second revition

12th Standard

Maths

Date : 06-Feb-23

Reg.No. : ☐☐☐☐☐☐

Exam Time : 03:00:00 Hrs

Total Marks : 90

I. CHOOSE THE CORRECT ANSWER

20 x 1 = 20

1) If |adj(adj A)| = $|A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

2) If z = x + iy is a complex number such that |z+2| = |z−2|, then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

3) If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

(a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

4) If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

5) The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line 3x−4y = m at two distinct points if

(a) 15< m < 65 (b) 35< m < 85 (c) −85 < m < −35 (d) −35 < m < 15

6) If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(a) 1 (b) -1 (c) 2 (d) 3

7) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

8) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25}$ radians /sec (b) $\frac{4}{25}$ radians /sec (c) $\frac{1}{5}$ radians /sec (d) $\frac{1}{3}$ radians /sec

9) The maximum slope of the tangent to the curve $y = e^x \sin x$, x ∈ [0, 2π] is at

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

10) If v (x, y) = log ($e^x + e^y$), then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d) 1

11) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) 0.3xdx m^3 (b) 0.03x m^3 (c) $0.03x^2$ m^3 (d) $0.03x^3$ m^3

12) The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

13) The order of the differential equation of all circles with centre at (h, k) and radius ‘a’ is

(a) 2 (b) 3 (c) 4 (d) 1

14) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

(a) i + 2n, i = 0,1,2... n (b) 2i- n, i = 0,1,2... n (c) n - i, i = 0,1,2... n

(d) 2i + 2n, i = 0, 1, 2...n

15) Which one of the following is a binary operation on N?

(a) Subtraction (b) Multiplication (c) Division (d) All the above

16) The system of linear equations x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = has a unique solution if __________

(a) k ≠ 0 (b) -1 < k < 1 (c) -2 < k < 2 (d) k = 0

17) The amplitude of $\frac{1}{i}$ is equal to _______

(a) 0 (b) $\frac{\pi}{2}$ (c) -$\frac{\pi}{2}$ (d) π

18) If ∝, β, ४ are the roots of $9x^3$-7x+6 = 0, then ∝ β ४ is __________

(a) $\frac{-7}{9}$ (b) $\frac{7}{9}$ (c) 0 (d) $\frac{-2}{3}$

19) If $sin^{-1}x - cos^{-1}x = \frac{\pi}{6}$ then ___________

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{-1}{2}$ (d) none of these

20) If * is defined by a * b = $a^2 + b^2$ + ab + 1, then (2 * 3) * 2 is ____________

(a) 20 (b) 40 (c) 400 (d) 445

II. ANSWER ANY SEVEN QUESTIONS (QUESTION NUMBER 30 IS COMPULSORY) 7x 2 =14

21) Find the inverse of the non-singular matrix A = $\begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.

22) Prove the following properties z is real if and only if z = $\bar{z}$

23) Construct a cubic equation with roots 1, 2 and 3

24) For what value of x, the inequality $\frac{\pi}{2} < cos^{-1}(3x-1) < \pi$ holds?

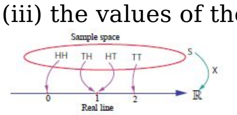
25) Find the general equation of a circle with centre (-3, -4) and radius 3 units.

26) A particle is acted upon by the forces $(3\hat{i} - 2\hat{j} + 2\hat{k})$ and $(2\hat{i} + \hat{j} - \hat{k})$ is displaced from the point (1, 3, -1) to the point (4, 1, -λ). If the work done by the forces is 16 units, find the value of λ.

27) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

28) Evaluate the following definite integrals:
$\int_3^4 \frac{dx}{x^2-4}$

29) Suppose two coins are tossed once. If X denotes the number of tails,
(i) write down the sample space
(ii) find the inverse image of 1
(iii) the values of the random variable and number of elements in its inverse images



A mapping X(.) from S to R

30) Construct the truth table for the following statements.
¬(p ∧ ¬q)

III. ANSWER ANY SEVEN QUESTIONS (QUESTION NUMBER 40 IS COMPULSORY) 7x 3 =21

31) If A = $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

32) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form

33) Solve the equation x^3-3x^2- 33x + 35 = 0.

34) Find the equation of the ellipse in each of the cases given below:
length of latus rectum 8, eccentricity = $\frac{3}{5}$, centre (0, 0) and major axis on x -axis.

35) Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

36) A particle moves so that the distance moved is according to the law s(t) = $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero.

37) Find df for f(x) = x^2 + 3x and evaluate it for
x = 3 and dx = 0.02

38) Evaluate the following
$\int_0^{\pi/4} sin^6 2x \, dx$

39) Solve $\frac{dy}{dx} + 2y = e^{-x}$

40) Prove that q → p ≡ ¬p → ¬q

IV. ANSWER ALL THE QUESTIONS 7x 5 = 35

41) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z}$ -1 = 0, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z}$ - 2 = 0, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z}$ + 1 = 0

42) Show that $\left(2 + i\sqrt{3}\right)^{10} - \left(2 - i\sqrt{3}\right)^{10}$ is purely imaginary

43) If p and q are the roots of the equation lx^2+ nx + n = 0, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}$ = 0.

44) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

45) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$
(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

46) Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

47) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where s(t) $=128t - 16t^2$.
(1) Compute the maximum height of the particle reached.
(2) What is the velocity when the particle hits the ground?

48) Find the angle between $y = x^2$ and $y = (x - 3)^2$.

49) A right circular cylinder has radius r =10 cm. and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10. 1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

50) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = $2\pi\sqrt{\frac{1}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1

51) Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$, y = 0 and x = 1.

52) Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C.
Find
(i) The temperature of water after 20 minutes
(ii) The time when the temperature is 40°C
$\left[log_e \frac{11}{15} = -0.3101; log_e 5 = 1.6094\right]$

53) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

Find
(i) the value of k
(ii) P(2 ≤ X < 5)
(iii) P(3 < X)

54) Show that $\neg(p \to q) \equiv p \wedge \neg q$