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Class – 12 Mathematics

Unit – 1: Application of Matrices and Determinants

Important 5 Mark Questions:

1. If A = $\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. (Example 1.12)
2. Decrypt the received encoded message $[2 \quad -3]$ $[20 \quad 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters A-Z respectively, and the number 0 to a blank space. (Exercise 1.1 – Q.No.15)
3. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations. (Example 1.19)
4. Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method. (Example 1.21)
5. Find the inverse of A = $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ by Gauss-Jordan method. (Exercise 1.2 – Q.No.3 (ii))
6. If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x-y+z = 4, x-2y-2z = 9, 2x+y+3z = 1. (Example 1.24)

7. The prices of three commodities A, B and C are ₹ x, y and c per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.) (Exercise 1.3 – Q.No.5)

8. In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy -coordinate system in the vertical plane and the ball traversed through the points (10,8),(20,16),(40,22), can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0) (Example 1.26)

9. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹ 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had? (Exercise 1.4 – Q.No.5)

10. The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2+bt+c, 0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.) (Example 1.28)

11. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual

income is ₹5000. The income from the third bond is ₹800 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.) (Exercise 1.5 – Q.No.3)

12. A boy is walking along the path $y = ax^2+bx+c$ through the points (-6,8),(-2,-12), and (3,8). He wants to meet his friend at $P(7,60)$. /will he meet his friend? (Use Gaussian elimination method.) (Exercise 1.5 – Q.No.4)

13. Test for consistency of the following system of linear equations and if possible solve: $x+2y-z = 3$, $3x-y+2z = 1$, $x-2y+3z = 3$, $x-y+z+1 = 0$. (Example 1.29)

14. Test for consistency of the following system of linear equations and if possible solve: $x-y+z = -9$, $2x-2y+2z = -18$, $3x-3y+3z+27 = 0$. (Example 1.31)

15. Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x+y+z = a$, $x+2y+3z = b$, $3x+5y+7z = c$. (Example 1.33)

16. Investigate for what values of λ and μ the system of linear equations $x+2y+z = 7$, $x+y+\lambda z = \mu$, $x+3y-5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (Example 1.34)

17. Find the value of k for which the equations $kx-2y+z = 1$, $x-2ky+z = -2$, $x-2y+kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution (Exercise 1.6– Q.No.2)

18. Investigate the values of λ and μ the system of linear equations $2x+3y+5z = 9$, $7x+3y-5z = 8$, $2x+3y+\lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions (Exercise 1.6– Q.No.3)

19. Test for consistency and if possible, solve $2x+2y+z = 5$, $x-y+z = 1$, $3x+y+2z = 4$ (Exercise 1.6– Q.No.1 (iii))

20. Determine the values of ʎ for which the following system of equations (3ʎ-8)x+3y+3z = 0, 3x+(3ʎ-8)y+3z = 0, 3x+3y+(3ʎ-8)z = 0 has a non-trivial solution. (Example 1.38)

21. By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$ (Example 1.39)

22. If the system of equations $px+by+cz = 0$, $ax+qy+cz = 0$, $ax+by+rz = 0$ has a non-trivial solution and p ≠ a, q ≠ b, r ≠ c, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c} = 2$. (Example 1.40)

23. Solve the system of homogenous equations $2x+3y-z = 0$, $x-y-2z = 0$, $3x+y+3z = 0$ (Exercise 1.7– Q.No.1(ii))

24. Determine the values of ʎ for which the following system of equations x+y+3z = 0, 4x+3y+ ʎz = 0, 2x+y+2z = 0 has (i) a unique solution (ii) a non-trival solution. (Exercise 1.7– Q.No.2)

25. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$
(Exercise 1.7– Q.No.3)

Life is a Good Circle,
you Choose The Best Radius....

Unit – 2: Complex Numbers

Important 5 Mark Questions:

1. Find the value of the real numbers x and y, if the complex number (2+i)x+(1-i)y+2i-3 and x+(-1+2i)y+1+i are equal. (Example 2.2)
2. Find the values of the real numbers x and y, if the complex numbers (3-i)x-(2-i)y+2i+5 and 2x+(-1+2i)y+3+2i are equal. (Exercise 2.2 – Q.No.3)
3. Show that (i) $(2+i\sqrt{3})^{10}+(2-i\sqrt{3})^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. (Example 2.8)
4. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If v = 3-4i and w = 4+3i, find u in rectangular form. (Exercise 2.4 – Q.No.4)
5. Prove the following properties: (i) z is real if and only if $z = \bar{z}$
   (ii) $Re(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$ (Exercise 2.4 – Q.No.5)
6. Find the least value of the positive integer n for which $(\sqrt{3}+i)^n$ (i) real
   (ii) purely imaginary. (Exercise 2.4 – Q.No.6)
7. Show that (i) $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary (ii) $\left(\frac{19-7i}{9+i}\right)^{12} - \left(\frac{20-5i}{7-6i}\right)^{12}$ is real. (Exercise 2.4 – Q.No.7)
8. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1+z_2+z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$. (Example 2.12)
9. Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$. (Example 2.15)

10. If $\left|z-\frac{2}{z}\right|$ =2, show that the greatest and least value of |z| are $\sqrt{3}$ + 1 and $\sqrt{3}$ - 1 respectively. (Exercise 2.5 – Q.No.10)

11. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1+z_2+z_3| = 1$, show that $|9z_1z_2+4z_1z_3+z_2z_3| = 6$.
(Exercise 2.5 – Q.No.7)

12. If the area of the triangle formed by the vertices z, iz, and z+iz is 50 square units, find the value of |z|. (Exercise 2.5 – Q.No.8)

13. Given the complex number z=3+2i, represent the complex numbers z, iz, and z+iz in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle. (Example 2.18)

14. If z = x+iy is a complex number such that Im $\left(\frac{2z+1}{iz+1}\right)$ = 0, show that the locus of z is $2x^2+2y^2+x-2y = 0$. (Exercise 2.6 – Q.No.2)

15. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form. (Example 2.26)

16. If z = x+iy and arg $\left(\frac{z-1}{z+1}\right)$ = $\frac{\pi}{2}$, show that $x^2+y^2 = 1$. (Example 2.27)

17. Find the rectangular form of the complex number $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$. (Exercise 2.7 – Q.No.2 (i))

18. Find the rectangular form of the complex number $¿\frac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}¿$ (Exercise 2.7 – Q.No.2 (ii))

19. If cosα + cosβ + cos γ = sinα + sinβ + sin γ = 0, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and
(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$ (Exercise 2.7 – Q.No.5)

20. If $z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$. (Exercise 2.7 – Q.No.6)

21. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$ (Example 2.29)

22. Simplify (i) $(1+i)^{18}$ (ii) $(-\sqrt{3}+3i)^{31}$ (Example 2.31)

23. Find the cube roots of unity. (Example 2.32)

24. Find the fourth roots of unity. (Example 2.33)

25. Solve the equation $z^3+8i = 0$, where $z \in \mathbb{C}$. (Example 2.34)

26. Find all cube roots of $\sqrt{3} + i$. (Example 2.35)

27. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1+i\sqrt{3}$, then find z_2 and z_3. (Example 2.36)

28. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin((\alpha+\beta)$

(Exercise 2.8 – Q.No.4)

29. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when

(i) $\theta = \frac{\pi}{3}$ (ii) $\theta = \frac{2\pi}{3}$ (iii) $\theta = \frac{3\pi}{2}$ (Exercise 2.8 – Q.No.9)

30. Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1\pm i)$. (Exercise 2.8 – Q.No.10)

Unit – 3: Theory of Equations

Important 5 Mark Questions:

1. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio 3:2. (Exercise 3.1 – Q.No.6)
2. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. (Exercise 3.1 – Q.No.9)
3. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away. (Exercise 3.1 – Q.No.12)
4. Write Complex Conjugate Root Theorem (P – 107 Theorem – 3.2)
5. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root. (Example 3.10)
6. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root. (Exercise 3.2 – Q.No.4)
7. Prove that a straight line and parabola cannot intersect at more than two points. (Exercise 3.2 – Q.No.5)
8. If $2+i$ and $3-\sqrt{2}$ are roots of the equation $X^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots. (Example 3.15)
9. Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression. Assume a, b, c, d $\neq$ 0. (Example 3.20)
10. If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^3+2p$. Assume p, q, r $\neq$ 0. (Example 3.21)
11. Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots. (Exercise 3.3 – Q.No.4)

12. Find all zeros of the polynomial $X^6-3x^5-5x^4+22x^3-39x^2-39x+135=0$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros. (Exercise 3.3 – Q.No.5)
13. Solve the equation $(x-2)(x-7)(x-3)(x+2)+19=0$ (Example 3.23)
14. Solve the equation $(2x-3)(6x-1)(3x-2)(x-12)-7=0$ (Example 3.24)
15. Solve $(x-5)(x-7)(x+6)(x+4)=504$ (Exercise 3.4 – Q.No.1(i))
16. Solve the equation $7x^3-43x^2=43x-7$. (Example 3.27)
17. Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$ (Example 3.29)
18. Solve: $8x^{3/2n} - 8x^{-3/2n} = 63$ (Exercise 3.5 – Q.No.3)
19. Solve: $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$. (Exercise 3.5 – Q.No.4)
20. Solve the equations $6x^4-35x^3+62x^2-35x+6=0$. (Exercise 3.5 – Q.No.5(i))
21. Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$. (Exercise 3.5 – Q.No.6)
22. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution. (Exercise 3.5 – Q.No.7)
23. Discuss the nature of the roots of the following polynomials. (i) $x^{208}+1947x^{1950}+15x^8+26x^6+2019$ (ii) $x^5-19x^4+2x^3+5x^2+11$ (Example 3.31)
24. Discuss the maximum possible number of positive and negative zeros of the polynomials x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graphs. (Exercise 3.6 – Q.No.2)
25. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has at least 6 imaginary solutions. (Exercise 3.6 – Q.No.3)

Unit – 4: Inverse Trigonometric Functions

Important 5 Mark Questions:

1. Find the domain of $\sin^{-1}(2-3x^2)$ (Example 4.4)

2. Find the domain $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (Exercise 4.1 – Q.No.6 (i))

3. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$ (Example 4.7)

4. Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$

(Exercise 4.2 – Q.No.5 (iii))

5. Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

(Exercise 4.2 – Q.No.6 (i))

6. Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

(Exercise 4.2 – Q.No.8 (ii))

7. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$
   Show that $x^2 + y^2 + z^2 + 2xyz = 1$ (Example 4.22)

8. Solve. $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$, a, b > 0

(Exercise 4.5 – Q.No.9 (ii))

9. Solve. $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$ (Example 4.29)

10. Find the value of $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$ (Example 4.18)

11. Find the number of solution of the equation
$$\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$$
(Exercise 4.5 – Q.No.10)

12. Prove that $\tan^{-1}(x) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \quad |x| < \frac{1}{\sqrt{3}}$
(Exercise 4.5 – Q.No.7)

13. Find the value of $sin^{-1}[\sin 10]$ (Example 4.17 (iv))

14. Find the value of $\tan\left(cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right)$

(Exercise 4.3 – Q.No.4 (i))

15. Find the value of $\cos\left(sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$

(Exercise 4.3 – Q.No.4 (ii))

16. Evaluate $\sin\left(sin^{-1}\left(\frac{3}{5}\right) + sec^{-1}\left(\frac{5}{4}\right)\right)$ (Example 4.20)

17. If $a_1, a_2, a_3 \ldots\ldots a_n$ is an A.P with common difference d,

Prove that $\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]$
$= \frac{a_n-a_1}{1+a_1a_n}$ (Example 4.23)

18. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, *if* $6x^2 < 1$. (Example 4.27)

19. Prove that $sin^{-1}\frac{3}{5} - cos^{-1}\frac{12}{13} = sin^{-1}\frac{16}{65}$ (Exercise 4.5 – Q.No.4 (ii))

20. Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(Exercise 4.5 – Q.No.5)

21. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x+y+z = xyz$

(Exercise 4.5 – Q.No.6)

22. Solve $sin^{-1}\frac{5}{x} + sin^{-1}\frac{12}{x} = \frac{\pi}{2}$ (Exercise 4.5 – Q.No.9 (i)), *9(ii)*

Unit – 5: Two Dimensional Analytical Geometry II

Important 5 Mark Questions:

1. Find the equation of the circle passing through the points (1, 1), (2, -1) and (3, 2). (Example 5.10)
2. If the equation $3x^2+(3-p)xy+3y^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.
   (Exercise 5.1 – Q.No.12)
3. Derive the equation of a parabola in standard form. (Page No. 183)
4. Derive the equation of an Ellipse in standard form. (Page No. 186)
5. Derive the equation of a Hyperbola in standard form. (Page No. 189)
6. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is (2, 3) and a directrix is $x = 7$. Also find the length of the major and minor axes of the ellipse. (Example 5.19)
7. Find the foci, vertices and length of major and minor axis of the conic
   $4x^2+36y^2+40x-288y+532=0$. (Example 5.20)
8. For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and foci. Also prove that the length of latus rectum is 2. (Example 5.21)
9. Find the vertex, focus, equation of directrix and length of the latus rectum of $x^2-2x+8y+17=0$ (Exercise 5.2 – Q.No.4(iv))

10. Find the vertex, focus, equation of directrix and length of the latus rectum of $y^2 - 4y - 8x + 12 = 0$ (Exercise 5.2 – Q.No.4(v))
11. Identify the type of conic and find centre, foci, vertices and directrices of $\frac{x^2}{25} - \frac{y^2}{144} = 1$ (Exercise 5.2 – Q.No.5(iii))
12. Identify the type of conic and find centre, foci, vertices and directrices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ (Exercise 5.2 – Q.No.5(iv))
13. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$. (Exercise 5.2 – Q.No.6)
14. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis. (Exercise 5.2 – Q.No.7)
15. Identify the type of conic and find centre, foci, vertices and directrices of $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$. (Exercise 5.2 – Q.No.8(iii))
16. Identify the type of conic and find centre, foci, vertices and directrices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ (Exercise 5.2 – Q.No.8 (v))
17. Identify the type of conic and find centre, foci, vertices and directrices of $9x^2 - y^2 - 36x - 6y + 18 = 0$. (Exercise 5.2 – Q.No.8(vi))
18. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y+5 = 0$ at $(1, -3)$. (Example 5.27)
19. Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$. (Example 5.28)
20. Find the equations of the two tangents that can be drawn from

(5, 2) to the ellipse $2x^2 + 7y^2 = 14$. (Exercise 5.4 – Q.No.1)

21. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse
$x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.
(Exercise 5.4 – Q.No.3)

22. Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$.
(Hint: Use parametric form) (Exercise 5.4 – Q.No.5)

23. Find the equations of the tangent and normal to hyperbola
$12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$. (Exercise 5.4 – Q.No.6)

24. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.
(Exercise 5.4 – Q.No.7)

25. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that
$t_2 = -\left(t1 + \frac{2}{t1}\right)$.
(Exercise 5.4 – Q.No.8)

26. Two coast guard stations are located 600km apart at points A(0,0) and B(0,600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.
(Example 5.39)

27. Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure 5.68 the parabola and hyperbola share focus F_1 which is 14m above the vertex of the parabola. The hyperbola's second focus F_2 is 2m above the parabola's vertex. The vertex of the hyperbolic mirror is 1m below F_1. Position a coordinate system with the origin at the centre of the hyperbola and

with the foci on the y-axis. Then find the equation of the hyperbola. (Example 5.40)

28. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?
(Exercise 5.5 – Q.No.2)

29. An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1.2m from the vertex
 (a) Position a coordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry and find an equation of the parabola.
 (b) Find the depth of the satellite dish at the vertex.
 (Exercise 5.5 – Q.No.4)

30. Parabolic cable of a 60m portion of the roadbed of a suspension bridge is positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. (Exercise 5.5 – Q.No.5)

31. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.
(Exercise 5.5 – Q.No.6)

32. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the

end of the pipe. How far beyond this vertical line will the water strike the ground? (Exercise 5.5 – Q.No.8)

33. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection. (Exercise 5.5 – Q.No.9)

34. Points A and B are 10km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it. (Exercise 5.5 – Q.No.10)

'Mathematics is the Queen of Science'

Unit – 6: Applications of Vector Algebra

Important 5 Mark Questions:

1. Prove by vector method that cos(α + β) = cos α cos β - sin α sin β
(Example 6.3)
2. Prove by vector method that sin(α – β) = sin α cos β – cos α sin β (Example 6.5)
3. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.
(Example 6.7)
4. In triangle ABC, the points D, E, F are the midpoints of the sides BC, CA and AB respectively. Using vector method, show that the area of Δ DEF is equal to $\frac{1}{4}$ (area of Δ DEF).
(Example 6.8)
5. If G is the centroid of a Δ ABC, prove that (area of Δ GAB) = (area of Δ GBC) = (area of Δ GCA) = $\frac{1}{3}$ (area of Δ ABC). (Exercise 6.1 – Q.No.8)
6. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
(Exercise 6.2 – Q.No.5)
7. If $\vec{a} = -2\hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} - 5\hat{j} + \hat{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$. State whether they are equal.
(Example 6.22)
8. If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that

$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$. (Exercise 6.3 – Q.No.4 (i))

9. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = l\vec{a} + m\vec{b} + n\vec{c}$, find the values of l, m, n. (Exercise 6.3 – Q.No.7)

10. The vector equation in parametric form of a line is $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + t(2\hat{i} - \hat{j} + 3\hat{k})$. Find (i) the direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line. (Example 6.25)

11. Find the direction cosines of the straight line passing through the points (5, 6, 7) and (7, 9, 13). Also find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points. (Exercise 6.4 – Q.No.4)

12. Find the coordinates of the foot of the perpendicular drawn from the point (-1, 2, 3) to the straight line $\vec{r} = (\hat{i} - 4\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + \hat{k})$. Also, find the shortest distance from the given point to the straight line. (Example 6.37)

13. Show that the lines $\vec{r} = (6\hat{i} + \hat{j} + 2\hat{k}) + s(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (3\hat{i} + 2\hat{j} - 2\hat{k}) + t(2\hat{i} + 4\hat{j} - 5\hat{k})$ are skew lines and hence find the shortest distance between them. (Exercise 6.5 – Q.No.2)

14. Show that the straight lines x+1=2y=-12z and x=y+2=6z-6 are skew and hence find the shortest distance between them. (Exercise 6.5 – Q.No.5)

15. Find the foot of the perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also, find the equation of the perpendicular. (Exercise 6.5 – Q.No.7)

16. If a plane meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (u, v, w) find the equation of the plane. (Exercise 6.6 – Q.No.6)

17. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (0, 1, -5) and parallel to the straight lines $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + s(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$. (Example 6.43)

18. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$
(Exercise 6.7 – Q.No.1)

19. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane 2x+6y+6z = 9.
(Exercise 6.7 – Q.No.2)

20. Find parametric form of vector equation and Cartesian equations of the plane passing through the point (2, 2, 1), (1, -2, 3) and parallel to the straight line passing through the points (2, 1, -3) and (-1, 5, -8). (Exercise 6.7 – Q.No.3)

21. Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point (1, -2, 4) and perpendicular to the plane x+2y-3z= 11 and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$. (Exercise 6.7 – Q.No.4)

22. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 8$.
(Exercise 6.7 – Q.No.5)

23. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points (3, 6, -2), (-1, -2, 6) and (6, -4, -2). (Exercise 6.7 – Q.No.6)

24. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\hat{i} - \hat{j} + \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$. (Exercise 6.7 – Q.No.7)

25. Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.

(Exercise 6.8 – Q.No.2)

26. If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

(Exercise 6.8 – Q.No.4)

27. Find the equation of the plane passing through the intersection of the planes 2x+3y-z+7 = 0 and x+y-2z+5 = 0 and is perpendicular to the plane x+y-3z-5 = 0. (Example 6.54)

28. Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$. (Example 6.55)

29. Find the coordinates of the point where the straight line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + t(3\hat{i} + 4\hat{j} + 2\hat{k})$ intersects the plane x-y+z-5 = 0

(Example 6.56)

30. Find the equation of the plane passing through the line of intersection of the planes x+2y+3z = 2 and x-y+z+11 = 3, and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1). (Exercise 6.9 – Q.No.2)

31. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4, 3, 2) to the plane x+2y+3z =2. (Excercise 6.9- Qo No. 8)

Unit – 7: Applications of Differential Calculus

Important 5 Mark Questions:

1. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?
   (1) At what time the particle is at rest?
   (2) At what time the particle changes direction?
   (3) Find the total distance travelled by the particle in the first 2 seconds.
(Example 7.6)

2. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second, at what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes. (Example 7.7)
3. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A *is* 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing? (Example 7.10)
4. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
(i) How long does the camera fall before it hits the ground?
(ii) What is the average velocity with which the camera falls during the last 2 seconds?
(iii) What is the instantaneous velocity of the camera when it hits the ground?
(Exercise 7.1 – Q.No.2)

5. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?
(Exercise 7.1 – Q.No.8)
6. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,

(i) How fast is the top of the ladder moving down the wall?
(ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? (Exercise 7.1 – Q.No.9)

7. Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t$, $t \in \mathbb{R}$ (Example 7.13)

8. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally. (Example 7.18)

9. Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$. (Exercise 7.2 – Q.No.6)

10. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally (Exercise 7.2 – Q.No.10)

11. Write the Maclaurin series expansion of the function

$\tan^{-1}(x)$; $-1 \le x \le 1$ (Exercise 7.4 – Q.No.1(v))

12. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$. (Exercise 7.4 – Q.No.2)

13. Evaluate the limit $\lim\limits_{x\to 0}\left(\frac{\sin x}{x^2}\right)$(Example 7.36)

14. Evaluate: $\lim\limits_{x\to 1^-}\left(\frac{\log(1-x)}{\cot(\pi x)}\right)$. (Example 7.38)

15. Evaluate: $\lim\limits_{x\to\infty}(1+2x)^{1/2\log x}$ (Example 7.44)

16. Evaluate: $\lim\limits_{x\to 1} x^{1/1-x}$ (Example 7.45)

17. Evaluate: $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ (Exercise 7.5 – Q.No.10)

18. If an amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is

$A = A_0\left(1+\frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is an $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$

(Excercise 7.5 - Q. No. 12)

19. Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$. (Example 7.56)

20. Find the absolute extrema of the function on given closed interval $f(x) = 2\cos x + \sin 2x$; $[0, \frac{\pi}{2}]$ (Exercise 7.6 – Q.No.1 (IV))

21. Find the intervals of monotonicities and hence find the local extremum for the function $f(x) = \sin x \cos x + 5$, $x \in (0, 2\pi)$

(Exercise 7.6 – Q.No.2 (v))

22. Determine the intervals of concavity of the curve $f(x) = (x-1)^3$.

$(x-5)$, $x \in \mathbb{R}$ and, points of inflection if any. (Example 7.57)

23. Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.

(Example 7.60)

24. Find the intervals of concavity and points of inflexion for the function $f(x) = \sin x + \cos x$, $0 < x < 2\pi$. (Exercise 7.7 – Q.No. 1(ii))

25. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. (Exercise 7.7 – Q.No.3)

26. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is which cut produces the box of maximum volume? (Example 7.62)

27. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from (1, 1). (Example 7.63)

28. A steel plant is capable of producing x tones per day of a low - grade steel and y tones per day of a high – grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed price of low – grade steel is half that of high – grade steel, then what should be optimal productions in low – grade steel and high – grade steel in order to have maximum receipts? (Example 7.64)

29. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum? (Exercise 7.8 – Q.No.5)

30. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1, 80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material? (Exercise 7.8 – Q.No.6)

31. Prove that among all the rectangles of the given perimeter, the square has the maximum area. (Exercise 7.8 – Q.No.8)

32. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume. (Exercise 7.8 – Q.No.10)

33. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone. (Exercise 7.8 – Q.No.12)

34. Find the asymptotes of the curve $f(x) = \frac{2x^2-8}{x^2-16}$ (Example 7.68)

35. Sketch the curve $y = f(x) = x^2 - x - 6$. (Example 7.69)

36. Sketch the curve $y = f(x) = x^3 - 6x - 9$. (Example 7.70)

37. Sketch the curve $y = \frac{x^2-3x}{x-1}$. (Example 7.71)

38. Sketch the graph of the function $y = \frac{3x}{x^2-1}$. (Example 7.72)

39. Sketch the graph of the function $y = \frac{x^2+1}{x^2-4}$. (Exercise 7.9 – Q.No.2 (iii)

40. Sketch the graph of the function $y = \frac{x^3}{24} - \log x$

(Exercise 7.9 – Q.No.2 (v)

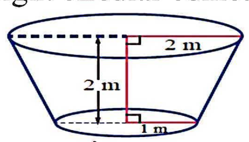
' Achieve Your Target Plan Well '

Unit – 8: Differentials and Partial Derivatives

Important 5 Mark Questions:

1. The radius of a circle plate is measured as 12.65 cm instead of the actual length 12.5 cm. Find the following in calculating the area of the circular plate:
   (i)Absolute error (ii) Relative error (iii) Percentage error
   (Exercise 8.1 – Q.No.4)
2. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.
   (Exercise 8.1 – Q.No.6)
3. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}$, $0 \le x \le 9$. What is the approximate number of words learned when x changes from
   (i) 1 to 1.1 hour? (ii) 4 to t.1 hour? (Exercise 8.2 – Q.No.9)
4. Consider $f(x, y) = \frac{xy}{x^2+y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that f is not continuous at (0, 0) and continuous at all other points of $\mathbb{R}^2$ (Example 8.9)
5. Consider $g(x, y) = \frac{2x^2y}{x^2+y^2}$ if $(x, y) \neq (0, 0)$ and $g(0, 0) = 0$. Show that g is continuous on $\mathbb{R}^2$ (Example 8.10)
6. Show that $f(x, y) = \frac{x^2-y^2}{y^2+1}$ is continuous at every $(x, y) \in \mathbb{R}^2$
   (Exercise 8.3 – Q.No.6)
7. Let $g(x, y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0,0) = 1$. Show that g is continuous at (0, 0). (Exercise 8.3 – Q.No.7)
8. Let $f(x, y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x, y) \in \mathbb{R}^2$. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$. (Example 8.13)

45. Find the volume of the spherical cap of height h cut of from a sphere of radius r. (Example 9.64)
46. Find the volume of the solid formed by revolving the region bounded by the parabola y = x^2 , x – axis, ordinates x = 0 and x = 1 about the x – axis. (Example 9.65)
47. Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b about the major axis. (Example 9.66)
48. The region enclosed between the graphs of y = x and y = x^2 is denoted by R. Find the volume generated when R is rotated through 360^0 about x – axis. (Exercise 9.9 – Q.No.4)
49. Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in fig. (Exercise 9.9 – Q.No.5)



50. A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration. (Exercise 9.9 – Q.No.6)

' Life is like Riding a Biycle you keep Moving Result in Sucuess '

Unit – 10: Ordinary Differential Equations

Important 5 Mark Questions:

1. Express each of the following physical statements in the form of differential equation.
   (i) Radium decays at a rate proportional to the amount Q present.
   (ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5, 00,000 and the population.
   (iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature.
   (iv) A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of 400 per year. (Exercise 10.2 – Q.No.1)
2. Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin. (Example 10.6)
3. Find the differential equation of the family of (i) all non-vertical lines in a plane (ii) all non-horizontal lines in a plane. (Exercise 10.3 – Q.No.1)
4. Form the differential equation of all straight lines touching the circle $x^2+y^2=r^2$. (Exercise 10.3 – Q.No.2)
5. Find the differential equation of the family of circles passing through the origin and having their centres on the x-axis. (Exercise 10.3 – Q.No.3)
6. Find the differential equation of the family of all the parabolas with latus rectum 4a and whose axes are parallel to the x-axis. (Exercise 10.3 – Q.No.4)
7. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin. (Exercise 10.3 – Q.No.6)
8. Find the differential equation of the curve represented by $xy = ae^{x}+be^{-x}+x^2$. (Exercise 10.3 – Q.No.8)

9. Show that y = 2(x^2-1) +C$e^{-x^2}$is a solution of the differential equation $\frac{dy}{dx}$+2xy - 4x^3= 0. (Example 10.9)
10. Show that y = a cos (log x) +b sin (log x), x > 0 is a solution of the differential equation x^2y^n+xy'+y = 0. (Example 10.10)
11. The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through (2, 5). Find the equation of the curve.
(Exercise 10.4 – Q.No.3)
12. Show that y = ae^{-3x} + b, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2}$+3$\frac{dy}{dx}$ = 0.
(Exercise 10.4 – Q.No.6)
13. Show that the differential equation representing the family of curves y^2 = 2a(x+$a^{\frac{2}{3}}$), where a is a positive parameter is $\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$. (Exercise 10.4 – Q.No.7)
14. Find the particular solution of (1+x^3) dy - x^2ydx = 0 satisfying the condition y (1) = 2. (Example 10.12)
15. Solve $y' = sin^2$(x – y + 1). (Example 10.13)
16. Solve: $\frac{dy}{dx} = \sqrt{4x + 2y - 1}$. (Example 10.14)
17. Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$. (Example 10.15)
18. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by M$\frac{dV}{dt}$ = F- kV, where k is a constant. Express V in terms of t given that V = 0 when t = 0. (Exercise 10.5 – Q.No.1)
19. The velocity v, of a parachute falling vertically satisfies the equation v$\frac{dv}{dx}$ = g$\left(1 - \frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x. (Exercise 10.5 – Q.No.2)
20. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1, 0). (Exercise 10.5 – Q.No.3)
21. Solve the differential equation (ydx – xdy) cot ($\frac{x}{y}$) = ny^2dx.
(Exercise 10.5 – Q.No.4 (vi))

22. Solve the differential equation $\frac{dy}{dx}$ -x$\sqrt{25-x^2}$ = 0
(Exercise 10.5 – Q.No.4 (vii))

23. Solve the differential equation $\frac{dy}{dx} = tan^2$(x+y)
(Exercise 10.5 – Q.No.4 (x))

24. Solve $(x^2 - 3y^2)$ dx + 2xydy = 0. (Example 10.17)

25. Solve (y+$\sqrt{x^2+y^2}$) dx – xdy = 0, y (1) = 0. (Example 10.18)

26. Solve (2x + 3y) dx + (y - x) dy = 0. (Example 10.19)

27. Solve (1+2$e^{\frac{x}{y}}$) dx + 2$e^{\frac{x}{y}}(1-\frac{x}{y})$ dy = 0. (Example 10.21)

28. Solve the differential equation
$\left[x + ycos\left(\frac{y}{x}\right)\right]$dx = x cos$\left(\frac{y}{x}\right)$dy (Exercise 10.6 – Q.No.1)

29. Solve the differential equation (x^3+y^3)dy - x^2ydx = 0
(Exercise 10.6 – Q.No.2)

30. Solve the differential equation y$e^{\frac{x}{y}}$ dx = $\left(xe^{\frac{x}{y}} + y\right)$dy
(Exercise 10.6 – Q.No.3)

31. Solve the differential equation 2xydx+ $(x^2$+2$y^2)$dy = 0
(Exercise 10.6 – Q.No.4)

32. Solve the differential equation (y^2 - 2xy) dx = (x^2 - 2xy) dy
(Exercise 10.6 – Q.No.5)

33. Solve the differential equation x$\frac{dy}{dx}$ = y - x$cos^2(\frac{y}{x})$
(Exercise 10.6 – Q.No.6)

34. Solve the differential equation
$\left(1+3e^{\frac{y}{x}}\right)$ dy + 3$e^{\frac{y}{x}}$ $\left(1-\frac{y}{x}\right)$dx = 0, given that y = 0 when x = 1
(Exercise 10.6 – Q.No.7)

35. (x^2+y^2) dy = xy dx. It is given that y (1) = 1 and y (x_0) = e.
Find the value of x_0. (Exercise 10.6 – Q.No.8)

36. Solve $[y(1 - x\tan x) + x^2\cos x]$dx – xdy = 0.
(Example 10.23)

37. Solve (1 +x^3) $\frac{dy}{dx}$ + 6x^2y = 1 +x^2. (Example 10.25)

38. Solve the Linear differential equation $(1 - x^2)\frac{dy}{dx}$ - xy = 1. (Exercise 10.7 – Q.No.2)

39. Solve the Linear differential equation $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$ (Exercise 10.7 – Q.No.4)

40. Solve the Linear differential equation $(y - e^{\sin^{-1} x})\frac{dx}{dy} + \sqrt{1 - x^2} = 0$ (Exercise 10.7 – Q.No.7)

41. Solve the Linear differential equation $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$ (Exercise 10.7 – Q.No.8)

42. Solve the Linear differential equation $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$ (Exercise 10.7 – Q.No.9)

43. Solve the Linear differential equation $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$ (Exercise 10.7 – Q.No.10)

44. Solve the Linear differential equation $\frac{dy}{dx} = \frac{sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3}y$ (Exercise 10.7 – Q.No.12)

45. Solve the Linear differential equation $\frac{dy}{dx} + \frac{3y}{x} = \frac{1}{x^2}$, given that y = 2 when x = 1 (Exercise 10.7 – Q.No.15)

46. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t. (Example 10.30)

47. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3, 00, 000 to 4, 00,000. (Exercise 10.8 – Q.No.2)

48. The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance

and L, the coefficient of induction. Find the current i at time t when E = 0. (Exercise 10.8 – Q.No.3)

49. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei has undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years? (Exercise 10.8 – Q.No.6)

50. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180^0F, and 10 minutes later it was 160^0F. Assume that constant temperature of the kitchen was 70^0F.

(i) What was the temperature of the coffee at 10.15 A.M.?

(ii) The woman likes to drink coffee when its temperature is between 130^0F and 14 0^0F between what times should she have drunk the coffee? (Exercise 10.8 – Q.No.8)

51. A tank initially contains 50 litres of pure water. Starting at time t = 0 a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time t > 0.
(Exercise 10.8 – Q.No.10)

Unit – 11: Probability Distributions

Important 5 Mark Questions:

1. Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10, and (iv) the number of elements in inverse image of X. (Example 11.2)

2. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹15 for each red ball selected. If X denotes the winning amount, find the values of X and number of points in its inverse images. (Exercise 11.1 – Q.No.4)
3. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images. (Exercise 11.1 – Q.No.5)
4. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred. (Example 11.5)
5. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours. (Example 11.6)
6. If the probability mass function f(x) of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

   Find (i) its cumulative distribution function; hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$ (Example 11.7)
7. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.
(ii) Find the cumulative distribution function.
(iii) Find $P(3 \le X < 6)$ (iv) Find $P(X \ge 4)$ (Example 11.8)

8. Find the probability mass function f(x) of the discrete random variable X whose cumulative distribution function F(x) is given by

$$F(x)=\begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \le x < -1 \\ 0.60 & -1 \le x < 0 \\ 0.90 & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \ge -1)$. (Example 11.9)

9. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$(iii) $P(X \le 4)$ (iv) $P(3 < X)$ (Example 11.10)

10. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii) the cumulative distribution function (iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$ (Exercise 11.2 – Q.No.2)

11. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls. (Exercise 11.2 – Q.No.3)

12. Suppose a discrete random variable can only take the values 0, 1 and 2. The probability mass function is defined by

$$f(x)=\begin{cases} \dfrac{x^2+1}{k}, & \text{for } x=0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k (ii) cumulative distribution function (iii) P $(X \geq 1)$. (Exercise 11.2 – Q.No.4)

13. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Find (i) the probability mass function (ii) P $(X < 1)$ and (iii) P $(X \geq 2)$. (Exercise 11.2 – Q.No.5)

14. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0 \\ \frac{1}{2} & \text{for } 0 \leq x < 1 \\ \frac{3}{5} & \text{for } 1 \leq x < 2 \\ \frac{4}{5} & \text{for } 2 \leq x < 3 \\ \frac{9}{10} & \text{for } 3 \leq x < 4 \\ 1 & \text{for } 4 \leq x < \infty \end{cases}$$

Find (i) the probability mass function (ii) P $(X < 3)$ and (iii)P $(X \geq 2)$. (Exercise 11.2 – Q.No.7)

15. If X is the random variable with probability density function f(x) given by,

$$f(x)=\begin{cases} x-1, & 1\le x<2 \\ -x+3, & 2\le x<3 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the distribution function F (x) (ii) P (1.5 ≤ X ≤ 2.5)
(Example 11.12)

16. The probability density function of random variable X is given by $f(x)=\begin{cases} k & 1\le x\le 5 \\ 0 & \text{otherwise} \end{cases}$ Find (i) Distribution function (ii) P(X < 3) (iii) P (2 < X < 4) (iv) P (3≤ X)
(Example 11.14)

17. The probability density function of X is

$$f(x)=\begin{cases} x & 0<x<1 \\ 2-x & 1\le x<2 \\ 0 & \text{otherwise} \end{cases}.$$

Find (i) P(0.2 ≤ X < 0.6) (ii) P (1.2 ≤ X < 1.8) (iii) P(0.5 ≤ X < 1.5)
(Exercise 11.3 – Q.No.2)

18. The probability density function of X is given by

$$f(x)=\begin{cases} k\,e^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$$

Find (i) the value of k (ii) the distribution function (iii) P(X < 3) (iv) P (5 ≤ X) (v) P (X ≤ 4) (Exercise 11.3 – Q.No.4)

19. If X is the random variable with probability density function f(x) given by,

$$f(x)=\begin{cases} x+1, & -1\le x<0 \\ -x+1, & 0\le x<1 \\ 0 & \text{otherwise} \end{cases}$$

Then find (i) the distribution function F(x) (ii)P(-0.5 ≤ X ≤ 0.5)
(Exercise 11.3 – Q.No.5)

20. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.
(Exercise 11.4 – Q.No.4)

21. The probability density function of the random variable X is given by

$$f(x)=\begin{cases}16xe^{-4x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

Find the mean and variance of X. (Exercise 11.4 – Q.No.7)

22. Find the binomial distribution for each of the following.
    (i) Five fair coins are tossed once and X denotes the number of heads.
    (ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared. (Example 11.19)

23. A multiple choice examination has ten questions; each question has four distracters with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer. (Example 11.20)

24. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. (Example 11.22)

25. Using binomial distribution find the mean and variance of X for the following experiments

(i)A fair coin is tossed 100 times, and X denotes the number of heads.
(ii)A fair die is tossed 240 times, and X denotes the number of times that four appeared. (Exercise 11.5 – Q.No.3)

26. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items? (Exercise 11.5 – Q.No.5)

27. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
(i)exactly 10 will have a useful life of at least 600 hours;
(ii)at least 11 will have a useful life of at least 600 hours;
(iii)at least 2 will not have a useful life of at least 600 hours.
(Exercise 11.5 – Q.No.6)

28. The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$ (iii) $P(X \geq 2)$. (Exercise 11.5 – Q.No.7)

29. If X B (n, p) such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation of X.
(Exercise 11.5 – Q.No.8)

30. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.
(Exercise 11.5 – Q.No.9)

Excercise	Example
9.3- 1(iv), (vi) 2(i), (iv), (vii), (viii), (ix), (xi) 9.4-3 9.5- 1(ii) 9.7 -1(ii) 9.8-3,6,8,10.	9.2,9.4,9.10,9.15, 9.19,9.21,9.22,9.30, 9.51to 9.63.

12.Discrete Mathematics:

1. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $m * n = m + n - mn;\ m, n \in \mathbb{Z}$
2. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.
3. Verify (i) closure property, (ii) commutative property, (iii)associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0,1, 2, 3, 4, 5, 6, 7,8, 9,10\}$
4. (i) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A ? If so, examine the commutative and associative properties satisfied by $*$ on A.
   (ii)Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A ? If so, examine the existence of identity, existence of inverse properties for the operation on A
5. Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.
6. State and prove Distributive Laws on propositions.