.

10 EXERCISE 6.9

3. Find the angle between the line $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(6\hat{i}+3\hat{j}+2\hat{k}\right)=8$

4. Find the angle between the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}-2\hat{k}\right)=3$ and $2x-2y+z=2$.

Example 6.47

Find the acute angle between the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=11$ and $4x-2y+2z=15$.

Example 6.48

Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x-y+z=5$.

11 EXERCISE 6.9

8. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x+2y+3z=2$.

Example 6.55

Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot(\hat{i}+2\hat{j}+4\hat{k})=38$.

Sun Tuition Center

CHAPTER-7 APPLICATIONS OF DIFFERENTIAL CALCULUS

1 EXERCISE 7.1

2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground?

4. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.

5. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres.

2 EXERCISE 7.3

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :
   (i) $f(x) = x^2 - x,\ x \in [0,1]$

4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
   (i) $f(x) = x^3 - 3x + 2,\ x \in [-2,2]$

3 EXERCISE 7.3

6. A race car driver is kilometer stone 20. If his speed never exceeds 150 km/hr, what is the maximum kilometer he can reach in the next two hours.

7. Suppose that for a function $f(x), f'(x) \le 1$ for all $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$.

10. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

4 EXERCISE 7.4

1. Write the Maclaurin series expansion of the following functions:
   (i) e^x (ii) $\sin x$ (iii) $\cos x$
   (iv) $\log(1-x)$; $-1 \le x < 1$ (v) $\tan^{-1}(x)$; $-1 \le x \le 1$

2. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$.

5 EXERCISE 7.5

3. $\lim_{x \to \infty} \frac{x}{\log x}$ 6. $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

Example 7.42

Evaluate : $\lim_{x \to \infty} \left(\frac{e^x}{x^m} \right), m \in N$.

6 EXERCISE 7.6

1. Find the absolute extrema of the following functions on the given closed interval.

(iii) $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$; $[-1,1]$

7 EXERCISE 7.6

2. Find the intervals of monotonicities and hence find the local extremum for the following functions:

(iii) $f(x) - \frac{e^x}{1-e^x}$

Example 7.51

Find the intervals of monotonicity and hence find the local extrema for the function $f(x) = x^{\frac{2}{3}}$.

Example 7.56

Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$.

8 EXERCISE 7.7

1. Find intervals of concavity and points of inflexion for the following functions:

(iii) $f(x) = \frac{1}{2}(e^x - e^{-x})$

Example 7.58

Determine the intervals of concavity of the curve $y = 3 + \sin x$.

9 EXERCISE 7.8

1. Find two positive numbers whose sum is 12 and their product is maximum.
2. Find two positive numbers whose product is 20 and their sum is minimum.

10

EXERCISE 7.9

1. Find the asymptotes of the following curves :

(i) $f(x) - \frac{x^2}{x^2-1}$ (ii) $f(x) - \frac{x^2}{x+1}$

(iv) $f(x) - \frac{x^2-6x-1}{x+3}$ (v) $f(x) - \frac{x^2+6x-4}{3x-6}$

CHAPTER-8 DIFFERENTIAL AND PARTIAL DERIVATIVES

1 EXERCISE 8.1

1. Let $f(x)=\sqrt[3]{x}$. Find the linear approximation at $x=27$. Use the linear approximation to approximate $\sqrt[3]{27.2}$.

2. Use the linear approximation to find approximate values of

$$\text{(i) } (123)^{\frac{2}{3}} \qquad \text{(ii) } \sqrt[4]{15} \qquad \text{(iii) } \sqrt[3]{26}$$

Example 8.1

Find the linear approximation for $f(x)=\sqrt{1+x}, x\geq -1$, at $x_0=3$. Use the linear approximation to estimate $f(3.2)$.

Example 8.2

Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

2 EXERCISE 8.1

6. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T=2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

7. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

3 EXERCISE 8.2

6. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

11. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

Example 8.7

If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

4 EXERCISE 8.3

1. Evaluate $\lim\limits_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y)=\dfrac{3x^2-xy}{x^2+y^2+3}$.

2. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3+y^2}{x+y+2}\right)$. If the limit exists.

3. Let $f(x,y)=\dfrac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for $(x,y)\neq(0,0)$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

4. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{e^x\sin y}{y}\right)$, if the limit exists.

Example 8.8

Let $f(x,y)=\dfrac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y)\in\mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$.

5 EXERCISE 8.4

3. If $U(x,y,z)=\dfrac{x^2+y^2}{xy}+3z^2y$, find $\dfrac{\partial U}{\partial x},\dfrac{\partial U}{\partial y}$, and $\dfrac{\partial U}{\partial z}$.

4. If $U(x,y,z)=\log(x^3+y^3+z^3)$, find $\dfrac{\partial U}{\partial x}+\dfrac{\partial U}{\partial y}+\dfrac{\partial U}{\partial z}$.

Example 8.15

Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

6. EXERCISE 8.5

1. If $w(x,y)=x^3-3xy+2y^2, x,y\in\mathbb{R}$, find the linear approximation for w at $(1,-1)$.

Example 8.17

Let $U(x,y,z)=x^2-xy+3\sin z, x,y,z\in\mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$.

7 EXERCISE 8.5

4. Let $V(x,y,z)=xy+yz+zx, x,y,z\in\mathbb{R}$. Find the differential dV.

Example 8.16

If $w(x,y,z)=x^2y+y^2z+z^2x, x,y,z\in\mathbb{R}$, find the differential dw.

8. EXERCISE 8.6

1. If $u(x,y)=x^2y+3xy^4$, $x=e^t$ and $y=\sin t$, find $\dfrac{du}{dt}$ and evaluate it at $t=0$.

Example 8.20

Let $g(x,y)=2y+x^2, x=2r-s, y=r^2+2s, r,s\in\mathbb{R}$. Find $\dfrac{\partial g}{\partial r},\dfrac{\partial g}{\partial s}$.

9. EXERCISE 8.7

4. If $u(x,y)=\dfrac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{3}{2}u$.

Example 8.21

Show that $F(x,y)=\dfrac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

CHAPTER -9 APPLICATION OF INTEGRAL CALCULUS

1 **Example 9.7**

Evaluate : $\int_0^1 [2x]\,dx$ where $[\cdot]$ is the greatest integer function.

Example 9.14

Evaluate : $\int_0^{1.5} \left[x^2\right] dx$, where $[x]$ is the greatest integer function.

2 EXERCISE 9.3

2. Evaluate the following integrals using properties of integration :

(ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\cos x + \tan^3 x + 1)\,dx$

(x) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$

Example 9.26

Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$.

Example 9.29

Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$.

3 EXERCISE 9.4

3. $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

4. $\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx$

Example 9.34

Evaluate : $\int_{-1}^{1} e^{-\lambda x}(1-x^2)\,dx$.

4 EXERCISE 9.7

Evaluate the following (iii) $\int_0^{\frac{\pi}{4}} \sin^6 2x\,dx$ (iv) $\int_0^{\frac{\pi}{6}} \sin^5 3x\,dx$

(v) $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^4 x\,dx$ (vi) $\int_0^{2\pi} \sin^7 \frac{x}{4}\,dx$

Example 9.37

Evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right) dx$

Example 9.38

Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.

5 EXERCISE 9.7

Evaluate the following (ii) $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$

2. If $\int_0^{\infty} e^{-\alpha x^2} x^3 dx = 32, \ \alpha > 0$, find α

6 **Example 9.50**

Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

Example 9.52

Find the area of the region bounded by $x-$axis, the sine curve $y = \sin x$, the lines $x = 0$ and $x = 2\pi$.

7 **EXERCISE 9.9**

1. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$, $y = 0$ and $x = 1$.

2. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = e^{-2x}$ $y = 0$, $x = 0$ and $x = 1$

Example 9.62

Find the volume of a sphere of radius a.

CHAPTER -10 DIFFERENTIAL EQUATION

1 **EXERCISE 10.3**

7. Find the differential equation corresponding to the family of curves represented by the equation $y - Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

Example 10.3

Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

2 **EXERCISE 10.3**

4. Find the differential equation of the family of all the parabolas with latus rectum $4a$ and whose axes are parallel to the x-axis.

Example 10.5

Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

3

EXERCISE 10.3

6. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

Example 10.6

Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

4

EXERCISE 10.4

4. Show that $y - e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 - 0$.

8. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y - 0$.

Example 10.7

Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

Example 10.8

Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.

5

EXERCISE 10.5

4. **Solve the following differential equations:**

(i) $\frac{dy}{dx} - \sqrt{\frac{1-y^2}{1-x^2}}$ (ii) $ydx + (1+x^2)\tan^{-1}x\, dy - 0$

Example 10.11

Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$.

Example 10.12

Find the particular solution of $(1+x^3)dy - x^2ydx = 0$ satisfying the condition $y(1) = 2$.

CHAPTER -II PROBABILITY DISTRIBUTION

1

EXERCISE 11.2

1. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

3. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

Example 11.6

A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

2

EXERCISE11.3

1. The probability density function of X is given by $f(x)=\begin{cases} k\,x e^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$.

Find the value of k.

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}(x^2+x) & 0\le x<1 \\ 1, & 1\le x<\infty \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3\le X\le 0.6)$

Example 11.13

If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & x<0 \\ x, & 0\le x<1 \\ 1, & 1\le x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2\le X\le 0.7)$.

4

EXERCISE 11.4

2. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

Example 11.17

Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance.

EXERCISE 11.4

5. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. It is known that the pdf of X is

$$f(x)=\begin{cases}\frac{1}{30} & 0<x<30\\ 0 & \text{elsewhere}\end{cases}$$

Obtain and interpret the expected value of the random variable X.

6. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}.$$

Find the expected life of this electronic equipment.

7. The probability density function of the random variable X is given by

$$f(x)=\begin{cases}16xe^{-4x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

find the mean and variance of X.

Example 11.18

Find the mean and variance of a random variable X, whose probability density function is

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\ge 0\\ 0 & \text{otherwise}\end{cases}$$

EXERCISE 11.5

2. The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

3. Using binomial distribution find the mean and variance of X for the following experiments
   (i) A fair coin is tossed 100 times, and X denote the number of heads.
   (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

9. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

Example 11.20

A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

CHAPTER- 12 DISCRETE MATHEMATICS

1

EXERCISE 12.1

2. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

3. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$.

Example12.1

Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary):

(i) $a * b = a + 3ab - 5b^2; \forall a, b \in \mathbb{Z}$ (ii) $a * b = \left(\frac{a-1}{b-1}\right), \forall a, b \in \mathbb{Q}$

2

8. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

9. Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$

10. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent

11. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

Example 12.16

Construct the truth table for $(p \,\bar{\vee}\, q) \wedge (p \,\bar{\vee}\, \neg q)$.

Example 12.17

Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$