

2RM

SECOND REVISION TEST - 2023

12 - Std

MATHEMATICS



Time : 3.00 Hrs

Marks : 90

PART - I

Note : 1) Answer all the questions. 2) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$ then $B =$

- a) $\left(\cos^2 \frac{\theta}{2}\right) A$ b) $\left(\cos^2 \frac{\theta}{2}\right) A^T$ c) $(\cos^2 \theta) I$ d) $\left(\sin^2 \frac{\theta}{2}\right) A$

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

- a) 1 b) 2 c) 4 d) 3

3. If Z is a non zero complex number such that $2iz^2 = \frac{1}{z}$ then $|Z|$ is

- a) $\frac{1}{2}$ b) 1 c) 2 d) 3

4. A zero of $x^3 + 64$ is

- a) 0 b) 4 c) 4i d) -4

5. The value of $\sin^{-1}(\cos x)$, $0 \leq x \leq \pi$ is

- a) $\pi - x$ b) $x - \frac{\pi}{2}$ c) $\frac{\pi}{2} - x$ d) $x - \pi$

6. The radius of the circle $3x^2 + by^2 + 4bx - 6by + by^2 = 0$ is

- a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

7. The distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

- a) 0 b) 1 c) 2 d) 3

8. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then

$\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

- a) 81 b) 9 c) 27 d) 18

9. The slope of the normal to the curve $f(x) = 2 \cos 4x$ at $x = \frac{\pi}{12}$ is

- a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

10. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

- a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

11. The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is

- a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{2}{3}$

12. The order and degree of the differential equation $\sqrt{\sin x} (dx + dy) = \sqrt{\cos x} (dx - dy)$ are

- a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1

13. If the probability mass function of a random variable is defined given below

x	-2	-1	0	1	2
f(x)	k	2k	3k	4k	5k

then the value of K is

- a) $\frac{1}{15}$ b) $\frac{1}{10}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$

14. Which of the following is a binary operation on N.

- a) Subtraction b) multiplication c) division d) all of these

15. The number of rows in a truth table of a compound statement there involved 3 simple statements is

- a) 9 b) 8 c) 6 d) 3

16. If $z = x + iy$ the rectangle form of $\operatorname{Re}(-iz)$ is

- a) x b) y c) x + iy d) -x + iy

17. All the roots of the polynomial $x^{20} + x^{18} + x^{16} + x^{12} + 10$ are

- a) real roots b) complex roots but not real c) positive real roots d) negative real roots

18. The principal value of $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

- a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

19. Area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

- a) 12π b) 24π c) 36π d) 48π

20. The integrating factor for the differential equation $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$ is

- a) $\sin x$ b) $\cos x$ c) $\sin^2 x$ d) $\cos^2 x$

PART - II

7 x 2 = 14

Note : i) Answer any seven questions. 2) Questions number 30 is compulsory.

21. If $A \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$ find A.

22. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ are same type of boolean matrices then find $A \vee B$ and $B \wedge A$

23. Find the value of $\left| \frac{2+i}{-1+2i} \right|$.

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24. Find the principal value of $\sin^{-1} \left(\frac{-1}{2} \right)$.

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25. Find the vertices of $9x^2 - 16y^2 = 144$.

26. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$. Complete the maximum height of the particle reached.

27. Evaluate : $\int_0^{\pi/2} \sin^{10} x \, dx$.

28. Find dy for the function $y = \frac{(1-2x)^3}{3-4x}$.

29. If the probability density function for the random variable x is given by

$$f(x) = \begin{cases} k, & 1 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases} \text{ then find the value of } K.$$

30. Determine whether the three vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

PART - III

7 x 3 = 21

Note : i) Answer any seven questions. ii) Questions number 40 is compulsory.

31. Solve by determinant method $5x - 2y + 16 = 0$, $x + 3y - 7 = 0$.

32. If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$.

33. Simplify $\left(\frac{1+i}{1-i} \right)^3 - \left(\frac{1-i}{1+i} \right)^3$ into rectangular form.

34. A concrete bridge is designed as a parabolic arch. The road over bridge is 40m long and the maximum height of the arch is 15. Write the equation of the parabolic arch.

35. prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$.

36. Product of two positive numbers is 20 and their sum is minimum, find the numbers.

37. Find the approximate value of $\sqrt{9.2}$ by linear approximation method without using calculator.

38. Construct a differential equation from the given equation $y = A \cos x + B \sin x$ by eliminating the constants A and B .

39. Two unbiased coins are tossed at the same time, find probability mass function for getting heads.

40. A binary operation * is defined on Q by $a * b = \frac{a+b}{2}$, $\forall a, b \in Q$ verify whether * satisfies closure property, commutative closure property, commutative property and associative property.

PART - IV

Note : Answer all questions.

7 x 5 = 35

41. a) Solve the system of linear equations, by Cramer's rule $x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$. **(OR)** b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

42. a) If $2 + i$ and $3 - \sqrt{2}$ are two roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ find the roots. **(OR)**

b) Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right)$.

43. a) Find the equation of circle passing through (1, 0), (-1, 0) and (0, 1). **(OR)** b) If $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ are intersecting orthogonally prove that

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$$

44. a) If $\vec{a} = \vec{i} - \vec{j}$, $\vec{b} = \vec{i} - \vec{j} - 4\vec{k}$, $\vec{c} = 3\vec{j} - \vec{k}$ and $\vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$ then verify that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \ \vec{c} \ \vec{d}] \vec{b} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a} . \text{ (OR)}$$

b) A right circular cylinder has radius $r = 10$ cm and height $h = 20$ cm suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, Calculate the relative error and percentage error.

45. a) Find the non parametric vector equation and Cartesian equation of a plane passing through a point (1, -2, 4) perpendicular to the plane $x + 2y - 3z = 11$ and parallel to

the straight line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$. **(OR)**

b) Find the area of the region enclosed by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

46. a) Find the dimensions of a largest rectangle which is inscribed in a circle of radius 10 cm. **(OR)**

b) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years in how many years will the population become triple?

47. a) If the probability density function of a random variable x is given by

$$f(x) = \begin{cases} ke^{-\frac{x}{3}}, & x > 0 \\ 0, & x < 0 \end{cases}$$

then find (i) The value of k. ii) Distribution function

iii) $P(x < 3)$ iv) $P(5 \leq x)$ v) $P(x \leq 4)$. **(OR)**

b) Let $A = Q \setminus \{1\}$, a binary operation * is defined on A by $x * y = x + y - xy$. verify whether * has closure property on A? Then verify whether * satisfies commutative law, associative law, identity law, and inverse law.