

DHARMAPURI DISTRICT SECOND REVISION TEST - 2023

12 - Std

Time - 3.00 Hours

MATHEMATICS

Reg.No

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Marks : 90

Part - I

Note : 1. Answer all the questions. 2. Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. 20 X 1 = 20

1. The value of $\sin^{-1}(\cos x)$, $0 \leq x < \pi$ - is
 - a) $\pi - x$
 - b) $x - \frac{\pi}{2}$
 - c) $\frac{\pi}{2} - x$
 - d) $x - \pi$
2. If $\sin^{-1} \frac{x}{5} + \cos^{-1} \frac{5}{4} = \frac{\pi}{2}$ then the value of x is
 - a) 4
 - b) 5
 - c) 2
 - d) 3
3. If $p(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with Foci $F_1(3, 0)$ and $F_2(-3, 0)$, then $PF_1 + PF_2$ is
 - a) 8
 - b) 6
 - c) 10
 - d) 12
4. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
 - a) $\tan^{-1} \left(\frac{3}{4} \right)$
 - b) $\tan^{-1} \left(\frac{4}{3} \right)$
 - c) $\frac{\pi}{2}$
 - d) $\frac{\pi}{4}$
5. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is
 - a) 1
 - b) $\sqrt{2}$
 - c) $\frac{3}{2}$
 - d) 2
6. The equation of the directrix of the parabola $y^2 = x + 4$ is
 - a) $x = \frac{15}{4}$
 - b) $x = -\frac{15}{4}$
 - c) $x = -\frac{17}{4}$
 - d) $x = \frac{17}{4}$
7. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$ then $\frac{| \text{adj } B |}{| C |} = \dots\dots\dots$
 - a) $\frac{1}{3}$
 - b) $\frac{1}{9}$
 - c) $\frac{1}{4}$
 - d) 1
8. The value of $\left(\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right)^{10}$ is
 - a) $\text{cis } \frac{2\pi}{3}$
 - b) $\text{cis } \frac{4\pi}{3}$
 - c) $-\text{cis } \frac{2\pi}{3}$
 - d) $-\text{cis } \frac{4\pi}{3}$
9. The value of $\frac{\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}}{\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}}$ is
 - a) 1
 - b) -1
 - c) i
 - d) $-i$
10. If $P(A) = P([A/B])$ then the system $Ax = B$ of linear equation is
 - a) consistent and has unique solution
 - b) consistent
 - c) consistent and has infinitely many solution
 - d) inconsistent

11. The order of the differential equation of all circles with centre at (h,k) and radius 'a' is
 a) 2 b) 3 c) 4 d) 1
12. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
 a) $xy = k$ b) $y = kx$ c) $y = k \log x$ d) $\log y = kx$
13. If in 6 trials, x - is a binomial variable which follows the relation $p(x = 4) = p(x = 2)$ then the probability of success is
 a) 0.125 b) 0.25 c) 0.375 d) 0.75
14. According to the rational root theorem which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?
 a) -1 b) $5/4$ c) $4/5$ d) 5
15. Which one of the following statement has the truth value T?
 a) $\sin x$ is an even function.
 b) Every square matrix is non-singular
 c) The product of complex number and its conjugate is purely imaginary.
 d) $\sqrt{5}$ is an irrational number.
16. In Z - we define $a * b = a + b + 1$ The identity element with respect to $*$ is
 a) 1 b) 0 c) -1 d) 2
17. If $\int_0^x f(t) dt = x + \int_x^1 f(t) dt$ then the value of $f(1)$
 a) $1/2$ b) 2 c) 1 d) $3/4$
18. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to
 a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u
19. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{j} + \vec{k}$, $\vec{c} = \vec{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$ then the value of $\lambda + \mu$ is
 a) 0 b) 1 c) 6 d) 3
20. The value of $\int_0^1 (\sin^{-1} x)^2 dx$ is
 a) $\frac{\pi^2}{4} - 1$ b) $\frac{\pi^2}{4} - 2$ c) $\frac{\pi^2}{4} + 1$ d) $\frac{\pi^2}{4} - 2$

Part - II

Note : 1) Answer any seven questions. 2. Question No.30 is compulsory.

7 X 2 = 14

21. If $\text{adj}(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ find A

22. Show that $|Z-2-i| = 3$ represents a circle and find the centre and radius.
23. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.
24. Find the principal value of $\operatorname{cosec}^{-1}(-1)$
25. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$
26. Evaluate: $\lim_{x \rightarrow 0^+} x \log x$
27. Let $g(x) = x^2 + \sin x$, calculate the differential dg.
28. Evaluate: $\int_0^1 x^3(1-x)^4 dx$
29. Compute $p(x=k)$ for the binomial distribution $n=6, p=1/3, k=3$
30. Construct truth table for $\neg(p \wedge \neg q)$

Part - III

Note : 1. Answer any seven of the following. 2. Question No.40 is compulsory. 7 X 3 = 21

31. Find the rank of the matrix

$$\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix} \text{ by row reduction method.}$$

32. Show that the equation $x^9 - 5x^4 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
33. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$ Also find the coordinates of the point of contact.
34. Find the magnitude and the direction cosines of the torque about the point $(2, 0, -1)$ of a force $2\vec{i} + \vec{j} - \vec{k}$ whose line of action passes through the origin.
35. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$, $[-3, 2]$
36. Assuming $\log_{10} e = 0.4343$ find an approximate value of $\log_{10} 1003$.
37. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
38. Solve: $\frac{dy}{dx} + 2y = e^{-x}$
39. If $X \sim B(n, p)$ such that $4P(x=4) = P(x=2)$ and $n = 6$ Find the distribution, Mean and standard deviation of x .
40. Show that the points $1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Part - IV

41. a) Solve : $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ by cramer's rule. (or)

b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{2}$ show that $x^2 + y^2 + 3x - 3y + z = 0$

42. a) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (or)

b) Solve $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$

43. a) on lighting a rocket cracker if gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

b) $\vec{a} = \vec{i} - \vec{j}$, $\vec{b} = \vec{i} - \vec{j} - 4\vec{k}$, $\vec{c} = 3\vec{j} - \vec{k}$ and $\vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$.

44. a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm (or)

b) If $u = \text{Sec}^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$.

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45. a) Prove that $\int_0^{\pi/4} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$ (or)

b) Expand $\log(1+x)$ as a Maclaurin's series up to 4 non - zero terms for $-1 < x \leq 1$.

46. a) Find the mean and variance of a random variable x, whose probability density

function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$ (or)

b) Prove that $p \rightarrow (-qvr) \equiv -pv(-qvr)$ using truth table.

47. a) Find the parametric, non-parametric vector and cartesian form of the equations of the plane passing through the three non-collinear points (3,6,-2), (-1,-2,6) and (6,4,-2) (or)

b) Solve : $(1 + 2e^{x/y})dx + 2e^{x/y}(1 - \frac{x}{y})dy = 0$
